This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

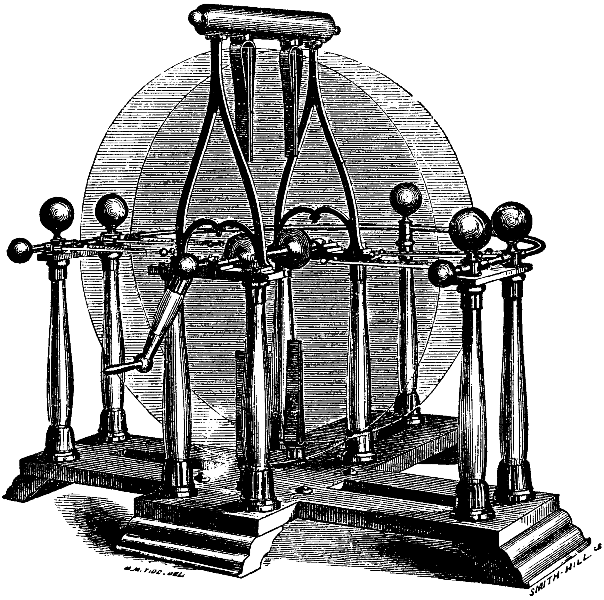
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



(Fig. 575.)

Double Plate Electrical Machine, made for the University of Mississippi, by Ritchie.
See § 837.

PRINCIPLES

OF

PHYSICS,

OR

NATURAL PHILOSOPHY;

DESIGNED FOR THE

Use of Colleges and Schools.

BY

BENJAMIN SILLIMAN, JR., M. A., M. D.,

PROFESSOR OF GENERAL AND APPLIED CHEMISTRY IN YALE COLLEGE.

SECOND EDITION,

REVISED AND REWRITTEN.

With Seven Hundred and Twenty-Two Illustrations.

NEW YORK:
IVISON, BLAKEMAN, TAYLOR, & COMPANY,
138 & 140 GRAND STREET.
CHICAGO: 133 & 135 STATE STREET.
1871.

Entered, according to Act of Congress, in the year 1860, by
H. C. PECK & THEO. BLISS,
in the Clerk's Office of the District Court of the Eastern District of Pennsylvania.

MEARS & DUSENBERY, STEREOTYPERS.

CAXTON PRESS OF
SHERMAN & CO., PHILADELPHIA.

PREFACE TO THE SECOND EDITION.

Many important changes have been made in the present edition, designed to adapt the work more fully to the wants of the higher seminaries, where mathematical demonstrations are required of the classes in Natural Philosophy. With this view, the two first Parts have been almost wholly rewritten, and upon a different plan of arrangement. Some subjects which were perhaps too fully treated in the first edition,—as, for example, Crystallography,—have been reduced, while others have been expanded to meet the just proportions of a harmonious treatment. These remarks apply also to *Part Third* (the Physics of Imponderable Agents), and especially to Optics and Heat. In the latter chapter some topics have been omitted which are more appropriately treated in Chemistry.

The mathematical demonstrations, while they are designed to be as simple as possible consistent with exactness, are believed to be as full and rigorous as are demanded in institutions where only geometric and algebraic methods are used. Analytical methods have not been introduced, as the book was not designed for the comparatively limited number of colleges where the higher mathematics are employed in teaching Physics.

The questions at the foot of the pages in the first edition, have been omitted, to gain space for a considerable number of practical problems (mostly original,) designed to exercise the student in the application of the principles and formulæ found in the text. To aid in the solution of these, and to assist the teacher in the construction of additional problems, numerous physical Tables have been added in the Appendix.

The plan of using two kinds of type, resorted to in the first edition, has been continued with more particularity in this The book is thus adapted to the use of the general reader, and to students who seek only a knowledge of general principles.

These changes and additions, the author believes, entitle this edition more fully to the encomiums bestowed on the first by many of the ablest physicists and most experienced teachers in this country. By the liberality of the publishers, numerous additions have been made to the wood-cuts, while new designs, in numerous cases, replace those of less beauty in the first edition.

The design has been, in this edition, to give to all the departments of physical science a just proportion of space, in harmony with the general scope of the book. The subject of Mechanics and Machines (upon which so many excellent special treatises exist) has, therefore, been condensed into a smaller proportionate space than it usually occupies in American treatises on Natural Philosophy; while such fundamental subjects as Motion, Force, Gravitation, Elasticity, Tenacity, and Strength of Materials, are considered at more length.

The author has freely availed himself of all the sources of information within his reach. A list of the works chiefly used in the preparation of this edition is appended—to which should be added the chief foreign journals, and transactions of learned societies—which have been resorted to for the original memoirs quoted on a great variety of topics. He is also particularly indebted for good counsel to many scientific and personal friends, the influence of whose criticisms on the first edition they will find frequently in the present. More than to all others is he indebted to Dr. M. C White, of New Haven, for his constant attention, both in the preparation of new matter and in the revision of the press.

He also takes pleasure in again acknowledging his obligations to Prof. C. H. Porter, of Albany.

For a final revision of the sheets, and the detection of a number of errors which had escaped previous proof-readers, the author is indebted to Mr. Arthur W. Wright, Assistant Librarian of Yale College.

Fuller references have been added, especially to American authorities; and the author hopes no apology is required for the frequent references to the American Journal of Science, which is supposed to be a work accessible to all American teachers, while the European journals are rarely so; and references to these would, therefore, be of little practical use to the great majority of readers of such a treatise as this.

As no table of errata is given (all errors thus far discovered being corrected), the author will esteem it a great favor if any person using the book will communicate to him direct any errors of fact or figures which may be discovered.

New Haven, *October* 15, 1860.

LIST OF THE PRINCIPAL WORKS USED IN PREPARING THIS EDITION.

Cooke. Chemical Physics. Boston, 1860.

Daguin. Traité de Physique, tom. I., II., and III. Paris and Toulouse, 1855–1859.

De la Rive. Treatise on Electricity, 3 volumes. London, 1853–1858.

Ganot. Traité de Physique. Paris, 1859.

Goodwin. A Collection of Problems and Examples. Cambridge, England, 1851.

Jamin. Cours de Physique, tom. I. and II. Paris, 1858–1859.

Kahl. Mathematische Aufgaben aus der Physik. Leipzig, 1857.

Miller. Chemical Physics. London, 1855.

Müller. Lehrbuch der Physik und Meteorologie. Braunschweig, 1857.

Potter. An Elementary Treatise on Optics, 2 Parts. London, 1851.

Potter. An Elementary Treatise on Mechanics. London, 1855.

Potter. Physical Optics. London, 1856.

Wernicke. Lehrbuch der Mechanik. Braunschweig, 1858.

FROM PREFACE TO THE FIRST EDITION.

THIS hand-book has been prepared with a view to give a fair exposition of the present condition of the several departments of Physics. * * * * * * Accuracy of statement, fullness of illustration, conciseness of expression, and a record of the latest and most reliable progress of science in these departments, have been the leading objects in its preparation.

Only those who have attempted to harmonize and present in due proportion the whole of so vast a subject as this, in a compendious form, can fully appreciate the labor and difficulties which attend it.

Without claiming for the present volume any credit more than belongs to a faithful digest and compilation from the best authorities in modern science, it is hoped that it will be found suited to the wants of a large class of both teachers and students. No pains have been wanting to secure accuracy both in fact and mechanical execution. The publishers have spared no expense to illustrate the book with a profusion of wood cuts. Many of these are original designs, or are reduced from larger drawings by photography—and others have been selected with care from the best standard authors. * * * * * * Whenever it was possible, reference has been had to original memoirs in Journals and Transactions, and in this way many errors current in works of inferior authority have been corrected. With but few exceptions, references to foreign memoirs have been omitted in the text, as their insertion could profit only a very small number of readers, and might seem pedantic. Not so with respect to names of discoverers of important principles and phenomena. A great number of names of these will be found in the text, in their proper places, and not unfrequently the dates of birth, or death, or both, are given.

Every teacher must have observed that an abstract principle is often fixed in the memory by the power of associated ideas, when it is connected with a date or item of personal interest, as the attention is awakened by the dramatic far more than by the didactic. Hence it has been thought judicious to introduce numerous important dates in the history of science.

* * * * * * * *

It gives me great pleasure to acknowledge many obligations to Prof. CHARLES H. PORTER, M. A., M. D., of Albany (some years my assistant), for his constant and most important assistance in the compilation and editing of this book. Preoccupied as my own time has been, I should not at times have found it possible to proceed without his valuable assistance and excellent judgment. Dr. M. C. WHITE, of this town, has also rendered me important aid, especially in OPTICS, and in the revision of the press.

* * * * * * * *

NEW HAVEN, CONN., *Oct.* 15, 1858.

CONTENTS.

PART FIRST.

PHYSICS OF SOLIDS AND FLUIDS.

CHAPTER I.

INTRODUCTION.

Matter—Observation and experiment, 1—Law, theory, and hypothesis—Inductive philosophy—Force, 2—The properties of matter are general, or specific—The changes in matter are physical, or chemical, 3—Physical and chemical properties of matter—Physics and chemistry—Vitality, 4—Light, heat, and electricity, 5.

CHAPTER II.

GENERAL PRINCIPLES.

§ 1. **Definitions and General Properties of Matter**—I. ESSENTIAL PROPERTIES—The essential properties of matter—Magnitude or extension, 5—Impenetrability—The three states of matter—II. ENGLISH AND FRENCH SYSTEMS OF MEASURES—Units of measure, 6—English units of length—The French system of measures, 7—III. ACCESSORY PROPERTIES OF MATTER—Divisibility, 8—Minute division in the animal and vegetable kingdoms, 9—Atoms, molecules—Compressibility, 10—Expansibility—Physical pores, 11—Sensible pores—Mobility, 12—Inertia—Action and reaction, 13.

§ 2. **Of Motion and Force**—I. MOTION—Varieties of motion, 13—Time and velocity—Uniform motion—Variable motion, 14—Motion uniformly varied, 15—Compound motion, 16—Parallelogram of velocities—II. OF FORCES—Definition of force—Forces are definite quantities—Weight—Unit of force—Dynamometers, 17—Equilibrium—Statical and dynamical forces—Direction of force, 19—Measure of forces—Mass—Propositions in regard to forces, 20—Momentum, 21—III. COMPOSITION OF FORCES—System of forces

—Components and resultant—The parallelogram of forces, 22—Examples of the composition of motion and force—Parallel forces—Resultant of unequal parallel forces, 25—Resultant of two parallel forces acting in opposite directions—Couples, 26—Two forces not parallel and applied to different points—The resolution of forces—Example of the resolution of force, 27—IV. CURVILINEAR MOTION—CENTRAL FORCES—Of curvilinear motion—Centrifugal and centripetal forces, 28—Examples of the action of centrifugal force—Experimental demonstration of the effects of centrifugal force, 29—The centrifugal drying machine for laundries, 31—Analysis of the motion produced by central forces, 32—Bohnenberger's apparatus—Parallelogram of rotations, 33—The gyroscope, or rotascope, 35—Problems on weights and measures—Problems on motion, 37.

CHAPTER III.

GRAVITATION.

§ 1. **Direction and Centre of Gravity**—Definition—Law of universal gravitation, 38—Direction of terrestrial attraction—Centre of gravity, 39—Point of application of terrestrial attraction, 40—Centre of gravity—Corollaries, 41—Centre of gravity of regular figures, 42—Centre of gravity lying without the body—Equilibrium of solids supported by an axis—Equilibrium of solids placed upon a horizontal surface, 43—Centre of gravity in bodies of unequal density in different parts—Equilibrium of bodies supported in more than one point, 44.

§ 2. **Laws of Falling Bodies**—Gravity is a source of motion—The laws of falling bodies are five, 45—Whole space described by a falling body, 47—Verification of the laws of falling bodies—Atwood's apparatus, 48—Morin's apparatus—Application of the laws of falling bodies, 50—Descent of bodies on inclined planes, 51—Descent of bodies on curves—Brachystochrone, or curve of swiftest descent—Action and reaction of a falling body, 52.

§ 3. **Measure of the Intensity of Gravity**—I. PENDULUM—The pendulum—Properties of the simple pendulum, 53—Isochronism of the pendulum, 54—Formulæ for the pendulum—Propositions respecting the simple pendulum, 55—The physical or compound pendulum—Centre of oscillation, 56—Application of the pendulum to the measurement of time, 57—Cycloidal pendulum—Physical demonstration of the rotation of the earth by means of the pendulum, 58—The pendulum applied to the study of gravity, 59—Use of the pendulum for measuring the force of gravity—Value of g in these experiments, 60—Seconds pendulum—II. MODIFICATIONS OF TERRESTRIAL GRAVITY AND THEIR CAUSES.—The intensity of gravity varies with the latitude, 61—Influence of the earth's figure upon gravity, 62—Exact dimensions of the earth—Sensible weight varies in different localities—Effect of the earth's rotation on gravity, 64—Demonstration, 65—Variation of gravity above the earth and below its surface—Below the earth's surface, 66.

§ 4. **Mass and Weight**—Mass—Weight—Density—Specific weight, 67—French system of weights, 68—English and American system of weights—Estimation of the density of the earth by experiment, 69—The inference, 71.

§ 5. **Motion of Projectiles**—Projectiles, 71—The ballistic pendulum—Problems—Falling bodies, 73—Descent of bodies on inclined planes—Central forces—Pendulum and Gravity, 74—Flight of Projectiles, 75.

CHAPTER IV.

THEORY OF MACHINERY.

§ 1 **Machines**—Principle of virtual velocities, 75—Machine, power, weight—Equilibrium of machines, 76—Utility of machines—Relation of power to weight, 77—Adaptation of the power to the weight in machinery—Vis viva, or living force, 78—Illustrations of vis viva—Impact and its results—Impact considered with reference to momentum, 80—Impact considered with reference to vis viva, 81—Pressure produced by impact—Destructive effects of impact, 82.

§ 2. **Mechanical Powers**—The lever, 83—Conditions of equilibrium in the lever—Compound levers—Application of the lever, 84—The scale beam—The steelyard, 85—Examples of compound levers, 86—Roberval's counter platform balance, 87—The wheel and axle, 88—Trains of wheel-work, 89—Analysis of a train of wheel-work, 90—The pulley—Fixed pulley—Movable pulley—Compound pulleys, 91—The inclined plane—Application of the power parallel to the length of the inclined plane, 93—Application of the power parallel to the base of the inclined plane, 94—Application of the power in some direction not parallel to any side of the plane—The wedge—Application of the wedge, 95—The screw, 96—Mechanical efficiency and applications of the screw—The endless screw, 97.

§ 3. **Strength and Power**—Animal strength—Strength of men—Horse-power machines, 98—Table of the comparative strength of men and other animals—Steam-power, 99—Perpetual motion, 100.

§ 4. **Impediments to Motion**—Passive resistances—Sliding friction—Starting friction—Friction during motion, 101—Coulomb's apparatus for determining starting friction—Results of Coulomb's experiments on starting friction, 102—Rolling friction—Coulomb's apparatus for determining rolling friction—Results of Coulomb's experiments on rolling friction, 104—Mr. Babbage's experiment—Advantages derived from friction—Rigidity of ropes—Resistances of fluids, 105—Actual and theoretical velocities—Ballistic curve, 106—PROBLEMS—Vis viva—The lever—Wheel and axle, 107—The pulley—Inclined plane—The screw—Resistance, 108.

PART SECOND.

THE THREE STATES OF MATTER.

CHAPTER I.

MOLECULAR FORCES.

Cohesion and repulsion—Repulsion, 109—Examples of cohesion among solids, 110—Cohesion in liquids and between liquid gases and solids, 111—Between gases and solids, 112.

CHAPTER II.

OF SOLIDS.—MOLECULAR FORCES ACTING BETWEEN PARTICLES OF LIKE KINDS.

§ 1. **Properties of Solids**—The characteristic properties of solids—Structure of solids, 113.

§ 2. **Crystallography**—Conditions of crystallization, 114—Amorphism—Crystalline forms—Definitions, 115—Systems of crystals, 118—Modified forms, 120—Compound crystals, 121—Cleavage—Determination of crystalline forms, 122.

§ 3. **Elasticity**—Elasticity of solids, 122—Elasticity of tension and compression, 123—Coefficient of elasticity, 124—Elasticity of flexure, 125—Applications—M. Bourdon's metallic barometer, 127—The Aneroid barometer—Elasticity of torsion, 128—Coulomb's laws of torsion, 129—Torsion of rigid bars—Limit of elasticity, 130—Change of density produced by tension, 131.

§ 4. **Strength of Materials**—Laws of tenacity, 131—Johnson's results—Tenacity of vegetable and animal substances—Resistance to pressure in columns, 132—The lateral or transverse strength of materials, 133—Practical applications, 134—General estimate of the strength of beams—The Britannia Tubular Bridge, 136—The Victoria Tubular Bridge—Limits of magnitude, 137.

§ 5. **Properties of Solids depending on a permanent displacement of their Molecules**—Malleability—Ductility—Hardness, 138—Brittleness—Hardening, Temper, Annealing, 139—Colors of tempered steel—Tempering by a bath—Temper of glass—Prince Rupert's Drops, 140—Tempering copper and bronze—Hammering, 141—Changes of structure affecting the mechanical properties of metals, 142.

§ 6. **Collision of Solid Bodies**—Motion communicated by collision, 142—Direct impact of elastic bodies—Modulus of elasticity, 143—Velocity of elastic bodies after direct impact, 144—Scholium, 145—Transmission of shock through a series of elastic balls—Experimental illustration of elasticity—

PROBLEMS—Elasticity of tension—Elasticity of flexure—Tenacity—Transverse strength, 146—Impact of elastic bodies, 147.

CHAPTER III.

OF FLUIDS.—HYDRODYNAMICS.

§ 1. **Hydrostatics**—I. DISTINGUISHING PROPERTIES OF LIQUIDS—Definitions—Fluids—Hydrodynamics—Mechanical condition of liquids—Elasticity of liquids—Compressibility, 148—Elasticity—Consequences, 150—II. TRANSMISSION OF PRESSURE IN LIQUIDS—Liquids transmit pressure equally in all directions, 151—The Bramah hydrostatic press, 152—Uses in the arts—Pressure of a liquid on the bottom of a vessel, 153—Upward pressure, 154—Pressure on the sides of a vessel—Pascal's experiment with a cask, 155—The water bellows, or hydrostatic paradox—Total pressure on the walls, 156—Total pressure on the bottom and sides of a vessel, 157—The centre of pressure, 158—Pressures vary as the specific gravities of liquids—III. EQUILIBRIUM OF LIQUIDS—The conditions of equilibrium in liquids, 159—Equilibrium of liquids when freed from the influence of gravity—The experiment of Plateau—Equilibrium of a liquid in communicating vessels, 160—Equilibrium of liquids of different densities in communicating vessels—Demonstration—The spirit level, 161—Artesian wells—IV. BUOYANCY OF LIQUIDS—Theorem of Archimedes, 162—Another demonstration of Archimedes' principle—Floating bodies, 164—Examples—Equilibrium of floating bodies, 165—Neutral equilibrium—Unstable equilibrium, 166—Stable equilibrium—The metacentre, 167—V. DETERMINATION OF SPECIFIC GRAVITY—The problem stated—Methods—Specific gravity by the hydrostatic balance, 168—Examples, 169, 170—Specific gravity bottles. 170—Example—Specific gravities by hydrometers or areometers—Nicholson's hydrometer or areometer, 171—Gay Lussac's and Beaumé's hydrometers, 172.

§ 2. **Hydraulics**—I. MOTION OF LIQUIDS—Definition—Pressure of liquids upon the containing vessel, 173—Appearance of the surface during a discharge—Theoretical and actual flow, 174—Reaction of the escaping vein—Barker's mill, 175—Flow—Theorem of Torricelli—Deductions from the Torricellian theorem, 176—Demonstration of the theorem of Torricelli, 177—The inch of water—Constitution of liquid veins, 178—Escape of liquids through short tubes—Escape of liquids through long tubes—Formulæ, 180—Jets of water—Pressure exerted by liquids in motion, 181—Velocity of rivers and streams—Stream measurers, 182—II. WATER-POWER AND WATER-WHEELS—Water-wheels—The overshot wheel—The undershot wheel, 183—The breast-wheel—Boyden's American turbine, 184.

*MOLECULAR FORCES ACTING BETWEEN PARTICLES OF UNLIKE KINDS.

I. CAPILLARITY—Observation—Definition, 187—General facts in capillarity, 188—Form of the surface—Cause of the curve of liquid surfaces by the contact of solids, 189—Experimental illustrations, 190—Influence of the curve on capillary phenomena, 191—Law of the elevation and depression of liquids in capillary tubes, 192—Depression of mercury in capillary tubes—Ascent of liquids in capillary tubes, 193—Laws of the equilibrium of liquids between parallel or inclined laminæ, 194—Movement of drops of liquid in conical tubes or between laminæ—Attraction and repulsion of light floating bodies, 195—Escape of liquids from capillary tubes—Flow of liquids from capillary tubes—II. OSMOSE OR OSMOTIC FORCE—Osmose, exosmose, endosmose, 196—Endosmometer—Necessary conditions—Materials for septum, 197—Direction of the current—Organic solutions—Inorganic solutions—Endosmose of gases—Theories of endosmose, 198—PROBLEMS ON HYDRODYNAMICS—Elasticity of liquids—Hydrostatic pressure, 199—Buoyancy of liquids—Floating bodies, 200—Motion of liquids—Capillarity, 201.

CHAPTER IV.

OF ELASTIC FLUIDS, OR GASES.

Pneumatics—I. DISTINGUISHING PROPERTIES OF GASES—Definitions—Pneumatics—Gases—Vapors—Tension—Expansion of gases, 202—Mechanical condition of gases—II. PROPERTIES COMMON TO BOTH LIQUIDS AND GASES—Gases transmit pressure equally in all directions, 203—The atmosphere, 204—Atmospheric pressure, 205—Buoyancy of air, 206—Impenetrability of air—Inertia of air—III. BAROMETERS AND BALLOONS—Torricellian vacuum—Measure of atmospheric pressure, 207—Pascal's experiments, 208—Construction of barometers—Apparatus illustrating the principle of the barometer, 209—Height of the barometric column at different elevations—Cistern barometer—Fortin's barometer, 210—The syphon barometer, 211—Wheel barometer—Causes of error—Correction for capillarity, 212—Correction for temperature—Variations of the barometric height, 213—Variations observed in barometers are of two kinds—Relation between barometric changes and the weather, 214—Rules by which coming changes are indicated—Measure of heights by the barometer, 215—Balloons, 216—Construction and filling of balloons, 217—Parachute—IV. COMPRESSIBILITY OF GASES—Mariotte's law, 218—Experimental verification of Mariotte's law, 219—Experiments of Despretz—Experiments of Regnault, 220—Results obtained by Regnault—General conclusions on the compressibility of gases, 221—V. INSTRUMENTS DEPENDING ON THE PROPERTIES OF GASES—Manometers—Manometer with free air—Manometer with compressed air, 222—Bellows—Bellows with a continuous blast, 223—Furnace blowers—Escape of compressed gases, 224—Pneumatic inkbottle—The syphon, 225—Intermittent syphon—Tantalus' vase—Intermittent springs, 226—Intermittent fountain—Air-pump, 227—Degree of exhaustion—Compressing machine—Water-pumps—Suction-pumps, 229—Suction and lifting pump—Forcing-pump, 230—Rotary pump—Fire-engine, 231—Hiero's fountain—Hydraulic ram, 232—Chain-pump—Archimedes' screw, 233—PROBLEMS ON PNEUMATICS—Atmospheric pressure, 234—Barometer and balloons—Mariotte's law, 235.

CHAPTER V.

OF UNDULATIONS.

§ 1. **Theory of Undulations**—Origin of undulations—Progressive undulations, 236—Mechanical illustration of undulations—Stationary undulations—Isochronous vibrations—Phases of undulations, 237—Nodal points, 238.

§ 2. **Undulations of Solids**—Solid bodies—Forms of vibration, 238—Vibration of cords—Laws of the vibration of cords, 239—Vibrations of rods—Paths of vibration, 240—Vibration of elastic plates—Nodal lines—Determination of the position of the nodal lines, 241—Laws of the vibration of planes

—Method of delineating nodal lines—Nodal figures, 242—Vibration of membranes, 243.

§ 3. **Undulations of Liquids**—Production of waves—Progressive undulations in liquids, 244—Stationary waves, 245—Depth to which waves extend—Reflection of waves—Waves propagated from the foci of an ellipse—Waves propagated from the focus of a parabola, 246—Circular waves reflected from a plane, 247—Combination of waves—Interference in an ellipse—Undulations of the waters of the globe, 248.

§ 4. **Undulations of Elastic Fluids**—Waves of condensation and rarefaction—Undulations of a sphere of air, 249—Mechanical illustration—Velocity and intensity of aerial waves, 250—Interference of waves of air—Intensity of waves of air expanding freely—PROBLEMS ON THE LAWS OF VIBRATIONS, 251.

CHAPTER VI.

ACOUSTICS.

§ 1. **Production and Propagation of Sound**—Acoustics—Sound—Sound a sensation—Key-note of nature—Noise—Musical sounds, 252—All bodies producing sound are in vibration, 253—Sound propagated by waves—Co-existence of sound-waves—Sound is not propagated in a vacuum, 254—Sound is propagated in all elastic bodies—Hearing—Time is required for the transmission of sound—The velocity of all sounds is the same, 255—Velocity of sound in air, 256—Velocity of sound in different gases and vapors—Calculation of distances by sound—Velocity of sounds in liquids—Velocity of sounds in solids, 257—Interference of sound—Acoustic shadow, 258—Distance to which sound may be propagated, 259—Reflection of sound—Echo, 260—Repeated echoes—Change of tone by echo, 261—Whispering galleries—Refraction of sound, 262—The speaking-trumpet, 263—Ear-trumpet—The siren, 264—Savart's toothed wheel—Music halls, 266.

§ 2. **Physical Theory of Music**—Qualities of musical sounds—Tone or pitch—Intensity or loudness—Quality—Unison—Melody—Harmony, 267—Musical scale—Gamut, 268—The sonometer or monochord—Relative number of vibrations corresponding to each note, 269—Absolute number of vibrations corresponding to each note, 270—Length of sonorous waves—Interval, 271—Compound chords—Perfect concord—A new musical scale, 272—Transposition—Temperament, 274—Beating—Diapason, tuning-fork, 275—Sensibility of the ear, 276.

§ 3. **Vibration of Air contained in Tubes**—Sonorous tubes—Mode of vibrating, 276—Mouth-pipes—Reed-pipes, 277—Gas jet—Musical instruments, 278—Vibration of air in tubes—Laws of Bernoulli—Demonstration, 279—Construction of musical instruments, 280—Vibrating dams, 281.

§ 4. **Vocal and Auditory Apparatus**—I. OF VOICE AND SPEECH—Voice and speech—The vocal apparatus of man, 281—The larynx—The glottis, 282—Mechanism of the voice—Range of the human voice, 283—Ventriloquism, stuttering, &c., 284—Production of sounds by inferior animals—Birds—Insects—II. THE EAR—HEARING—Auditory apparatus of man—The external ear, 285—The middle ear, tympanum, or tympanic cavity, 286—The internal ear, 287—Functions of the different parts of the ear—Natural diapason, 288—Organs of hearing in the lower animals—PROBLEMS IN ACOUSTICS—Velocity of sound, 289—Physical theory of Music, 290.

PART THIRD.

PHYSICS OF IMPONDERABLE AGENTS.

LIGHT, HEAT, AND ELECTRICITY.

CHAPTER I.

LIGHT, OR OPTICS.

§ 1. **General Properties of Light**—Optics—Light—Nature of light—Theories, 291—Sources of light—Phosphorescence, 292—Relation of different bodies to light—Rays, pencils, and beams of light—Visible bodies emit light from every point, 293—Propagation of light in a homogeneous medium—Velocity of light—Foucault's apparatus for measuring the velocity of light, 294—Fizeau's method—Results—No theory of light is entirely satisfactory—Properties of light, 296—Amount of light reflected at different angles of incidence, 298—Internal reflection—Total reflection, 299—Irregular reflection—Diffused light—Umbra and penumbra, 300—Images produced by light transmitted through small apertures—Intensity of light at different distances—Photometers, 301—Bunsen's photometer—Rumford's photometer—Silliman's photometer, 302.

§ 2. **Catoptrics, or Reflection by Regular Surfaces**—I. Mirrors and specula—Mirrors—Specula are metallic reflectors—Forms of mirrors—II. Reflection at plane surfaces—Reflection by plane mirrors, 303—Images formed by plane mirrors, 304—Images multiplied by two surfaces of a glass mirror—Images formed by light reflected by two plane mirrors, 305—Multiplicity of images seen by means of inclined mirrors—Deviation of light reflected by two mirrors, 306—Kaleidoscope, 307—Hadley's sextant—III. Reflection at curved surfaces—Concave and convex spherical mirrors, 308—Foci of concave mirrors for parallel rays—Foci of diverging rays, 309—Converging rays—Virtual focus—Secondary axes—Oblique pencils—Rule for conjugate foci of concave mirrors—Convex spherical mirrors, 311—Images formed by concave mirrors, 312—Virtual images—Formation of images by convex mirrors, 313—General rule for constructing images formed by mirrors Spherical aberration of mirrors—Caustics, 314.

§ 3. **Dioptrics, or Refraction at Regular Surfaces**—I. Definitions—Prisms and lenses, 314—II. Refraction at plane surfaces—Refraction by prisms, 315—Method of determining the index of refraction, 316—Plane glass—Light passing through parallel strata of different media, 317—Pencils of light refracted at plane surfaces, 318—Pencils of light transmitted through plane glass—III. Refraction at curved surfaces—Principles determining the foci of lenses, 320—Small pencils of light refracted at a spherical surface, 321—Refraction at a concave surface, 322—Action of a double convex lens upon small pencils of light, 323—Conclusions deduced—Analysis, 324—Action of a double concave lens upon small pencils of light—Conclusions deduced from analysis, 326—Rules for determining the foci of lenses—Combined lenses, 327—Oblique pencils—The optical centre of a lens, 328—Images formed by lenses—Spherical aberration of lenses, 329—Accurate estimate of spherical aberration, 330—Aberration of sphericity; distortion of images, 331.

§ 4. **Chromatics**—Analysis of light—Spectrum—Primary colors—Recomposition of white light—Analysis of colors by absorption, 332—Complementary colors—Properties of the solar spectrum, 333—Fraunhofer's dark lines in the solar spectrum, 334—Lines in light from different sources, 335—Fixed lines in the spectra from various colored flames—Intensity of luminous, calorific, and chemical rays—Refraction and dispersion of the solar spectrum—Kalychromatics, 336—Chromatic aberration—Achromatism, 337—Formulæ for achromatism, 338.

§ 5. **Vision**—Structure of the human eye, 339—Action of the eye upon light, 340—Inversion of the image formed in the eye—Optic axis—Optic angle—Visual angle, 341—The brightness of the ocular image—Conditions of distinct vision—Background, 342—Sufficiency of illumination—Distance of distinct vision—Visual rays nearly parallel, 343—Adaptation of the eye to different distances—Appreciation of distance and magnitude—Aerial perspective, 344—Single vision with two eyes—Double vision—Binocular vision, 345—Near-sightedness—Long-sightedness, 346—Duration of the impression upon the retina—Optical toys—Thaumatrope—Time required to produce visual impressions—Appreciation of colors—Color blindness, 347—Chevreul's classification of colors, and chromatic diagram, 348—Examples—The study of colors, 349.

§ 6. **Optical Instruments**—Magnifying glasses, 349—The magnifying power of a lens, 350—The simple microscope—Raspail's dissecting microscope—The compound microscope, 351—The telescope—The telescope used by Galileo, 352—The opera-glass—Night-glasses—The astronomical telescope——Eye-pieces—The positive eye-piece—The negative eye-piece, 353—The terrestrial eye-piece—Reflecting telescopes—Sir William Herschel's telescope, 354—Lord Rosse's telescope—Achromatic telescopes, 355—Equatorial mountings for telescopes, 356—The Cambridge telescope with equatorial mountings, 357—The visual power of telescopes, 358—Achromatic object-glasses for microscopes—Lister's aplanatic foci, and compound objectives, 359—Aberration of glass cover corrected—The compound achromatic microscope, 361—Solid eye-piece—Visual power of the achromatic microscope, 362—The mechanical arrangement of the microscope, 363—The magic lantern—The solar microscope, 364—The camera obscura—Wollaston's camera lucida, 365—Photography—Railway illumination, 366—The Fresnel lens—Sea-lights, 367—Revolving lights, 368—The telestereoscope, 369—The stereoscope, 370—The stereomonoscope, 372.

§ 7. **Physical Optics**—I. INTERFERENCE, DIFFRACTION, FLUORESCENCE, &c. Interference of light, 373—Facts at variance with theory—Interference colors of thin plates, 374—Newton's rings—Length of luminous waves or vibrations, 375—Diffraction, 376—Fluorescence—Epipolic dispersion—Phosphorescence, 377—Colors of grooved plates—II. OPTICAL PHENOMENA OF THE ATMOSPHERE—The rainbow, 378—A secondary rainbow, 380—Spurious rainbow—Fog-bows—Halos—Coronas—Parhelia—Atmospheric refraction—Looming—The mirage, 381—III. POLARIZATION OF LIGHT—Direction of luminous vibrations—Transmission of luminous vibrations, 382—Change produced by polarization of light—Resolution of vibrations, 383—Light polarized by absorption—Polarization by reflection, 384—Polarization by metallic reflection—Polarization by refraction—Polarization by successive refractions, 385—Partial polarization—Double refraction, 386—Positive and negative crystals—Polarization by double refraction—Nicol's single image prism, 387—Polarizing instruments—Colored polarization, 388—Rotary polarization—Arago's chromatic polariscope, 389—Colored rings in crystals—Polarization by heat, and by compression, 390—Magnetic rotary polarization—Atmospheric polarization of light—The eye a polariscope, 391—The practical applications of polarized light—PROBLEMS IN OPTICS—Velocity and intensity of light, 392—Reflection of light—Refraction of light, 393—Optical Instruments—Polarization of light, 394.

CHAPTER II.

HEAT.

§ 1. **Nature of Heat**—Heat—Its nature—Temperature—Heat and cold, 395—Action of heat on matter, 396.

§ 2. **Measurement of Temperature**—I. THERMOMETERS—Thermometers—Construction of mercurial thermometers—Thermometer tubes, 396—Filling the tube with mercury—Standard points in the thermometer—Graduation, 397—Freezing point—Boiling point, 398—Different thermometric scales—Fahrenheit's scale—Centigrade scale, 399—Reaumur's scale—Comparison and conversion of thermometric scales—House thermometers, 400—Tests of a good thermometer—Sensibility of thermometers—Displacement of the zero point, 401—Limits of the mercurial thermometer—Spirit thermometers—Other liquid thermometers, 402—Defects inherent in mercurial thermometers, 403—Standard thermometer, 405—II. SELF-REGISTERING THERMOMETERS—Maximum and minimum thermometers, 406—Rutherford's maximum and minimum thermometer—Negretti and Zambra's maximum thermometer, 407—Walferdin's maximum thermometer—Metastatic thermometer, 408—III. METALLIC THERMOMETERS AND PYROMETERS—Breguet's metallic thermometer—Saxton's deep-sea metallic thermometer, 409—Saxton's reflecting pyrometer—Wedgewood's pyrometer—Daniell's pyrometer, 410—Draper's pyrometer—Estimation of very high temperatures—IV. THERMOSCOPES—Thermoscopes—Air thermometers, 411—Leslie's differential thermometer—Howard's differential thermometer—Rumford's thermoscope—Thermo-multiplier, 412.

§ 3. **Expansion**—I. EXPANSION OF SOLIDS—Linear expansion—Pyrometers—Cubical expansion, 413—Relation between cubical and linear expansion—Expansion of crystals—Coefficient of expansion, 414—The capacity of hollow vessels is increased by the expansion of their walls—The amount of expansion in solids, 415—The ratio of expansion increases with the temperature—Amount of force exerted by expansion, 416—Common phenomena produced by the expansion of solids, 417—Fire regulators—Unequal expansion of solids, 418—Compensating pendulums—Harrison's gridiron compensating pendulum—Graham's compensating pendulum, 419—Mr. Henri Roberts' compensating pendulum—Martin's compensating pendulum—Compensating balance wheels of watches and chronometers, 420—II. EXPANSION OF LIQUIDS—General statement—Apparent and absolute expansion, 421—Correction of the observed height of the barometer for temperature—Apparent expansion of mercury, 422—Amount of expansion of liquids, 423—Expansion of liquids above their boiling points—Curves of expansion of liquids—The amount of force exerted in the expansion of liquids, 424—Expansion of water—Maximum density of water, 425—Effects of the unequal expansion of water—Maximum density of different aqueous solutions, 426—The volume of water at different temperatures—III. EXPANSION OF GASES—General statement—Gay Lussac's laws for the expansion of gases by heat—Result of Regnault's experiments upon the expansion of gases, 427—Formulæ for computing changes of volume in gases, 428—Formulæ expressing general relation between volume, temperature, and pressure—Relation between expansibility and compressibility—Density of gases, 429.

§ 4. **Communication of Heat**—I. CONDUCTION—Modes in which heat is communicated—Conduction of heat, 430—Determination of the conductibility of solids, 431—Conductibility of metals, &c.—Conductibility of crystals—Conductibility of wood, 432—Vibrations produced by conduction of heat—Conductibility of liquids—Conductibility of gases, 433—Relative conductibility of solids, liquids, and gases—Examples and illustrations of the different conductibility of solids—Examples drawn from the animal and vegetable kingdoms, 434—The conducting power of substances in a pulverized or fibrous state—Clothing, 435—II. CONVECTION—Convection—Convection in liquids—Currents in the ocean, 436—III. RADIATION—Radiation of heat—Radiant heat is but partially absorbed by the media through which it passes, 437—Intensity of radiant heat—Law of cooling by radiation, 438—Universal radiation of heat—Apparent radiation of cold, 439.

§ 5. **Action of different Bodies upon Heat**—I. SURFACE ACTION—Reflection of heat—Conjugate mirrors—Determination of reflective power, 440—Absorptive power—Emissive or radiating power, 441—Causes which modify the emissive, absorbent, and reflective powers of bodies—Applications of the powers of reflection, absorption, and radiation, 442—II. DIATHERMANCY—Transmission of radiant heat—Melloni's apparatus, 443—Influence of the substance of screens, 444—Influence of the material and nature of the source—Other causes which modify the diathermanic power of bodies, 445—Thermochrosy—Applications of the diathermancy of bodies—Refraction of heat, 446—Polarization of heat, 447.

§ 6. **Calorimetry**—Calorimetry—Unit of heat, 447—Specific heat—Method of mixture—Method by fusion of ice, 448—The method of cooling—Specific heat affected by change of state—Specific heat of gases, 449—Specific heat of gases under a constant volume, 450—Specific heat determined by the laws of acoustics, 451—Relation between the specific heat and atomic weight of elements and compounds, 452.

§ 7. **Liquefaction and Solidification**—Latent heat, 452—Liquefaction and congelation are always gradual—Freezing mixtures, 453—Laws of fusion and latent heat of fusion—Peculiarities in the fusion of certain solids,—Refractory bodies—Solution—Saturation, 454—Laws of solidification—Elevation of temperature during solidification—Change of volume during solidification, and its effects, 455—Freezing of water—Absolute zero, 456.

§ 8. **Vaporization and Condensation**—Vaporization, 456—Formation of vapors in a vacuum, 457—Saturated space or maximum tension of vapors—Dalton's law of the tension of vapors—The tension of vapors in communicating vessels unequally heated—Temperature and limits of vaporization, 458—Circumstances influencing evaporation—Dew-point—Ebullition, 459—Circumstances influencing the boiling point, 460—The culinary paradox—Frank-

lin's pulse glass—Useful applications in the arts—Measurement of heights by the boiling point—Hypsometer, 461—High pressure steam—Production of cold by evaporation, 462—Twining's ice machine—Latent heat of steam, 463 -Latent and sensible heat of steam at different temperatures--Mechanical force developed during evaporation, 464—Liquefaction—Distillation—Distilling apparatus--Physical identity of gases and vapors--Theory of the liquefaction and solidification of gases, 465--Methods of reducing gases to liquids —Later researches of Faraday—Thilorier's and Bianchi's apparatus for condensation of gases, 466—Properties of liquid and solid gases—Latour's law, 467--Density of vapors, 468.

§ 9. **Spheroidal condition of Liquids**--Spheroidal state—Illustration of the spheroidal state--Noticeable phenomena connected with the spheroidal state, 468—Spheroidal state produced upon the surface of liquids—A liquid in a spheroidal state is not in contact with the heated surface beneath, 469—A repulsive action is exerted between the spheroid and the heated surface—The causes which produce the spheroidal form, 470—Freezing water and mercury in red-hot crucibles—Remarkable phenomena connected with the spheroidal state—Explosions produced by the spheroidal state, 471—Steam boiler explosions—Familiar illustrations of the spheroidal state, 472.

§ 10. **The Steam-Engine**—Historical—The eolipile—First steamboat—Savary's engine, 473—Papin's steam cylinder—Newcomen's engine, 474—The atmospheric engine—Watt's improvements in the steam-engine, 475--The low pressure or condensing engine, 476—The high pressure engine—The cut off, 477—Steam boilers—Mechanical power of steam—Horse-power, 478—Evaporating power and value of fuel, 479.

§ 11. **Ventilation and Warming**--I. VENTILATION—Currents in air and gases, 480--Draught in chimneys, 481—Reversed draughts and smoky chimneys --Products of respiration and combustion, and necessity for ventilation, 482 -—The quantity of vapor given off by the body--The quantity of air required for good ventilation—Products of gas illumination, 483--The actual ventilation of buildings is a practical problem—Stone's ventilating shaft, 484—Cold currents produced by ice—Refrigerators—Cowls--Emerson's ventilators, 485 —The supply of fresh air in dwellings—II. WARMING--The artificial temperatures demanded in cold climates—The open fire, 486--Hot air furnaces, 487 —Heating by hot water—Perkins' high pressure hot water apparatus, 488 —Gold's steam heaters—The radiators, 489—The boiler of Gold's steam heater, 490.

§ 12. **Sources of Heat**—Sources of heat—I. MECHANICAL SOURCES OF HEAT —Friction, 490—Quantity of heat produced by friction—Circumstances which vary the quantity of heat developed by friction--Illustrations and application of the heat developed by friction, 491—Compression—Percussion—Capillarity, 492--II. PHYSICAL SOURCES OF HEAT—The sun—Quantity of heat emitted by the sun—Extremes of natural temperature, 493--Terrestrial radiation--Origin of terrestrial heat—Atmospheric electricity, 494—III. CHEMICAL SOURCES OF HEAT--Chemical combination—Combustion--On the cause of the heat generated by combustion--The amount of heat developed by chemical action, 495 —The pyrometrical heating effect of a substance, 496--Relative value of fuel —Combinations in the humid way--Animal heat; warm and cold-blooded animals, 497—The cause of animal heat--Temperature of vegetables, 498.

§ 13. **Correlation of Physical Forces**--I. MECHANICAL EQUIVALENT OF

HEAT—Relations of heat and force, 498—Unit of measurement, the foot-pound—Determination of the mechanical equivalent of heat—Results of Joule's experiments, 499—Conclusions deduced from Joule's experiments.—II. DYNAMICAL THEORY OF HEAT—The dynamical theory of heat—Motions of the molecules—Changes in the state or volume of bodies, 500—III. ANALOGY OF LIGHT AND HEAT—Vibrations producing heat and light—Impressions of light and heat, 501—Bodies become luminous by incandescence, 502—Heat and light produced by chemical and mechanical action—Dilatation and change of state, 503—Change of state produced by heat—Quality of heat changed by absorption and radiation, 504—Difference between quantity and intensity of heat—Conclusion, 505—PROBLEMS ON HEAT—Thermometers, 506—Expansion—Specific heat, 507—Tension of vapors—Ventilation and warming, 508.

CHAPTER III.

ELECTRICITY.

General statement 509

§ 1. **Magnetic Electricity**—I. PROPERTIES OF MAGNETS—Lodestone—Natural magnets—Artificial magnets, 509—Magnetic needles—Distribution of the magnetic force—Polarity, 510—Magnetic phantom—Magnetic curves—Magnetic figures, 511—Anomalous magnets—Attraction and repulsion—Magnetism by contact, 512—Magnetism in bodies not ferruginous—II. MAGNETIC INDUCTION OR INFLUENCE—Induction, 513—Theoretical considerations—Theory of two fluids, 514—Coercitive force—III. TERRESTRIAL MAGNETISM—Magnetic needle—Directive tendency, 515—The mariner's compass, 516—The astatic needle—Magnetic meridian—Declination or variation, 517—Variation chart—Isogonal lines, 518—Daily variations of the magnetic needle—The annual variation—Dip or inclination, 520—The action of the earth's magnetism—Dipping needle, 521—Inclination map, or isoclinal lines, 522—Magnetic intensity, 523—Isodynamic lines, 524—The inductive power of the earth's magnetism—System of simultaneous magnetic observations, 525—Lines of magnetic force—Atmospheric magnetism, 526—Notions of the origin of the earth's magnetism—IV. PRODUCTION OF MAGNETS—Artificial magnets, 527—The circumstances affecting the value of magnets—Magnets by touch, 528—Horse-shoe magnets, 529—Magnets by electro-magnetism—Compound magnets, 530—Magnetism of steel by the sun's rays—To deprive a magnet of its power, 531.

§ 2. **Statical or Frictional Electricity**—I. ELECTRICAL PHENOMENA—Definitions, 531—The chief sources of electrical excitement—Electrical effects—Attraction and repulsion, 532—Vitreous and resinous, or positive and negative electricities—Electroscopes, 533—Conductors of electricity—Good conductors—Bad conductors—Insulation, 534—The earth is the great common reservoir—Theories of electricity, or electrical hypothesis—Franklin's single-fluid hypothesis, 535—The hypothesis of Symmer, or Du Fay, 536—Electrical tension—Electrical currents are either momentary or permanent—Path and velocity of electric currents, 537—II. LAWS OF ELECTRICAL FORCES AND DISTRIBUTION OF ELECTRICITY UPON THE SURFACE OF BODIES—Coulomb's laws—Torsion electrometer, 538—Demonstration of the first law—Demonstration of the second law—Sir Wm. S. Harris—Proof-plane, 539—Electricity resides only on the outer surfaces of excited bodies, 540—Distribution of electricity—The power of points 541—The loss of electricity in excited bodies—III. INDUCTION OF ELECTRICITY—Electrical influence or induction, 542—The laws of induction—Induction is an act of contiguous particles, 543—The attractions and repulsions of light bodies, 544—Electrometers—Cavallo's, Volta's, and Bennett's—IV. ELECTRICAL MACHINES—The electrophorus, 545—The cylinder electrical machine, 546—Amalgam—Ramsden's plate machine, 547—The American plate electrical machine—Large electrical machine—Ritchie's double plate machine, 548—The Tylerian machine, 549—Care and management of electrical machines—Electricity from steam, 550—Other sources of electrical excitement, 551—Theory of the electrical machine—Experimental illustrations of electrical attractions and repulsions—The insulating stool, 552—Henly's electroscope—Electrical bells—Volta's hail-storm—The electrical wheel—By a candle flame, 553—V. ACCUMULATED ELECTRICITY AND ITS EFFECTS—Disguised or latent electricity—The condenser of Æpinus, 554—The discharge of the condenser—Volta's condensing electroscope, 556—Dr. Hare's single gold-leaf electrometer—The Leyden jar, 557—Electricity in the Leyden jar resides on the glass, 558—The electric battery—Discharge in cascade, 559—The universal discharger—The electric spark, 560—Kinnersley's thermometer—The color of the electrical spark—The electrical discharge in a vacuum, 561—Difference between the positive and negative spark—The diamond jar—Scintillating tube and magic squares, 562—Effects of the electric discharge—Physiological effects, 563—Inflammation of combustibles—Chemical union effected by electricity, 564—Volta's electrical lamp—The mechanical effects of the electrical discharge, 565—The chemical effects—Ozone—VI. ATMOSPHERIC ELECTRICITY—Franklin's kite, 566—Free electricity in the atmosphere, 567.

§ 3. **Dynamical Electricity**—I. GALVANISM AND VOLTAISM—Discovery of galvanism, 568—The galvanic fluid—Origin of Volta's discovery—Contact theory, 569—Volta's pile, or the Voltaic battery, 570—Distinction between voltaism and galvanism, 571—Quantity and intensity—Simple voltaic couple, 572—Electro-positive and electro-negative—Amalgamation, 573—II. BATTERIES WITH ONE FLUID—Voltaic batteries—Trough batteries—Hare's calorimotor, 574—The deflagrators of Dr. Hare—Smee's battery, 575—The sulphate of copper battery—III. DRY PILES—Dry piles of Zamboni and De Luc, 576—Bohnenberger's electroscope—IV. BATTERIES WITH TWO FLUIDS—Daniell's constant battery, 577—Grove's nitric acid battery, 578—Carbon battery, 579—Other forms of voltaic battery—V. POLARITY, RETARDING POWER, AND NOMENCLATURE OF THE VOLTAIC PILE—Polarity of the compound circuit, 580—Grouping the elements of a pile—Electrical retarding power of the battery—Ohm's law, 581—Formulæ of electric piles, 582—Intensity given by many couples, 583—Effect of increasing the number of couples in a battery—Effect of enlarging the plates of a battery, 584—Effect of enlarging the couples and increasing their number—Faraday's nomenclature, 585—VI. THE EFFECTS OF THE VOLTAIC PILE—1. *Physical effects*—The voltaic spark and arch, 586—Regulators of the electric light, 587—Dubosq's photo-electric lantern—Properties of the electric light, 588—Heat of the voltaic arch—Deflagration, 589—Measurement of the heat of the voltaic current—2. *Chemical effects of the pile*—Historical, 590—Electrolysis of water—Voltameter, 591—Laws of electrolysis—Electrolysis of salts, 592—Electro-metallurgy—The electrotype, 593—Crystallization from the action of feeble currents, 594—Examples—Deposit of metallic oxyds and Nobili's rings, 595—3. *Physiological effects of the pile*—The physiological effects of the voltaic pile—The magnetic effects of the pile—VII. THEORY OF THE PILE—Three views, 596—Volta's contact theory, 597—The chemical theory—Laws of the disengagement of electricity by chemical action—The quantity of electricity required to produce chemical action, 598—Polarization and transfer of the elements of a liquid—Chemical affinity and molecular attraction distinguished—Peschel's molecular theory of the pile, 599.

§ 4. **Electro-Dynamics**—I. ELECTRO-MAGNETISM—General laws, 600—Œrsted's discovery, 601—The electric magnetic current moves at right angles to the course of the conjunctive wire—Galvanometers or multipliers, 602—The tangents or sine compass galvanometer—Rheostat, 604—Ampère's electro-magnetic discoveries and theory—Mutual action of electric currents, 605—De La Rive's floating current—Roget's oscillating spiral—Helix, solenoid, or electro-dynamic spiral, 606—Directive action of the earth—Magnetizing by the helix, 608—Arago's original experiment—Electro-magnets, 609—Page's revolving electro-magnet, 610—Power of electro-magnets, 611—Vibrations and musical tones from induced magnetism—Electro-magnetic motions and mechanical power, 612—Conversion of magnetism into heat—II. DIAMAGNETISM—Action of magnetism on light, 614—Diamagnetism, 615—III. ELECTRIC TELEGRAPH—Historical, 616—The earth circuit, 618—Varieties of electro-telegraphic communication—The electro-mechanical form—The electro-chemical telegraphs—Morse's recording telegraph, 619—House's electro-printing telegraph, 620—The electro-chemical telegraph, 621—Autograph telegraphic messages—Submarine telegraphs—The Atlantic cable, 622—Electrical clocks and astronomical records—Fire-alarm, 623.

§ 5. **Electro-dynamic Induction**—I. INDUCED CURRENTS—Currents induced from other currents—Volta-electric induction, 623—Induced currents of different orders, 625—Extra-current, or the induction of a current on itself—Page's vibrating armature and electrotome, 626—Induced currents from the earth's magnetism—Conversion of dynamic into static electricity—The induction coil, 627—Effects of the induction coil—The luminous effects, 629—The chemical effects—Light of these currents in a vacuum—Gassiot's cascade in vacuo, 630—Rotation of the electric light about a magnet—Applications, 631—II. MAGNETO-ELECTRICITY—Currents induced by magnets, 632—Clark's magneto-electric apparatus, 633—Identity of electricity from whatever source, 634.

§ 6. **Other Sources of Electrical Excitement**—Universality of electrical excitement, 634—I. THERMO-ELECTRICITY—Thermo-electricity, 635—Thermo-electric motions—Cold produced by electrical currents—Melloni's thermo-multiplier—II. ANIMAL ELECTRICITY—The galvanic current, 636—Electrical animals—Electricity of plants, 638.

APPENDIX.

CHAPTER I.

METEOROLOGY.

§ 1. **Climatology**—Climates, seasons—Influence of the sun, 639—Meteorological observations—Mean temperature, 640—Variations of temperature in latitude—Variations of temperature in altitude, 641—Limit of perpetual snow—Isothermal lines, 642.

§ 2. **Aerial Phenomena**—General consideration of winds—Propagation of winds—Anemoscopes, 643—Anemometers—The velocity of winds—Formulæ, 644—Regular winds—Periodical winds, 545—Variable winds—General direction of winds in the higher latitudes, 646—Physical properties of winds—Hot winds—Hurricanes, or cyclones, 647—Tornadoes or whirlwinds—Water-spouts, 649—General laws of storms, 650.

§ 3. **Aqueous Phenomena**—Humidity of the air—Absolute humidity—Relative humidity, 651—Hygrometers—Saussure's hygrometer—Daniell's hygrometer—August's psychrometer or hygrometer of evaporation, 652—Fogs or mists—Dew—Cause of dew—Circumstances influencing the production of dew, 653—Substances upon which dew falls—Frost, 654—Clouds—Classification of clouds, 655—Rain, 656—Distribution of rain—Days of rain, 657—Annual depth of rain—Snow—Colored snow, 658—Hail, 659.

§ 4. **Electrical Phenomena**—Free electricity of air—Thunder-storms, 659—The geographical distribution of thunder-storms—The origin of thunder-clouds—Thunder, 660—Lightning—Classes of lightning—Zigzag or chain lightning—Sheet lightning—Heat lightning—Ball lightning, 661—Volcanic lightning—Return stroke—Lightning-rods, 662—Protective power—Aurora borealis—Appearance of auroras—Remarkable auroras, 663—Height of auroras, 664—Frequency of auroras—Geographical distribution of auroras, 665—Magnetic disturbances during auroral displays—Effect of the aurora on telegraphic wires, 666—Reversal of polarity in the auroral current—PROBLEMS ON ELECTRICITY, 667.

Addenda—Note to § 369: Uniform musical pitch—Note to § 343: The velocity of all sounds not the same—Note to § 392: Sounds produced by insects, 668.

CHAPTER II.

PHYSICAL TABLES.

TABLE I.—Measures and weights (Cooke) . . . . . . PAGE 669
II.—For converting French decimal measures and weights into English measures and weights (Gmelin) 672
III.—Expansion of solids 674
IV.—Expansion of liquids 675

TABLE V.—Expansion of gases (Regnault) . . . . . PAGE 676
VI.—Radiating power according to Provostaye, Desains, and Melloni 676
VII.—Conducting power of metals and building materials . . 677
VIII.—Absorptive power of different bodies (Provostaye & Desains) 678
IX.—Absorptive power for heat from different sources (Melloni) 678
X.—Diathermancy of different liquids (Melloni) 679
XI.—Specific heat (Regnault and others) 679
XII.—Freezing mixtures 680
XIII.—Diathermancy of different solids (Melloni) 681
XIV.—Tension of vapors at equal distances above and below the boiling point of their respective liquids 681
XV.—Melting points and latent heat of fusion of different bodies . 682
XVI.—Boiling point of water under different pressures (Regnault) 682
XVII.—Boiling points of liquids (Kopp & Pierre) 683
XVIII.—Boiling point of water at different places and their elevation above the sea 683
XIX.—Boiling point of water at different atmospheric pressures (Regnault) 684
XX.—Liquefaction and solidification of gases (Faraday) . . 684
XXI.—Aqueous vapor in a cubic foot of saturated air at different temperatures (Guyot, condensed) 685
XXII.—Latent and sensible heat of steam (Regnault) 686
XXIII.—Specific gravity of solids and liquids 686
XXIV.—Volume and density of water (Kopp) 687
XXV.—Comparison of the degrees of Baumé's hydrometer, with the real specific gravity 688
XXVI.—Tension, volume, and density of aqueous vapor (Regnault, Peclet) 689

ANSWERS TO PROBLEMS 691
INDEX 699

PART FIRST.

PHYSICS OF SOLIDS AND FLUIDS.

CHAPTER I.

INTRODUCTION.

1. **Matter.**—Matter is that which occupies space, and is the object of sense. Our knowledge of the material world is founded upon experience, or the evidence of our senses; and the conviction that the same causes will always produce the same effects.

A definite and limited portion of matter, whether it be a particle of dust or a planet, is called a *body*. The different kinds of matter, as water, marble, gold, or diamond, are called *substances*. Numberless as are the various substances known to man, they are all composed of a limited number of simple bodies called *elements*.

2. **Observation and experiment.**—By observation we become acquainted with those changes, in the condition and relations of bodies, which occur spontaneously in the ordinary course of nature; but the knowledge thus acquired is limited when compared with the results of *experiment*. By the use of proper apparatus we can repeat natural phenomena under varied conditions; and, among all the attendant circumstances, we can determine what are accidental, and what are essential to any given effect.

Phenomena.—A phenomenon, in the sense in which this word is used in science, is any event taking place in the ordinary course of nature. Thus the changes of the seasons, the fall of rain or dew, the burning of a fire, and the death of an animal, are more truly phenomena of nature than those more rare or alarming events to which in a vulgar sense this word is usually confined.

3. **Law, Theory, and Hypothesis.**—In conducting an experiment, we are taught to trace with certainty the *connection* between different phenomena; to classify effects of the same kind and refer them to their common cause; in fine, to deduce from many experiments the governing principle, or *law of nature,* in obedience to which they are produced, and to unite both facts and principles into a *theory,* or comprehensive view of the whole subject. Such theories are a fruitful source of new experiments and new discoveries.

The terms *law, theory,* and *hypothesis* are often used interchangeably, and are all designed to express the various degrees of perfection attained in any department of human knowledge, towards the understanding of the thoughts of God as expressed in the phenomena of the physical world. An *hypothesis* is a guess or assumption, designed to aid further investigation, and bears the same relation to a theory or law as the scaffolding bears to the perfect building. A theory is the most perfect expression of physical truth, and is deduced from both laws and principles that have been established on independent testimony.

That a theory should rise to the highest expression of the laws of nature, it must account not only for all known phenomena falling under it, but for all possible cases with their irregularities and variations. Thus the law of gravitation, as developed by Newton from terrestrial phenomena, has been found strictly universal in its application; not only meeting all known facts in celestial mechanics, but, outstripping observation, it has foretold events which have been subsequently confirmed, or which it still requires centuries of years to verify.

4. **Inductive Philosophy.**—When individual experience is enlarged by the experience of other inquirers and other times, and the combined knowledge of many is so arranged as to be comprehended by one, the system becomes a *science* or philosophy of nature. Because its principles are founded upon a comparison and analysis of facts, a system of this kind is also called *Inductive* Philosophy.

Inductive philosophy is of modern origin. Galileo (born in 1564) was the first to commence a course of experimental researches; and Bacon (born in 1561), in his immortal work, *Novum Organum,* showed that this was the only road to an accurate knowledge of nature. The ancients were ignorant of the principles and methods of inductive science. Their explanations of natural phenomena were based on *assumed* causes; they are therefore confused and contradictory, and often in direct opposition to experience.

5. **Force.**—From the axiom that every event must have a cause, the mind naturally passes to the recognition of certain powers or *forces* in nature adequate to account for the observed phenomena. Thus we

refer the fall of bodies to the earth to the force of gravitation—the strength of materials to the force of cohesive attraction—the directive power of the compass-needle to the earth's magnetism—the evaporation of water to the action of heat—the combustion of a fire to the action of oxygen on the elements of the fuel, or to the force of chemical affinity.

Man exercising his volition walks, or strikes a blow—examples of the mysterious connection between spirit and matter, of the conscious exercise of mechanical force. By the use of a lever or screw he transmits or multiplies his force at will—by experiment he learns that he can also, by suitable appliances, call into action, where he pleases, certain other forces, otherwise dormant, which he calls *chemical*, or *physical*, according as they do, or do not, involve an essential change in the nature of the materials employed. Both his consciousness and experience inform him that all these manifestations of force result from the voluntary but mysterious action of mind upon matter. He is thus led to the unavoidable conclusion that those great phenomena of nature, over which he has no control, must have their origin also in the volitions of a SUPREME RULER. FORCE and WILL thus become related terms, and we are compelled to regard the forces of nature, as they are usually styled, as only the outward and visible manifestations of the mind of GOD.

In Physics the term force is often used for the unknown cause of a known effect.

6. **The properties of matter are general, or specific.**—The attentive consideration of any sort of matter will show us the existence of two sorts of properties in it—namely, general properties and specific properties. Gold, for example, occupies space and possesses weight, but so also does all matter, whether solid, liquid, or gaseous; these properties are *general*. But gold has a peculiar color and lustre, is unchangeable by the action of causes which destroy the identity of nearly all other sorts of matter, has a definite and peculiar crystalline form, and weighs about nineteen times as much as a like bulk of water. These are qualities peculiar to gold, and by which we always recognize it. They are its *specific properties.*

7. **The changes in matter are physical, or chemical.**—Water is changed by heat to steam or vapor, by loss of heat (cold) it is reduced to a solid. By the ceaseless action of these natural causes, it perpetually changes its place and condition. It returns to the earth, from its distillation in the great alembic of the atmosphere, as dew, mist, rain, hail, or snow, and by gravity seeks to gain a place of rest in the great ocean. But in all its changes of state and position it is

still the same substance. A bar of iron, by contact with a lodestone, acquires new properties, which we call magnetic, but its color, form, and weight remain unchanged. A glass tube or plate of resin rubbed by dry silk or fur becomes electrical, in virtue of which property it will attract or repel light bodies. These changes, which do not destroy the specific identity of the substance, are termed *physical changes.*

But in a damp atmosphere the iron bar is soon covered with rust, from the action of oxygen (one of the gases of the air) upon the iron. The same change follows the action of water alone. In this latter case the water is decomposed, and with great activity if a dilute acid is present. The oxygen of the water combines with the iron, while the hydrogen escapes as a gas, and thus the *specific identity* of both substances is destroyed. Such changes, destructive of *specific identity,* are called *chemical changes.*

8. **Physical and chemical properties of matter.**—The changes of matter just noticed correspond to its physical and chemical properties. Gold possesses certain specific properties, depending solely on its physical qualities; its density, lustre, color, form, malleability, and its high point of fusion, are all qualities of gold which can never be lost without an essential change of its nature, and are therefore termed physical properties. Exposed however to the action of chlorine and certain other agents, gold loses its specific identity, and becomes, as it were, a new substance, while the same change passes equally upon the agent by whose efficiency the transmutation is effected. Such changes of matter, involving an essential loss of specific identity, depend on the *chemical properties* of matter.

9. **Physics and Chemistry.**—It is plain that the distinctions just pointed out are fundamental, in the nature of things, and that out of them spring two entirely distinct, although nearly related, branches of human knowledge, namely, PHYSICS and CHEMISTRY; the former is more frequently called, in this country, *Natural Philosophy;* a term too comprehensive in its general significance for an exact definition. Now as all substances possess both physical and chemical properties, it is plain that a thorough knowledge of either branch involves some familiarity with the other. But the natural order of knowledge consists in obtaining first a familiarity with the general properties and laws of matter, and subsequently the specific properties. Physical knowledge therefore naturally precedes chemical.

10. **Vitality,** or the principle of life, is recognized as a distinct force in nature, controlling both physical and chemical forces; by its action inanimate or unorganized matter is transformed into animate and organized existences. Thus, out of air, water, and a few mineral

substances, all living forms, both animal and vegetable, are built up by the chemistry of life. After a life of definite duration, they die, and their structures dissolve again into the inanimate bodies out of which they grew. They are subject to the general laws of matter, but these laws are often modified, and sometimes directly opposed by the action of that unknown power which we call the *principle of life*. The description of organized bodies constitutes the science of Natural History.

11. **Light, Heat, and Electricity** are terms employed to distinguish certain phenomena, or forces in nature, connected with, or growing out of the changes of matter, physical or chemical, or both. They are supposed by most physicists to be dependent on the existence of certain hypothetical fluids, or on the vibrations of an assumed ethereal medium. As these fluids, or forces, are without weight or other sensible properties of ordinary matter, they are termed, by many writers, the imponderable agents, or simply imponderables. What the spirit is to the animal body these mysterious agents are to lifeless matter.

CHAPTER II.

GENERAL PRINCIPLES.

§ 1. Definitions and General Properties of Matter.

I. ESSENTIAL PROPERTIES.

12. **The essential properties of matter** are (1) *magnitude*, or extension, (2) *impenetrability*. We cannot conceive of matter without magnitude, and it is equally clear that the space occupied by any given particle of matter cannot, at the same time, be occupied by any other particle.

All the other general properties of matter, however universal they may be, have been made known to us by observation and experiment, and are not essential to the fundamental notion of the existence of matter. The accessory or non-essential properties of matter are, 1, Divisibility, 2, Compressibility, 3, Expansibility, 4, Porosity, 5, Mobility, and, 6, Inertia.

13. **Magnitude or extension.**—Extension is the property which

every body possesses of occupying a portion of space. The amount of space so occupied by a body is called its *volume.*

Every body has three dimensions, length, breadth, and thickness, the external boundaries of which are surfaces and lines. The exact measurement of these three dimensions is the foundation of all exact knowledge in experimental science, and demands the adoption of certain arbitrary units of comparison.

14. **Impenetrability.**—The power of a body to exclude all other bodies from the space occupied by itself is called *impenetrability.* This property is possessed by all forms of matter. Air may be compressed indefinitely, perhaps, but the mechanical force required for its compression is at once the evidence and the measure of its impenetrability.

A stone dropped into the water displaces its own bulk of the fluid, but does not penetrate its particles. A nail driven into a board only displaces certain particles of the wood, whose resistance or elasticity imparts to the nail its power of adhesion.

The union of these two properties, extension and impenetrability gives exactness to our fundamental notion of matter. Neither alone will suffice to produce a body. The image in a mirror is not a body, for behind the mirror, where the image appears, is the wall, or perhaps another body. The shadow of any object in the sunlight has extension, but, as it is not impenetrable, it is not a body.

15. **The three states of matter.**—Matter is presented to our senses in three unlike physical states, viz., *solid, liquid,* and *gaseous.* The last two states are more comprehensively called *fluids.* These three physical conditions of matter represent the opposite action of the forces of attraction and repulsion. But as these interesting relations, and the physical laws governing them, are fully discussed under their appropriate heads, it is needless to do more than refer to them here. (146.)

II. ENGLISH AND FRENCH SYSTEMS OF MEASURES.

16. **Units of measure.**—In order to determine with accuracy the *volume* of solids and the *area* of surfaces or the *length* of lines, some arbitrary unit of extension must be adopted. Of the three geometric degrees of extension the unit of length is the only one which need be arbitrary, since by squaring it we may measure surfaces, and by cubing it we can measure solids. In early times the weight of grains of wheat, ("thirty-two of which, from the midst of the ear, were, A. D. 1266, declared to be equal to an English penny, called a sterling,") or the length of "barley corns" (three to an inch) gave the rude basis of legal units of weight and measure in England, and, long after, by adoption in the United States.

17. **English units of length.**—The *yard* is the English unit of length, adopted both in Great Britain and America. It appears to have had its origin about A. D. 1120, in the reign of Henry the First, "who ordered that the *ulna*, or ancient ell (which corresponds to the modern yard) should be made of the exact length of his own arm, and that the other measures of length should be based upon it." The yard is divided into thirty-six inches.

In 1824 it was enacted by the English Parliament, that if at any time the standard yard should be lost, defaced, or otherwise injured, it should be restored by making a new standard yard, bearing the same proportion to a pendulum vibrating seconds of mean time in the latitude of London, in a vacuum and at the level of the sea, as 36 inches bears to 39.1393 inches, the latter being the length of the pendulum vibrating seconds at London.

In 1834 the Parliament House was destroyed by fire, and with it the standard yard. The measurement of the seconds' pendulum, as given above, was subsequently found to be incorrect, and the commissioners appointed to consider the steps to be taken to restore the lost standard, recommended the construction of four standard yards from the best authenticated copies of the old standard. These duplicates (a copy of which exists in the U. S. Mint) are the basis of English and American standards of length.

The subdivisions and multiples of the yard are given in Table I., at the end of this volume.

Nearly all the English units of surface are squares whose sides are equal to the units of length. The square and cubic inch are the units most frequently employed for scientific purposes.

The measures of capacity are related to those of length, by the determination that a gallon contains 277.274 cubic inches. (§ 101.)

Where volume can be calculated from linear measurements, it is usual to estimate it in cubic yards, cubic feet, or cubic inches. In this way earth-work and masonry are measured.

18. **The French system of measures** originated with the great revolution in France, when all regard for ancient institutions was repudiated. A commission of the members of the Academy of Sciences was appointed, who developed a decimal system, which was at once adopted. They proposed that the ten-millionth part of the quadrant of a meridian of the globe should be assumed as the basis of a new metrical system. This was called a *metre*, and subdivisions and multiples of this unit were made on the decimal system. The metre is equivalent to 39.37079 English inches, or 39.36850535 American inches. Later determinations have shown that the length of the

standard metre is not precisely the one ten-millionth part of a quadrant. Thus it appears that the metre of France is a standard of measure not less arbitrary than the English yard.

The French metre is subdivided into tenths, called *decimetres;* hundredths, or *centimetres;* and thousandths, or *millimetres.** The names of the multiples are as follows: the *decametre,* ten metres; the *hectometre,* one hundred metres; and the *kilometre,* one thousand metres. This last length is equal to about two-thirds of an English mile, and it is the ordinary road-measure in France.

The French units of surface are squares, whose sides are equal to the units of length. The common French measure of land is the square decimetre, which is called an *are.*

The measures of capacity are connected with those of length by means of the *litre,* which is a cubic decimetre, (or a cube measuring 3.937 English inches on the side). It is equal to 1.765 Imperial pints, or somewhat more than 1$\frac{3}{4}$ English pints. (See Table I.)

The cubic metre is the measure of bulky articles, and has received the name of *stere.* The *stere,* as well as the *litre,* and the *are,* have decimal multiples and subdivisions, named like those of the metre.

The connection of the system of weights with those of capacity and length is explained in § 100.

III. ACCESSORY PROPERTIES OF MATTER.

19. **Divisibility.**—By mechanical means matter may be reduced to an extreme degree of comminution. By chemical means, and the processes of life, this subdivision is carried very much farther. A few illustrations of each of these kinds of divisibility will suffice.

Gold is beaten into leaves so thin that one million of leaves measure less than an inch in thickness. A bar of silver may be gilded, and then drawn into wire so fine that the gold, covering a foot of such thread, weighs less than $\frac{1}{6000}$ of a grain. An inch of this wire, containing $\frac{1}{72,000}$ of a grain, may be divided into 100 equal parts distinctly visible, and each containing $\frac{1}{7,200,000}$ of a grain of gold. Under a microscope magnifying 500 times, each of these minute pieces may be again subdivided 500 times, each subdivision having to the eye the same apparent magnitude as before, and the gold on each, with its original lustre, color, and chemical properties unchanged, represents $\frac{1}{3,600,000,000}$ part of the original quantity.

Dr. Wollaston, by a very ingenious device, obtained platinum wire for the micrometers of telescopes, measuring only $\frac{1}{30,000}$ of an inch in diameter. Though platinum is nearly the heaviest of known bodies, a mile of such wire would weigh only a grain, and 150 strands of it would together form a thread only as thick as a filament of raw silk.

---

* The smaller measures are named by *Latin,* the larger by *Greek* numbers.

A grain of copper dissolved in nitric acid, to which is afterwards added water of ammonia, will give a decided blue color to 392 cubic inches of water. Now each cubic inch of the water may be divided into a million particles, each distinctly visible under the microscope, and therefore the grain of copper must have been divided into 392 million parts.

One hundred cubic inches of a solution of common salt will be rendered milky by a cube of silver, 0·001 of an inch on each side, dissolved in nitric acid, and the magnitude of each particle of silver thus represents the one-hundred billionth part of an inch in size. To aid the student in forming an adequate conception of so vast a number as a billion, it may be added that to count a billion from a clock beating seconds, would require 31,688 years continuous counting, day and night.

Minute division in the animal and vegetable kingdoms.—The blood of animals is not a uniform red liquid, as it appears to the naked eye, but consists of a transparent colorless fluid, in which float an innumerable multitude of red corpuscles, which, in animals that suckle their young, are flat circular discs, doubly concave, like the spectacle glasses of near-sighted persons. In man, the diameter of these corpuscles is the 3500th of an inch, and in the musk-deer, only the 12,000th of an inch, and therefore a drop of human blood, such as would remain suspended from the point of a cambric needle, will contain about 3,000,000 of corpuscles, and about 120,000,000 might float in a similar drop drawn from the musk-deer.

But these instances of the divisibility of matter are far surpassed by the minuteness of animalcules, for whose natural history we are indebted chiefly to the researches of the renowned Prussian naturalist, Ehrenberg. He has shown that there are many species of these creatures, so small that millions together would not equal the bulk of a grain of sand, and thousands might swim at once through the eye of a needle. These infinitesimal animals are as well adapted to life as the largest beasts, and their motions display all the phenomena of life, sense, and instinct. Their actions are not fortuitous, but are evidently governed by choice, and directed to gratify their appetites and avoid the dangers of their miniature world. The stagnant waters of the earth (and sometimes the atmosphere) everywhere are populous with them, to an extent beyond the power of the imagination to conceive their numbers. Their silicious skeletons are found in a fossil state, forming the entire mass of rocky strata, many feet in thickness and hundreds of square miles in extent. The polishing-slate near Bilin, in Bohemia, contains in every cubic inch about 41,000 millions of these animals. Since a cubic inch of this slate weighs 220 grains, there must be in a single grain 187 millions of skeletons, and one of them would, therefore, weigh about the one 187-millionth of a grain. The city of Richmond, Va., has been shown by Prof. Bailey to rest on a similar deposit of silicious animalcules of exquisite form. It is impossible to form a conception of the minute dimensions of these organic structures, and yet each separate organ of every animalcule is a compound of several organic substances, each in its turn comprising numberless atoms of carbon, oxygen, and hydrogen. It is plain from these examples that the actual magnitude of the ultimate molecules of any body is something completely beyond the reach equally of our senses to perceive, or of our intellects to comprehend.

20. **Atoms, Molecules.**—The ultimate constitution of matter has divided the opinions of philosophers from the earliest period of science. Two hypotheses have prevailed; the one, that matter is composed of irregular particles without fixed size or weight, and divisible without limit; the other, that "matter is formed of solid, massy, impenetrable, movable particles, so hard as never to wear or break in pieces" (Newton), and which, being wholly indivisible, have a certain *definite* size, figure, and weight, which they retain unchangeably through all their various combinations. These ultimate and unchangeable particles are called *atoms* (meaning that which cannot be subdivided).

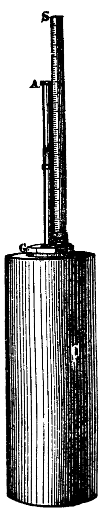
While there is no mathematical objection to the assumption that matter is infinitely divisible (since no mass can be conceived of, so small that it cannot be mentally subdivided), physics and more particularly chemistry have shown, from the mutual relations of bodies, that their constituent particles possess definite and limited magnitudes.

The term *molecule* (a little mass) is more commonly applied to what, in chemistry, are sometimes called divisible atoms; *i. e.*, to a group of two or more atoms, *e. g.*, the molecule of water is composed of at least two atoms, one of hydrogen and one of oxygen, forming together a chemical compound. The phenomena of crystallization show us that molecules possess different properties at different points of their surfaces. While their form is unknown, it is assumed that they touch each other not at all, or only at a few points, leaving spaces between them which bear a large ratio to their own bulk. From this fact result the two general properties of compressibility and expansibility, which are next to be considered.

21. **Compressibility.**—Diminution of volume in solids, by mechanical means, and by loss of heat, is a fact long familiar to all who are

conversant with constructions. Even columns of stone, and arches, supporting heavy loads, are found to diminish sensibly from their original dimensions by pressure alone. Metals are compressed by coining. In liquids it was long believed there was no compressibility, but in reality liquids possess this property to a greater extent than solids.

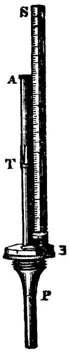
1



Saxton's modification of Perkins's original apparatus serves to demonstrate the compressibility of water. A strong metallic vessel, C, fig. 1, is filled with water, and closed by a close fitting screw plug, B, fig. 2; and a perfectly polished cylindrical piston of steel, A, passes water-tight through the steel collar, P. When the vessel is thus prepared, it is placed in a larger vessel capable of enduring great pressure, which is also filled with water.

Pressure to any extent desired is then applied by means of a hydraulic press. It is evident that if the water undergoes diminution of volume when subjected to pressure, the piston A must be forced into the cylinder to a corresponding extent. The index T having been placed at 0 of the scale S, if it is found, after the experiment, above that point, as in fig. 2, it is evidence of a corresponding descent of the piston, due to compression of the water contained in the cylinder C. On removing the pressure the elasticity of the water restores the original bulk. Water is found, by this experiment, to yield about fifty millionths of its volume for each atmosphere of pressure, *i. e.*, for a pressure of fifteen pounds to a square inch.

2



In air and all gases we see the property of compressibility very apparent. The air syringe is an instrument in which a portion of air is compressed before a solid piston, with the evolution of so much heat as to set fire to tinder.

The return of gases and liquids to their original bulk on removal of the condensing force is due to a property termed *elasticity*. This quality exists in many solids, if not in all, and its consideration will be resumed hereafter.

22. **Expansibility.**—The expansion and contraction of all bodies by heat and cold is a fact sufficiently familiar. Upon it is based the construction of all instruments for reading changes of temperature, for a description of which the reader is referred to the chapter on heat.

23. **Physical pores.**—The facts connected with the compressibility of matter, and its change of form by heat, indicate clearly that the atoms of matter (assumed to be unchangeable) are not in contact. The spaces existing between them are called *physical pores*, on the existence of which depends the property of *porosity*. Many chemical phenomena illustrate the existence of this property. If equal measures of alcohol and water, or of water and sulphuric acid are mixed, the

bulk of the resulting liquid is sensibly less than the sum of the two liquids before they were mingled. This shrinkage can result only from the insinuation of the particles of one substance among the pores of the other.

The great amount of heat developed, during these experiments, is a significant fact. These molecular or physical *pores* of bodies are no more sensible to our organs than the atoms themselves, and are permeable only to light, heat, and electricity.

24. **Sensible pores.**—It is important to distinguish the molecular porosity just described from those sensible openings which give to certain substances the property generally known as porosity. The pores of organic bodies, as of wood, skin, and tissues, are only capillary openings, or canals, for the passage of fluids. Nearly all animal and vegetable substances present these sensible pores. The familiar pneumatic experiment—the *mercurial rain*—is an illustration of the porosity of wood. Many minerals and rocks are porous. Common chalk and clay are familiar examples. Hydrophane is a kind of agate, opaque when dry, but translucent when wet from absorption of water. Even gold, and other metals, under great pressure, as in the experiments of the Florentine academicians in 1661, are found to exude water.

25. **Mobility.**—We constantly see bodies changing their place by *motion*, while others remain in a state of *rest*. The capacity of change of place, or of being set in motion, constitutes what is called *mobility.*

We recognize motion only by comparing the body moving with some other body at rest. If that rest is real then the motion is *absolute,* but if it is only apparent then the motion is only *relative.* Thus, on board ship, or on a rail car, the passenger appears to change his place in reference to objects about him. But all these objects are equally in motion with himself.

All motion on the earth's surface is relative, because the globe itself is impelled by a double movement—of revolution on its own axis, and of translation about the sun.

Rest is also *absolute* or *relative.* Absolute when the body occupies really the same point in space—relative when it preserves the same apparent distance from surrounding objects regarded as fixed, but which are not in reality so. A ship sailing six miles an hour against a current of the same velocity appears to persons on her deck to be advancing with reference to the surrounding waves; but, viewed from the shore, or by comparison with objects on shore, she appears at rest. Absolute rest is of course unknown on the earth, since every terrestrial object partakes of the double motion already noticed, and it is doubtful if any part of the universe is in absolute rest, seeing that the sun itself with the whole solar system is carried around with a rapid motion of translation in space about a central sun.

26. **Inertia.**—No particle of matter possesses within itself the power of changing its existing state of motion or rest. Matter has no spontaneous power either of rest or motion, but is equally susceptible to each, according as it may be acted on by an external cause. If a body is at rest, a force is necessary to put it in motion; and conversely, it cannot change from motion to rest without the agency of some force. A body once put in motion will continue that motion in an unchanging direction with unchanging velocity until its course is arrested by external causes. This passive property of matter is called *inertia.* Descartes first gave definite expression to this law in his "Principles."

When we are told that a body at rest will for ever remain so, unless it receives an impulse from some external power, the mind at once assents to a statement which embodies the results of our constant experience. But it requires some reflection in one who for the first time considers the subject, to admit that bodies in motion will continue to move for ever, unless arrested by external forces. Casual observation seems to contradict the assertion. On the earth's surface we know of no motion which does not require force to maintain as well as produce it.

We may observe, however, that all such moving bodies meet with constant obstruction from friction, and the resistance of the air; and that as one or both of these are diminished, the motion becomes prolonged and continuous.

The familiar apparatus called the *wind-mill in vacuo* is a good illustration of the tendency to continued motion due to inertia—the usual causes of arrest of motion being here greatly diminished.

The planets furnish the only example of constant motion. These celestial bodies, removed from all the casual resistances and obstructions which disturb our experiments at the earth's surface, roll on in their appointed orbits with faultless regularity, and preserve unchanged the direction and velocity of the motion which they received at their creation.

27. **Action and reaction.**—It follows as a necessary consequence of the inertia of matter, that when a body, M, in motion strikes another body, M', at rest, the action of M in imparting motion to M' is exactly equaled by the power of M' to destroy motion in M. Hence the law that *action and reaction are always equal and opposite.* It is here assumed that the bodies impinging are entirely devoid of elasticity, and so related that after collision they shall move on as one body. It is also true for elastic bodies. See § 181.

§ 2. Of Motion and Force

I. MOTION.

28. **Varieties of motion.**—We distinguish the following varieties in the motion of a body.

a—A motion of *translation*, or *direct motion*, in which all the points of a body move parallel to each other.

b—A motion of *rotation*, as of a wheel on an axis, where the different parts of a body move at the same time in different directions. *Oscillation* or *vibration*, as in a pendulum, is only a particular case of rotation.

c—A combination of translation and rotation, as in the motions of the earth.

The direction of motion is represented by a straight line drawn from the point where motion commences, to the point towards which the body is propelled. The direction is *rectilinear*, when it is constantly the same, and *curvilinear*, when it varies every moment.

29. **Time and velocity.**—As all the phenomena of nature may be referred to motion, so the succession of natural phenomena gives us the idea of duration, or *time*. Day and night, months, and the order of the seasons, are nature's units of time; but in physics the invariable unit of time is the duration of a single oscillation of a pendulum, called a *seconds' pendulum*, the time of oscillation being *a second*. The length of such a pendulum at London is 39·14056 English inches. The distance passed over by a moving body, in a unit of time, is its velocity, represented by V in physical formulæ. This symbol obviously involves both time and velocity.

30. **Uniform motion.**—A body moving over equal spaces in equal times is said to have *uniform motion*. It follows from the property of *inertia* that a body in motion, if left to itself, will continue its motion uniformly both in time and direction.

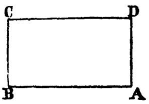
Proposition I. *The distance* PASSED *over, in uniform velocity, is proportioned to the time.* This follows directly from the definition of velocity. Denoting by D the distance passed over, and by T the number of seconds, we have

$$D = V \times T, \quad V = \frac{D}{T}, \quad \text{and } T = \frac{D}{V}.$$

The first expression is called the formula for uniform motion, and the two others serve to calculate the velocity, the distance and time being known; or the time, the distance and velocity being given.

It follows that if we represent T by one of the longer sides (A B) of a parallelogram, A B C D, fig. 3, and V by one of the shorter sides (B C) of the same parallelogram, then the area of the parallelogram A B C D represents the distance passed over by a moving body in the number of seconds denoted by T.

3



31. **Variable motion.**—In varying motion the distances passed

over in successive seconds are unequal. In this case, the velocity at any given instant, is the relation between the distance traversed and the time, considering the time infinitely small; or the distance that would be traversed in a unit of time, supposing the motion at the given instant to be continued uniform for the unit of time.

32. **Motion uniformly varied.**—When the velocity of a body increases by a constant quantity in a given time, it is said to be *uniformly accelerated.* The increase of velocity in a second is called its *acceleration,* which will be represented by v. *Uniformly retarded motion* is where the velocity of the body diminishes by a uniform quantity in each second of time.

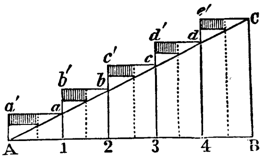
Proposition II. *The change of velocity in uniformly varying motion, at the end of any given time, is proportional to that time.*

Let u be the initial velocity, that is, the velocity at the instant from which the time is computed, v the acceleration and V the velocity at the end of t seconds, then $V = u \pm vt$. The sign + corresponds to the case of uniformly accelerated motion, and the sign — to that of uniformly retarded motion. In the last case the velocity becomes null when $u = vt$, that is at the end of a number of seconds represented by $\frac{u}{v}$. The above formula in fact involves this proposition. If we then make $u = 0$, that is, if it be assumed that the motion starts from a state of repose, we shall have at the end of the time t, $V = vt$. This is what we announced in stating that the velocity acquired, at the end of a given time, is proportional to that time.

Proposition III. *In uniformly accelerated motion the distances passed over, by a body starting from a state of rest, are proportional to the squares of the times employed.*

Representing the time by the line A B, fig. 4, and the velocity at the end of the given time by the line B C, divide the time A B into minute equal parts, A–1, 1–2, 2–3, 3–4 — — — — — The velocities acquired during the times represented by A 1, A 2, A 3, A 4, will be represented by the lengths of the several lines, 1 *a*, 2 *b*, 3 *c*, 4 *d*, — — — — which are proportioned to these times. Suppose, however, that during each minute portion of time A–1, 1–2, 2–3, 3–4 — — —, the velocity is constant, and equal to that attained at the end of each interval; the motion being uniform, the distances passed over during these several subdivisions of time will be represented (30) by the areas of the parallelograms 1 *a′*, 2 *b′*, 3 *c′*, — — — —, and the distance passed over at the end of the time A B, by the sum of these parallelograms. This sum

4



differs from the area of the triangle A B C by all that passes the line A C. It is evident that if the time A B had been divided into a larger number (say double) of equal parts, the sum of the parallelograms would have been less in excess of the triangle. The diminution is indicated by the areas shaded in the figure. In proportion therefore as the subdivisions of the time A B are more numerous, the sum of the parallelograms will differ less from the area of the triangle A B C. Finally, when the number of divisions becomes infinite, that is, when the velocity varies in a uniform manner, the distance passed over during the time A B will be represented by the surface of the triangle A B C.

But that area $= \frac{1}{2}$ A B $\times$ B C. Substituting A B $= t$, B C $= V$, and recalling the value of $V = vt$, we have for the distance passed over $D = \frac{1}{2}$ A B $\times$ B C or $D = \frac{1}{2}\, vt^2$, a formula involving the proposition stated above. This elegant demonstration is due to Galileo, who discovered the laws of uniformly varying motion.

Corollaries. 1*st.* The last formula shows that the distance passed over, in uniformly accelerated motion, by a body which starts from a state of repose, is equal to the distance it would pass over with a uniform mean velocity. 2*d.* It follows also, from the same formula, if we represent by a the distance through which a body moves in the first second, we can easily find the following values for the distances it will move through during each succeeding second, and also the whole distance it will have passed through at the end of each second.

Times,	1	2	3	4	5	n
Successive distances,	a	$3a$	$5a$	$7a$	$9a$	$(2n-1)a$
Whole distances,	a	$4a$	$9a$	$16a$	$25a$	n^2a.

The coefficients in the last series are as the squares of the times, while those in the second series are as the odd numbers, and are deduced from the last series by subtracting from each of its terms the one next preceding it. 3*d.* The distance passed over during a given time, in uniformly accelerated motion, is equal to one half the distance which would be traversed during the same time by a uniform motion, with the velocity acquired at the end of the given time; that is, the velocity at the end of the time t is equal to vt, and the distance which it traverses during the time t is $\frac{1}{2}vt^2$, according to the formula. 4*th.* To determine the velocity acquired in terms of the distance passed over, it is necessary to eliminate t from the equations $V = vt$ and $D = \frac{1}{2}vt^2$, which gives $V = \sqrt{2\, v\, D}$.

The velocity is said to be due to the distance, (D,) an expression which should not be literally interpreted.

33. **Compound motion.**—A body moving along a right line may

partake of two or more motions, in which case its path will be the resultant of the combination of these motions. Such a motion is called a *compound motion.* Daily observation confirms this statement, which will be sufficiently illustrated when we consider the results of *compound forces.*

34. **Parallelogram of velocities.**—The composition and resolution of velocities will be more readily explained and illustrated when considering the forces in which motion has its origin, as they follow the same laws.

II. OF FORCES.

35. **Definition of force.**—By *force,* as used in mechanics, we mean any cause producing, or modifying, motion. All known forces, under this definition, have their origin in three causes; namely, 1st, *gravitation,* or the mutual attraction of bodies for each other; 2d, the unknown cause of the phenomena of *light, heat,* and *electricity;* and 3d, *life,* or the mysterious agency producing the motions of animals.

The study of forces and their effects constitutes the science of *mechanics.*

36. **Forces are definite quantities.**—As we readily conceive of one force as greater than another, so we understand that forces are equal when, operating in opposite directions, they mutually balance each and produce equilibrium. The same may be true of the action of two, three, or more equal forces, forming, by their union, double, triple, or any higher combination of force.

To determine a force with precision we must consider three things; 1st, the *point of application;* 2d, the *direction;* 3d, the *intensity,* or energy with which the force acts.

It is usual to represent forces, like other magnitudes, by lines of definite lengths. Any line may be chosen as the *unit of force.* The direction of a line will then represent the direction of the force, starting from the point of application; and its length will represent the magnitude, or *intensity,* of the force, expressed by the number of times that it contains the unit of force. A force is therefore defined in each of its three elements by a line, being thus brought within the limits of number, geometry, and mathematical analysis.

37. **Weight; Unit of Force; Dynamometers.**—Where a body is left free to the action of gravity, but is held immovably by some obstacle, the pressure or tension which it exerts on the point of support is called its *weight.* It is important to distinguish carefully between the words *weight* and *gravity.* The latter signifies the general cause which produces the fall of all bodies to the earth, while the former

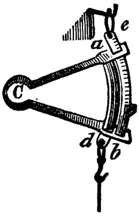
means only the result of the action of that general cause in the case of a particular body.

The common unit of force is the pound avoirdupois.

The weight of a body may be rendered sensible by the use of an instrument called a dynamometer, or measurer of force.

One of the most simple of these is represented in fig. 5; it consists of a steel spring, *a C b*. The metallic arc *a d* is fixed near the end of the limb *C a*, and passes freely through an opening in the other limb. The graduated arc *b e*, is fixed, in like manner, in the limb *C b*. The amount of the force exerted at the points *e* and *d*, determines the degree to which the two limbs will approach, and is represented on the graduated arc in pounds and ounces, the graduation of the arc being the result of actual trial, by hanging known weights upon the hook, and observing the positions marked by the index.

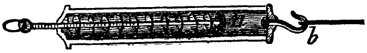
5



Many forms of dynamometer exist, of which the spring balance, or Le Roy's dynamometer, is the most familiar.

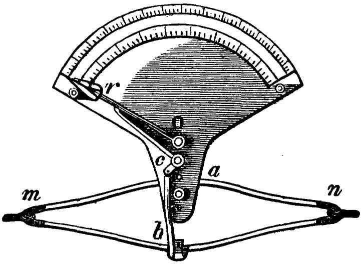
Le Roy's dynamometer, or spring balance, fig. 6, consists of a steel spring, coiled within a cylindrical tube. The end *n* of the spring is attached to a rod of metal, graduated in pounds and ounces, which is drawn out more, as the applied force, *b*, is greater.

6



Reynier's dynamometer, fig. 7, consists of a steel spring, *m a n b*, of which the part *a* and *b* approach each other, when a force is exerted at the points *m* and *n*. An arc graduated in lbs. is attached to the spring at the point *a*, and carries a needle, *r o*, worked by a lever, *c*. The position of this needle upon the arc, indicates the amount of the force exerted. If it is wished to determine the strength or force exerted by any animal or machine, as the strength exerted by a horse in drawing a plow through the ground, it is only necessary to attach one end of the instrument, as *n*, to the plow, and have the horse attached to the other end, *m*. The degree which is marked by the pointer, when the horse moves, represents in lbs. the force exerted.

7



Another dynamometer is often used, similar in construction to the last, only that the force is exerted at the points *a* and *b*, fig. 7, which are made to approach by a contrivance similar to that shown in fig. 5.

It is evident that, in this last arrangement, the force is applied in the most favorable position for producing the maximum effect in collapsing the spring; while in Reynier's dynamometer, the force is applied where only the minimum effect is produced, and the instrument is therefore generally employed for determining only very considerable forces.

38. **Equilibrium.**—When all the forces acting on a body are mutually counterbalanced, or neutralized, they, and the body on which they act, are said be in *equilibrium*. The word *repose* and *equilibrium* are to be carefully distinguished, however, as signifying different conditions of a body. Repose implies simply a state of rest, without involving any idea of motion. Equilibrium signifies the state of a body which, submitted to action of any number of forces, is still in the same condition as if these forces did not act. By the definition of inertia (26) a body may be in motion without being submitted to the action of any force, and it may even continue its motion undisturbed, although it becomes subject to forces producing equilibrium, since such forces mutually neutralize each other. Equilibrium does not therefore include the idea of immobility, and thus the words repose and equilibrium have significations essentially unlike.

Equilibrium may exist without any point of support or apparent resistance. A balloon in the air, or a fish in the water, are examples, but the balloon and fish are balanced by counteracting forces hereafter explained. What are familiarly known as examples of stable or unstable equilibrium are only special cases of the action of the force of gravity, to be explained in their proper place.

39. **Statical and Dynamical forces.**—*Statics* is the science of equilibrium. It considers the relations existing between the three conditions (36), which are involved in the case of every force, in order that equilibrium may result. Archimedes was the author of this portion of mechanical science.

Dynamics considers the motions which forces produce. The foundations of this part of mechanical science were laid by Galileo in the early part of the seventeenth century.

Hydrostatics and *Hydrodynamics* are the principles of statics and dynamics applied to the phenomena of rest and motion in fluids.

The distinction between statics and dynamics is so far artificial that the same force may, according to circumstances, produce either pressure or motion, without any change in the nature of the force.

40. **Direction of force.**—It is self-evident that the direction in which a force is applied must determine the direction in which the

body receiving the force will move, if motion results, or of the resulting pressure, if the body is not free to move.

Moreover, *the action of a force upon a body is independent of its state of rest or motion.* Daily experience and observation confirm this statement, which is also susceptible of experimental proof.

It follows: 1st. That if two or more forces act upon a body at the same time, each of these forces produces the same effect as if it acted alone, since the effect which each produces is not dependent upon any motion which the others are capable of producing in the same body.

2d. Therefore, a body under the influence of a force, constant both in direction and intensity, moves with a constantly accelerated velocity; for as in each second the variation of velocity, v, is the same, in t seconds it will be $= vt$; *i. e.*, at the end of 2 seconds it will be $2v$, of 3 seconds $3v$, and so on. In other words, it is proportional to the time. Reciprocally, 3d. A body moving in a right line with a uniform acceleration is actuated by a force of constant intensity acting in the direction of its motion.

41. **Measure of forces. Mass.**—In mechanics forces are usually measured by their effects rather than by weight. The effects of a force depend, other things being equal, on the *mass* of the body acted on.

The *mass* of a body is the quantity of matter the body contains, and is proportional, in the same substance, to the number of its molecules. *Masses are equal* when, after receiving for an equal time the impulse of an equal and constant force, they acquire equal velocities.

Since we know forces only by their effects, that is by the amount of motion or pressure they produce, let us look for a just measure of any given force in the amount of motion which it causes. The following four propositions will render this subject clear.

42. **Propositions in regard to forces.**—PROPOSITION I. *Two constant forces are to each other as the masses to which in equal times they impart equal velocities.*

Consider, for example, n equal forces f, f, f, parallel to each other, acting upon n equal masses m, m, m. These masses receive equal velocities, and consequently preserve the same relative positions, and we readily conceive of them, therefore, as bound together to form one mass equal to $n \times m$. This compound mass $(n \times m)$, in order that it may possess the same velocity, v, which any single mass, m, receives from f, must be acted on by the force $n \times f$.

PROPOSITION II. *Two constant forces are to each other as the velocities which they impress, during the same time, upon two equal masses.*

Suppose two forces F and F' to be commensurable, and let l be their common measure, so that $F = nl$ and $F' = n'l$. Represent also by u the velocity which the force l imparts at the end of a given time to the common mass. The force nl will impart to this mass the velocity $V = nv$, since each force equal to l acts as if it were alone; in like manner the force $n'l$ will impart the velocity $V' = n'u$ to the same mass. Thence follows this proportion,

$$nl : n'l = nu : n'u, \text{ or } F : F' = V : V' \text{ which was to be proved.}$$

If the forces compared are not commensurable, we must take l infinitely small.

PROPOSITION III. *Two constant forces are to each other as the products of the masses by the velocities which they impart to these masses in the same time.*

Let $F\ F'$ be two forces acting on the two masses $M\ M'$, and imparting to them, at the end of the same time, the velocities V and V'; also consider f another force able to impart to the mass M, in the same time, the velocity V'; comparing the forces F and f, which in the given time impart to equal masses $M\ M$ unequal velocities V and V', it follows from Proposition II. that $F : f = V : V'$.

Comparing the forces f and F', which impart to unequal masses M and M' equal velocities V' and V', it follows from Proposition I. that:—$f : F' = M : M'$.

Multiplying the two propositions term by term we have $F : F' = MV : M'V'$, which was to be proved.

From these principles it follows, that the measure of any force is obtained by selecting some *unit of force* to serve as a term of comparison for all other forces; such a force acting on a unit of mass, during one second (the unit of time), should impart to it a velocity or acceleration of one foot, one yard, one metre, or any other arbitrary measure, as one foot per second, which latter measure we adopt.

By proposition second we can then find the relation that any force F, acting on a mass of matter during one second, will bear to the unit of force. For if F' is the unit of force in the proportion $F : F' = MV : M'V'$, then, by the definition, both M' and V' are equal to unity, and we have $F = MV$. That is, according to the definition, F contains the unit of force as many times as there are units in the product of the number M into the number V. Assume, for example, that the mass moved is equal to six units of mass, and the *acceleration* v in a unit of time is equal to ten feet, then the intensity of the force is equal to sixty, *i. e.*, sixty times the unit of force. Hence we deduce the following:—

PROPOSITION IV. *The measure of a force is the product of the mass moved by the acceleration, or velocity, imparted in a unit of time.*[*]

43. **Momentum.**—The momentum of a moving body is its amount of motion, or its tendency to continue in motion. The momentum of a body is equal to its mass multiplied by its velocity. When a force acts upon a body, free to move, it produces its effect as soon as motion is diffused among all the molecules, and the force is then transferred into the substance of the moving body. In consequence of the inertia

---

[*] In all the propositions relating to velocity as a measure of force, there is supposed to be no resistance to motion; the force acting only to overcome the inertia of the body.

of matter (26), if the moving body should meet no resistance, it would continue to move with the same velocity, and in the same direction, for ever.

The expression Mv represents the intensity of the force which has set the body in motion, and MV represents the amount of force that is at any time accumulated and retained by the inertia of the moving body. In either case the moving body is supposed to encounter no resistance from any other object.

It is a fundamental principle in mechanics that the same force acting upon different bodies in similar circumstances imparts velocities in the inverse ratio of their quantities of matter. If the same force, in the absence of resistance, successively projected balls whose masses were as the numbers 1, 2, 3, &c., it would impart to them the velocities 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., so that a mass ten times greater would acquire a velocity of only $\frac{1}{10}$. The product of each of these masses into its velocity is the same, for $1 \times 1 = 1$, $2 \times \frac{1}{2} = 1$, &c.

When a moving body encounters resistance, depending not only upon inertia, but also upon other properties of matter, the effects produced depend upon the **rapidity with** which the force, expressed by momentum, is brought to act upon the opposing body. This class of effects is, therefore, proportioned to momentum, multiplied by velocity. This product MV^2 is called *vis viva*, the application of which to practical mechanics will be explained hereafter (111). By the principle that action and reaction are equal (27), we know that when a musket is discharged the force of the explosion reacts upon the musket with the same intensity as it projects the ball. According to the principles of momentum, the weight of the gun, multiplied by the velocity of the recoil, must be equal to the weight of the ball, multiplied by the velocity of its projection, yet the recoil of the gun is received by the sportsmen with perfect impunity, while the moving ball deals death or destruction to opposing objects.

III. COMPOSITION OF FORCES.

44. **System of forces. Components and resultant.**—Whatever may be the number and direction of forces acting upon one point, they can impart motion or pressure in only one direction. We therefore assume, that there is a single force which can produce the same action as the system of forces, and may replace them. This is called the *resultant*, and the forces to whose effect it is equivalent, are termed the *components*. The components and resultant may be interchanged without changing the condition of the body acted on, or the mechanical effect of the forces themselves.

A force is therefore mechanically equivalent to the sum of its components. On the other hand, any number of forces are mechanically equivalent to their resultant. As we know forces only by their effects in producing motion or pressure, any forces which produce equal motions or pressures are equal. We shall proceed to illustrate this proposition in a few particular cases.

45. **The parallelogram of forces.**—It has already been stated that

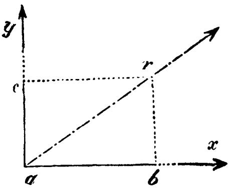
forces may be represented by lines, both in direction and intensity; also, that two forces acting on the same or equal masses of matter, are to each other as their velocities, or $F' : F'' = V' : V''$. The same principles will therefore apply to the combinations of forces that apply to the combinations of velocities or of motions.

When several forces act on a body, they may be arranged in three ways, according to their direction. The forces may act,

1. All in one direction;
2. In exactly opposite directions; or
3 At some angle.

In the first case, the resultant is the sum of all the forces, and the direction is unaltered. In the second, the resultant is the difference of the forces, and takes the direction of the greater. If opposite forces are equal, the resultant is nothing, and no motion is produced. In the third case, a resultant is found to two forces, whether equal or unequal, by the parallelogram of forces, according to the following law. *By any number of forces acting together for a given time, a body is brought to the same place as if each of the forces, or one equal and parallel to it, had acted on the body separately and successively for an equal time.*

8



Suppose two forces, at right angles to each other, act simultaneously on the point a, fig. 8, one in the direction $a\ x$, and the other in the direction $a\ y$. Let one force be such that, in a given time, as a second, it will move the point from a to b, while the other will, in the same time, move it from a to c, then by the joint action of both forces it will be impelled to r in the same time. The first force, by its separate action, would impel the body to b in one second, and if it were then to cease, the second force, or one equal and parallel to it, would impel the body to r in the same time; or the body might be carried from a to c, and from c to r; in either case the result is the same.

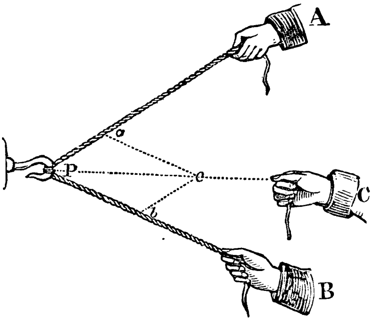
If $a\ b = X$, $a\ c = Y$, and $a\ r = R$, then $R = \sqrt{X^2 + Y^2}$. If we call the angle $r\ a\ b = a$ cos. $a = \frac{X}{R}$, and sin. $a = \frac{Y}{R}$.

Again, suppose the forces act at an oblique angle; let the point P, fig. 9, be acted on by two forces in the directions P A and P B. On P A, measure P a, containing as many units of length as the force A contains units of force; and on P B take P b in the same manner. Complete the parallelogram P a c b; the diagonal P c will represent the direction of a single force C, equivalent to the combined effect of A and B and P *c* will contain as many units in length as C contains units of force.

In the same manner a resultant may be found for three or any number of motive forces, by compounding them, two by two, successively.

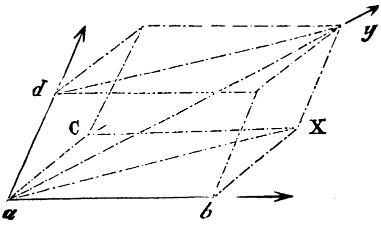
In the triangle P *a c*, fig. 9, the sides *a* P and *a c* = P *b*, are known, and also the angle P *a c*, which is the supplement of the angle *a* P *b*, formed by the directions of the forces. We may therefore calculate the side P *c*, that is, the intensity of the resultant, and the angle *a* P *c*, which determines its direction.

9



Let the point *a*, fig. 10, be subjected to the forces whose magnitudes and directions are represented by the lines *a b*, *a* C, and *a d*. We first take any two which lie in the same plane, as *a b* and *a* C, and find their resultant *a* X; and compounding this with the third force *a d*, we find *a y*, which will represent the magnitude and direction of the general resultant of all three forces.

10



The resultant of any number of forces can therefore be determined by geometrical construction, or calculated from well known geometrical principles.

This system of compounding forces is called the *parallelogram of forces*, and applies equally to the combination of velocities or motions.

In order that the body may move in the straight line *a r* (fig 8), the two forces must act in the same manner. They may be instantaneous impulses, which will cause uniform motion; or both may act continuously and uniformly, so as to produce a uniformly accelerated motion; or, both forces may act with a constantly varying intensity, increasing or diminishing at the same rate, and the body will still move in a straight line. But if one force is instantaneous and the other constant, or one constant and the other variable, or both varying by different laws, then the body will move in a curve; but in every case it will reach the point *r* in the same time that it would have passed from *a* to *b*, or from *a* to *c*, by the separate action of either force.

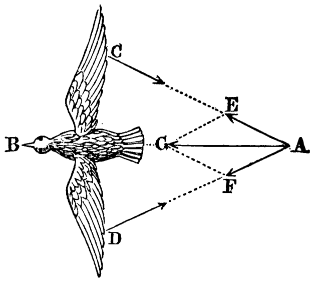
If the three forces *a b*, *a* C, and *a d*, all pass through the same point, we may construct a parallelopipedon, as shown in fig. 10, and *a y*, the diagonal of the parallelopipedon, will represent the direction and intensity of the resultant force. This method of compounding forces is called the parallelopipedon of forces.

Examples of the composition of motion and force are of constant and familiar occurrence.

A man in swimming, impels himself in a direction perpendicular to his feet and hands, and if the forces are equal on each side, he will move in a resultant line, passing through the centre of his body. Another instance is the flight of birds. While flying, their wings perform symmetrical movements, and strike against the air with equal force.

In the case of flying birds, the resistance of the air is perpendicular to the surface of the wings, and may be represented, fig. 11, by C A and D A, at right angles to their surface. Neither of these pressures tends to impel the bird straight forward, but it moves in their *resultant;* for if the wings are equally extended, and act with equal force, the lines C A and D A make equal angles with A B, passing through the centre of the bird, and hence their diagonal, or A G, the diagonal of equal parts of them, will coincide with A B, and the bird will fly directly forward.

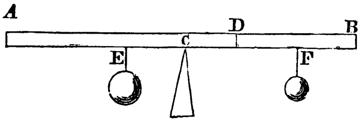
11



46. **Parallel forces. Resultant of unequal parallel forces.**—Two forces, acting side by side, produce the same effect as if they were in the same straight line. Two horses drawing a cart is an example. Hence the resultant of two parallel forces acting in the same direction is equal to their *sum*, and is parallel to them, and when they are *equal*, is applied midway between them.

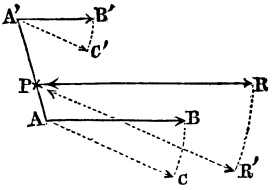
If the parallel forces are *unequal*, the point of application of the resultant may be found by the following experiment. Let A B, fig. 12, a bar of uniform thickness and density, be balanced on its centre C. We may suppose the bar to be divided into two, A D and D B, of unequal lengths, which might also be balanced on their centres E and F. Now we have two parallel and unequal forces—the weight of A D and the weight of D B—whose resultant is not midway between their points of application, E and F, but passes through C, which is nearer E than F in the exact ratio that the force at E exceeds that at F; for the weights of the two bars are as their lengths, and C E measures half the length D B, and C F half the length of A D; so that C E is to C F *inversely* as the weight at E is to the weight at F. The truth of this conclusion may be tested by suspending at E and F two additional weights which have the same ratio to each other as A D to D B, and the equilibrium will be undisturbed.

12



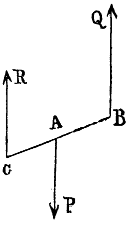
Hence the resultant of two parallel but unequal forces is equal to their sum, and its distances from them are inversely as their intensities. Thus, in fig. 13, if any two parallel forces act at A and A′, and their intensities are expressed by A B and A′ B′, then their resultant will be represented by P R, provided it acts at P, a point so situated that P A′ : P A = A B : A′ B′. The same will be true whatever be the common direction of the forces; if the positions of A B and A′ B′ are changed to A C and A′ C′, then P R must move to P R′, and equilibrium equally obtains.

13



47. **Resultant of two parallel forces acting in opposite directions.**—The resultant of two parallel forces, which act in opposite directions, is found by the same construction as before, but it is equal to the *difference* of the intensities of its components, and takes the direction of the greater. Its point of application, fig. 14, is in the prolongation of the line A B, at the point C, situated so that C B and C A are in the inverse ratio of the forces Q and P. The point C will be further removed as the difference between the forces P and Q are diminished, so that if the forces were equal, the resultant would be nothing, and situated at an infinite distance.

14



The general resultant of any number of parallel forces may be found by compounding them, successively, two by two, in the methods already prescribed.

48. **Couples.**—Whenever a body is solicited by two forces which

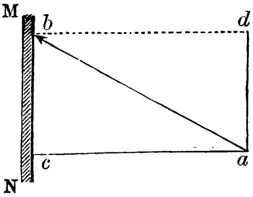
are equal, parallel, and acting in opposite directions, it is impossible to replace them, or produce equilibrium by a single force. Such a system of forces is called a *couple*, and its tendency is to produce revolution around an axis. In this case, the value of the resultant is evidently equal to zero, and the point of application is also at an infinite distance from the points of application of the two equal components.

49. **Two forces not parallel and applied to different points** may have a resultant, if they lie in the same plane. It is found by extending the lines of direction until they intersect. But if the forces are not parallel, and lie in different planes, then the directions, though infinitely prolonged, will never intersect, and they cannot have any single resultant, or be in equilibrium by any single force.

50. **The resolution of forces** is the converse of their composition. Since two or more forces can be replaced by a single force, so, commonly, we may substitute two or more forces for one; and since an infinite number of systems may have the same resultant, conversely, one force may be replaced in innumerable ways by a system of several forces. But if one of two required components is given in magnitude and direction, there can be but one solution, and the problem is definite.

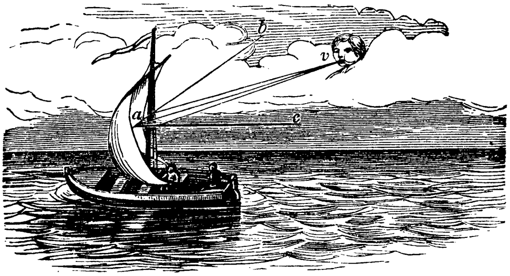
When a force acts upon a body at any other than a *right* angle, a part of its effect is lost. By resolving such an oblique force into two, one parallel, and the other perpendicular to the body, the latter component will represent the actual force produced. Let *a b*, fig. 15, represent a force acting under the angle *a b c* against the surface M N. Resolve *a b* into *a c* perpendicular to M N, and *a d* parallel with it; then *a c* will be the absolute effect of the force, and *a b*—*a c* is the loss.

15



Example of the resolution of force.—The sailing of a boat in a direction different from the wind is a most familiar illustration of these principles. For example: the wind blows in the direction *v a*, fig. 16, oblique to the sail, and to the course of the boat, and its force is resolved into two components, one acting in the direction *c a*, impelling the boat on its course in the line of least resistance, the other in the direction *b a*, acting to carry the boat laterally on in the line of greatest resistance. As the model of the boat allows it to advance freely through the water in the direction *c a*, while it offers great resistance to lateral motion, the force of the wind resolved in the direction *b a* produces little effect upon the motion of the boat, she being held to her course by the rudder. A skillful sailor can, by thus availing himself of the principles of resolution of force, sail his vessel on a course within five or six points* of being directly opposed to the wind which impels it.

16



IV. CURVILINEAR MOTION—CENTRAL FORCES.

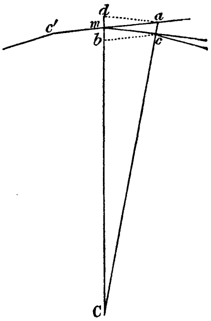
51. **Of curvilinear motion.**—It has been shown that a body acted upon by two forces will move in a direction which is the resultant of the two forces. If one of two forces acting upon a body is a continuous force, acting in a direction tending to turn the body out of the course it receives from the other, the resultant will not be a straight line, but a curve, having its concavity turned towards the direction of the continuous force. If at any instant the continuous force ceases to act, the body will continue to move in the direction in which it was moving when the constant force ceased to act. This direction will be a tangent to the curve at that point.

52. **Centrifugal and centripetal forces.**—Let us consider a material point moving with a uniform velocity in the circumference of a circle. The resultant of the forces acting on the particle will, therefore, by the necessity of the case, pass through the circumference of the circle. In this case the force which prevents the moving particle from darting off in a tangent to the circle is called the *centripetal*, or *centre-seeking*, force. This force at every instant arrests the tendency of the particle to fly away from the centre. The tendency of the particle to fly away from the centre is called the *centrifugal, or centre-flying*, force. These two forces are, together, termed *central forces*. They are necessarily antagonistic to each other at every instant of curvilinear motion.

---

* A circle is divided into four quadrants, and each quadrant into eight points, according to the phraseology of seamen.

17



To understand the antagonism of central forces, take mc, fig. 17 an infinitely small arc of the circumference, and making with the direction $c'ma$, of the preceding element $c'm$, an infinitely small angle amc. Join the extremities of the element mc with the centre of the circle by the radii mC, cC, and draw bc parallel to ma. Since mc is considered infinitely small, ac and mb are parallel to each other, and $macb$ is a parallelogram. Therefore it follows by the parallelogram of forces, that while the body was moving over the arc mc it would in the same time have passed over the space ma, in virtue of the velocity acquired in passing over $c'm$, if no other force intervened; while by reason of the central force acting from m to c, it would have passed over the space mb, had it not been for the original impulse $c'm$. This is the *centripetal force*, and, compounded with the original impulse, $c'm$, the particle follows the resultant mc.

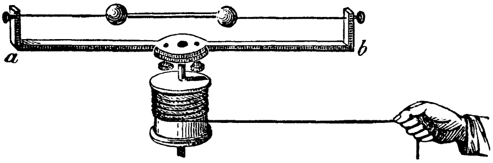
Draw now ad parallel to mc, forming the second parallelogram $mcad$. The velocity of the particle following the diagonal ma may be decomposed into the two components; one mc in the path of the circular motion, and the second md following the radius. This quantity (md) represents the *centrifugal force*, and as $md = ac = mb$, it follows that at each instant of the circular motion the centripetal and centrifugal forces exactly counterbalance each other, and that the sole resulting motion is in the arc mc. If at any instant the centripetal force ceases to act, ma, the resultant of the forces mc, md, will throw the particle in the path of the line $c'ma$ tangent to m. The term centrifugal force must not be understood to mean a force which would cause the body to fly directly from the centre, since as we have seen it must in that case move also in the tangent.

Examples of the action of centrifugal force.—A stone flies from a sling with a velocity equal to the force acquired by its revolution at the moment when, by releasing one of the strings from the finger, it flies off in a line tangent to the point of release. The water flies from a grindstone, or mud from a carriage wheel, whenever the centrifugal force due to the velocity of revolution is sufficient to overcome the force of adhesion. The rapidity of revolution may be sufficient in a grindstone to overcome the cohesion of the particles of the stone, when it bursts with a loud explosion, carrying death and destruction in its path. A pail filled with water may be whirled with such velocity that the centrifugal force overcomes the force of gravity, and the liquid is not spilled.

53. **Experimental demonstration of the effects of centrifugal force.**—The effects of centrifugal force may be illustrated by the apparatus represented in fig. 18. A wire is stretched upon a frame ab, connected with an upright shaft, which is made to revolve rapidly by means of a cord, as shown in the figure. Two perforated balls, united by a string,

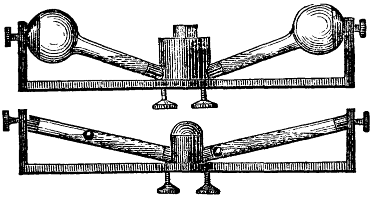
slide freely upon the horizontal wire. If the two sliding balls are of equal weight, and are placed at equal distances from the axis of rotation, still united by the string, they will retain the same position with any velocity of rotation which may be given to the apparatus; but if one of the balls is more distant from the axis than the other, it will draw the other along with it, and both balls will strike the support on the same side, provided the distance between the balls is less than half the line *a b*. If the two balls, united as before, and placed at equal distances from the axis, have unequal masses, the heavier ball will draw the other towards its own side of the apparatus. Two unequal balls, united in the same manner, will remain at rest, if their distances from the axis, on opposite sides, are in the inverse ratio of their masses. Admitting that the centrifugal force is proportional to the mass of the body, these experiments prove that the centrifugal force is proportional to the radius of the circle described, when the times of revolution are the same.

18



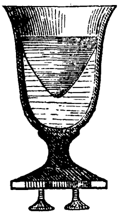
To demonstrate the effects of centrifugal force in liquids, the apparatus shown in the upper part of fig. 19 is attached by screws to the coupling at the top of the revolving shaft shown in fig. 18. Two flasks with long necks are placed obliquely, communicating with a reservoir filled with liquid placed at the middle of the bar which supports them. As the apparatus is rapidly revolved, the liquid rises into the flasks, and again descends when the motion is arrested. If the vase, fig. 20, containing water, is attached to the machine, and made to revolve, the surface of the water becomes concave, the water rising by the sides of the glass, the surface becoming more deeply concave as the motion becomes more rapid. The piece of apparatus shown at the bottom of fig. 19 carries two inclined tubes, one enclosing water and a metallic ball, the other water and a ball of

19



wood floating on its surface. As the rotation becomes rapid, the water rises to the top of the tubes, the ball of wood then descends, and takes a position on the inferior surface of the liquid, while the metallic ball advances through the liquid, and rises to the most elevated extremity of the tube which contains it, the liquid itself rising to the exterior end of the tube.

20



The different effect upon the two balls results from the fact that the metal has a greater mass than an equal volume of the liquid, while the contrary is true of the wood; the centrifugal force being in proportion to the mass, the tendency is to carry the denser substance to the greater distance from the axis of rotation.

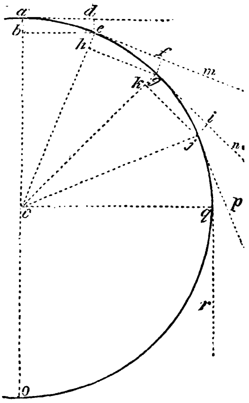
If a tube contains different liquids incapable of acting chemically upon each other, they will place themselves, during rapid rotation, in such an order that the denser fluid will be more distant from the axis, the outward tendency being directly proportional to the mass of matter in a given volume. These effects do not take place till, by the rapidity of revolution, the centrifugal force becomes greater than the force of gravity. The common circular or fan blowing machine is an example of the action of centrifugal force on bodies in a gaseous condition. A centrifugal pump has been devised acting in this manner. The fan-blower is also used as a ventilator, drawing its supply of air from the space to be ventilated, to supply that thrown out by the tangential opening.

The centrifugal drying machine for laundries consists of a very large upright cylinder, having a smaller cylinder within it. The circular chamber between the two cylinders is closed by covers, by opening which the linen to be dried can be introduced. The bottom of this chamber is pierced with holes like a sieve, through which the water expressed from the linen can flow off. A rapid rotation being given to this cylinder, the linen, by the effect of centrifugal force, is urged against the exterior surface of the cylinder, and is there squeezed with a force which increases with the rapidity of rotation, by the effect of which the water is pressed out of it, and escapes through the holes in the bottom. A rotation of 25 turns per second, or 1500 per minute, is given to these drying cylinders, by which the linen, however moist it may be, is rendered so nearly dry that a few minutes' exposure in the air renders it perfectly so. In large linen manufactories this machine produces a great saving of labor in the laundry department.

In the motion of the heavenly bodies we find the most wonderful examples of the action of central forces, acting as they do to prevent the moon from fall ing upon the earth, and the planets into the sun. The action of centrifugal force will be further considered in connection with terrestrial gravity.

54. **Analysis of the motion produced by central forces.**— Let a body placed at a, fig. 21, receive an impulse which would carry it to d in one second, or any small portion of a second, and let it be attracted towards c by a constant force which would move it to b in the same time that the first impulse would carry it to d, then by the principles of the composition of forces it will be found at the end of the given time at e, and if the attractive force should then cease it would continue to move in the direction em. If the attractive force continues, the body will be found at g at the end of the second period. As the central force is continually acting, the body will diverge more and more from the direction of the first force, and will describe a curve. As the attractive force acts always towards the central point c, the body will revolve around that point. If the relation of the two forces is such that ce, cg, &c., are each equal to ca, the curve of revolution will be a circle. In most other cases the curve of revolution will be an ellipse.

21



If the curve described is a circle, and we assume the arc ae to be very small, it will not sensibly differ from a straight line, and according to well known geometrical principles we shall have

$$ab : ae = ae : ao.$$

$$ab = \frac{ae^2}{ao}.$$

That is, *the centrifugal and centripetal forces of a body describing a circle with uniform velocity are directly proportional to the square of the velocity, and inversely as the diameter of the circle.*

The relation of these forces may be expressed differently. Considering ae as the space described in one second, it will be the velocity of the body; but in curvilinear, just as in rectilinear movement, the velocity is equal to the distance divided by the time, *i. e.*, equal to the circumference of the circle divided by the time of revolution. Let R represent the radius of the circle, T the time of revolution, v the velocity, and π the ratio of the circumference to the diameter,* and we shall have $ae = v = \frac{2\pi R}{T}$. Substituting this value of ae in the previous equation, and considering that $ao = 2R$, we shall have

$$ab = \frac{4\pi^2 R^2}{2RT^2} = \frac{2\pi^2 R}{T^2}.$$

Therefore the attractive force would generate in one second a velocity expressed by twice *a b*, the distance passed over, equal to $\frac{4\pi^2 R}{T^2}$. The angular velocity per second is expressed by the actual velocity, divided by the radius; calling this angular velocity *V*, we have

---

* The ratio of the circumference of a circle to its diameter is 3·14159, and this number is usually represented by the Greek letter π in mathematical formulæ.

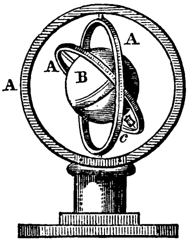
$$\frac{4\pi^2 R}{T^2} = V^2 R.$$

The centrifugal or (which is the same thing) the centripetal force varies directly as the radius of the circle of revolution multiplied by the square of the angular velocity. Also if two bodies move in different circles and in different times, their centrifugal and centripetal forces will be directly as the radii of their circles, and inversely as the squares of the times of revolution.

It is also evident that the centrifugal force is proportional to the mass of the body.

55. **Bohnenberger's apparatus.**—In consequence of the operation of the law of inertia, moving bodies preserve their planes of motion. This is true as well of planes of rotation as of planes in a rectilinear direction. By means of Bohnenberger's apparatus we may illustrate the tendency of rotating bodies to preserve their plane of rotation, and the invariability of the axis of the earth during its revolution. Bohnenberger's apparatus consists of three rings, A A A, fig. 22, placed one within the other; the two inner ones are movable, and connected by pins at right angles to each other, in the same way as the gimbals that support a compass. In the smallest ring there is a heavy metallic ball B supported on an axis, which also carries a little pulley *c*. The ball is set in rapid rotation by winding a small cord around *c*, and suddenly pulling it off. The axis of the ball will continue in the same direction, no matter how the position of the rings may be altered; and the ring which supports it will resist a considerable pressure tending to displace it.

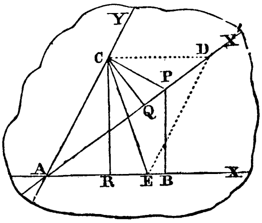
22



56. **Parallelogram of rotations.**—It has been shown (48) that rotary motion is produced by two equal parallel forces acting in opposite directions. If two new equal parallel forces act upon the same body, tending to produce rotation about another axis situated in the same plane, the compound resultant will tend to produce rotation about a third axis, situated in the same plane, between the directions of the other two.

Let the irregular body shown in fig. 23, while rotating about the axis A X be suddenly acted upon by forces tending to produce rotation about the axis A Y. Suppose the parts of the body lying between A X and A Y to be impelled in opposite directions by the two rotary forces.

23



Take any point, as P, and drawing from P perpendiculars upon the two axis, let $PC = y$ and $PB = x$, also let v represent the angular velocity about the axis A X, and v' the angular velocity about the axis A Y, then will $vx - v'y$ express the resultant force exerted upon the point P. Now since, if the point P were taken in the axis A X, we should have $vx = 0$, and if P were taken in A Y we should have $v'y = 0$, it is evident that P may be so taken that $vx - v'y = 0$, or $vx = v'y$. Lay off on A X a distance A E, such that $AE : AC = v : v'$, construct the parallelogram A C D E, and draw the diagonal A D; then $AE : AC = DC : DE = v : v' = y : x$. But every point on the line A D X′ will have the same relation to the axis A X and A Y. Hence every point in this line will remain at rest, and A X′ becomes the resultant axis of rotation, in virtue of the forces previously tending to produce rotation about the two component axes.

To determine the velocity of rotation about the resultant axis A X′, take any point, as C, on the axis A Y. At this point $v'y = 0$, and the point C has no tendency to move except that given by the moment vx about the axis A X. Draw the perpendiculars C R and C Q upon A X and A X′ respectively. Represent C R by r, and C Q by r'', and denote the angular velocity about A X′ by v''. Now as the distance passed over by the point C during any instant depends only on the moment vx, it will be the same whether the rotation takes place about A X or AX'; hence $vr = v''r''$, $\therefore\ v'' = \frac{rv}{r''}$. The triangles ACD and ACE are equal, being each one-half the parellelogram $ACDE$, hence $AD \times CQ = AE \times CR$, and $AD \times r'' = vr$, hence $AD = \frac{vr}{r''}$. Comparing this equation with the value of v'' found above, we find that $v'' = AD$.

From the above reasoning it appears that,—

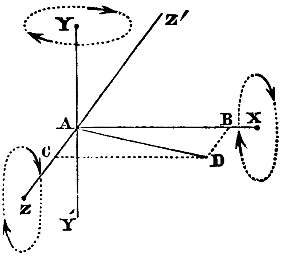
Where a body is acted upon by two systems of forces, tending to produce rotations about two separate axes, lying in the same plane, the resultant motion will be rotation about a new axis, represented in direction by the diagonal of a parallelogram, the sides of which will be represented by the component axes of rotation, and the magnitudes of the sides by the forces tending to produce rotation about those axes. The velocity of rotation about the new axis represented in direction by the diagonal of the parallelogram, will be measured by the length of the diagonal.

From the same principles it follows that, If a body is acted upon by three systems of forces, tending to produce rotation about three different axes, all passing through the same point, the resultant motion will be rotation about a new axis, represented in direction by the diagonal of a parallelopipedon formed on the original axes as adjacent edges, with the magnitude of those edges corresponding to the respective forces acting about those axes, and the velocity of the new rotation will be represented by the length of the diagonal of the parallelopipedon.

In describing rotary motions it is customary to speak of the motion as *right-handed* or positive, or as *left-handed* or negative.

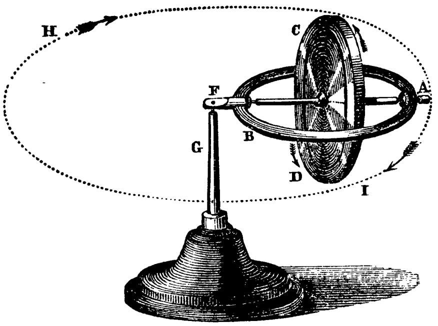
Let A X, A Y, A Z, fig. 24, be three rectangular axes, passing through the point A, which is called the origin of co-ordinates. Distances measured from A towards either X, Y, or Z, are called positive, and distances measured in the opposite directions are called negative. If a body revolves about either of these axes, or about any axis drawn through A, in such a direction as to appear, to an eye placed beyond it, and looking towards A, to move in the same direction as the hands of a watch, when we look at the dial, such motion is called right-handed, or positive rotation. If rotation takes place in the opposite direction, it is called left-handed, or negative. If a body revolves in the directions shown in either part of fig. 24, the rotation is called right-handed, or positive. If the three axes, A X, A Y, A Z, were brought towards each other till they coincide, these rotations would all coincide in direction.

24



57. **The gyroscope, or rotascope,** is an instrument exhibiting some remarkable results of the combination of rotary motions, and which also shows, as in Bohnenberger's apparatus, the tendency of rotating bodies to preserve their plane of rotation. A common form of the rotascope, fig. 25, consists of a metal ring, A B, inside of which is placed a metallic disk, C D, loaded at its edge, and which turns independently of the ring, upon the axis. Motion is communicated by means of a cord, wound around the axis of the disk and suddenly drawn off. If, when the disk is rotating rapidly, it be placed on the steel pin F, supported on the column G, it seems indifferent to gravity, and instead of dropping it begins to revolve in a horizontal orbit, H I, about the vertical axis F G, in a direction corresponding with the movement of the lower part of the disk. This hori-

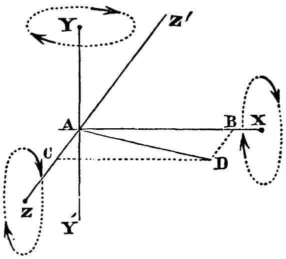
25



zontal revolution gradually increases in velocity, and the free end of the disk, in some circumstances, vibrates upward and downward, in spiral curves. When the rotation of the disk is considerably diminished by friction, gravity gradually prevails over the supporting power, and at length the disk falls, in a descending spiral curve.

To explain the movement of the gyroscope, let the axis Y A Y′, fig. 24, correspond to the standard. A being the point where one end of the axis of the revolving disk is supported. Let A X correspond to the horizontal axis around which the disk of the gyroscope revolves, and let Z A Z′ be another horizontal axis, at right angles with A X and A Y. Let a right-handed rotation be given to the disk of the gyroscope about the axis A X, as indicated by the arrows. While the force of gravity causes the free end of the axis to descend, there will also be commenced a right-handed rotation about the axis A Z. Both these rotations may be supposed to have constant velocities for an infinitely short interval of time. Let v denote the angular velocity about A X, and v' the angular velocity about A Z. Lay off on A X a distance A B $= v$, and on A Z a distance A C $= v'$, complete the parallelogram A B D C; then the diagonal, A D, will represent the resultant axis of rotation, and the length of this diagonal will represent the velocity of rotation about the new axis. As the disk can only rotate about the new axis by moving towards D, there will thus be commenced a new movement of rotation, which we may call a horizontal revolution about the perpendicular axis. This will be a right-handed rotation about A Y.

24 (*bis.*)



We now have to consider rotation about three axes, all passing through the axis A. To construct the position of the resultant axis for the second instant, lay off on A Z, considered as perpendicular to A D and to A Y, a distance to represent the angular velocity due to gravity, and on A Y the angular velocity in the orbit, acquired in passing from the position A X to A D; A D represents the velocity with which the disk rotates. Construct a parallelopipedon on these three lines, and draw its diagonal through A. This diagonal will evidently give a direction that will continue the horizontal revolution already commenced, and also an upward tendency in opposition to gravity. The same process of construction might be continued for every instant the rotation continues. It is evident that the horizontal rotation would continually increase in velocity, and the tendency to lift the end of the axis of the disk would also increase, were it not continually counteracted by the action of gravity. As the velocity of the rotation of the disk is continually retarded by friction, the lifting power exerted against gravity diminishes until the disk gradually descends with a spiral revolution, and the instrument is brought to rest.

If the disk of the gyroscope were made to rotate in the opposite direction, or left-handed, the motion around the axis Z A Z′, caused by gravity, would also be left-handed when considered from the point Z′, and the resultant A D′ would lie on the opposite side of A X. This would also give a left-handed rotation about the perpendicular A Y, and the resultants would all be found in the solid angle A X, A Y, A Z′, and again the tendency would be to counteract the depressing force of gravity. Thus we should have the rotating disk supported, as experience shows it is, in whichever direction it is caused to rotate.

If the weight of the gyroscope were counterbalanced (as it is frequently constructed) by an equal weight on the opposite side of the vertical axis, it can readily be seen that it would have no horizontal rotation. If the counterbalancing weight were so great as to raise the disk upward, the horizontal revolution would be performed in the opposite direction.

Problems on Weights and Measures.

1. (*a.*) How many English yards are contained in 135 French kilometres? (*b.*) Reduce 2·5934 centimetres to English inches. (*c.*) What part of an English inch is 1 millimetre? (*d.*) Reduce 3 centimetres to inches.

2. (*a.*) Reduce 4 feet 7 inches to French measure. (*b.*) Reduce 225 rods to French measure of length. (*c.*) Reduce 13 miles to French kilometres. (*d.*) Reduce 5 yards to French metres.

3. (*a.*) Reduce 3 pints to litres and cubic centimetres. (*b.*) Reduce 5 litres to English measure. (*c.*) Reduce 7 gallons to litres and decimal parts. (*d.*) Reduce 7 cubic centimetres to English pints.

4. Reduce, by means of the Table II.,—(*a.*) 25 inches to decimal parts of a metre. (*b.*) 139 centimetres to American inches. (*c.*) 75 feet to metres. (*d.*) 5 kilometres to American yards.

5. Reduce, by means of the table at the end of the book,—(*a.*) 7¼ pints to cubic centimetres. (*b.*) 10 gallons to cubic centimetres. (*c.*) 735 cubic centimetres to gallons.

Problems on Motion.

6. A body passes uniformly over a distance of 200 yards in 1 hour and 6 minutes: what is the numerical value of its velocity, according to the usual conventions concerning the units of space and time?

7. A body is observed to describe uniformly a feet in n seconds; supposing the unit of time to be 1 minute, what must be the unit of distance in order that the numerical value of the body's velocity may be 1?

8. A man walks with a velocity represented by 2, and he finds that he walks 7 miles in 2 hours; if 1 foot be the unit of length, what is the unit of time?

9. A particle is moving with such a velocity, and in such a direction, that

the resolved parts of its velocity in the directions of two lines perpendicular to each other, are respectively 3 and 4; determine the direction and velocity of the particle's motion.

10. A is a more powerful and a heavier man than B; the greatest weights which they can lift are as 8 : 7, and their own weights are as 7 : 6. Which is likely to be the faster runner of the two?

11. Supposing a body acted upon by a force capable of causing an acceleration of 3 feet per second: what will be its velocity, and the distance passed over, at the end of 1 minute?

12. What velocity will be acquired by a body moving 100 feet, under the influence of a force capable of causing an acceleration of 2 feet per second?

13. If a body starting with an initial velocity of 125 per second is found to come to rest after the end of 5 seconds, what is the amount of retardation, and what distance has the body passed over?

14. If a body weighing 100 pounds moves with a velocity of a mile in 7 seconds, what must be the weight of a body moving 3 feet per second, to have the same momentum as the former?

15. If a ship weighing 2000 tons strikes a pier with a velocity of 6 inches per second, what velocity must be given to a 64 pound shot, in order that it may strike an obstacle with the same momentum?

16. Uniform force is defined as that which generates equal velocities in equal times; would it be correct to define it as that which generates equal velocities while the body moves through equal spaces?

CHAPTER III.

GRAVITATION.

§ 1. Direction and Centre of Gravity.

58. **Definition.**—The fall of unsupported bodies to the earth, and the pressure exerted by bodies at rest on the earth's surface, is due to the *force of gravity.* The amount of this force seen in the downward pressure of any body is called its *weight.* Weight is due to the earth's attraction for the body weighed. This is only a particular case of a general force in nature, by reason of which all bodies in the universe attract each other, thereby maintaining the order and stability of the heavenly bodies.

59. **Law of universal gravitation.**—The law of gravitation is stated as follows: *Every particle of matter attracts every other particle in the* DIRECT *ratio of its mass, and in the* INVERSE *ratio of the square of its distance.*

Let M and M' represent two masses, D and D' the distances. Take G and g the absolute gravities of the two masses at a given distance, and $g : G'$, the ratio of the force of gravity at distances D and D', then

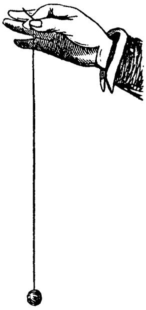
$$G : g = M \quad : M',$$
$$g : G' = D'^2 \quad : D^2, \text{ compounding these proportions we have}$$
$$G : G' = MD'^2 : M'D^2, \quad \text{hence} \quad G : G' :: \frac{M}{D^2} : \frac{M'}{D'^2}.$$

Or the force of gravity of different bodies, at different distances, is directly as the masses and inversely as the squares of the distances.

This law was discovered in 1666, by Sir Isaac Newton. Reflecting on the power which causes the fall of bodies to the earth, and bearing in mind that this power is sensibly the same on the highest mountain as in the deepest valleys, he conceived that it might extend far beyond this earth, and even exert its influence through all space. He assumed, in conformity to the relation already discovered by Kepler, between the times of revolution of the planets and their distances from the sun, that this force must diminish in the inverse ratio of the square of the distance. His first results were unsatisfactory, because (as afterwards appeared) he used in his calculations an erroneous value of the earth's radius. But in June, 1682, he received Picard's new measurement of the arc of the meridian in France, from which it appeared that the radius of the globe was nearly one seventeenth greater than had been previously supposed. Armed with these new data, Newton resumed his calculations, and in 1687 published his great work, the "*Principia*," in which he develops the consequences of his great discovery of the laws of gravitation.

60. **Direction of terrestrial attraction. Centre of gravity.**— As the direction of a force is the direction of the motion or pressure caused by it, (40), it follows that a body falling under the influence of gravity moves on a line, which would pass, if extended, through a point sensibly near the centre of the globe. This point in the globe is called *its centre of gravity.* Therefore the direction of the force of gravitation is in the line uniting the centre of gravity of the earth with that of the attractive body. The *plumb line*, fig. 26, gives this direction. Here a mass of lead is suspended by a string, and when it is at rest, it is evident without a mathematical demonstration, that the direction of the pressure, and hence that of the force of gravitation, coincides with that of the plumb line. This direction is called the *vertical*, and a direction perpendicular to it is called the *horizontal.* Such is the surface of a liquid at rest.

26



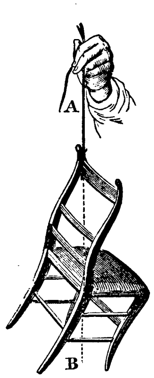
It is plain, on the slightest reflection, that as the plumb line coincides at every point of the earth's surface very nearly to the *radius* of the same points, that several plumb lines placed near each other, are sensibly parallel to each other (but are not mathematically so), because the distances between them are almost insensible compared to the length of a radius of the earth. The convergence is only one minute to a geographical mile.

The rotation of the earth does not affect the direction of a falling body, because it had, before it fell, the same velocity as the earth itself. Thus a bullet dropped from the mast-head of a ship, sailing never so rapidly, falls to the deck on precisely the same spot as if the ship were motionless—in virtue of the principle of the composition of motion, (45). Nevertheless, if bodies fall from a very great height, there is an easterly deviation, as Newton announced, because at the point of departure, on a circumference sensibly larger than at the surface of the earth, the body has a horizontal velocity sensibly greater than the latter. For a height of 550 feet, calculation indicates a deviation of 0·108 inch, and experiment gave Reich in the deep mines of Freyberg 0·110 inch.

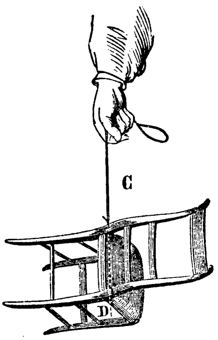
61. **Point of application of terrestrial attraction.**—As every particle of a body is equally attracted by the earth, there must be as many points of application for this force as there are particles of matter in the body. Hence from the principle (46) for finding the resultant of a system of parallel forces, it follows that if a body be supported by a flexible cord from a fixed point, it cannot remain at rest unless the resultant of all the parallel forces which gravity exerts on it passes through the point of support.

It is thus possible to determine experimentally the position of the resultant of the system of parallel forces which gravity exerts upon a body.

27



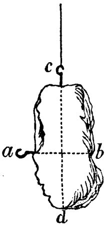
28



For example, in fig. 27, the chair is held by a string attached to one arm, and the resultant of the forces exerted by gravity is in the line A B, in which the chair comes to rest. But if the chair is supported from another point, as in fig. 28, it will come to rest in a new position, and the resultant will now be found also in a new position, namely in the line C D, and so for every new point of suspension we can by experiment demonstrate a different position for the resultant.

62. **Centre of gravity.**—As there is in every solid body a point about which, when it is suspended, its molecules are equally distributed in every direction, all the attractions exerted upon them may be replaced by a single resultant force applied at this point. But whatever position the body may assume, the resultant of its parallel forces of gravity will always pass through the same point. This common point, of intersection of the resultants of gravity in any body, is called the *centre of gravity.* As we may find the resultants experimentally, so also is the centre of gravity of any solid easily found. If any irregular solid is suspended, as in fig. 29, its centre of gravity will lie in the line *c d*, prolonged through its axis. It will also lie in the line *a b*, by which the body is a second time suspended, and being found in both lines, it must necessarily be at their intersection. A correct conception of the important relations of the centre of gravity lies at the foundation of the whole science of mechanics, and especially of equilibrium.

29



63. **Corollaries.**—(1.) The centre of gravity must be regarded as the point of application of the resultant of the forces which gravity exerts upon the molecules of any body. This is proved by the fact that the point of application is any point on the line of the resultant, and that the centre of gravity is a point common to all the resultants.

(2.) When the centre of gravity is supported, the body remains at rest. Conceive the irregular mass, fig. 29, to be sustained on an axis passing through *a b* or *c d*, the body will remain at rest in whatever position of revolution it may be, on either of these axes, since the whole intensity of the forces of gravity is expended in pressure against the points of support.

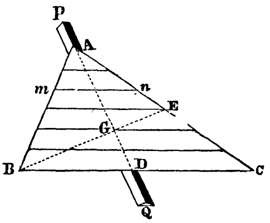
(3.) The sum of all the attractions exerted by any mass of matter may be conceived as proceeding from its centre of gravity. Newton has demonstrated that a particle of matter placed without a hollow sphere is attracted in precisely the same manner as if the whole mass of the sphere were collected at its centre, and constituted a single particle there. The same must be true of solid spheres, since they may be regarded as made up of a great number of hollow spheres, having the same centre.

The principle that action and reaction are equal and opposite (27), applies perfectly to the mutual attractions of the masses of matter. Hence follows the somewhat startling inference that the earth must rise to meet a falling body. This is unquestionably true, but since the mass of the earth is almost infinitely greater than that of any body falling on its surface, its motion must likewise be almost infinitely small, since the velocity v', acquired by the earth at the end of one second is as much less than the velocity v, acquired by the falling body, as the mass of the body (m) is less than the mass of the earth (M), or $v' : v = m : M$.

64. **Centre of gravity of regular figures.**—In case of solids which have a regular figure, and uniform density, it is not necessary to resort to experiment. In such bodies, the centre of gravity coincides with the centre of figure, and to find it is a question purely geometrical. The truth of this assertion may be shown, if we suppose a plane or line to be divided into two equal and similar parts, so that its molecules are arranged two by two, with respect to the dividing line. Take any two molecules similarly situated, on opposite sides of the division, their attractions will be equal and opposite; and so also of every other pair; therefore, the resultant of the system must be at the point of division, and the centre of gravity is there also.

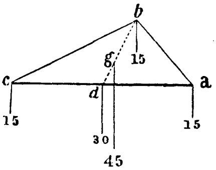
The centre of gravity of a circle, or sphere, is at the centre of each; of a parallelogram or parallelopiped, at the intersection of the diagonals; and of a cylinder at the middle point of its axis. To find the centre of gravity of a triangle, fig. 30, draw a line A D, from the vertex to the middle point of the base; it will divide equally all the lines, as *m n*, drawn parallel to the base. If the triangle is placed so that the line A D may be exactly over the edge of the prism P Q, each one of the rows of molecules composing the figure, as *m n*, will be in equilibrium on the edge of the prism, since it is supported at its centre. The same will be true when they are united, and the triangle will not tend more to one side than another; hence its centre of gravity must be in the line A D, and for a like reason, also in the line B E (situated similarly to A D), and therefore at their intersection G. It may be shown geometrically that the point, thus found, divides the line joining the summit and the middle of the base, into two parts, of which the one nearest the vertex is double that nearest the base.

30



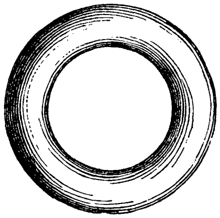
SUPPORT OF A TRIANGULAR MASS AT ITS ANGLES.—If it were required to support a triangular block of marble at its angles, we may find what part of the weight will be sustained by each support, by applying the foregoing principles. The weight of the block, fig. 31, which we will suppose to be 45 lbs., is a force applied to its centre of gravity, *g*. We have stated that the distance *b g* is twice the distance *g d*, and hence we may resolve the vertical force of 45 lbs., acting at *g*, into two others; one of 15 lbs. at *b*, and the other of 30 lbs. at *d;* but the last force, since it acts at the middle point of *a c*, may also be resolved into two others of 15 lbs. each, acting the one at *a*, and the other at *c*. Hence the weight of the triangle is equivalent to three equal forces acting vertically at its angles; and the three points of support sustain equal pressures, whatever may be the form of the triangle.

31



65. **Centre of gravity lying without the body.**—The centre of gravity is not, necessarily, in the body itself, but may be in some adjoining space. This is evidently true of the solid ring, fig. 32, and generally of any hollow vessel, of whatever form.

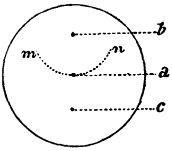
32



Of a compound body, the centre of gravity is easily found by composition of forces, when the weights and centres of gravity of the parts are known.

66. **Equilibrium of solids supported by an axis.**—A solid is in equilibrium when its centre of gravity is supported. This is according to § 63. But this condition may be fulfilled in different ways, according to the method of support. If a disk of uniform density, fig. 33, is supported by an axis, passing through the centre *a*, which is also its centre of gravity, it will be in that sort of equilibrium which is called *indifferent*, because it has no tendency to revolve, either to the right or left, but remains at rest in all positions. If the axis passes through *b*, the disk will be in *stable* equilibrium; for if it is turned about its axis, the centre of gravity will move in the arc *m n*, and being no longer vertically below the axis, it will not be directly supported by it, but tends always to return to its former position. If the axis is at *c*, the equilibrium is *unstable;* for, if the centre of gravity is in the least removed from a position vertically above the axis, it cannot return, but it will describe a semicircle in its descent, until it comes to rest exactly below the point of support.

33



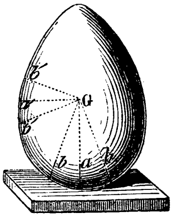
In general terms, therefore, a body attached to an axis may be in stable, unstable, or indifferent equilibrium, according as its centre of gravity is below, above, or upon the axis.

67. **Equilibrium of solids placed upon a horizontal surface.**—In bodies placed upon a horizontal surface, the centre of gravity, as in

those which are suspended, tends to descend, and if the bodies are free to move, they will rest in one of the positions of equilibrium just named. If rays are drawn from the centre of gravity to every part of the surface, some of these rays will be oblique, and some perpendicular, or *normal* to the surface, whatever may be the external form of the body; and among the normal rays, there is generally a longest and a shortest ray. If the body rests upon the plane, at the extremity of one of the normal rays, its centre of gravity is evidently in the vertical line, drawn through the point of contact, and the body is in equilibrium. But if it rests at the extremity of an oblique ray, the centre of gravity is not supported, since it is not in the vertical of the point of contact, and the body falls.

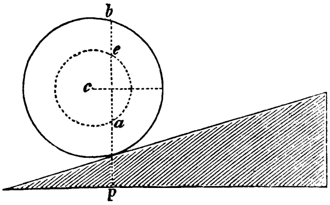
If the normal ray at the point of contact is neither longest nor shortest, but simply *equal* to the adjacent rays, the equilibrium is indifferent. Such is the case with a sphere, placed on a level plane; it rests in every position, for its centre of gravity cannot fall lower than it is. But this position cannot be assumed by a body not strictly spherical. For example, if an egg rests at the extremity of a longest descending ray, *a*, as in fig. 34, it will be in unstable equilibrium, since motion to either side tends to lower the centre of gravity, and enable it to fall; but if it rests at the extremity of a shortest ray, *a′*, it will be in stable equilibrium, since any motion sideways must *raise* the centre of gravity, and it will, therefore, fall back to its original position.

34



68. **Centre of gravity in bodies of unequal density in different parts.**—If the density of a body is unequal in different parts, its centre of gravity will be external to its centre of magnitude, and the body can come to rest in only two positions, when the centre of gravity is at the highest, and at the lowest place in the vertical of the point of contact. If a cylinder of this description were placed upon an inclined plane, as in fig. 35, it would be in equilibrium when its centre of gravity was at either *e* or *a*; if at *e*, and the cylinder were moved a little to the right, the centre of gravity would fall through the arc *e a*, but at the same time the cylinder itself would perform the apparent contradiction of ascending the plane.

35



69. **Equilibrium of bodies supported in more than one point.**—

When a body is supported by two points, the vertical, from its centre of gravity, ought to fall on the centre of the line which connects them. If a body has four points of support, as a common table, the vertical should fall upon the intersection of their diagonals.

In carriages, if the vertical falls in a different manner, the load is improperly distributed, and the carriage will be liable to upset, in passing over an uneven road.

A body resting on a base more or less extended, will be in equilibrium only when the vertical from the centre of gravity falls within the area of the base; and the body will stand firmer in proportion as the centre of gravity lies lower, and the base is broader. A pyramid is, therefore, the most stable of all structures.

The singular feats exhibited by children's toys, and by rope-dancers, depend on the facility with which the centre of gravity is shifted.

§ 2. Laws of Falling Bodies.

70. **Gravity is a source of motion.**—In discussing the laws of motion, we have already cited gravitation as a source of uniformly accelerated motion. We have now to consider the laws of falling bodies, and in doing so we shall have occasion to recapitulate some of the ground already passed over.

71. **The laws of falling bodies are five**, as follows:

THE FIRST LAW IS—*The velocity of a falling body is independent of its mass.*

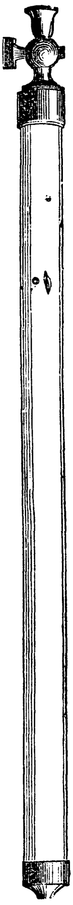
Galileo (born 1564), who first demonstrated the laws of falling bodies, at Pisa, argued that if the molecules of a body were separated from each other, each molecule would fall with the same velocity, since each is solicited by the same force; and if we conceive these molecules reunited into a mass, each particle still acts alone, and hence it is of no importance whether the particles are many or few. The velocity of the mass will be that of one of its particles—and, consequently, is independent of the mass.

THE SECOND LAW IS—*The velocity of a falling body is independent of the nature of the body.* Experiment alone can confirm this law, which at its first statement appears to be contradicted by common observation.

For example: a gold coin falls swiftly, and in a straight line, but a piece of paper descends in a devious course, and with a slow, hesitating motion. The popular explanation is, that the coin is heavy and the paper light; but this cannot be the true reason, since, when the gold is beaten out into thin leaves, its weight remains the same, but the time of its fall is very much prolonged.

The differences in the time and manner of falling are caused solely by the resistance of the air; which resistance varies, according to the shape and volume of the body, and not according to its mass, or the number of particles contained in it. This conclusion is established by the guinea and feather experiment, first devised by Newton, who used a glass tube 10 feet long, arranged as in fig. 36, with a stopcock for removing the air by an air-pump. Bodies of unlike density, as a coin and piece of paper, within the tube will, when it is suddenly inverted, be seen to fall with equal rapidity, and strike the bottom together; but after admitting the air, the one will descend swiftly, and the other will be retarded, just as it happens when they fall under ordinary circumstances. A piece of stiff paper, cut to the exact size of a coin and placed on it, will fall with it, if care is taken to drop the two quite horizontally, and without disturbing the position of the paper on the coin. This simple experiment illustrates the law as well as the vacuum tube, the resistance of the air being all met by the coin. Thus, when no resistance modifies the effects of gravity, it attracts all bodies with the same energy, and gives them the same velocity, whatever may be their weight, and whatever the kind of matter of which they are composed.

36



THE THIRD LAW IS—*The velocity acquired by a body falling freely from a state of rest is proportional to the times, and follows the order of the natural numbers* 1, 2, 3, &c. This is the case of a uniformly accelerated motion, (32).

THE FOURTH LAW IS—*The whole spaces passed over by a falling body, starting from a state of rest, are proportional to the squares of the times employed in falling—while the spaces fallen through in successive times increase as the odd numbers* 1, 3, 5, 7, &c. The velocity of a body when it begins to fall, is nothing; but from that moment it regularly increases. Let us represent the velocity acquired at the end of the 1st second by g; then the average velocity during the same time will be,

$$\frac{0+g}{2}=\tfrac{1}{2}g,$$

the arithmetical mean between 0, the starting velocity, and v, the final velocity. A body moving at this rate, will traverse the same space in one second which it would have fallen in one second; let this space $= s$; then the space being equal to the product of the velocity and the time, $\frac{1}{2}g \times 1 = s$, or $g = 2s$; that is, the final velocity acquired by a body falling one second, is double the space through which it has fallen. It has been ascertained that in latitude 45° this space is about $16\frac{1}{12}$ feet, (16·08538 feet, see 90) and $g = 32\frac{1}{6}$ feet.

In the 2d second, the body starts with a velocity of $g = 32\frac{1}{6}$ feet, and acquires, at the close, the velocity of $2g = 64\frac{1}{3}$ feet. The space fallen during the same time is $48\frac{1}{4}$ feet; viz. $32\frac{1}{6}$ feet by the velocity acquired during the first second, and $16\frac{1}{12}$ feet by the gradual action of gravity in this second only. Or as before, the space described by the body during the 2d second, is equal to the space it would have fallen with the mean velocity between its initial and final velocities; *i. e.* with the velocity

$$\frac{g + 2g}{2} = \frac{3g}{2} = 3s;$$

the space, therefore, $= 3 \times 16\frac{1}{12} = 48\frac{1}{4}$ feet.

In the same way we find that the velocity acquired at the end of the 3d second, will be $3g = 96\frac{1}{2}$ feet; and in the same time the body will have fallen, with the mean velocity,

$$\frac{2g + 3g}{2} = \frac{5g}{2}$$

through a space of $5s = 5 \times 16\frac{1}{12} = 80\frac{5}{12}$ feet.

A falling body, therefore, descends, in the 2d second of its fall, through three times, and in the 3d second, through five times the space fallen in the first second. Or in the words of the 4th law, *the spaces increase as the odd numbers.*

Whole space described by a falling body.—We have seen that the time of falling, and the final velocity, increase in the same ratio; and that the average velocity of any fall, is exactly half the final velocity, and the whole distance fallen is the same as if the body had moved at a uniform rate, with a mean velocity; hence, any increase in the time of falling is attended by a corresponding increase of the average velocity during the whole fall. But the whole space described in any fall is jointly proportional to the time, and the average velocity; if, therefore, the time is doubled, the body falls not only twice as long, but also twice as fast, and it must descend through four times the distance. Again, if a body falls three times as long as another, it also falls with three times the average velocity, and descends, altogether, through nine times the distance. The times being represented by the order of the natural numbers, 1, 2, 3, &c., the spaces are represented by their squares, 1, 4, 9, 16, &c.

THE FIFTH LAW IS—*A body falling freely from a state of rest acquires, during any given time, a velocity which would, in the same time, carry it over twice the space already traversed.*

We have seen that a body falling for any time, acquires a final velocity which is double the average velocity of the fall; if, therefore, the action of gravity were suspended at the end of any given time, and the body continued to move with its acquired velocity, it would, in the same time, traverse twice the distance it had already fallen. For instance, the space fallen through in three seconds is 144¾ feet, and the final velocity is 96½ feet; now a body falling uniformly, for three seconds, with this velocity, would pass through a space of $3 \times 96\frac{1}{2} = 289\frac{1}{2} = 2 \times 144\frac{3}{4}$ feet.

TABLE EXPRESSING THE LAWS OF FALLING BODIES.—The following table expresses the 2d, 3d, and 4th laws: (See 32.)

Times,	1,	2,	3,	4,	5.
The final velocities,	2,	4,	6,	8,	10.
The space for each time,	1,	3,	5,	7,	9.
The whole spaces,	1,	4,	9,	16,	25.

Let $D =$ the distance, $t =$ the time, $V =$ the final velocity, and $g =$ the velocity acquired during the first second, then from the foregoing laws we may deduce the following equations, by which practical questions are readily solved.

$$(1.)\ V = gt, \text{ whence } (2.)\ t = \frac{V}{g}.$$

$$(3.)\ D = \tfrac{1}{2}gt^2, \text{ whence } (4) t = \sqrt{\frac{2D}{g}}.$$

By substituting in (3) the value of t (2)

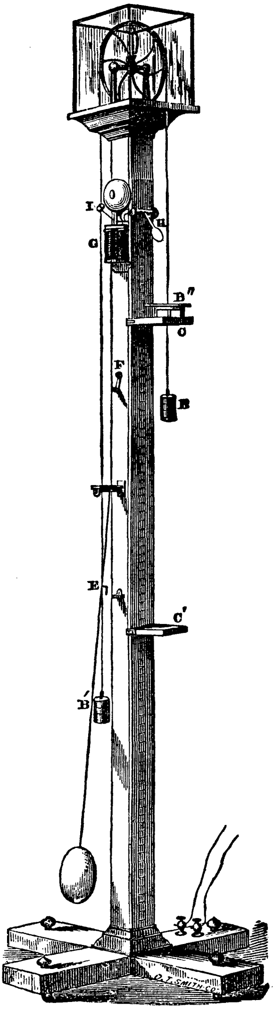
$$D = \tfrac{1}{2}g \times \frac{V^2}{g^2} = \frac{V^2}{2g}.$$

And substituting (4) in (1),

$$V = g\sqrt{\frac{2D}{g}} = \sqrt{2gD}.$$

72. Verification of the laws of falling bodies; Atwood's Apparatus.—It is evident that the third and fourth laws of falling bodies cannot be verified by direct experiment; both because the results of such a trial would be disturbed by the resistance of the air, and because the velocity of the fall is too great to be followed by the eye. But there are mechanical contrivances by which the intensity of the force of gravity may be diminished, without changing its nature. We can cause a falling body to descend so slowly that the resistance of the air becomes imperceptible, and all the circumstances of the fall may then be observed with entire precision. Galileo used an inclined plane to verify his laws, but a far more exact form of apparatus is now in use for this purpose, called, from the name of its inventor, Atwood's apparatus

37



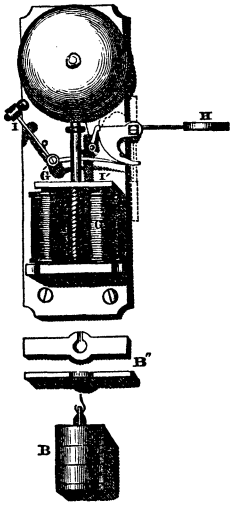
This apparatus, fig. 37, is composed of a vertical column, about eight feet in height, surmounted by a large wheel, moving with the least possible friction, and upon which is suspended a fine silk cord, carrying equal weights, B B′, at its extremities. A scale of feet and inches is marked on the standard, parallel to the path of one of the weights, to measure the spaces through which it falls, and the corresponding times are shown by the seconds pendulum. To insure the simultaneousness of the fall with the beat of the pendulum, the weight is set in motion by the fall of the tablet H, which is released by the action of an electro-magnet, G, (seen on a large scale in fig. 38), acting as will presently be explained. The weights B and B′ being equal, the force of gravity has no effect upon them, and they remain at rest in any position. But let one of them, as B, be increased by a small additional weight, B″, and the equilibrium will be immediately disturbed. The weight of B″ being the only disturbing force, the motion produced is of the same kind as the motion of a body falling freely, but the rate of acceleration and the space fallen through, are each as much less as the mass of B″ is less than the combined masses of B″ + 2 B.

The form and relation of B and B″ are more clearly shown in the enlarged scale of fig. 38. For example; let B″ be a quarter of an ounce, and the weights B and B′ be each 24 ounces, or 96 quarter ounces. The whole mass to be moved, by the action of gravity upon B″ only, is 193 times the weight of B″, and therefore the velocity imparted, and the space fallen through, must be 193 times less than the velocity and space of B″ falling freely.

In the apparatus here figured, the beats of the seconds pendulum are announced by the bell, whose hammer, I, is struck by the electro-magnet, G, as neatly constructed by Ritchie. A point, E, on the pendulum rod, just touches a drop of mercury as it passes the vertical line, and thus completes for an instant the circuit of a voltaic battery, whose terminal wires are shown at the base of fig. 37.

The electro-magnet then acts, and its armature, G′ I′, moves to the poles of G, thus releasing, first the tablet H, and with it the weight B, and next announcing the successive seconds upon the bell, at each swing of the pendulum. No clock-movement is required to secure the accurate beats of the seconds pendulum during the short period of a single experiment. Now B, with B″ attached to it, will fall from the tablet H, at the instant the first sound of the bell is heard, following the first swing of the pendulum. At the instant of the next ring, the weight will be seen to have fallen exactly 1 inch; during the second beat, through three inches more; during the third beat, through 5 inches; during the fourth beat, through 7 inches, &c., according to the fourth law.

38



In the same experiment it appears, that the whole space fallen through at the end of the 1st second, is 1 inch; at the end of the 2d second, 4 inches; at the end of the 3d second, 9 inches; at the end of the 4th second, 16 inches, &c., according to the first clause of the 4th law.

To demonstrate the 3d and 5th laws, it is necessary to arrest the accelerating force at a given moment. This is accomplished by giving to B″ the form of a slender bar, as shown in fig. 38, long enough to be caught by the sliding stage C, while B continues its course with a uniform velocity, from the time it ceases to be acted on by the gravity of B″. The velocity at the end of any second is determined by the space traversed during the next second. If the stage C is fixed at the distance of one inch from the top of the scale, B″ will be detached at the end of the first second, and B will descend uniformly through two inches during each succeeding second. If the ring is fixed at the fourth division, the bar will strike at the end of two seconds, and B will pass on at the rate of four inches per second. By the use of this simple and ingenious apparatus, a most satisfactory experimental demonstration of the truth of Galileo's laws of falling bodies is obtained.

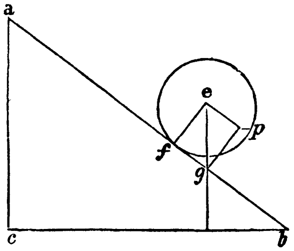
Morin's Apparatus.—Another apparatus for the verification of these laws has been contrived by Morin, a French physicist, in which the velocity of the falling body is not retarded, and the error from atmospheric resistance is made inconsiderable by the use of a large weight. The falling body carries a crayon which marks a line on a rapidly revolving vertical cylinder, covered with paper, and divided by vertical and horizontal lines, representing respectively time and distance. The line drawn by the crayon carried by the falling body is a portion of a parabola, a curve whose distance from each division on its vertical axis is as the ratio of the squares of the successive divisions on that line.

73. **Application of the laws of falling bodies.**—The laws of falling bodies apply equally to every motion produced by a uniform force or pressure.

In every such motion, the velocities are proportional to the times elapsed since the motion began; the final velocity is twice the average velocity; the spaces described in equal successive times, increase as the odd numbers; and the whole spaces increase as the squares of the times in which they are described. But in each instance the velocity acquired, and the space described in a given time, will be different; and the rate of acceleration will never be so rapid as in the case of a body falling freely, because in no other instance will the force be so great in proportion to the quantity of matter moved.

74. **Descent of bodies on inclined planes.**—When a body is placed upon an inclined plane, it descends, as just explained, with a uniformly accelerated motion, but its velocity is less than that of a body falling freely. The weight of the body, or its gravitation, represented by the line *e g*, fig. 39, is resolved (50) into two components, one of which, *ef* (or *p g*), is perpendicular to the plane, and produces pressure only, and the other, *ep* (or *fg*), is parallel to the plane, and is the cause of the accelerated motion. The triangles *efg* and *abc* being similar, their corresponding sides are proportional, and we have

39

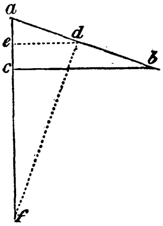


$$fg : eg = ac : ab;$$

that is, the rate of acceleration on an inclined plane, is to that of a body falling freely as the height of the plane is to its length.

The final velocity depends on the height of the plane. In fig. 40, let *a c*, the height of the plane, be ⅓ of its length *a b*; then, that part of the weight of the body which produces motion, is ⅓ of the whole force, and the velocity acquired, and the space traversed in one second, by the action of this force, would be ⅓ of the velocity and space of a body falling freely. Let the line *a f* represent $16\frac{1}{12}$ feet, and take *a d*, equal to ⅓ of *a f*; then, a body starting from *a* would arrive at *d* in one second, or, falling freely, it would reach *f* in the same time.

40



Draw the horizontal line *e d*; the ratio of *a e* to *a d* is the same as the ratio of *a c* to *a b*; that is, *a e* is equal to ⅓ of *a d*; and *a d* having been taken equal to ⅓ of *a f*, *a e* is $\frac{1}{9}$ of *a f*. Since the spaces increase as the squares of the times, the body that would fall to *f* in one second would fall to *e* in ⅓ of a second; and (3d law) the velocity acquired at *e* would be ⅓ of the velocity acquired at *f*. But we have already seen, that the velocity acquired by a body descending to *d*, is ⅓ of the velocity acquired by the same body falling to *f*, in the same time; hence the velocity of a body descending the inclined plane to *d*, is equal to that of a body falling freely to *e*; and generally—

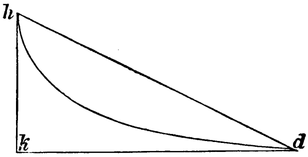
The velocity acquired at any given point on an inclined plane, is proportional to the vertical distance of that point below the point of departure.

From this it also follows that the average velocities are the same in descending all planes of the same height; and, therefore, the times of descent are proportional to their lengths.

75. **Descent of bodies on curves.**—It was shown by Galileo that all bodies starting from a horizontal plane with equal initial velocities arrive at the level of a second horizontal plane with equal velocities, whatever kind of curve they may have passed over. This is a general truth of which the statement at the close of the last paragraph is only a particular case. From these principles it follows that the velocity acquired in descending any regular curve is the same as would be acquired in falling freely by gravity through the same vertical height. But the time of descending a curve, concave upwards, is less than is required to descend an inclined plane between the same points, while for descending a curve convex upwards a greater time is required.

76. **Brachystochrone; or curve of swiftest descent.**—it was demonstrated by J. Bernoulli, and is confirmed by experiment, that a body descending a *cycloid*, whose base is horizontal, reaches its goal in less time than by any other path between the same points. The cycloid is a plain curve, described by a point on the circumference of a wheel, rolling on a level surface without slipping. The curve *h d* in the triangle *h k d*, fig. 41, is part of a cycloid.

41

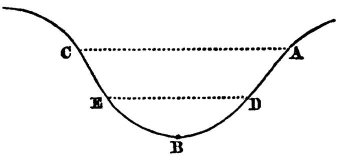


At first it would seem that the straight line *h d*, being an inclined plane, would be the brachystochrone, since it is shorter than the cycloid joining the same points, but the latter descends very rapidly at first, and so the falling body acquires near its starting point a much higher velocity than it would on the inclined plane. This increased velocity it adds to each of its subsequent movements, and though its velocity on arriving at *d* is no greater than if it had passed down the inclined plane, it arrives there in a shorter time than it could by any other path. Another curious property of the cycloid is, that a body will descend from *h* to *d* in this curve, in the same time it would descend to *d* from any intermediate point in the cycloid.

77. **Action and reaction of a falling body.**—On arriving at the bottom of a plane or curve, a body will have acquired (5th law) a velocity, such as would carry it, in the same time, over a distance equal to twice the length of the descent, or cause it to ascend another similar curve. The ascent of the body being opposed by the constant force of gravity, will be retarded at a rate which exactly corresponds with its

previous acceleration. On the double curve A B C, fig. 42, the body will have equal velocities at any two points at the same level, as at E and D; and the velocity being nothing when the body has arrived at C, it will descend again and mount to A, the point from which it first started. This alternate movement being caused by the constant force of gravity, would continue for ever, and furnish an instance of perpetual motion, were it not for the resistance of the air and friction, by which the body is gradually brought to rest at B.

42



The pendulum is an example of a body alternately ascending and descending a very small circular curve.

§ 3. Measure of the Intensity of Gravity.

I. PENDULUM.

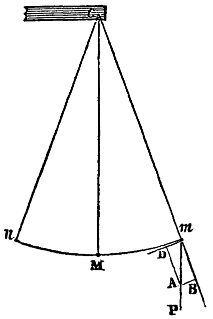
78. **The pendulum.**—Any body suspended by a flexible cord, or wire, from a fixed point of support, is a pendulum. A plumb line is a pendulum, and when it is at rest, as we have seen (60), it shows the exact vertical, and indicates the direction of the force of gravity. But if it is moved from the perpendicular into any other position, and left to fall, the pendulum swings in a vertical plane, and rises on the other side of the vertical to a height equal to that from which it had fallen. The cause of these alternate movements is gravity, and the motion is called an *oscillation.*

To aid in the study of the movements of the pendulum, mathematicians distinguish between the *simple,* or mathematical pendulum, and the *compound*, or *physical* pendulum.

79. **Properties of the simple pendulum.**—The simple pendulum consists, by mathematical conception, of a single heavy particle of matter, suspended at the extremity of a line, without weight, inextensible, and perfectly flexible. Such an instrument is purely ideal, and is conceived of only as a convenient means of investigating the laws of the real, or physical pendulum.

Let C M, fig. 43, be a *simple pendulum,* in a vertical position, and consequently in equilibrium. If it is moved to the position C *m*, the weight *m* P, acting at the point *m*, is decomposed into two components, *m* B acting in the direction C *m*, and consequently destroyed by the resistance of the point of support, and *m* D perpendicular to C *m*, which solicits the return of *m* to the position of equilibrium. This last component is equal to g sin. a; calling g the accelerating force of gravity, represented by m P, and a the angle m C M, or D A m, which is the same thing.

43



It is plain that the component m D must diminish with the angle a, that is in proportion as the pendulum approaches the point of equilibrium C M. The accelerated velocity of its fall is therefore not a case of uniform acceleration, since it becomes null when the pendulum is vertical.

The pendulum does not however rest at M, but, in virtue of its acquired velocity (momentum), it rises through an equal ascending arc, M n, with a retarded motion, since the component of gravity, tangent to the arc described, is now turned in the opposite direction—so that this component diminishes the velocity at each point of M n, by a quantity equal to the increase of velocity acquired at the corresponding points of m M, and equidistant from M. Thus the acquired velocity is entirely destroyed when the pendulum has passed over the arc M n, equal to M m. At n it rests for an inappreciable instant, after which it returns again to M, mounts to m, and would thus continue moving for ever like the ball rolling in a double curve (77), supposing it met no resistance from friction and the air.

Each swing, from n to m, or m to n, is called an *oscillation*, and one half the angle n C m, or one half the arc n m, which measures it, is called the *amplitude* of the oscillation. The time occupied in describing the arc m n is the time or duration of an oscillation. The *angle of elongation* n C M, or M C m, measures the deviation of the pendulum from the vertical.

80. **Isochronism of the pendulum.**—From the last section it is evident that the movements of the pendulum, on each side of the vertical, are made in equal times. But it is also true that the duration of an oscillation is always the same, in the same locality, and provided the angle of elongation, n C M, fig. 43, does not exceed 4° or 5°. Within this limit the time of oscillation is sensibly the same, and the pendulum requires as much time to describe an arc of one-tenth of a degree as one of ten degrees. The explanation of this curious and most remarkable fact is to be found in the varying length of the component D m, fig. 43, which increases with the angle of elongation. Hence, the greater length of arc is exactly compensated by the greater velocity with which the pendulum describes it. This is what is meant by the

isochronism of the pendulum—from two Greek words, meaning *equal times* This isochronism is not, however, absolute, unless the amplitude of the oscillation is infinitely small.

81. **Formulæ for the pendulum.**—The property of isochronism, and the other properties of the simple pendulum, when the amplitude is infinitely small, are comprised in the formula—

$$T = \pi\sqrt{\frac{l}{g}}:$$

T representing the duration of an oscillation, l the length of the pendulum, π the relation between the circumference and diameter of a circle (equal to 3·1416), and g the accelerating force of gravity.

If the amplitude of the oscillations is not infinitely small, the formula becomes, for ordinary limits,

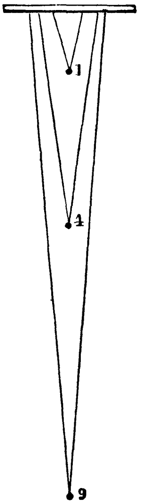
$$T = \pi\sqrt{\frac{l}{g}}\left(1 + \frac{a^2}{16l^2}\right).$$

where a is one-half the length of the arc *n m*, fig. 43. It requires the aid of the higher mathematics to demonstrate these formulæ fully, but we may deduce from them the following important propositions:

82. **Propositions respecting the simple pendulum.**—1st. *Oscillations of small amplitude are made in times sensibly equal.* By substituting in the first formula $l = 39{\cdot}14056$ inches, as determined by experiment, for the seconds pendulum at London, and let $T = 1^s$, we shall find the accelerating force of gravity, $g = 32{\cdot}175$ feet. Substituting these values of g and l in the second formula, and also $a = 3{\cdot}1416 \div 90 = 0{\cdot}0349$, we shall have for the time of vibration when the elongation is four degrees on each side of the vertical $T = 1{\cdot}000076$, which differs from the time of vibration, when the arc of vibration is infinitely small, by only seventy-six millionths of a second.

2d. *The duration of an oscillation in pendulums of different lengths, is proportional to the square root of the length of the pendulum.* This law may be demonstrated experimentally by comparing pendulums of different lengths. If the lengths are in the ratio of 1, 4, 9, then the times of oscillation will be as 1, 2, 3, respectively. Let three such pendulums, arranged as in fig. 44, commence to oscillate at the same time; it will be found that the one-foot pendulum makes two oscillations for each oscillation of the four-foot pendulum, and three oscillations for each one made by the nine-foot pendulum. The time of oscillation, and the length of the pendulum being known, we may determine by this law, 1st, the length of a pendulum which would oscillate in any proposed time; and, 2dly, the time of oscillation of a pendulum of any proposed length. For the times of oscillation are as the square roots of the lengths, or, what is the same thing, the lengths are as the squares of the times.

44



Or mathematically, by substituting in the first equation

81) $C = \sqrt{\frac{\pi^2}{g}}$, which is a constant quantity at any given place, the equation becomes $T = C\sqrt{l}$. For a pendulum of any other length, as l', we have $T' = C\sqrt{l'}$ and comparing the two

$$T : T' = \sqrt{l} : \sqrt{l'} : \quad \text{and also}$$
$$l : l' = T^2 : T'^2.$$

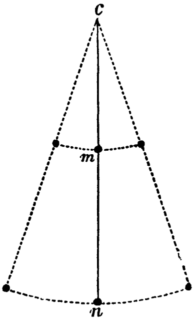
3d. In a pendulum of invariable length the duration is inversely proportional to the square root of the intensity of gravity. Hence,

$$T : T' = \frac{1}{\sqrt{g}} : \frac{1}{\sqrt{g'}} = \sqrt{g'} : \sqrt{g}.$$

where g' and g represent the intensity of gravitation at two places.

83. **The physical or compound pendulum:—Centre of oscillation.**—The simple pendulum, as already remarked, is only an intellectual conception, and cannot be realized in experiment. Practically, we employ for the physical pendulum a heavy body, suspended by an inflexible rod from a fixed point. The axis of suspension is usually a knife edge of steel, resting on polished agate planes, or hard steel. In the physical pendulum the rod has weight as well as the ball; and nearly all the material points of both rod and ball are placed at different distances from the point of suspension. Let us examine the oscillations of any two of these material points, *m* and *n*, fig. 45. If they were suspended by separate threads, then, according to the 3d law, *m* would oscillate more rapidly than *n*; but if they are suspended by the same inflexible wire, they must move together, and make their oscillations in the same time. The first accelerates the second, and the second retards the first, so that their common velocity is intermediate between the velocity of either of them, oscillating alone. Such a compensation takes place in every oscillating body, and between the particles which are nearer and those more remote from the point of suspension, there is always a point so situated that it is neither accelerated nor retarded, but oscillates exactly as if it were suspended alone, at the end of a thread.

45



without weight. This remarkable point is called the *centre of oscillation;* and its distance from the point of suspension is the *length* of the pendulum. This is equal to the length of a simple pendulum which would oscillate in the same time as the physical pendulum.

The position of the centre of oscillation depends upon the form, magnitude, and density of the several parts of the pendulum, and the position of its axis. If the rod of the pendulum is thick in proportion to the ball, its centre of oscillation will be higher than in a contrary arrangement. It is always below the centre of gravity, although whatever raises or lowers the centre of gravity will change the centre of oscillation in the same direction. *Whatever may be the positions of the point of suspension, and the centre of oscillation, they are always interchangeable; i. e.*, if the pendulum is suspended by its centre of oscillation, these two points exchange their functions, and the oscillations are made in the same time as before. It is by an experiment of this kind, that the centre of oscillation, and consequently the length of a pendulum, is determined. This remarkable property of the compound pendulum was first demonstrated by Huyghens.

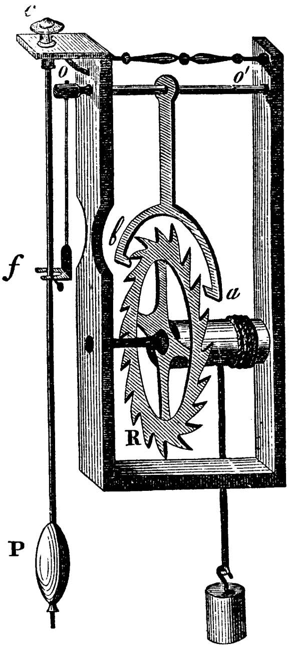
84. **Application of the pendulum to the measurement of time.**—Galileo, to whom we owe the discovery of so many important physical laws, discovered also the properties of the pendulum. When a choir boy in the great Cathedral of Pisa (from whose bell tower—the *leaning tower* of Pisa—he demonstrated long afterward the laws of falling bodies), and not yet eighteen years of age, his attention was arrested by the great regularity of the movements of a lamp suspended by a chain from the ceiling of the cathedral. This observation led him to the discovery in question. Although Galileo attempted to employ the pendulum to measure time, it was the great Dutch philosopher, Christian V. Huyghens, to whom we are indebted for the invention (in 1656) of the *clock escapement*, by means of which the pendulum is made to perform its proper function as a time-keeper.* This apparatus is seen in fig. 46.

An oblique-toothed wheel, R, called a *rachet* wheel, is moved by a weight and cord. This motion is controlled by a piece, $a\ b$, called the anchor escapement, placed above the wheel so as to oscillate on its axis of suspension, $o\ o'$, at right angles to the wheel. The oscillatory motion is imparted to the anchor $a\ b$ by the pendulum c P, which is made to communicate with the axis $o\ o'$ by the crotchet $o\,f$. If the pendulum is vertical the apparatus is at rest, for then the pallet *b* of the escapement holds the wheel by the point of one of its teeth. If, however, the pendulum is then moved to the left, the wheel is released and moves forward by force of the weight, until the other point of the escapement *a* again arrests its motion; but the return swing of the pendulum in its turn disengages *a*, and the wheel R revolves the space of another tooth, when it is again caught on *b*, and so on. The motion of the wheel R is thus made up of small equal advances, succeeding each other regularly with the oscillations of the pendulum. The points of the teeth on the *rachet* wheel, and also the points of the escapement anchor, are carefully formed to offer the least possible friction and resistance to motion.

---

* "The Arabian astronomers, and more especially En-Junis, at the close of the tenth century and during the brilliant epoch of the Abbassidian Califs, first employed these vibrations" (of the pendulum) "for the determination of time." (*Humboldt's Cosmos, Vol. 5, p.* 19.)

46



The pendulum and escapement of a clock are so arranged as to prevent the clock from running down, except by the regular and measured velocity indicated by the movement of the pendulum and escapement. On the other hand, the escapement is so constructed that the rachet wheel imparts to it a slight pressure at every swing of the pendulum, sufficient to counteract the retarding force of friction to which the pendulum is subject. The train of wheel-work, of which the clock is composed, serves to record the vibrations of the pendulum, and indicate at once to the observer the progress of time.

85. **Cycloidal pendulum.**—Owing to the resistance of the air, and to friction, a pendulum unconnected with other machinery has the amplitude of its vibrations gradually diminished, and vibrations varying greatly in amplitude, vary very sensibly in the time in which they are performed. To make vibrations of different amplitude absolutely isochronous, Huyghens conceived the idea of making the pendulum describe a cycloid, which, it will be remembered, is the curve of swiftest descent (76). The vibrations of such a pendulum would, in theory, be absolutely isochronous; but the mechanical difficulties in the way of adapting the pendulum to motion in this curve forbid its adoption.

A long, heavy pendulum vibrating in a very small circular arc, is found in practice the most perfect, and is for this reason generally used in astronomical clocks. The sources of error in the clock arising from inequalities of temperature will be considered in the chapter on heat.

86. **Physical demonstration of the rotation of the earth by means of the pendulum.**—Mr. Leon Foucault, in 1851, executed the first physical demonstration that had been made of the rotation of the

earth upon its axis. This remarkable and most interesting experiment consists in suspending a heavy ball to a long and flexible wire, and allowing the whole to vibrate freely, in the manner of a pendulum. Under these circumstances it will be found, in these latitudes, that the plane of vibration gradually changes its position, turning slowly from east to west, or with the motion of the hands of a watch.

The connection between the motion of the pendulum plane and the earth's rotation, may be easily understood. A pendulum set in motion will continue in the same plane of vibration, however the point of suspension may be rotated. This may be proved by holding in the fingers a pendulum, made of a simple ball and string, and causing it to vibrate. Upon twirling the string between the fingers, the ball will rotate on its axis, without, however, affecting at all the direction of its vibrations. The reason for this is obvious; the swinging pendulum, when about to return (after an outward oscillation) from its point of rest, is made to move from that point by gravity alone, and can, therefore, fall in but one direction.

If a pendulum were oscillating at either of the poles of the earth, the plane of revolution, as it would not change with the revolution of the earth, would mark this revolution, by seeming to revolve in a contrary direction, and in 24 hours it would make apparently the whole circuit of 360 degrees. But, at the equator, the plane of vibration is carried forward by the revolution of the earth, and so undergoes no change with reference to the meridians. Between the equator and the poles, the time required for the pendulum to make 360°, varies according to the latitude, being greater the further from the poles.

The observed rate of motion of the plane of vibration nearly coincides with that indicated by calculation. Thus, at New Haven (N. lat. 41° 18½′), the calculated motion, per hour, was 9·928°, the observed motion was 9·97°. (C. S. Lyman.)

The greatest length of the pendulum wire hitherto employed was that of 220 feet, in the Pantheon at Paris. At Bunker Hill Monument it was 210 feet long; at New Haven 71 feet. The weight of the ball employed (usually lead), has varied from 2 to 90 pounds. The longer the wire, and the heavier the ball of the pendulum, the greater will be the probability of accurate results, for when the mass of the body is great, and its motion slow, the resistance of the air will have but comparatively little effect on the direction of the vibration.

87. **The pendulum applied to the study of gravity.**—By the pendulum we ascertain more accurately than by any other method the truth of the first law of falling bodies; viz., that gravity acts equally upon matter of every description (71). Newton, and more recently

Bessel, verified this law, by using a pendulum having a hollow ball, which was filled, successively, with various substances—metals, ivory, meteoric stones, wool, feathers, liquids, &c.—that could not be otherwise submitted to trial. This experiment affords the most precise and unmistakable evidence that gravity (g) acts on all bodies in the same manner. Since π is a constant quantity, the formula for the pendulum shows that if T and l do not vary, g remains also constant.

88. **Use of the pendulum for measuring the force of gravity.**—The value of the term g for any place may be easily obtained mathematically (the length of a pendulum which oscillates in a given time (T) being known), by transposing the formula for the pendulum (81); thus we have for the intensity sought—

$$g = \frac{\pi^2 l}{T^2},$$ and assuming T equal unity, then l is the length of a seconds pendulum, and we have

$$g = \pi^2 l.$$

Experimentally, we may determine the intensity of gravity at any place, by counting with exactness the number of oscillations made at the place of observation in a given time, by a pendulum whose length is known, and then dividing the time by the number of oscillations. Any error in observing the time of a single oscillation is thus greatly diminished, by subdivision, and by a sufficient number of repetitions this error may be reduced to a quantity too small for consideration.

It was thus that Borda and Cassini, in 1790, measured with great accuracy, the intensity of gravity at the Observatory in Paris, using a pendulum composed of a platinum ball, suspended by a fine platinum wire, upon knife edges of steel, resting on agate planes. The whole was about four metres long, and its oscillations were counted, not directly, but by means of an ingenious comparison with the motions of a clock pendulum, placed a few metres behind, marking by a telescope the occurrence of a coincidence in the vertical position of the two pendulums, and then observing the number of seconds before a coincidence occurred again. The pendulums were inclosed in glass cases, to avoid currents of air.

89. **Value of g in these experiments.**—After carefully eliminating the errors of experiment due to the influence of the air (the con sideration of which would lead us too far into the refinements of this subject for our limited space), Borda and Cassini found for the intensity of gravity at Paris $g = 9{\cdot}8088$ metres, equal to 32·1798 feet. This value has been confirmed by Arago, Biot, and others, and slightly corrected by Bessel, by considering the loss of weight in air due to the motion of the pendulum, giving the quantity $g = 9{\cdot}8096$ metres.

Seconds pendulum.—On the other hand, when we know the accelerating force of gravity, g, at any given place, it is easy to calculate the length of the simple pendulum vibrating seconds, assuming the oscillations to be infinitely small. Thus in the formula for the pendulum (81), making $T = 1^s$, and using for g the value determined for the place, we have $l =$ at Paris 0·993866 metre = 39·127 inches, and corrections being made for the interference of the air, this quantity, as determined by Bessel, is 0·993781 metre = 39·12367 inches.

II. MODIFICATIONS OF TERRESTRIAL GRAVITY AND THEIR CAUSES.

90. **The intensity of gravity varies with the latitude.**—Very numerous observations made with the pendulum, on different parts of the earth's surface, have shown that the force of gravity is by no means the same at all places, and particularly that it increases in going from the equator toward either pole. This result is observed in the increasing length of the pendulum vibrating seconds, since by § 88, g is proportional to l, the pendulum must be longer, as the force of gravity is greater, to preserve the same time in oscillation. The value of g for any latitude is obtained with approximate accuracy by the formula $g = 32{\cdot}17076\ (1 - 0{\cdot}00259,\ \cos.\ 2\lambda)$, in which λ is the latitude of the place, and 32.17076 feet the value of g at latitude 45°. By substituting for λ, successively 0° and 90°, we obtain at the equator $g = 32{\cdot}0874377$ feet, and at the poles $g = 32{\cdot}254083$ feet.

The following table of the variation in the length of the seconds pendulum, with the latitude, is condensed from a large list in Saigey. (*Physique du Globe, p.* 132, *t.* 2.)

Places observed.	Latitudes.	Length of seconds pendulum in American inches.*	Names of observers.
Spitzbergen, ...	79° 49′ 58″ N.	39·2161492	Sabine.
Greenland, ...	74° 32′ 19″ "	39·204339	"
St. Petersburg, . . .	59° 56′ 31″ "	39·1704818	Lutké.
Paris, ...	48° 50′ 14″ "	39·1299322	Biot.
New York, ...	40° 42′ 43″ "	39·1023743	Sabine.
Jamaica, W. I., . . .	17° 56′ 07″ "	39·0362352	"
St. Thomas, W. I., . .	0° 24′ 41″ "	39·0216688	"
Maranham, ...	2° 31′ 35″ S.	39·0126141	Foster.
Rio Janeiro, ...	22° 55′ 22″ "	39·0452899	Basil Hall.
Cape of Good Hope, .	33° 55′ 56″ "	39·0795405	Fallows.
Cape Horn, ...	55° 51′ 20″ "	39·1567028	Foster.
N. Shetland, ...	62° 56′ 11″ "	39·1807176	"

---

* In reducing Saigey's table of lengths of the seconds pendulum at different localities, to American inches, the French metre has been taken at 39·36850535 inches, as adopted by the United States Coast Survey.

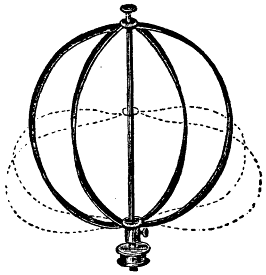
Numerous observations on the U. S. Coast Survey, and elsewhere, show that the value of g is by no means rigorously the same at all points on the same parallel; a discrepancy to be explained only by supposing an inherent difference in the constitution of the earth's crust at different places.

The variation of gravity with change of latitude is due to two causes. 1st. To the flattening of the earth at its poles. 2d. To the centrifugal force created by the revolution of the earth upon its axis. The last cause also, beyond doubt, induced the flattening of the poles in the earlier history of our planet.

91. **Influence of the earth's figure upon gravity.**—Until 1666 the perfect sphericity of the earth had not been questioned, although in the preceding century the flattening of the planet Jupiter at the poles had been observed. Subsequently (in 1672), Richer, sent by the Academy of Paris to Cayenne, remarked that his pendulum no longer beat seconds at the latter place, until it was shortened a line and a quarter from its length at Paris. This observation at once indicated a less force of gravity at Cayenne than at Paris, and suggested doubts respecting the sphericity of the earth. Huyghens attributed this diminution of the force of gravity to centrifugal force, and conceived that the earth must be bulged out at the equator.

Huyghens and Newton, assuming that the earth had become solid from an originally fluid mass, whose particles attracted each other, subject to the laws of hydrostatics and of the centrifugal force, arrived at the conclusion, from mathematical calculation, that the earth's figure was that of an oblate spheroid, whose polar diameter was about 26 miles less than its equatorial. Laplace reached almost the same conclusion, by calculating the effect of the equatorial mass on the motions of the moon. The effect of the centrifugal force upon a yielding mass, may be shown by the apparatus, fig. 47. Two circles of wire, or flexible metallic ribbons, are attached below to an axis, and above to a sliding ring, and being rapidly rotated by the whirling table, the circles flatten in the direction of the axis, and bulge at the equator, as shown by the dotted lines.

47

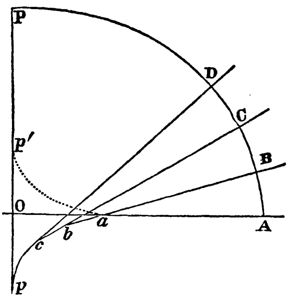


But it was only by the actual measurement of an arc of meridian that the exact figure of the earth became known. This important geodesic operation was undertaken, by La Condamine and others, in 1736 in Peru, by order of the French government, and was less accurately performed by Picard in France, in 1669. This operation led to the conclusion (since demonstrated by numerous similar measurements), that the successive arcs on the same meridian, comprised between two verticals forming an angle of 1°, become larger and larger as we advance toward the poles. Consequently, the equatorial radius is greater than the polar, and the plumb line will point to the centre of the earth only in one of those radii.

The astronomers Mason and Dixon, who, in 1764–6, established the boundaries between Pennsylvania, Delaware, and Maryland, afterwards, in 1768, re-measured a line of 538,067 feet, with great accuracy, very near the meridian, for the purpose of determining the value of an arc. Four-fifths of this (434,011$\frac{64}{100}$) was one unbroken line, without triangulation, on a vast level plain. They used rods of fir, frequently compared with a standard brass measure at a fixed temperature. They found the length of a degree of latitude to be 363,763 English feet. (Phil. Trans. 1768.)

The general results may be illustrated by the following diagram.

48



Let the line A P, fig. 48, represent a quadrant of a meridian, of which O P is the polar, and O A the equatorial radius. Let us take stations on this meridian, one degree distant from each other, commencing from the equator, and from each station prolong the direction of the plumb line until it intersects the plumb line similarly produced from the previous station; *a b c* are three such points, and it is plain that the intersections of the plumb lines from each of the ninety verticals on the quadrant, would together evolve the curve *a b c p,* and the same if the stations were infinite. Objects on different parts of the earth's surface are not attracted to a common centre of gravity. The centre of gravity for any point, A, B, C, P, on the quadrant, A P, must lie in the corresponding points, *a, b, c, p,* where the respective normals cut the evolute *a p*. At A, for example, the attraction of gravity acts as if it originated at *a,* for B at *b,* for C at *c,* &c. But the *intensity* of gravity is greater at B than at A, at C than at B, and so on.

The revolution of the evolute *a p* on its axis O *p* will evidently generate a surface (called a *locus*), in which will be found the centres of gravity for all points on the upper hemisphere, and a similar surface may be produced for all points on the lower hemisphere by the revolution of the curve *a p′*.

Evidently, therefore, a body placed at the equator will be very differently affected by the force of gravity, from what it would if placed at the poles. The amount of flattening at the poles is about $\frac{1}{300}$ of the equatorial radius, or, accurately, $\frac{1}{302{\cdot}4}$; that is, the polar radius is so much shorter than the equatorial—exactly 21·319 kilometres, equal 13·246483 miles; or in the diameter nearly 26½ miles (26·492966). In an exact model of the earth 15 inches diameter, it would be represented by $\frac{1}{20}$ of an inch; a quantity too small to be detected by the eye or hand.

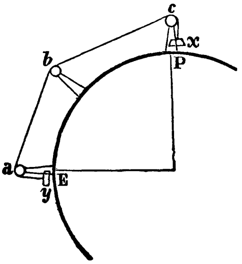
92. **Exact dimensions of the earth.**—According to the latest calculations the exact dimensions of the earth, as given by Kohler, when reduced to American standard measures, are as follows:

Volume of the earth,	259,756,014,917	cubic miles.
Surface of the earth,	196,881,077	square miles.
Length of a quadrant,	6213·99609	miles.
Mean radius (lat. 45°),	3955·94978	"
Equatorial radius,	3962·57302	"
Polar radius,	3949·32654	"

Difference between the last two dimensions 13·24648 miles. The equatorial swelling, or that portion of the earth which lies outside of a perfect sphere, whose circumference is described by the polar radius, is $\frac{1}{157}$ part of the whole volume of the earth. Two verticals include an angle of 1″ when they are 101·7 feet distant from each other, and they will inclose a sector of 1′ when they are distant from each other 1·15 miles.

93. **Sensible weight varies in different localities.**—The same body is sensibly lighter at the equator than at the poles of the earth, in the ratio of 194 to 195. This difference cannot be detected by the balance, because the thing weighed is counterpoised by an equal standard weight, under the same circumstances; and if both are removed to another station, their weight, if changed, will be changed equally, and a body and its counterpoise once adjusted, will continue to balance each other wherever they are carried. It is not in this sense that 194 lbs. at the equator will weigh 195 lbs. at the poles; but if we conceive a body, y, suspended by a cord, imagined without weight, passing over a pulley at the equator, as in the annexed figure, 49, and connected by other pulleys, all without friction, with x, another equal weight, at the poles; then, although the weights would counterpoise each other in a balance, they would not in this situation, but the polar weight would preponderate, and y would require to be increased by $\frac{1}{194}$th part, to restore the equilibrium.

49



The above phenomena are readily demonstrated by the spring-balance, or dynamometer (37.)

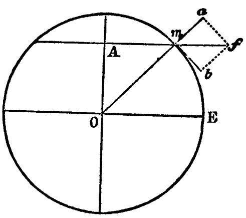
94. **Effect of the earth's rotation on gravity.**—Newton and others have determined, by calculation, that the increase of weight, due to the spheroidal form of the earth, is $\frac{1}{568}$, when a body is transported from the equator to the poles; yet the difference of weight is found experimentally to amount to the much more considerable quan

tity of $\frac{1}{194}$ part of the total weight of the body. This large difference is accounted for by the centrifugal force, which is nothing at the poles, and regularly increases towards the equator, where it is greatest, and in the same ratio diminishes the weight of bodies on the earth's surface. The earth revolves once in 24 hours, but if it revolved seventeen times more rapidly than it now does, or in 1h. 24m. 25s., the centrifugal force would balance the force of gravity, and bodies at the equator would have no sensible weight. If the velocity of revolution was farther increased the oceans would be thrown off like water from a grindstone, and all loose materials would fall into space.

Demonstration.—By the laws of centrifugal force it follows that the observed weight of any substance on the earth's surface is the difference between the earth's attraction and the centrifugal force developed by the revolution of the earth. By § 54 the centrifugal force at the equator $= C = \frac{4\pi^2 R}{T^2}$; R being the equatorial radius, and T a diurnal revolution. If G represent the attraction of the earth, and g the weight of a body at the equator, then (1) $g = G - \frac{4\pi^2 R}{T^2}$.

Let m, fig. 50, be a material particle taken on any parallel, and represent A m, the radius of this parallel, by r, the centrifugal force at this point, $mf = c = \frac{4\pi^2 r}{T^2}$. But as this force does not act in the direction of gravity, decompose it into two others, one of which, $m\,b$, being at right angles to gravity, has no effect upon it, and the other $m\,a$ acting directly against gravity. Let m O E, the latitude of the place, which is equal to $a\,m\,f$, be designated by L, then in the right-angled triangle $a\,m\,f$, $m\,a$ is equal to $mf \times$ cosine of $a\,m\,f = c$ cos. L. In the triangle A m O, A m $= r = R$ cos. L. It follows that the vertical component $m\,a = \frac{4\pi^2 R}{T^2}$ cos.^{2} L. The force of gravity at m is then,

50



$$(2)\ g = G - \frac{4\pi^2 R}{T^2} \text{ cos.}^2 L.$$

The diminution of gravity due to the centrifugal force is therefore proportional to the square of the cosine of the latitude. At the pole, where $L = 90°$, $g = G$. At the equator $L = 0$, and g is found by the formula. In the first formula (1) the value of the second term of the equation being very small in relation to the first, gives very nearly $g = G\left(1 - \frac{4\pi^2 R}{gT^2}\right)$, by placing G as a common factor, and replacing it in the denominator of the second term by g.

Taking for the mean radius of the earth $R = 20{,}887{,}413$ feet, and $g = 32{\cdot}1798$ feet, and $T = 86{,}164$ seconds (the time of a revolution of the earth on its axis), we find for the value of $\frac{4\pi^2 R}{gT^2}$ very nearly $\frac{1}{289} = \frac{1}{17^2}$. If, therefore, the earth revolved 17 times faster than it does at present, making T seventeen times smaller, the second term in the parenthesis would become unity, and the value of g would be zero, or bodies at the equator would have no weight. The expression (1) enables us to calculate the attractive force at the equator, assuming as a starting point the value $g = 32{\cdot}09025$ feet as the value of gravity as indicated by the pendulum. We then find the attractive force at the equator $G = 32{\cdot}20147$ feet, and the centrifugal force at the equator $= 0{\cdot}111216$ feet.

95. **Variation of gravity above the earth and below its surface.**—By the law of gravitation it follows that as we rise above the earth the force of gravity must diminish. This diminution is sufficient to be appreciable at any considerable distance above the level of the sea; therefore, to compare the results of experiments relating to the force of gravity at different situations on the earth's surface, it is necessary to reduce all observations to a common standard—the sea level.

Representing by g' the intensity of gravity at any elevation h, and the earth's radius by R, neglecting the variation of the centrifugal force, we have

$$g : g' = (R + h)^2 : R^2;\ \text{hence } g = g' \frac{(R + h)^2}{R^2}.$$

The mean distance of the moon from the earth's centre is about sixty times the equatorial radius of the earth, and it completes its orbit (assumed to be circular) in 27·322 days. As, therefore, the intensity of the earth's attraction at the moon equals the centrifugal force (as is evident from physical astronomy), this force can be calculated by substituting for R sixty times the earth's radius, in the formula for centrifugal force. Substituting for T the time of a lunar revolution expressed in seconds, we find for the earth's attraction on the moon $g = \frac{4\pi^2 \times 60R}{T^2}$ $= 0.00679$ feet, which is about 3600 times less than the attraction of the earth for bodies on its surface at the equator (assuming for bodies as distant as the moon that the attraction of the earth is concentrated at its centre). This agrees with the law of gravitation, the square of 60 being 3600.

Below the earth's surface, assuming the earth to be a sphere, the force of gravity is proportional to the distance of the particle from its centre. It is plain that the force of attraction at any point beneath the surface is diminished by whatever part of the earth is above the particle, and the resultant force is the difference between the two components. Could a body be placed in empty space at the centre of the earth, it would be sustained there without any material support by the equal and opposite attractions. It can also be demonstrated mathematically that, if the earth were a hollow sphere of uniform density, a material particle would remain at rest at any point within it. It follows from this that:—*The attraction of the earth, for a particle of matter below its surface, is directly proportional to its distance from the centre of the earth.*

§ 4. Mass and Weight.

96. **Mass**.—The mass of a body is the quantity of matter which it contains; and since the *absolute weight* of a mass of matter is the sum of the attraction of gravitation upon all its molecules, it follows that, in the same place, the masses of bodies are to each other as their weights. Calling the mass M and the weight W, and the force of gravity g, for any given body, then $W = Mg$. We have already seen (41) that the masses of bodies may be compared by the forces required to impart to them equal velocities. Since gravity acts equally on matter of whatever description, this comparison may also be made by comparing their weights when otherwise under the same conditions.

97. **Weight.**—The term weight as used above, and always in scientific language, means the pressure exerted by a given mass, due to the force of gravity. This varies, as we have seen, with the force of gravity, and is not the same for the same mass at all parts of the earth's surface (93). The weight of any given kind of matter varies also with its mass. A mass of two, three, or ten times a given unit weighs two, three, or ten times as much as that unit at the same place, and hence we are very prone to confound the weight of a substance with its mass. On the surface of the earth this confusion of terms can lead, as we have seen (93), to an error of only about one two-hundredth part of the whole ($\frac{1}{194}$). That is, a mass of iron weighing 1000 lbs. on the equator would weigh 1005 lbs. at the pole. Such a mass of iron would weigh only 500 lbs. at a distance of 2000 miles below the surface of the earth, or 1650 miles above the earth, and only 160 lbs. on the moon, while it would weigh about 2600 lbs. on the planet Jupiter, and 28,000 lbs. if placed on the sun.

98. **Density.**—The density of a body is the mass comprised under a unit of volume, or $M = V \times D$, where the mass, M, of a body is equal to its volume, V, multiplied by its density, D;

$$\text{transferring, we have}\quad V = \frac{M}{D}.$$

This may be otherwise stated, thus—1st, the mass is proportional to the volume; 2d, for an equal volume the mass is proportional to the density; and, 3d, the density of the same mass is inversely proportional to the volume it occupies.

99. **Specific weight** is the weight contained in a unit of volume; this is also often called specific gravity. Representing specific weight by w,

and absolute weight by W, we have $W = V \times w$, hence, 1st, the weight is proportional to the volume; 2d, for an equal volume the absolute weight is proportional to the specific weight; and, 3d, for equal absolute weights the specific weight is inversely as the volume.

By the first formula we have $w = Dg$, whence w is the weight of the unit of volume, and D its mass. Replacing w by this value in the last formula, it becomes $W = V \times D \times g$.

Specific weight differs therefore from density exactly as weight differs from mass. Both weight and gravity vary with the latitude, and the unit accepted as a standard varies also, but when the same standards are employed, the numbers expressing the weights remain unchanged, and no sensible error results. The terms density and specific gravity have thus been used interchangeably for each other, although, speaking strictly, involving different quantities. The balance is the common instrument used to determine weights. It will be described under the lever, of which it is one form.

100. **French system of weights.**—As in measures (16), so in weights it is indispensable to assume some arbitrary standard unit. The French have assumed as their unit of weight, the pressure exerted by one cubic centimetre of pure water at its maximum density (39°.2 Fahrenheit), in a vacuum, and at the latitude of Paris. This unit is called a *gramme*, and it weighs (nearly) 15·433 grains English. The gramme is multiplied and divided decimally, and these multiples and subdivisions are named on the same plan with the parts of a metre: Thus we have,

1 Kilogramme	= 1000	grammes,	1 Gramme	= 1·000	gramme,
1 Hectogramme	= 100	"	1 Decigramme	= 0·100	"
1 Decagramme	= 10	"	1 Centigramme	= 0·010	"
1 Gramme	= 1	"	1 Milligramme	= 0·001	"

The kilogramme is the commercial unit of weight, and is rather less than 2¼ lbs. avoirdupois, being 15,432·42 English grains.

The French unit is of course a gramme only at Paris, and at higher or lower latitudes weighs (according to the principles before explained) more or less than a gramme. But this leads to no practical inconvenience, so long as a set of exact measurements made in one latitude are not brought into rigorous comparison with those made by the same standard in another latitude. The general acceptance of the French system among scientific men, and its special fitness for scientific research, owing to the very simple relation which exists between it and the system of measures already described, would seem to render the universal adoption of a decimal system of weights and measures for the United States one of the great desiderata still to be accomplished for our common country.

101. **English and American system of weights.**—In England, as in the United States, two distinct units of weight are in common use, leading to constant confusion, both of terms and quantities. These units, the *Troy pound* and the *Avoirdupois pound*, are entirely arbitrary. They are represented by certain masses of brass, declared by law to be the legal standards of the above names. These pounds are related to each other in the ratio of 144 to 175, and, excepting the grains, none of their subdivisions are alike. The troy pound contains 5760 grains divided among 12 ounces, and the avoirdupois pound contains 7000 grains divided among 16 ounces. The legal standard of weight in the United States is the *troy pound*, copied by Capt. Kater in 1827 from the English Imperial Troy pound, for the U. S. Mint at Philadelphia, where it now is. The avoirdupois pound is, however, the unit of weight in actual use in most commercial transactions. Kater's copy of the troy pound is a standard at 62° of Fahrenheit's thermometer and 30 inches of the barometer. A cubic inch of distilled water weighs in the air at 62° Fahrenheit and 30 inches barometric pressure 252·456 grains.

The English standard of weight is connected with that of measure by the parliamentary enactment, that 277·274 cubic inches shall constitute the *Imperial gallon* of 70,000 grains, or ten pounds of pure water at 62° F. and 30 inches barometric pressure.

The *American standard gallon* contains at 39°·83 F. (the maximum density of water adopted by Hassler) 58,372 grains of pure water at 30 inches barometric pressure. Tables for the comparison and reduction of the French, English, and American units will be found at the end of this volume.

102. **Estimation of the density of the earth by experiment.**—In the vicinity of a mountain a plumb-line is not truly perpendicular, but is drawn to one side by the lateral attraction of the mountain. The amount of this deviation is measured by observations on the zenith distances of a star, at two stations on opposite sides of the mountain, and on the same meridian. This deviation was first noticed near Mount Chimborazo in 1738, by the French Academicians engaged in measuring a meridian arc in Peru, where the deviation was 7″·5. In 1774, Maskelyne found a deviation of 5″·83, caused by the lateral attraction of Schehallien, an isolated mountain in Scotland. Hutton spent three years in ascertaining the mean attraction of one thousand stations on this mountain; a labor rewarded by the Royal Society of London. Estimating the mean density of the rocks of Schehallien at 2·5 to 3 2, as determined by Playfair, the mean density of the earth was determined to be over five times the density of water. The accurate inves-

tigation of this problem was one of the highest importance in astronomy, since it furnished the means of determining the mean density of the earth, by comparing its attraction with the attraction of a part of its mass, whose density could be ascertained by direct experiment.

This problem is solved with much greater precision, by the famous experiment of Cavendish, in which the earth's attraction is compared with that of a mass of lead.

Cavendish's determinations of the density of the earth were made, in 1798, by means of an apparatus suggested by the Rev. John Michell.

"Michell's apparatus was a delicate torsion balance, consisting of a light wooden arm, suspended in a horizontal position, by a slender wire 60 inches long, and having a leaden ball, about 2 inches in diameter, hung at either extremity. Two heavy spherical masses of metal were then brought near to the balls, so that their attractions conspired in drawing the arm aside. The deviation of the arm was observed; and the force necessary to produce a given deviation of the arm, being calculated from its time of vibration, it was found what portion of the weight of either ball was equal to the attraction of the mass of metal placed near it. From the known weight of the mass of metal, the distance of the centres of the mass, and of the ball, and the ascertained attraction, it is easy to determine the attraction of an equal spherical mass of water, upon a particle as heavy as the ball placed on its surface. Now the attraction of this sphere will have to that of the earth the same ratio as their densities; and as the attraction of the earth is equal to the weight of the ball, it follows, that as the calculated attraction is to the weight of the ball, so is the density of water to the earth's density, which is thus determined." (*Wilson's Life of Cavendish.*)

A comparison of about two thousand experiments with an improved form of this delicate apparatus, conducted by Mr. Francis Bailey, in 1842, determined the mean density of the earth to be 5·6604 times that of water. It is worthy of remark that Newton, whose *guesses* were often worth more than the researches of less sagacious men, had conjectured the earth's density to be between 5 and 6 times the density of water.

The calculation is conducted thus. Let L be half the length of the horizontal arm of wood. G the attraction of the masses of lead, and t the time of an oscillation—neglecting the effect of torsion—Then, according to the theory of the pendulum (81),

$$t = \pi\sqrt{\frac{L}{G}}.$$

Take l for the length of a simple pendulum oscillating in the same time (t) by gravity, and we have

$$t = \pi\sqrt{\frac{l}{g}}; \quad \text{or } L : G = l : g, \text{ and } l : L = g : G.$$

Calling the attraction of the unit of mass upon the unit of mass at the unit of distance a; the mass of each sphere of lead m; d the distance from the centre of this sphere to that of the attracted sphere when in the position of equilibrium;

and, lastly, M the mass and R the mean radius of the earth; and we have, according to the laws of attraction,

$$G = \frac{am}{d^2}, \quad g = \frac{aM}{R^2}.$$

By substituting these values of G and g in the last proportion, it becomes

$$l : L = d^2M : R^2m, \quad \text{or}$$

$$M : m = lR^2 : Ld^2,$$

which fixes the ratio between the mass of the earth (M) and that of one cf the masses of lead (m), as given by the balance. The volume of the earth being represented by V, and its mean density by D, we have by (98) $M = V \times D$, from which, M and V being known, D is deduced.

The inference, unavoidable, from these facts is, that the interior parts of the earth must be much more dense than the superficial crust. Granite and other rocks on the earth's surface have an average density of about 2·5. This remarkable fact may be explained, partly by remembering that the interior parts of the earth sustain the enormous pressure of the surface portions, and partly by the hypothesis of primitive fluidity, which authorizes the belief that the more dense portions of the planet would seek the lowest place, and the lighter parts the surface.

§ 5. Motion of Projectiles.

103. **Projectiles** *are bodies thrown into the air by some momentary force.* They are therefore subject to two forces, one the projectile force, which is momentary, the other the constant force of gravity.

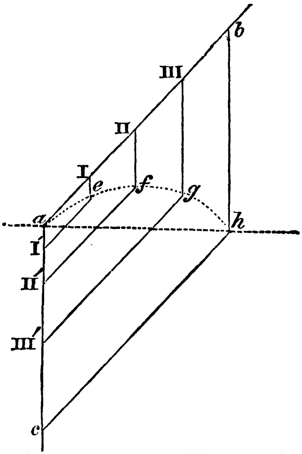
When a body is projected vertically upward, it rises with a uniformly retarded motion, the action of gravity diminishing the velocity of ascent, at every instant, until the projectile force is expended, when the body commences to descend, and passing every point in its downward path at the same rate as in its upward flight, it acquires at the end of its fall a velocity equal to that with which it was projected.

In the same manner when a body is projected vertically downwards its path is the same as that of a body falling freely, but the space traversed, and also the velocity, are resultants of the sum of the two forces. These are simple cases under the laws of uniformly accelerated or retarded motion already considered (32).

If the direction of the projectile is not perpendicular, then the path of the projectile must be a curve (51).

Thus, if a cannon-ball is shot in the direction $a\ b$ (fig. 51), with a velocity which would carry it through the space a I in one second, then, by the laws of inertia, it would continue in this line, passing through equal spaces in equal times. If it was acted upon by gravity alone, it would move in the vertical $a\ c$, through the spaces I′ II′ III′, in corresponding seconds. But, while it is projected in the direction *a b*, it is subject also to the action of gravity, and, like any other body, must fall through the vertical space of $16\frac{1}{12}$ feet during the 1st second; at the end of that time, therefore, it will be found at *e*, instead of at I. In the same manner, at the end of the 2d and 3d seconds, it will be at *f* and *g*, instead of II and III; and at the end of four seconds the body will arrive at *h*, the result by the parallelogram of forces being exactly the same as if it had been first carried by the projectile force in the line *a b* during four seconds, and then allowed to fall during four seconds by the action of gravity over $b\,h = a\,c$. Since the action of the projectile force is only momentary, while the effect of gravity is constantly increasing, the body will not describe the diagonals of the parallelograms, *a e*, *a f*, &c., but a curve, which in mathematics is called a parabola, indicated by the dotted line connecting *a e f g* and *h*.

51

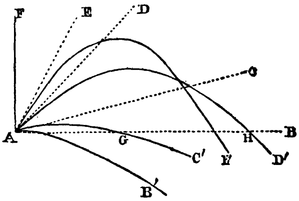


By a similar construction, we find the path of a body projected horizontally, or obliquely downwards, in which cases the projectile will describe one-half of a parabola. In every case the path of the projectile is a complete or partial parabola, whose axis is in the direction of gravity; and its vertical distance below the line of projection at any given moment, is always equal to the space it would have fallen freely during the time since it was projected.

By the principle of parallelogram of velocities, it is evident that in the time the body would describe the curve *a e f g h*, it would, without the action of gravity, describe the line $a\,b = vt$, v being the velocity of projection, and t the time of flight. Let a be the angle of elevation *b a h*, v' the vertical velocity, and v'' the horizontal velocity, then $v' = v \sin. a$, and $v'' = v \cos. a$. The vertical velocity would evidently be spent in one-half the time of flight, and an equal descending velocity would be acquired at the time of striking the point *h*, hence, $v' = v \sin. a = \frac{1}{2}gt$, and $t = \frac{2v \sin. a}{g}$ = the time of flight. The horizontal range will equal the horizontal velocity v'' multiplied by the time of flight $= v \cos. a \times \frac{2v \sin. a}{g} = \frac{2v^2 \sin. a \cos. a}{g} = \frac{v^2 \sin. 2a}{g}$.

This value of the horizontal range *a h* is evidently the greatest for any value of v, when $\sin. 2a = 1$, or $a = 45°$; and, for elevations equally above and below 45°, the horizontal range will be equally diminished; that is, the horizontal range will be the same for an elevation of 40° as for 50°, and the same for an elevation of 30° as for 60° Fig. 52 shows the form of the curves described by projectiles at

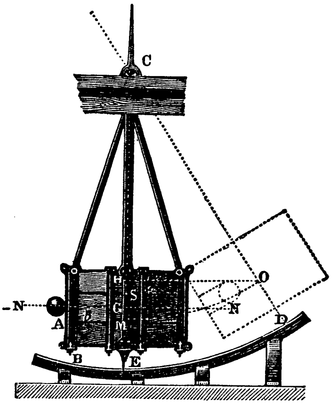
52



angular elevations of 0°, 15°, 45°, 60°, and 90° (A B, A C, A D, A E, A F). The dotted lines show the angles of projection, and the smooth lines, with corresponding letters, show the paths described by the projectiles. The effect upon the flight of projectiles produced by resistance of the air, will be considered hereafter.

104. **The ballistic pendulum** is an instrument employed to measure the velocity of projectiles. A heavy mass of wood and iron, shown at *b*, fig. 53, is suspended at C, on a shaft three or four yards in length over a graduated arc B E D. The ball, fired in the direction N N, strikes the ballistic pendulum at A, and, penetrating the heavy mass, imparts to it a velocity which is determined by comparison of the arc E D described by the pendulum, and the time in which the whole mass is found to vibrate. S is supposed to be the centre of gravity, M the centre of oscillation, C G the arm of impact, and M H the perpendicular height through which the pendulum rises. From these data the velocity of the ball at the moment of impact can be calculated

53



Problems.—Falling Bodies.

17. If a stone is dropped into a well, and it is seen to strike the water at the end of 3 seconds, what is the depth of the well?

18. A body is projected upward with a velocity which will carry it to the height of 64 feet 4 inches; after how long a time will it be descending with half the original velocity?

19. Find the velocity with which a body must be projected upwards from the foot of a tower, so as to meet half way another body let fall at the same time from the top of the tower.

20. A balloon is ascending vertically with a given velocity, and a body is let fall from it, which reaches the ground in t seconds: find the height of the balloon at the moment of the body leaving it.

21. A body is observed to fall the last a feet of its descent from rest in t seconds: find the height from which it fell.

22. A body has fallen through the distance of half a mile; what was the distance described in the last second?

23. A body is projected upwards with a velocity of 64½ feet in a second; how far will it ascend before it begins to return?

24. A stone dropped from a bridge strikes the water in $2\frac{1}{2}$ seconds; what is the height of the bridge? Also if the stone be projected downwards with a velocity of 3 feet per second, in what time will it strike the water?

25. A stone thrown horizontally from the summit of a high cliff is seen to strike the ground at the end of 5 seconds; what is the height of the cliff above the point where the stone falls?

26. A body is projected vertically upwards, and the time between its leaving a given point and returning to it again is given; find the velocity of projection and the whole time of motion.

27. From what elevation must a body weighing 500 pounds fall, to strik with the same momentum as a body weighing 900 pounds falling from an elev tion of $64\frac{1}{3}$ feet?

Descent of Bodies on Inclined Planes.*

28. What time will be required for a body to descend an inclined plane whose length is 200 feet, and whose elevation is $64\frac{1}{3}$ feet.

29. What velocity will be acquired by a body descending a plane inclined at an angle of 30°, the perpendicular height being $145\frac{1}{2}$ feet?

30. If a railway train, with a speed of 30 miles per hour, arrives at a descending grade of 60 feet to the mile, and has no force applied to check its speed, what will be its velocity after running 3 miles on the grade?

31. If a train, moving at the rate of 25 miles an hour, arrives at a grade of 50 feet per mile, 2 miles in length, and no more steam is applied than before arriving at the grade, what will be the velocity of the train after ascending the grade?

Central Forces.

32. Find the force with which a body weighing 8 lbs. would stretch a string, 3 feet long, retaining it in a circle, when the body makes 3 revolutions per second.

33. What must be the weight of a body revolving 7 times per second in a circle 10 feet in diameter, in order that the centrifugal force of the revolving body may be equivalent to a weight of 1000 lbs.?

34. How many times must the revolution of the earth be increased to have the weight of bodies at the equator diminished one-half, calling the radius of the earth 4000 miles?

35. What must be the number of revolutions per second of a body weighing 17 lbs., revolving in a circle whose radius is 5 feet, that its centrifugal force may be the same as that of a body weighing 25 lbs., revolving 9 times per second in a circle whose radius is 3 feet?

Pendulum and Gravity.

36. What is the time of vibration at Paris of a simple pendulum whose length is 3 metres?

37. What is the force of gravity in a deep mine where the length of th seconds pendulum is found to be 38 inches?

38. What is the time of vibration of a simple pendulum 30 inches in length, where the accelerating force of gravity is 32 feet per second?

39. What is the time of vibration of a simple pendulum at Paris, the length of the pendulum being one metre, and the amplitude of vibration being $a = 9^\circ$?

---

* In these problems the retarding force of friction is not to be considered.

40. What is the accelerating force of gravity at New York? at Boston? at New Orleans? at Cape Horn? at Stockholm?

41. If the force of gravity at the earth's surface be regarded as unity, what will be the force of gravity at a distance below the surface equal to one-tenth part of the earth's radius?

Flight of Projectiles.*

42. What distance will a ball be thrown on a horizontal plane, if it is fired from a cannon with a velocity 700 feet per second at an angular elevation of 33°?

43. What is the greatest distance to which a ball can be thrown on a horizontal plane, if it leaves the mouth of the cannon with a velocity of 1000 feet per second?

44. If a ball leaves the cannon at an elevation of 30°, with a velocity of 800 feet per second, in what time will it strike the horizontal plane?

45. At what angle of elevation must a ball be fired that, with an initial velocity of 600 feet per second, it may strike a horizontal plane at a distance of two miles?

46. If a ball discharged from the mouth of a cannon, at an elevation of 35°, strikes the horizontal plane at a distance of three miles, what was its original velocity?

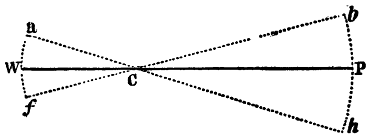
CHAPTER IV.

THEORY OF MACHINERY.

§ 1. Machines.

105. **Principle of virtual velocities.**—It was shown in § 46, that when a body, having a fixed point of support, is acted on by two parallel forces in the same direction, the forces will be in equilibrium, if they are to each other inversely as their distances from the supporting point. Thus in fig. 54, if an inflexible rod, supported at C, is acted on by two forces, W and P, such that

54



$$W : P = CP : CW,$$

then they will be in equilibrium. But as in every proportion the product of the first and last terms is equal to the product of the second and third; so instead of saying that the forces are inversely as their distances, the same thing is expressed by $W \times CW = P \times CP$. The principle may be otherwise illustrated thus:—Let the bar W P be made to oscillate gently about the point of support C. It is plain that the spaces described by the ends of the bar will be proportional to their distances from the axis; for the angles at the axis being equal, the arcs af and bh are directly proportional to their radii C W and C P. Hence

---

* In these problems no account is supposed to be taken of the resistance of the air.

$$W : P = bh : af;$$

That is, two forces are in equilibrium when they are to each other inversely as the spaces which they describe. The arcs being described in the same time, represent the velocities, and the principle is usually thus stated: *forces in equilibrium must be to each other inversely as their velocities.* The products, therefore, of the forces multiplied by their respective velocities, are equal:

$$W \times af = P \times bh.$$

These products are called the moments of the forces, and when these momenta are equal the forces are in equilibrium. If the movement is doubled, halved, or raised in any proportion, the efficacy of the force is similarly varied. Any arrangement by which two forces are brought into this relation to each other, constitutes *a machine.*

106. **Machine, Power, Weight.**—In extension of the statement last made, a machine is any arrangement of parts in an apparatus, by which force may be transmitted from one point to another, usually with some modification of its intensity or direction, and with reference to the performance of *mechanical work.*

The moving force in a machine is called *the power;* the place where it is applied is *the point of application;* and the line in which this point tends to move is *the direction of the power.* The resistance to be overcome is called *the weight,* and the part of the machine immediately applied to the resistance is *the working point.*

The moving powers and the resistances in mechanics are both extremely various; but of whatever kind they may be, they can always be expressed by equivalent weights, *i. e.*, such as being applied to the machine would produce the same effects.

107. **Equilibrium of machines.**—When the power and weight are equal, the machine is in equilibrium, and it may be at rest, or, as is usually the case, in a state of uniform motion. If a machine in this case is put into uniform motion, it must, by force of inertia, continue to move indefinitely; for the power and weight being equal, neither of these forces can stop or modify the motion, without some extraneous force, which is contrary to the supposition.

Thus, if an engine draws a railway train with uniform velocity, the power of the engine is in equilibrium with the resistance of the train. At starting the power is greater than the resistance, and the motion of the train is consequently accelerated, until the resistance becomes equal to the power, when equilibrium is again established. If any part of the power is now withdrawn, the power becomes less than the resistance, and the motion is consequently retarded until the train is brought to rest.

The mechanical energy, or moving force of the power, is found by multiplying its equivalent weight by the space through which it moves, or its velocity; and the value of the resistance is estimated in the same manner. As we have just seen, the relation between these moments determines the state of the machine.

108. **Utility of machines.**—It is sometimes said, in illustration of the usefulness of machines, that a great weight may be supported or raised by an insignificant power; but such statements, if literally understood, are obviously untrue. No machine, however ingenious its construction, can create any force, and therefore the working point can exert no more force than is transmitted to it from the source of power. Every machine has certain fixed points, which are arranged to support any required part of the weight, while the remainder of the weight, and that part only, is directly sustained by the power. This remainder cannot be greater than the power.

109. **Relation of power to weight.**—But if the weight is not only supported, but raised through a given space, then the power must move through a space as much greater than the weight moves through, as the weight itself is greater than the power; in other words, the power and weight must be inversely as their velocities. This inverse proportion is expressed when it is said, that power is always gained at the expense of time.

To raise 1000 lbs. to a height of one foot by a single effort, would require a force equivalent to 1000 lbs.; but the same thing may be accomplished by a power of 1 lb. acting for 1000 times successively, through a space of one foot. If a man by exerting his entire strength could lift 200 lbs. to a certain height, in one minute, no machine whatever can enable him to lift 2000 lbs. to the same height in the same time. He may divide the weight into ten parts, and lift each part separately; or by the intervention of a machine he may raise the whole mass together, requiring, however, ten minutes for the task.

On the other hand, it is often the object of a machine to move a small resistance by a great power.

In a watch, the moving force of the mainspring is very much greater than the resistance of the hands, revolving about the dial. In a locomotive engine, each full stroke of the piston moves the train through a space equal to the circumference of the driving wheel; if the length of stroke is one foot, and the circumference of the wheel 12 feet, then the velocity of the piston will be to the velocity of the train, as 2 to 12; consequently the power acting on the piston is greater than the resistance of the train, in the proportion of 12 to 2.

110. **Adaptation of the power to the weight in machinery.**—The use of machines is to adapt the power to the weight. If the intensity, direction, and velocity of the power, were the same as the intensity and direction of the resistance, and the velocity required to be given to it, then the power might be directly applied to the resistance, without the intervention of a machine. But if a small power is required to move a great resistance; or, if a power acting in one direction, is required to impart motion in another; or, to impart a velocity greater or less than its own, then it is necessary to employ a machine which will modify the effect of the power in the required manner. Besides these, the motion of the power may differ from the motion required in the resistance in a great variety of ways.

The power may have a reciprocating motion, as in the locomotive engine, and be required to produce a continuous motion in a straight line, as in moving a train upon a railway. Or, the power may have a rectilinear motion, as a stream, and be employed to produce the circular motion of the stones in a grist-mill, or the reciprocating motion of a saw, in a saw-mill.

In every class of machines, the relations existing between the power and the resistance, depend solely on the construction of the machine; but even a general account of the ingenious contrivances by which the moving force is regulated, modified, and adapted to the varying conditions and requirements of the resistance, would lead us far beyond the limits and design of this work.

111. **Vis viva, or living force,** is the power of a moving body to overcome resistance, or the measure of work which can be performed before the body is brought to a state of rest. The vis viva of a body is represented by MV^2, or the mass of the body multiplied by the square of its velocity.

When a body is projected vertically upwards, the height to which it will ascend is proportional to the square of its velocity. If W represent the weight of the body, and h the height to which it is elevated by a given impulse, the amount of work performed will be represented by Wh, but $W = Mg$ and $h = \frac{V^2}{2g}$, substituting these values of W and h, we have the work performed $= \frac{1}{2} MV^2$. Hence the work which can be performed by the accumulated power of a moving body is equal to one-half the mass multiplied by the square of the velocity.

Take the case of a pile-driver, in which a heavy mass of iron is elevated to a height of 30 or 40 feet, and is then suddenly allowed to fall; the resistance overcome in raising the driver is exactly proportional to the elevation to which it is raised, and the accumulated power of the stroke increases in the same ratio; hence it is evident that the vis viva, or power of overcoming resistance, must be truly represented by MV^2.

Again, in the case of a railway train moving with a velocity V, the greatest velocity attainable by a given power of steam; let v be the acceleration of velocity imparted to the train by the locomotive during the first second of its action, and M the mass of the moving train, including the locomotive. If the movement of the train were not retarded by friction, or some other opposing force, we should have $V = vt$, or the velocity, V, would go on constantly increasing; but such we know is not the case, for the train soon attains a maximum velocity, when the entire force of the locomotive is every instant expended in overcoming friction, and the train moves on with a momentum expressed by MV, but its vis viva is expressed by MV^2. If the force of steam were suddenly discontinued, the power of the moving train to ascend a grade, to overcome any obstacle, or to deal destruction to itself, or to any object with which it comes in collision, would still be proportional to *vis viva* or MV^2. Now suppose the velocity of the train to be doubled, so that $V' = 2V$. It is evident that in any given interval of time the train will pass over twice as many points of resistance as before, and as it passes each point at twice the previous velocity, it will encounter at every point twice as much resistance to motion as before. Hence to impart to the train a double velocity, a fourfold force is required; and the power of the train to overcome resistance will be proportional to its vis viva, MV'^2. This will be the true measure of the force which has imparted the velocity V', and which is now constantly expended in overcoming the resistance encountered by the moving train. The same principles determine the power expended, or work actually performed (resistance included), by any kind of machinery.

It may be necessary to explain more fully the distinction between momentum and vis viva, so that it may be readily understood when the one or the other is to be taken as the measure of force.

Momentum, MV, expresses the relation of force to inertia, or the amount of motion in a moving body. Vis viva, MV^2, is the measure of twice the amount of work which a moving body can perform before it is brought to rest. Vis viva is the measure of force required to maintain a constant motion, MV, against the resistance caused by the positive properties of bodies, as attraction, cohesion, repulsion. Momentum is the measure of the force required, without regard to time, to set a body in motion with a velocity V, when no other body interferes with its motion, as in the case of a body falling freely in a vacuum. In the case of the railway train, the mass of the train multiplied by its velocity is the measure of *useful* work performed in a unit of time, but it is not the measure of resistance overcome, or *actual* work performed, or of the force which has been expended in performing that work. The latter is measured by one-half the vis viva, or $\frac{1}{2} MV^2$.

Illustrations of vis viva.—Suppose a battering-ram weighing 4000 lbs. to be impelled with a velocity of 30 feet per second, its vis viva, $MV^2 = 4000 \times 30 \times 30 = 3{,}600{,}000$; yet a cannon ball weighing 64 lbs., flying with a velocity of 1000 feet per second, will have a power of dealing destruction more than seventeen times as great, for its vis viva equals 64,000,000. Calculations of this sort explain the origin of the terribly destructive power of the engines of modern warfare.

A railway train moving 50 miles an hour will possess more than six times the vis viva that it would have when going twenty miles an hour; and, therefore, it will possess more than six times the power of dealing destruction, either to itself or to an obstacle, at the former than at the latter rate. Thus the well known relation between speed and amount of damage, in case of accident, is readily accounted for, as also the enormous comparative cost of fuel, and wear and tear of trains of high speed.

The destructive power of hurricanes, which move from 60 to 100 miles an hour, is readily understood when we know that the power of dealing destruction increases in proportion to the square of the velocity.

112. **Impact and its results.**—When a body in motion encounters another, the velocity and momentum of both undergo certain changes, which depend on the elasticity of the bodies, and other physical circumstances.

Impact considered with reference to momentum.—*a*—When a body in motion strikes another at rest, it can continue to move only by pushing this body before it, and it must impart so much momentum that, after impact, both may move with a common velocity. If the masses of the two bodies are equal, it is evident that, after impact, the momentum will be equally divided between them, and their velocity will be one-half of the velocity of the moving body before collision. If the mass at rest is double the mass in motion, the common velocity will be one-third; and generally, when a moving body communicates motion to a body at rest, the velocity of the two united will be to that of the moving body as the mass of the latter is to the sum of the masses of both.

If a musket ball, whose weight is $\frac{1}{20}$ lb., and its velocity 1300 feet a second, strikes a suspended cannon ball weighing 48 lbs., it will put it in motion, and their common velocity will be to that of the bullet as $\frac{1}{20}$ is to $48 + \frac{1}{20}$, or as 1 is to 961; the velocity of the two is therefore $\frac{1300}{961}$, or about $1\frac{1}{3}$ feet a second.

b—Bodies moving in the same direction may impinge, if their velocities are different. If an inelastic body overtakes another, the first will accelerate the second, and the second will retard the first, until they have acquired a common velocity, when they will move on together

Since the bodies move in the same direction, there can be no increase or diminution of the total momentum by impact, but only a re-distribution. If they are equal in mass, their velocity, after impact, will be half the sum of their previous velocities.

If before impact, A had a velocity of 6, and B a velocity of 4, then their common velocity will be 5.

The two bodies may have unequal masses as well as velocities.

If the mass of A is 8, and its velocity 17, its momentum will be 136. If B has a mass of 6, and velocity of 10, its momentum will be 60. The sum 196 is the total momentum of the united masses after impact; and this sum divided by the sum of the masses gives 14, the common velocity.

c—If two equal bodies, moving with equal velocities in opposite directions, impinge on each other, their moments being equal, will be mutually destroyed, and the bodies will remain at rest. The force of the shock, in this case, is equal to that which either would sustain, if, while at rest, it were struck by the other with a double velocity. If the moments of the bodies are unequal, then, after impact, they will move together in the direction of the greater, and their joint momentum will be equal to the difference of their previous moments, and their velocity will be found by dividing that difference by the sum of the masses.

d—These laws may be shown experimentally by suspending two balls at the centre of a graduated arc, and producing impact according to the conditions described.

If two bodies moving in different lines impinge on each other, then, after contact, they will move together in the diagonal of that parallelogram whose sides represent their previous moments and directions.

From these principles it follows that, if two inelastic bodies, M and N, moving in the same direction, with velocities V and V', come in contact, their common velocity after impact will be expressed by the formula $V'' = \frac{MV + NV'}{M + N}$. When the bodies move in opposite directions, the velocity of the body having the greater momentum is to be taken as positive, and the other negative; the resultant velocity will be in the direction of the body which previously had the greater momentum.

Impact considered with reference to vis viva.—When a body in motion strikes another body at rest, which is free to move, the two bodies have a common velocity, $V'' = \frac{MV}{M + N}$. The vis viva after impact will be expressed by $(M + N)\ V''^2 = \frac{M^2 V^2}{M + N}$. Suppose the second body N to be a certain number of times, (represented by a,) greater than the first body M, then $N = a\,M$. The vis viva of the combined mass after impact will then become $\frac{M^2V^2}{M+N} = \frac{M^2V^2}{M(1+a)} = \frac{MV^2}{1+a}$. Hence:—

When a moving body strikes a body at rest, and the two move on together, the vis viva of the combined mass is as many times less than the vis viva of the first body before impact as the combined mass is greater than the first.

This principle shows how a man may receive the most violent strokes of a sledge-hammer, upon an anvil laid upon his chest, without the slightest injury, when, if only a light board were interposed between his person and the descending hammer, the stroke would be instantly fatal to life. The interposition of any heavy body wards off the force of a blow on the same principles.

Pressure produced by impact.—Beaufoy determined that a body of 1 lb. weight, with a velocity of 1 foot in a second, strikes with a pressure equal to 0·5003 lb. To find the pressure produced by the impact of any projectile, we have the general formula,

$$\text{Pressure} = 0{\cdot}5003\,MV^2.$$

If the body descends vertically, the weight of the body itself must of course be added to the direct effects of impact.

Destructive effects of impact.—The motion communicated to very large or immovable bodies, by an impact of small ones, is not lost, but becomes insensible from its enormous diffusion. Motion can be destroyed only by motion; friction and resistance disperse, but do not destroy it. An impact can act directly upon only a few of the molecules of the body to which it imparts motion.

The power which projects a bullet acts on only one-half its surface.

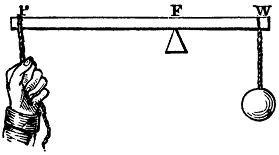
The motion must, therefore, be diffused from the parts struck to all the other parts of the body, before it can begin to move; and this diffusion requires *time*, which may be short indeed, but is not infinitely so. It happens, therefore, that a movable body, if struck by another moving with great velocity, may be penetrated or broken at the point of impact, without being itself put in motion. The part of the body which receives the blow is set in motion with such velocity that its particles are rent asunder before motion can be communicated to the mass of the body. Such effects appear incredible to persons unacquainted with the inertia of matter and its consequences.

A rifle ball may be fired through a pane of glass suspended by a thread, without shattering the glass, or even causing it to vibrate. A door half open may be perforated by a cannon ball without being shut by it. A soft missile, like tallow, or a light one, like a feather, will act with the force of lead, if sufficient velocity is given to it. Firing a tallow candle through a board is a well-known feat of showmen. In *ricochet* firing, a cannon ball, shot at an elevation of from 3° to 6°, rebounds from the surface of water, just as every boy has made flat stones skip from point to point on its surface.

§ 2. Mechanical Powers.

113. **The lever.**—A lever is any inflexible rod, fig. 55, resting on a point, F, called the *fulcrum*, and around which any two forces tend to turn it. Levers may be either straight, or bent; simple, or compound. It is usual to divide levers into three classes, according to the position of the fulcrum in relation to the power and weight.

55



56

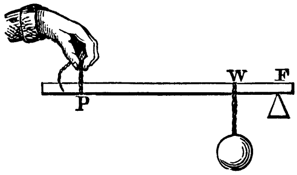
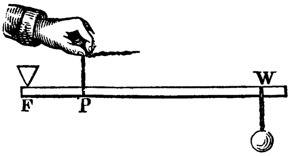


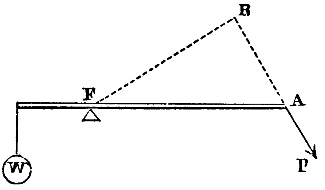
Fig. 55 is a lever of the *first class*, where the fulcrum is between the power and the weight. In the second class, fig. 56, the weight is between the fulcrum and the power. In the *third class*, fig. 57, the power is applied between the fulcrum and the weight.

57



The arms of a lever are the lines on each side of the fulcrum, at right angles to the direction of the power and weight. In the three figures just given, the levers being horizontal and the forces vertical, the arms of the lever are evidently, in each case, the portions into which it is divided. If, however, the lever is bent or is inclined to the direction of either, or both, of the forces, then the arms are the perpendiculars between the fulcrum and directions of the forces. Thus in fig. 58 the power acting in the direction B P, the moment of the power is not expressed by P × A F, but by P × B F. The distance from the fulcrum is called *the leverage.*

58



114. **Conditions of equilibrium in the lever.**—These conditions are: 1st, The lines of direction of the two forces must be in the same plane with the fulcrum; 2d, The two forces must tend to turn the lever in opposite directions; 3d, Whatever may be the class of the lever the weight and power will be in equilibrium when they are inversely as their distances from the fulcrum. Thus in either of the three figures above

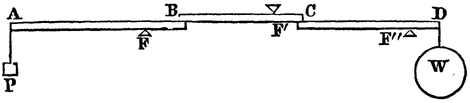
$$P : W = F W : F P \quad \text{or} \quad P \times F P = W \times F W.$$

Consequently the moment of the power, or its tendency to turn the lever will be augmented, either by increasing the power itself or its distance from the fulcrum.

The pressure on the fulcrum, when the power and weight are in equilibrium, is found by applying the principle of the composition of forces (46). In a lever of the first class, the resultant of the power and weight is a single force, equal to their sum, and passing through the fulcrum; consequently, the pressure will be equal to the sum of the power and weight. In a lever of the second or third class the resultant is equal to the difference of the power and the weight.

Compound levers.—When a small force is required to sustain a considerable weight, and it is not convenient to use a very long lever, a combination of levers, or a compound lever is employed. When such a system is in equilibrium, the power, multiplied by the continued product of the alternate arms of the levers, commencing from the power, is equal to the weight multiplied by the continued product of the alternate arms, commencing from the weight. For example, the system represented in fig. 59, consisting of three levers of the first class, will be

59



in equilibrium when

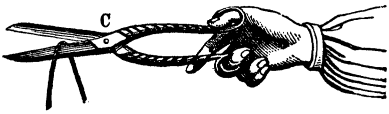
$$P \times A F \times B F' \times C F'' = W \times D F'' \times C F' \times B F.$$

If the long arms are 6, 4, and 5 feet, and each of the short arms 1 foot, then 1 lb. at A will sustain 120 lbs. at D; but if a simple lever had been used, the long arm being increased simply by adding these quantities, we should have gained a power of only $6 + 4 + 5 = 15$ to 1.

115. **Application of the lever.**—Machines and utensils in daily use offer us familiar examples of the three classes of levers.

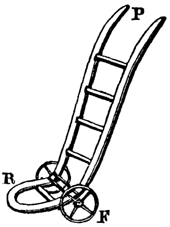
Of the *first class* we name the crowbar and poker, when used to raise the load on their points. Scissors, snuffers, and pincers are pairs of levers of this class, the point C, fig. 60, which connects them being the fulcrum. The power is applied at the handles, and the resistance is the object between the blades.

60

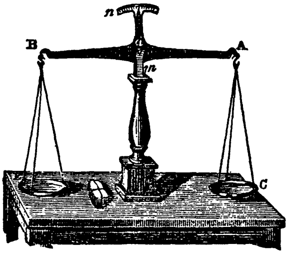


Another example of the bent lever is seen in the ordinary truck, fig. 61, used for moving heavy goods a short distance. In this machine, the axis of the wheels, F, is the fulcrum, against which the foot is placed, while the weight at R is raised off the ground by the hand, applied at P.

61



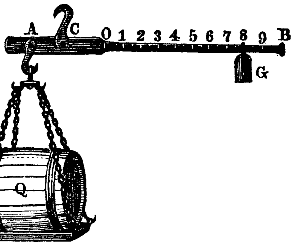
62



The scale beam, or balance, is one of the most useful applications of the first class of levers. The beam is a lever poised at its centre on a knife-edge of steel, *a*, fig. 62. From its ends A B are suspended the scale pans C E. The centre of gravity, *m*, is placed below the fulcrum, *a*, to secure a horizontal position of the beam when in equilibrium. If it coincided with the fulcrum the balance would rest equally well in all positions, and if it were above the fulcrum the beam would be upset by a slight disturbance.

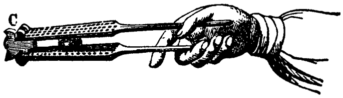
The steelyard is a lever of the first class, with unequal arms. The mass, Q, to be weighed, is attached to the short arm, A, fig. 63, and it is counterpoised by a constant weight, G, shifted upon the longer arm, marked with notches to indicate pounds and ounces, until equilibrium is obtained. It is evident that a pound weight at G will balance as many pounds at Q as the distance G C is greater than A C.

63



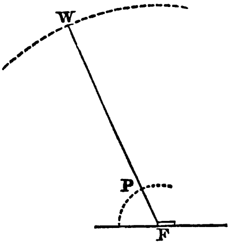
Levers of the second class occur less frequently. A pair of nutcrackers, with the fulcrum at the joint C, fig. 64, is a double lever of this class. An oar is another example; the water is the fulcrum, the boat is the weight, and the hand the power. A door moving on its hinges, and a wheelbarrow, are other examples of levers of the second class.

64



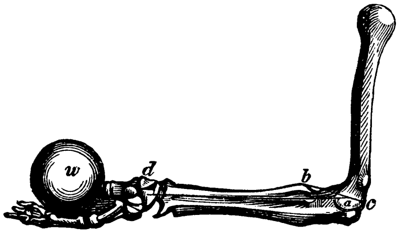
In levers of the third class, the power being nearer the fulcrum, is always greater than the weight. On account of this mechanical disadvantage, it is used only when considerable velocity is required, or the resistance is small. Fig. 65 represents such a lever, W F, moving on a hinge as a fulcrum; it is plain that the power P moves through a small arc, and the weight through a large one, and since they are described in the same time, the velocity of the power is less than that of the weight.

65



The common fire-tongs, sugar-tongs, and sheep-shears, are double levers of this class. The most striking illustrations of this class of levers are seen in the animal kingdom. The compact form and beautiful symmetry of animals depend on the fact that their limbs are such levers. The socket of the bone, *a*, fig. 66, is the fulcrum; a strong muscle, *b c*, attached near the socket is the power, and the weight of the limb and whatever resistance *w* may oppose to motion, is the weight. The fore-arm and hand are raised through a space of one foot, by the contraction of a muscle applied near the elbow, moving through less than $\frac{1}{12}$ that space. The muscle, therefore, exerts 12 times the force with which the hand moves. The muscular system is the exact inversion of the system of rigging a ship. The yards are moved through small spaces with great force, by hauling in a great length of rope with small force; but the limbs are moved through great spaces with comparatively little force, by the contraction of muscles through small spaces with very great force.

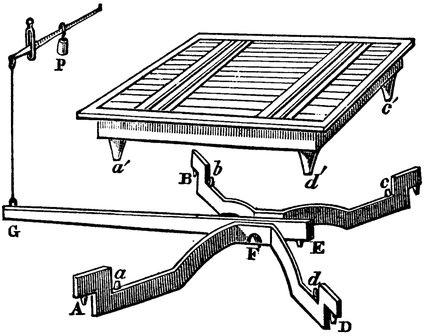
66



Examples of compound levers are familiar in the various platform scales, such as Fairbanks' and others. However various in form, they all depend upon the principles already explained.

The principle of the construction of the *weighing machine* is illustrated in fig. 67 It consists of a wooden platform, placed over a pit made in a location convenient for driving heavy loads upon it, and is so arranged as to move freely up and down without touching the walls of the pit. The platform rests upon four levers, A F, B F, C F, and D F, all converging toward the centre F, and each moving on a fulcrum at A, B, C, D, securely fixed in each corner of the pit. The platform rests on its feet, *a′ c′ d′*, which rest on steel points, *a b c d*. The four levers are supported at the point F, under the centre of the platform, by a long lever, G E, resting on a steel fulcrum at E, while its longer arm at G is connected with a rod, which is carried up and attached to the shorter arm of the steelyard, and is counterpoised by the weight P, which, by its position on the longer arm, indicates at once the weight of the load placed upon the platform.

67



As the four levers A, B, C, D, are perfectly equal and similar, and all act upon the same fulcrum F, the effect of the weight placed upon any part of the platform is the same as if it were concentrated at either of the points *a*, *b*, *c*, *d*.

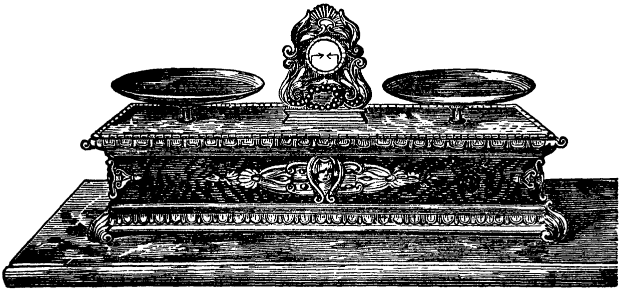
In order therefore to ascertain the conditions of equilibrium, we need only consider one of these levers, as A F. Suppose the distance from A to F to be 10 times as great as from A to *a*, a force of 1 lb. at F would balance 10 lbs. at *a*, or on any part of the platform. So, also, if the distance from E to G be 10 times greater than the distance from the fulcrum E to F, a force of 1 lb., applied so as to raise up the end of the lever G, would counterpoise a weight of 10 lbs. on F, therefore, 1 lb. tending to raise G, would balance 100 lbs. on the platform. If the poise, P, is placed 5 times as far from the fulcrum of the steelyard as the attachment of the rod connected with G, then 2 lbs. at P will balance 10 lbs. at G, or 100 lbs. at F, or 1000 lbs. on the platform. If the weight of P and the graduation of the steelyard are arranged on these principles, the weight of the heaviest loads on the platform may be determined with great facility.

Weigh-locks on canals, and many other applications of the compound lever, are arranged on the same principles.

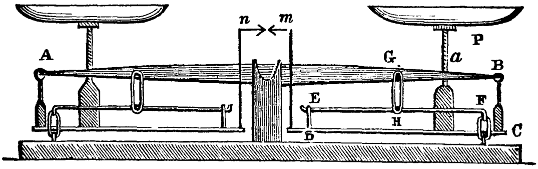
Roberval's counter platform balance.—The exterior appearance of this balance is shown in fig. 68, and its interior arrangement in fig. 69. The equilibrium of this system of levers is, like that of fig. 67, independent of the position of the load on the pans, and the mechanism is such that the pans move on a vertical stem with no deflection from a horizontal plane.

Each pan is supported by a system of three levers, A B, C D, E F, fig. 69. The lever D C, which supports the pan P, rises and falls equally at both ends with the motions of the beam A B, C being attached to the end of the beam B, and D being attached at E to the lever E F; E H being equal to G B, while F is securely attached to the base, E and B rise and fall equally, and D C is always

68



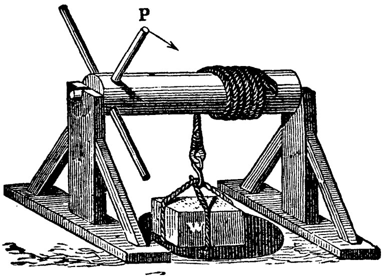
69



horizontal in every position of the beam A B. Since the lever D C preserves its horizontal position, the stem *a* supporting the pan P, moves vertically, whatever may be the position of the load on the pan. The indices *n m* are in the same horizontal line when the pans are in equilibrium. This system is named from the inventor, Mr. Roberval of Paris, and dates from about A. D. 1660.

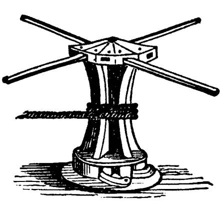
116. **The wheel and axle.**—The common lever is chiefly employed to raise weights through small spaces, by a succession of short intermitting efforts. After the weight has been raised it must be supported in its new position, until the lever can be again adjusted, to repeat the action. The wheel and axle is a modification of the lever, which corrects this defect; and, since it converts the intermitting action of the lever into a continuous motion, it is sometimes called the perpetual lever.

70

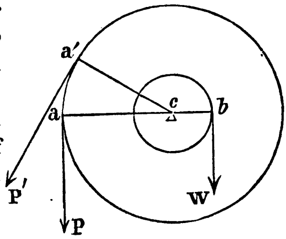


This machine consists of a cylinder called the axle, turning on a centre, and connected with a wheel of much greater diameter. The power is applied to the circumference of the wheel, and the weight is attached to a rope, wound around the axle in a contrary direction. Instead of the whole wheel, the power may be applied to a handle named a winch, or to one or more spokes inserted in the axle. When the axle is horizontal the machine is called a windlass, fig. 70, when it is vertical it forms the capstan, fig. 71, used on shipboard, chiefly to raise the anchor. The head of the capstan is pierced with holes, in each of which a lever can be placed, so that many men can work at the same time, exerting a great force, as is often necessary in raising an anchor, while the recoil of the weight is arrested by a catch at the bottom.

71



72



The law of equilibrium is the same as in the lever. Draw from the centre, or fulcrum *c*, fig. 72, the straight lines *c b* and *c a*, or *c a′*, to the points on which the weight and power act; *a c b*, or *a′ c b*, is evidently a lever of the first class, in which the short arm *c b* is the radius of the axle, and *c a* or *c a′*, the long arm, is the radius of the wheel. Hence,

$$P \times ac = W \times cb.$$

Or,

$$P : W = cb : ac.$$

That is to say, the wheel and axle are in equilibrium, when the power is to the weight as the radius of the axle is to the radius of the wheel.

In one revolution of the machine, the power moves through a space equal to the circumference of the wheel, and the weight moves through a space equal to the circumference of the axle; hence the power and weight are inversely as their velocities, or the spaces they describe.

117. **Trains of wheel-work.**—The efficiency of this machine is augmented by diminishing the thickness of the axle, or by increasing the diameter of the wheel. But if a very great power is required, either the axle would become too small to sustain the weight, or the wheel must be made inconveniently large. In this case a combination of wheels and axles may be employed. Such a system corresponds to the compound lever, and has the same law of equilibrium. The power

being applied to the first wheel transmits its effect to the first axle, this acts on the second wheel, which transfers the effect to the second axle, &c., until the force, transmitted through the series in this order, arrives at the last axle, where it encounters the resistance. In equilibrium, the power multiplied into the continued product of the radii of all the wheels, is equal to the weight multiplied into the continued product of all the axles.

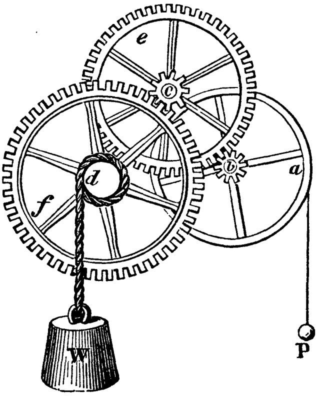
Trains of wheel-work are connected by an endless band, or by cogs raised on the surfaces of the wheels and axles. Cogs on the wheel are called teeth, and those on the axle are called leaves; the axle itself is named a shaft. The number of teeth on the wheels, and leaves on the pinions is proportional to their circumferences, and also to their radii. Hence, the number of teeth and leaves is substituted for the radii of the wheels and axles, and the law of equilibrium is stated as follows:

The power multiplied into the product of the number of teeth of all the wheels, is equal to the weight multiplied into the product of the number of leaves in all the pinions.

Analysis of a train of wheel-work.—A system of wheels is represented in fig. 73. If the number of leaves in *b*, the pinion of the first wheel, is one-sixth of the number of teeth on the second wheel, *e*, the wheel will be turned once by every six turns of the pinion. Let the second pinion, *c*, have the same relation to the third wheel, *f*; then the first wheel will revolve 36 times while the third revolves once; and the radius of *a*, the wheel to which the power is applied, being 3 times the radius of *d*, the axle which ssutains the weight, the velocity of the power is 3 × 36 = 108 times the velocity of the weight. Or,

$$P : W = 1 : 108.$$

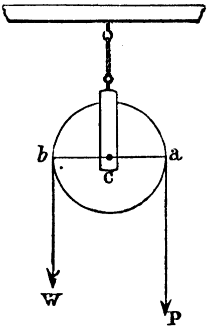
73



Combinations of wheel-work are employed either to concentrate or to diffuse force; either to set heavy loads in motion by means of a small power, or to produce a high velocity by exerting a considerable power. In the first case, the power is applied to the first wheel of the series, and is transmitted in the order already described. In the second instance, this arrangement must be reversed; the power must exert itself on the shaft, *d*, in order to produce rapid revolution of the last wheel. The crane for hoisting goods is an example of the first kind the watch is an instance of the second.

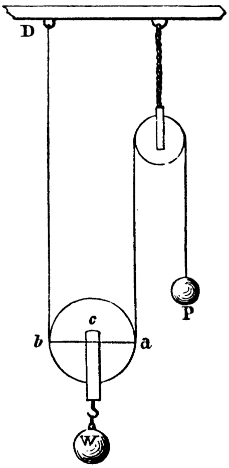
118. **The pulley.—Fixed pulley.**—The usual form of this machine is a small wheel, turning on its axis, and having a groove on its edge, to admit a flexible rope or chain. In the simple fixed pulley, fig. 74, there is no mechanical advantage, except that which may arise from changing the direction of the power. Whatever force is exerted at P, is transmitted, without increase or diminution (except from friction and the rigidity of the rope), to the resistance at the other end of the cord. From the axis C, draw C *a* and C *b*, radii of the wheel, at right angles to the direction of the forces; *a* C *b* represents a lever of the first class, with equal arms; hence, in equilibrium, the power and weight must be equal, and they describe equal spaces.

74



119. **Movable pulley.**—When the block or frame is not fixed, the pulley is said to be movable. The weight is suspended from the axis of the movable pulley, and the cord is fastened at one end, and passing over a fixed pulley, is acted on by the power at the other. In this arrangement, fig. 75, it is plain that the weight is supported equally by the power and the beam at D. For the pulley acts as a lever of the second class, whose arms are to each other as 1 : 2; the fulcrum is at *b*, *b c* is the leverage of the weight, and *b a* the leverage of the power. The diameter *b a* is twice the radius *b c*, therefore equilibrium will obtain when the power is equal to one-half of the weight: *i. e.*,

75



$$P : W = b\,c : b\,a = 1 : 2,$$

therefore,

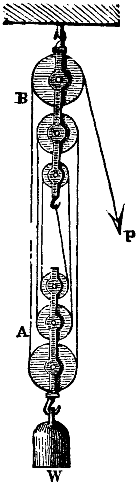
$$P = \frac{W}{2}.$$

To raise the weight one foot, each side of the cord must be shortened one foot, and the power, consequently, passes over two feet. The space traversed by the power is twice the space described by the weight.

120. **Compound pulleys.**—Sometimes compound pulleys are used, each consisting of a block which contains two or more single pulleys, generally placed side by side, in separate mortices of the block. Such an arrangement is shown in fig. 76. The weight is attached to the

movable block, and the fixed one only serves to give the power the required direction. It is easily seen that the power required at P is just the same as would be required at any point between A and B. The weight is divided equally among the pulleys of the movable block, and, of course, among the cords passing around them; and as the power required to sustain a given weight is diminished one-half by a single movable pulley, it follows that such a system will be in equilibrium when the power is equal to the weight divided by the number of cords, or by twice the number of movable pulleys.

76



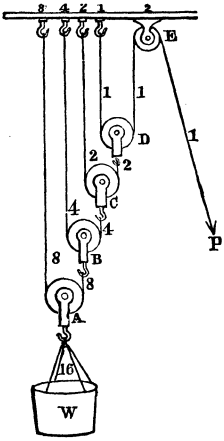
$$P : W = 1 : 2n, \quad \text{or,} \quad P = \frac{W}{2n}.$$

In this system, as in the single movable pulley, the space through which the weight is raised, is as much less than the space through which the power descends, as the weight is greater than the power.

$$P : W = \text{velocity of weight} : \text{velocity of power}.$$

If the power is moved through 6 feet, fig. 76, each division of the cord in which the movable block hangs will be shortened one foot, and the weight raised one foot.

77

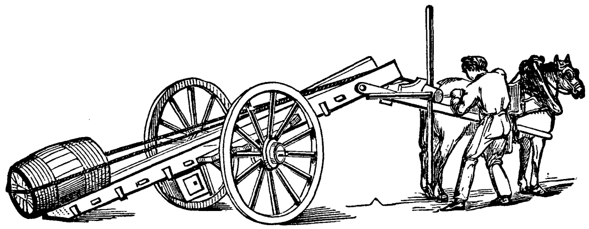


Another system of pulleys is represented in fig. 77. In this arrangement each pulley hangs by a separate cord, one end of which is attached to a fixed support, and the other to the adjacent pulley. The effect of the power is rapidly augmented, being doubled by each movable pulley added to the system. The numbers placed near the cords show what part of the weight is sustained by each, and by each pulley. Such a system, however, is of little practical use, on account of its limited range. In the common block system, fig. 76 (in practice the pulleys or sheaves of each block are placed side by side, to save room, here they are separated for sake of clearness), the motion may continue until the movable block touches the fixed one; but in this only till D and E come together, at which time A will have been raised only $\frac{1}{8}$ of that distance.

$$P : W = 1 : 2^n \quad \text{or} \quad P = \frac{W}{2^n}.$$

121. **The inclined plane.**—This mechanical power is commonly used, whenever heavy loads, especially such as may be rolled, are to be raised a moderate height. In this way casks are moved in and out of cellars, and loaded upon carts. The common dray is itself an inclined plane (as is clearly seen by inspecting fig. 78). Suppose a cask weighing 500 lbs. is to be raised 4 feet by means of a plank 12 feet long; it is plain, that while the cask ascends only four feet, the power must exert itself through 12 feet, and hence, $12 : 4 = 500 : 166\frac{2}{3}$, the force necessary to roll the cask.

78



In mechanics, the inclined plane is a hard, smooth surface, inclined obliquely to the resistance. The length of the plane is R S, fig. 79, S T its height, and R T its base. The power may be applied,

a—In a direction parallel to the length;

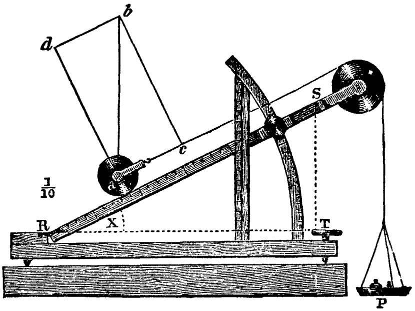
b—Or parallel to the base;

c—Or in any other direction.

In each case the conditions of equilibrium may be derived from those of the lever.

122. **Application of the power parallel to the length of the inclined plane.**—When a body is placed upon an inclined plane, fig 79, its weight, which is the resistance to be overcome, acts in the direction of the force of gravity, namely in the perpendicular *b a*. Let the power, P, act, by means of the cord, in the direction *a c*, parallel to the inclined plane R S, then from the point *a* draw *a d* at right angles with the inclined plane, and complete the parallelogram *a c b d*. The force of gravity will be resolved into two

79



other forces; one represented by bc causing pressure on the inclined plane; the other, represented by ca, tending to cause motion down the inclined plane. This latter force is to be balanced by the power applied to move the body. The body will therefore be sustained when

$$P : W = ac : ab;$$

and since the triangles abc and R S T are similar,

$$P : W = \mathrm{ST} : \mathrm{RS};$$

Or

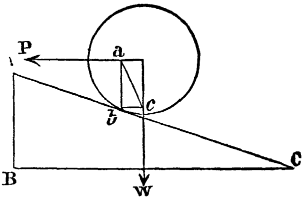
$$P = W \times \frac{\mathrm{ST}}{\mathrm{RS}}.$$

This may be illustrated by an apparatus constructed like that shown in the figure.

If the direction of the power is parallel to the inclined plane, equilibrium will obtain, when the power is to the weight as the height of the plane is to its length. While the weight is raised through a space equal to the vertical height of the plane, the power must move through a space equal to its length. If the length of a plane is 10 feet, and its height 2 feet, P must move 10 feet, while W is raised 2 feet; hence the power and weight are inversely as their velocities.

123. **Application of the power parallel to the base of the inclined plane.**—In the second case, let the power act in the direction of a P, fig. 80, parallel to B C, the base of the plane; and draw, the lines ba and bc perpendicular to the direction of the power and weight: then $a\ b\ c$ is a bent lever, having its fulcrum at b, and equilibrium will take place when

80



$$P : W = bc : ab;$$

and, the triangles abc and A B C being similar,

$$P : W = \mathrm{AB} : \mathrm{BC};$$

Or

$$P = W \times \frac{\mathrm{AB}}{\mathrm{BC}}.$$

If the direction of the power is parallel to the base of the plane, equilibrium will obtain when the power is to the weight as the height of the plane is to its base.

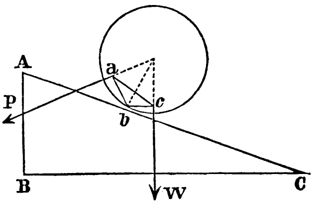
In this case the space described by the power is to the space described by the weight as the base of the plane is to its height.

124. **Application of the power in some direction not parallel to any side of the plane.**—Lastly, let the power act in some direction not parallel to any side of the plane; for example: in the direction *a* P, fig. 81, draw the lines *b c* and *b a* perpendicular to the directions of the two forces; then, as before,

$$P : W = bc : ab.$$

But as the triangles *a b c* and A B C are not similar, the proportion between the arms of the lever cannot be expressed by the sides of the plane.

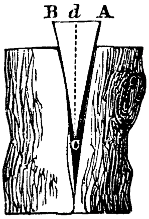
81



It follows from what has been said, that the effect of a given power is greater, as the height of the plane is diminished or its length increased; and that the effect is greatest when its direction is parallel to the length of the plane, for, if the power acts in any other direction, a part of its force is expended, either in increasing the pressure of the body on the plane, or in lifting the weight directly.

125. **The wedge.**—Instead of lifting a load by moving it along an inclined plane, the same result may be obtained by moving the plane under the load. When used in this manner, the inclined plane is called a wedge. It is customary, however, to join two planes base to base. In fig. 82, A B is called the back of the wedge, A C and B C its sides, and *d* C its length. The power is applied to the back of the wedge, so as to drive it between two bodies, and overcome their resistance.

82



The resistance may act at right angles to the length or to the sides of the wedge. In the first case it resembles an inclined plane, when the power is parallel to the base; and hence the forces will be in equilibrium when the power is to the resistance as the back of the wedge is to its length. In the second case it is similar to a plane when the power is parallel to the length; and therefore in equilibrium when the power is to the resistance as the back of the wedge to its side.

The power is supposed to move through a space equal to the length of the wedge, while the resistance yields to the extent of its breadth.

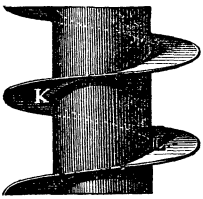
126. **Application of the wedge.**—As a mechanical power, the wedge is used only where great force is to be exerted in a limited space. In oil-mills, the seeds from which vegetable oils are obtained are sometimes compressed with enormous force by means of a wedge. It is everywhere employed to split masses of stone and timber. The edges of all cutting tools, as saws, knives, chisels, razors, shears, &c., and the points of piercing instruments, as awls, nails, pins, needles, &c., are modified wedges. Chisels intended to cut wood, have their edge at an angle of about 30°; for cutting brass, from 50° to 60°; and for iron, about 80° to 90° The softer or more yielding the substance to be divided

the more acute the wedge may be constructed. In general, tools which are urged by pressure, admit of being sharper than those which are driven by a blow.

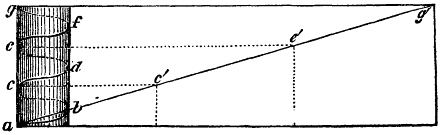
The theory of the wedge gives but very little aid in estimating its effects, as it takes no account of friction, which so largely modifies the results, and the proportion between pressure and a blow cannot be defined.

127. **The screw.**—This machine has the same relation to the ordinary inclined plane that a spiral staircase has to a straight one. This relation is shown in fig. 83, the dotted line K L marking the continuation of the spiral. The position of the different parts of an inclined plane upon a screw is shown in fig. 84. *a b c d e f g*, is the spiral course of the inclined plane upon the screw, and *a c′ e′ g′* are points in the straight inclined plane corresponding to similar letters on the threads of the screw, as would be seen by winding the plane around the cylinder. The thread of a screw projects from the surface of the cylinder, and is designed to fit into a spiral groove, cut in the interior of a block called the nut; a lever is also fixed in the head of the cylinder to which the power is applied. The combination of these parts forms the mechanical power technically called the screw.

83



84



In working the screw, the resistance acts on the inclined face of the thread, and the power parallel to the base of the screw. This corresponds to the case in which the direction of the power is parallel to the base of the inclined plane. Equilibrium will, therefore, take place when the power is to the resistance as the distance between the threads of the screw is to the circumference described by the power.

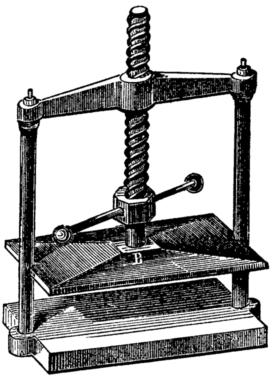
$$P : W = h : 2R\pi, \text{ and } P = W \times \frac{h}{2R\pi};$$

h being the distance between the threads, R the radius of the screw, or of the length of the lever attached to it, and π the ratio of the circumference of a circle to its radius.

During each revolution the power describes a circle, whose circumference depends on the length of the lever, but the end of the screw advances only the distance between two threads; thus in this, as in all cases of the use of machines, what is gained in power is lost in velocity

128. **Mechanical efficiency and applications of the screw.**— The mechanical efficiency of the screw is augmented, either by increasing the length of the lever, or by lessening the distance between the threads. If the threads of a screw are ½ of an inch apart, and the power describes a circle of 5 feet (120 half-inches) circumference, a power of 1 lb. will balance a resistance of 120 lbs.; if the threads are ¼ inch apart, 1 lb. will balance 240 lbs., the efficiency being doubled.

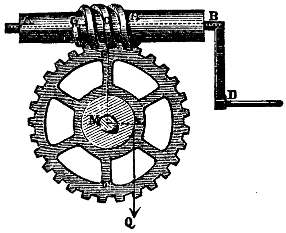
85



Fine screws are therefore more powerful than coarse ones. The applications of this most useful mechanical power are too numerous to mention, but no more frequent or important example of its use can be named than is seen in its use in presses of nearly all kinds. A good illustration of which is seen in the copying press, fig. 85.

129. **The endless screw** is a contrivance by which a slow motion is imparted to a wheel, as shown in fig. 86. The threads of the screw act upon the cogs of the wheel, and serve to move the weight Q, attached to the axis M L. If we call the radius of the circle described by the winch D B $= r$, and let $h =$ the distance between the threads of the screw, we shall have the power of the screw $= \frac{2\pi r}{h}$. Let $R =$ M F, and $R' =$ M L, and the power of the wheel and axle will $= \frac{R}{R'}$.

86



$$\text{Then } W : P = 2\pi r \times R : h \times R';$$

$$W = P \times \frac{2\pi r R}{h R'}.$$

Therefore the weight is to the power as the circumference of the circle described by the winch D, multiplied by the radius of the wheel, is to the distance between the threads of the screw multiplied by the radius of the axis.

§ 3. Strength and Power.

130. **Animal strength.**—The mechanical effect produced by men and animals is subject to extreme variation, according to the various circumstances under which it is applied. The effect produced is determined by multiplying the load (or weight) by the speed. There is always a certain relation between the elements, which will give the maximum effect; for the load may be so great that it will require all the strength of the animal to support it, and then he cannot move; or, again, the animal may have a speed of motion so great, that he cannot carry any load, however small.

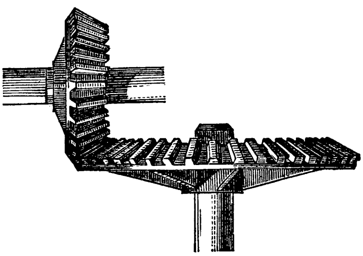
131. **Strength of men.**—It has been found that the strength of a man may be exerted, for a short time, most advantageously in raising a weight when it is placed between his legs. The greatest weight that can be raised in this manner, varies from 450 to 600 lbs.; its average amount does not however exceed 250 lbs.

The greatest mechanical effect from muscular force is obtained when the animal acts simply by raising his weight to a given height, and then is lowered by simple gravity, as upon a moving platform, the animal actually resting during the descent. In other words, the animal affords a convenient mode of raising a given weight (his own) to a certain height. Thus, if two baskets are arranged at each end of a rope hung over a pulley, and a weight to be raised is placed in one of the baskets, one or more men, whose weight is greater than that of the load to be raised, can, by getting into the empty basket, raise the weight as often as may be required. It has been found by experiment that, in this way, a man working eight hours can produce an effect equivalent to 2,000,000 lbs. raised one foot; while at a windlass an effect of only 1,250,000 lbs. is produced, and at a pile engine, only 750,000 lbs. In the tread-mill, the daily effect of men of the average strength is 1,875,000 lbs. raised one foot. Spade labor is one of the most disadvantageous forms in which human labor can be applied; the force exerted being always much greater than the weight of the earth raised. The muscular effect of the two hands of a man is about (50 *k*) 110¼ lbs., and for a female about two-thirds of this quantity.

132. **Horse-power machines.**—One of the most advantageous methods of applying the strength of animals, is by machines constructed upon the principle of the tread-mill. In practice, however, it has been found more convenient to apply horse-power to machinery by means of a large beveled or toothed wheel, fixed horizontally on a strong vertical axis, as in fig. 87. The horses are attached to projecting arms of this wheel, and as they move in their circular path, they

push against their collars, and make the wheel revolve. This beveled wheel acts on a beveled pinion attached to a horizontal shaft, in connection with the machinery to be set in motion. The maximum effect which a horse can exert in drawing is 900 lbs., but when he works continuously, it is much less. In the machine just mentioned, a horse of average strength produces as much effect as seven men of average strength working at a windlass. According to experiments made by Predgold, it appears that the average load which a single horse can draw, at the rate of 20 miles per day, in a cart weighing 7 cwt., is one ton of 1800 lbs.

87



133. **Table of the comparative strength of men and other animals.**—The following estimates of the relative strength of man and other animals, have been given by the authorities whose names are indicated, Coulomb's estimate of the labor of a man being in each case taken as the unit.

Carrying loads on the back, on a level road:

Horse,	according to	Brunacci,	4·8
“	“	Wessemann,	6·1
Mule,	“	Brunacci,	7·6

In drawing loads, on a level road, with a wheeled vehicle:

Man with a wheelbarrow, according to Coulomb,	10·0
Horses in four-wheeled wagon, “ “	175·0
“ in two-wheeled cart, according to Brunacci,	243·0
Mule, “ “ “ “	233·0
Ox, “ “ “ “	122·0

Hassenfratz gives the following comparative estimate:

In carrying loads on a level road.		In drawing loads on a level road.	
Man,	1·0	Man,	1·0
Horse,	8·0	Horse,	7·0
Mule,	8·0	Mule,	7·0
Ass,	4·0	Ass,	2·0
Camel,	31·0	Ox,	4 to 7·0
Dromedary,	25·0	Dog,	0·6
Elephant,	147·0	Reindeer,	0·2
Dog,	1·0		
Reindeer,	3·0		

134. **Steam-power.**—Water is converted into steam by the application of heat. Steam is an elastic, condensible vapor, capable of exert-

ing great force. During the conversion of a cubic inch of water into steam a mechanical force is exerted, which may be stated, in round numbers, as equivalent to a ton weight raised one foot high. The water is merely the medium by which the mechanical effects of heat are evolved. The real moving power is the combustible, the coal or wood, consumed in the evaporation of the water.

The maximum effects from a given weight of coal, in evaporating water, and consequent mechanical effect, have been obtained in Cornwall, England, where a bushel of coal, weighing 84 lbs., has produced a mechanical effect equivalent to 120,000,000 lbs. raised one foot. Probably 100,000,000, is the maximum mechanical effect attainable, in regular work, by the consumption of a bushel of coal.

As the maximum effect produced by man is 2,000,000, and that of a horse 10,000,000, it follows, that one bushel of coal consumed daily may perform the work of 50 men or 10 horses.

In the chapter on heat and the steam-engine, this subject will be more fully considered. It is introduced here only for the sake of the convenient standards of force it gives us.

The *dynamometer*, already described (37), is employed to measure the efficiency of any given mechanical power.

135. **Perpetual motion.**—Many visionary persons have convinced themselves of the possibility of constructing a machine to work continuously, with no new access of power. That such a machine is impossible, in the nature of things, is clear from the fact that no combination of parts in a machine can *create power*. A machine can only transmit force, subject to the various losses incident to friction and other resistances.

88

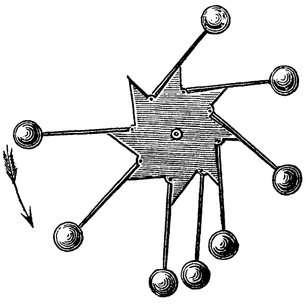


Fig. 88 shows one of the numerous forms of apparatus contrived in the vain effort to elude the laws of nature and produce a perpetual motion. The eight balls are hinged on points, in such a way that those on the falling side are held farther from the axis than those on the rising side. Not only does experiment show that such a machine *will not work*, but it is plain that the sum of the resistances (aside from friction, &c.) must equal the sum of the powers.

Nature, in cataracts, in the revolution of the earth and other heavenly bodies, furnishes us examples of *perpetual motion*. But in mechanics this term implies the assumed capacity of a machine to continue the performance of its work by some renewing force—as of a clock which should wind itself up in proportion as by gravity it runs down—a thing plainly impossible.

§ 4. Impediments to Motion.

136. **Passive resistances.**—Besides those resistances which a machine is designed to overcome, there are certain others which arise during the movement of the machine, and oppose its useful action by destroying more or less of the moving power. These forces are designated by the general name of *passive resistances*, or impediments to motion.

Several kinds are distinguished:

1st.—When we attempt to cause one body to slide over another, a resistance is experienced, so that it is necessary to use a certain degree of force to commence the sliding, and also to continue the motion after it has been begun. This is the resistance called *sliding friction*, or simply friction.

2d.—When a cylindrical body is rolled on a plane surface, the movement is opposed by a force called the *rolling friction.* It is seen, for example, in the rolling of carriage wheels on the ground.

3d.—The ropes and chains which enter into the composition of some machines, are supposed, in theory, to be perfectly flexible, but as they are not so, a considerable loss of power is caused by their stiffness, or *imperfect flexibility.*

4th.—The movements of all machines take place either in air or water, and the particles of these fluids which come in contact with the machine, are continually set in motion, which can only happen at the expense of the moving power. This is called the *resistance of fluids.*

137. **Sliding friction.**—If the surfaces of bodies were perfectly hard and smooth, they would slide upon each other without any resistance. But the most highly polished surfaces are, really (as they appear under the microscope), full of minute projections and cavities, which fit in each other when two surfaces are brought into contact. The force required to overcome the roughness and consequent adhesion of surfaces is the measure of friction. This quantity, divided by the weight of the body, forms a fraction which is called the co-efficient of the friction.

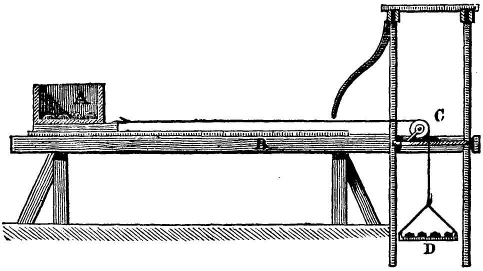
138. **Starting friction—Friction during motion.**—The amount of force necessary to commence the motion of two bodies, sliding on each other, is, in most cases, greater than the force required to continue the movement uniformly after it has been begun; hence this resistance is

distinguished into two kinds, starting friction, and friction during motion. They are also called statical and dynamical friction. Whewell proposes to name the former *striction,* reserving the word friction for the latter. However named, the laws of each can be determined only by experiment.

Coulomb's apparatus for determining starting friction.—Different observers are by no means agreed in respect to all the laws of friction; we shall here follow the results obtained in 1781, by the celebrated French philosopher and mathematician, Coulomb. In 1831, Morin, by command of the French government, repeated and enlarged the experiments of Coulomb, usually verifying his general conclusions.

The principal apparatus used by Coulomb is represented in fig. 89.

89



It consists of a horizontal table; a box, A, to receive the weights used to produce the different pressures; a pan, D, on which were placed the weights to drag the box along the table by means of a cord passing over a pulley. The box was mounted on slides, of the same substance on which the experiment was to be made, and corresponding slips of the same, or a different substance, were placed under the sliders on the table. The amount of the weight required to be placed in D, to move the box from a state of rest, is the measure of starting friction; and the weight necessary to continue the movements uniformly, is the measure of friction during motion.

Results of Coulomb's experiments on starting friction.—Without detailing the experiments, it will be sufficient to state their general results embraced in the following laws:—

Friction during movement is,

1st.—Proportional to the pressure exerted upon the sliding surfaces

2d.—Independent of the extent of the surfaces in contact.

3d.—Independent of the velocity of the movement.

4th.—Greater between surfaces of the same than surfaces of different materials

5th.—Greatest between rough surfaces, and is diminished by polishing, and usually by the use of suitable unguents.

Friction at starting is,

1st.—Proportional to pressure.

2d.—Independent of extent of surface.

3d.—Generally increased by polishing the surfaces.

The friction at starting, and during the movement, are the same, when the sliding surfaces are hard, like the metals; but if the bodies are compressible, like wood, the starting friction is much the greatest. When at least one of the surfaces is compressible, the resistance is not always the same, but varies according to the time the surfaces have been in contact. If wood slides on wood, the starting friction attains its greatest intensity in two or three minutes; but if the sliding surfaces are wood and metals, the greatest intensity is not reached for a much longer time, several hours, and sometimes several days. But after a certain time has elapsed, the starting friction is no longer augmented by lengthening the time of contact.

It appears strange at first, and contrary to our previous ideas, that the friction at starting, and during movement, should not be increased by enlarging the surfaces in contact, and *vice versa.* The explanation is this. Friction is proportional to pressure; if, therefore, two bodies have the same weight, and one has twice the surface of the other, the weight, being equally distributed on each surface, will be twice as great on each point of the surface of the first body as on each point of the second, and consequently the friction at each point of the first is twice the friction at each point of the second, and the whole friction must be the same for each body. This law, however, does not hold good in extreme cases.

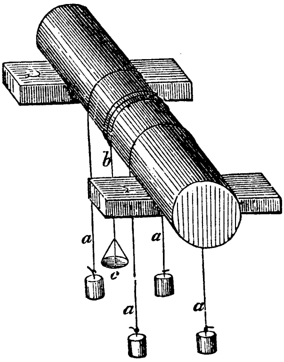
With the same pressure, the friction varies exceedingly according to the nature of the surfaces in contact. The following table shows the ratio of friction in several cases, the pressure being 100.

Surfaces in contact.	Ratio of friction to pressure.	
	At starting.	In motion.
Wood upon wood, ...	0·50	0·36
" " " with coating of soap, . .	0·36	0·14
" " " " " of tallow, .	0·19	0·07
" " metals, ...	0·60	0·42
" " " with coating of tallow, .	0·12	0·08
Leather bands on wood, ...	0·63	0·45
" " wet, ...	0·87	0·33
Metals on metals, ...	0·18	0·18
" " " with coating of olive oil,	0·12	0·07

139. **Rolling friction.**—The resistance experienced in rolling a cylinder along a plane surface is distinct in character from the friction produced in sliding the cylinder, and very much less in amount. In wood rolling on wood the proportion of resistance to pressure is from 16 to 1000, or 6 to 1000, while the sliding friction in the same case would be as 5 to 10, or 36 to 100, according to the kind of sliding friction. The resistance of rolling friction arises from a slight change of form produced in the body, and the surface on which it moves, and corresponding to the amount of pressure. The cylinder is flattened, and the plane depressed, so that the moving force is exerted in continually moving the body up a very minute inclined plane.

Coulomb's apparatus for determining rolling friction.—The apparatus employed by Coulomb, consisted of two bars, horizontal and parallel, with a space between them, fig. 90. A cylinder of the same, or a different substance, was placed transversely across the bars, and loaded with any required pressure by hanging strings upon it, carrying equal weights at their extremities. Another string, wound several times around the middle of the cylinder, carried a pan *c* to receive the weight necessary to produce motion. It is evident that this weight acted always at the extremity of the radius of the cylinder as a lever.

90



Results of Coulomb's experiments on rolling friction.—From the experiments were derived the following laws:—

The friction of rolling bodies is,

1st.—Proportional to pressure.

2d.—Independent of velocity, of the diameter of the cylinder, and of the extent of the surfaces in contact.

3d. Greater when the substances are the same than when they are different.

4th.—Not diminished by coatings of grease, but is so by the polish of the surfaces.

If the force which produces the movement, instead of being applied always at the same arm of the lever, fig. 90, were applied horizontally at the centre of the cylinder, or at the upper extremity of its vertical diameter, it would be inversely proportional to the diameter.

The friction of the axle of a wheel, whether the axle itself turns, or the wheel on the axle, is somewhat less than sliding friction, but obeys the same laws. The friction of axles may be reduced one-half or one-quarter its original amount by the use of proper unguents.

140. **Mr. Babbage's experiment.**—Mr. Babbage cites an instructive experiment to illustrate the decrease of friction. A block of stone weighing 1080 lbs. was drawn on the surface of a rock by a force of 758 lbs.; placed on a wooden sledge, it was drawn on a wooden floor by a force of 606 lbs.; when both wooden surfaces were greased, 182 lbs. was sufficient; and when the block was mounted on wooden rollers of three inches diameter, a force of only 28 lbs. was required to move it.

141. **Advantages derived from friction.**—The advantages arising from friction are vastly greater than the loss of power which it occasions. Without this property of matter it would be equally impossible to make or use machines, for nothing could be nailed, or screwed, or tied together, or grasped securely in the hand. From the difficulty of walking on very smooth ice, we may infer how useless would be the effort to move, if our feet met no resistance whatever.

142. **Rigidity of ropes.**—When ropes are used to transmit force, their stiffness occasions a considerable loss of power, amounting, in some combinations of pulleys, to two-thirds of the whole power. The amount of the loss from this cause is modified by many external circumstances, such as the dampness of the cordage, its quality, and the manner in which it is made. In general, the resistance of ropes is,

1st.—Proportional to the tension to which they are subjected.

2d.—It increases with the thickness, and is greatest in those that have been strongly twisted.

3d.—It is inversely proportional to the diameter of the wheel or cylinder around which the ropes are bent.

143. **Resistances of fluids.**—The resistance which a moving body meets in air and water, is an effect of the transfer of motion from the solid to the particles of the fluid. For the moving body must constantly displace a part of the fluid equal to its own bulk, and the motion thus communicated is so much loss of the motive power. When other circumstances are the same, the denser the medium the greater will be the resistance which it offers. Newton demonstrated that if a spherical body moves in a medium at rest, and whose density is the same as its own, it will lose half of its motion before it has described a space equal to twice its diameter. The resistance encountered by a body moving in water is 800 times greater than if it were moving with the same velocity in air; for water being 800 times more dense than air, the body must displace and communicate its own motion to 800 times as much matter in the same time.

The resistance also depends upon the extent and form of the surface which is directly opposed to the resistance; *i. e.*, at right angles to the direction of the motion. A body with a pointed, wedge-shaped, or curved surface, is less opposed than one whose surface is flat and broad.

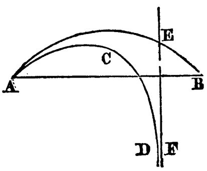
The resistance increases as the square of the velocity; for if the velocity is doubled, the loss of motion must be quadrupled, because there is twice as much fluid to be moved in the same time, and it has also to be moved twice as fast. Again, let the velocity be trebled, then the body will meet three times as many particles of the fluid in the same time, and communicate three times the velocity; therefore the resistance is $3 \times 3 = 9 = 3^2$.

Bodies having the same figure and density overcome the resistance of fluids more easily in proportion to their size. In cannon-balls, for example, the extent of surface to which the resistance is proportional increases as the square of the diameter, while the weight, or power to overcome resistance, increases as the cube of the diameter. If two balls have diameters in the ratio of 2 : 3, the resistances which they will encounter at the same velocity of projection, will be in the ratio of 4 : 9, and their moving force in the ratio of 8 : 27.

144. **Actual and theoretical velocities.**—In consequence of these impediments to motion, the actual movements of bodies are materially different from the theoretical motions explained in previous sections. The motion of falling bodies is very far from being uniformly accelerated, nor do all bodies fall with equal rapidity, as theory requires, and as was seen to be true in the guinea and feather experiment. The resistance of the air, which is very small at first, rapidly increases, and after a certain time becomes equal to the force of gravity, when the body will no longer be accelerated, but move uniformly through the remainder of its descent. The descent of bodies on inclined planes and curves deviates still more from uniformly accelerated motion, since the effect of friction is added to the resistance of the air.

145. **Ballistic curve.**—A still greater difference is observed between the actual and theoretical motions of projectiles (103). Instead of describing a parabola, A E B, fig. 91, the projectile actually describes the curve A C D, called the ballistic curve, which never attains so great a vertical height, or so long a range as the corresponding parabola, and which, toward the end of its course, continually approaches the perpendicular, E F. A four-pound shot, which flies 6437 feet in the air, would traverse in a vacuum a space of 23,226 feet.

91



As in the case of friction, the benefits resulting from this state of things overpay the disadvantages. Fish could not swim, nor birds fly, were it not for the resistances of the media they inhabit. The paddle-wheels of a steamer would not move it, nor its rudder guide its course, if they met no resistance to their movements. And we can very well dispense with a perfect theory of projectiles, if thereby the rain is prevented from descending with the destructive velocity of hail-stones.

Problems.—Vis Viva.

47. If a locomotive and train move 20 miles an hour, how much greater force will be required to move a train weighing three times as much 25 miles an hour?

48. If a locomotive weighing 30 tons will draw a train weighing 90 tons 15 miles an hour, at what velocity will it draw a train weighing 30 tons? The weight of the locomotive is to be added to the train in both cases.

49. What will be the relative destructive power of a hurricane moving 60 miles an hour, and another moving 90 miles an hour?

50. If a pile-driver weighing 1500 lbs. raised 20 feet strikes with a given force to overcome resistance, to what height must it be raised to give a shock two and a half times as great?

51. What is the comparative destructive power of a cannon-ball weighing 64 lbs. flying 1000 feet per second, and another ball weighing 200 lbs. flying 1500 feet per second?

The Lever.

52. Two weights, 3 and 4, balance on the extremities of a lever 4 feet long; find the fulcrum.

53. Four weights, 1, 3, 7, 5, are placed at equal distances on a straight lever. Determine the position of the fulcrum.

54. Two men carry a weight of 2 cwt. hung on a pole, the ends of which rest on their shoulders; what part of the load is borne by each man, the weight hanging 6 inches from the middle of the pole, the whole length of which is 4 feet?

55. A beam, 18 feet long, is supported at both ends; a weight of 18 cwt. is suspended at 3 feet from one end, and a weight of 12 cwt. at 8 feet from the other end; required the pressure at each point of support.

56. A uniform beam, 40 feet in length, the weight of which is 4 cwt., is supported by two props, A and B, 30 feet apart; a weight of 24 cwt. is then suspended on the beam at the distance of 10 feet from B, the beam projecting 8 feet over the prop A, and 2 feet over that at B; required the pressure on each of the props.

57. On a lever 3 feet in length a weight of 500 lbs. is suspended at one end, at $2\frac{3}{4}$ inches from its fulcrum; what weight at the other end will keep the lever in equilibrium, the lever being assumed to be without weight?

Wheel and Axle.

58. A power of 10 lbs. on a wheel the diameter of which is 10 feet, balances a weight of 300 lbs. on the axle; what is the diameter of the axle, the thickness of the rope on the wheel being one inch, and that of the rope on the axle two inches?

59. A weight of 2240 lbs. is sustained by a rope of 2 inches in diameter, going round an axle 4 inches in diameter; what weight must be suspended at the circumference of a wheel—radius 6 feet—by a rope of the same thickness, to obtain equilibrium?

60. In a combination of wheels and axles there are given the radii of the wheels, 20, 26, and 48 inches, and the radii of the pinions and axle, 4, 5, and 8 inches. If a power of 1 cwt. be applied to the circumference of the first wheel, what weight will it be able to sustain at the circumference of the axle, or last shaft?

61. The number of teeth in each of three successive wheels is 144, and the number of teeth in each of the axles or pinions is 6; what weight will this machine support on the last shaft with a power of 2 cwt. on the first wheel?

The Pulley.

62. In a system of pulleys, such as is shown in fig. 76, the number of movable pulleys being 5; required the weight, the power being 500 lbs., and the weight of the movable block and pulleys being 30 lbs.

63. What power at P, fig. 77, will be required to balance a weight, W, of 3 tons, the number and arrangement of the pulleys being as shown in the figure?

Inclined Plane.

64. What power acting as in fig. 79, will balance a weight of 300 lbs. on the inclined plane, the length of the plane being 25 feet, and the vertical height five feet?

65. On an inclined plane, whose base is 10 feet and height 3 feet, what power acting parallel to the base will balance a weight of 2 tons?

66. What power is required to draw a train of cars, weighing 40 tons, up a railway grade rising 1 foot in every 100 feet?

The Screw.

67. What weight can be raised by means of a screw having its threads one inch apart, by a power of 150 lbs. acting at a distance of 6 feet from the axis of the screw?

68. Supposing one-third of the power is lost in overcoming the friction of the screw, what power will be required, acting 3 feet from the axis of the screw, to raise 3 tons, the threads of the screw being 2 inches apart?

69. What power at the winch D, fig. 86, is required to raise a weight of 2 tons at Q, if the radius of the axle is 6 inches, the radius of the wheel 3 feet, the distance between the threads of the screw $\frac{1}{100}$ part of the circumference of the wheel, and the length of the winch D B = 2 feet?

Resistance.

70. If a mass of iron weighing 5 tons slides upon iron rails, what force is required to start it, and how much to keep it moving afterwards?

71. If a steam vessel of 1000 tons is moved through the water at 10 miles an hour, by an engine of 300 horse-power, what is the power of an engine required to propel another vessel, of the same model, of 2000 tons at the same speed?

72. If a ball, 6 inches in diameter, is discharged from a cannon at the rate of one mile in 7 seconds, how much greater force would be required to throw a ball of double the weight with the same velocity, taking into account the resistance of the air and the dimensions of the balls?

73. If an engine of 500 horse-power propels a vessel of 1500 tons 12 miles an hour, at what velocity will an engine of 600 horse-power propel a vessel of 2000 tons, built on the same model as the preceding?

PART SECOND.

THE THREE STATES OF MATTER.

CHAPTER I.

MOLECULAR FORCES.

146. **Cohesion and Repulsion.**—The three states of matter (15)—the solid, liquid, and gaseous—exist, as is assumed, in virtue of certain forces inherent in the particles of matter, and called *molecular forces.* These forces are either *attractive* or *repulsive,* drawing the particles of bodies toward each other, or tending to separate them. In solids the attractive force greatly overpowers the repulsive, and the particles of matter become therefore relatively fixed at certain distances from each other—not being in actual contact, but having, as we have seen (23), numerous pores between them. Heat may enlarge and cold diminish these pores, and with them the sensible magnitude of the solid; but the integral particles of the solid cannot be separated without the exercise of some exterior and greater force. When the particles of a body are not separated too far, they return again, upon the withdrawal of the constraining force, to their original position (Elasticity). This species of attraction existing between particles of the same kind is distinguished by the term *cohesion,* and when existing between particles of an unlike kind, it is called *adhesion.*

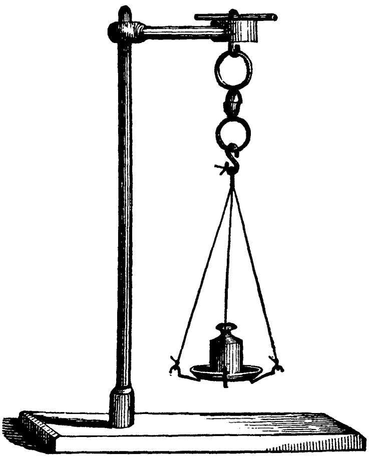
Repulsion.—If we admit, as the phenomena of porosity demand (23), that in spite of cohesive attraction, the particles of bodies do not actually touch each other, it follows that there must exist in the molecules of matter a second and counterbalancing force opposed to cohesion and—in solids—in equilibrium with it. This force is called *repulsion*. We shall presently revert to the evidence of its action on matter.

While the existence of these two molecular forces is in many cases capable of direct proof, the exact mode of their action is chiefly conjectural. It is, however, certain that the attractive forces act only at insensible distances. In this respect the molecular forces are to be distinguished from gravitation, which acts at all distances. Chemical attraction is also distinguished from the mechanical forces of cohesion and adhesion, by the important fact that its exercise is invariably attended by the loss of specific identity (7), and, of course, by the substitution of new qualities in the compound for those characteristic of its constituents.

Since the force of gravity is proportional to the mass, and inversely as the distance, if cohesion were merely the attraction of gravitation acting at insensible distances, the particles of a body situated at the centre of gravity of a large mass, should cohere more strongly than particles at a distance from the centre of gravity, or than the same particles when the mass is reduced to fragments, but no such difference has been observed, we must therefore conclude that gravity and cohesive attraction are essentially different forces.

147. **Examples of cohesion among solids.**—Cohesion, when once destroyed by mechanical violence, is not usually brought into exercise again by mere contact of the separated particles. Thus the broken fragments of a glass vessel, or of a stone, do not reunite at ordinary temperatures. Two hemispheres of tarnished lead will not adhere by their flat surfaces by mere pressure, but if the coating of oxyd is first removed by a sharp knife, and the two clean surfaces are then pressed together, with a slight wrenching motion, they will cohere strongly. Arranged as in fig. 92, two surfaces one inch in diameter will sustain ten pounds or more. This is only an example of welding at common temperatures, a process sufficiently familiar in hot iron. Wax, dough, india rubber, and other similar substances, offer examples of a like nature, provided clean surfaces be pressed together. Dust, or other foreign bodies, prevent this union. Even polished glass plates, allowed to remain long

92



in contact, if perfectly clean, and under pressure, have been known to cohere so strongly as to separate by fracture in any other direction sooner than in the line of junction. Boyle demonstrated that this fact is not accounted for by attributing the action to atmospheric pressure. He suspended a pair of adhesion plates in the vacuum of an air-pump, where, in the absence of atmospheric pressure, it required still a considerable weight to detach the surfaces.

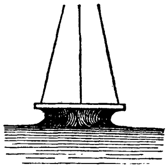
Adhesion is distinguished from cohesion by the fact that while the latter occurs between particles of a like kind, producing homogeneous bodies, the former takes place between particles of unlike kinds, producing heterogeneous bodies. Glue binding together pieces of wood is an example of adhesion. This species of mechanical attraction is, however, seen in its most important relations in the curious phenomena of capillarity, to be discussed hereafter.

The terms cohesion and adhesion are often used interchangeably, and, when the distinctions here pointed out are borne in mind, no evil will arise from this use of terms.

The force of cohesion among the particles of solids, when exerted under favorable conditions, produces the regular forms of crystals, to which we shall presently revert.

148. **Cohesion in liquids and between liquid gases and solids.**—The force of cohesion in liquids gives the spherical form to drops of rain and dew, and rounds the drop of water suspended from the end of a glass rod. If two drops of water or any other liquid approach each other near enough, they unite to form a larger spherical drop. A soap-bubble is only a large hollow sphere of water, whose outer film of liquid assumes and preserves its spherical form in virtue of cohesive attraction and the laws of liquid equilibrium. The soap, while it adds to the viscous condition of the water, really diminishes its cohesive force, as was shown by Prof. Henry. The force of cohesion in water may be directly measured by suspending a counterpoised disk of glass or metal from a scale pan, fig. 92^b adjusted to allow the disk just to touch the surface of the water. The weight required in the opposite pan, to separate the disk from the water, then becomes the measure of cohesion among the particles of water forming the outer circle of contact.

92^b



By this means Gay Lussac found that a disk of 4·362 inches in diameter required 982 grains to separate it from water, while from alcohol (density 0·819) and spirits of turpentine, 478·83 and 525 grains respectively, produced separation; all being at the temperature of 46° Fahr. The thickness and material of the disk made no difference in the result, showing that the force of cohesion was the only force to be overcome, and that this force was exerted at a distance less than the thickness of the film of liquid necessary to moisten the surface of the disk. It is also evident that the weights obtained do not represent the whole cohesive force in each case, since, from the circumstances of the trial, it can be only the outer circle of particles whose cohesion is overcome, and this being the largest circle, each succeeding row or line of particles yields readily to the same force.

Between liquids and solids the force of adhesion is modified by the phenomena of capillarity and surface attraction.

Between gases and solids cohesion is seen to exist when we attempt to wet the polished surface of a steel blade, or a clean surface of glass, with water. The liquid fails to wet the polished surface of the metal, &c., owing to the film of air adhering to it, due to the attraction of the solid for the air. If the blade is slightly heated, or its surface is roughened mechanically or by acids, this film of air is removed, and the blade is then wetted. (See Smee's battery: Electricity.)

Gases do not manifest cohesion among themselves, because the repulsive force overcomes it, but numerous examples of its exercise may be quoted besides that just named. The bubbles of gas escaping from aerated water adhere to the sides of a glass vessel from this cause. But above all is this seen in the power of recently ignited charcoal to absorb and retain gases. Owing to its numerous sensible pores, charcoal presents a very large surface in a small space. The more compact the wood the more numerous are these pores, and the more remarkable the consequent absorption of gas. Different gases are also very differently absorbed by it, depending on their condensibility and solubility. Thus, while only four or five volumes of common air are absorbed by charcoal, thirty volumes of carbonic acid, and eighty or ninety volumes of ammonia or chlorohydric acid gas, are absorbed by charcoal recently ignited. This curious and important property is easily illustrated over the mercurial trough, by using glass cylinders, filled with the various gases, to cover bits of charcoal placed on the mercury—the absorption commencing at once and advancing gradually for some hours.

We will consider the action of molecular forces, *first*, between molecules of the same kind, and, *second*, when acting between molecules of unlike kinds, to which are referred the phenomena of capillarity.

CHAPTER II.

OF SOLIDS.

MOLECULAR FORCES ACTING BETWEEN PARTICLES OF LIKE KINDS.

§ 1. Properties of Solids.

149. **The characteristic properties of solids**, now to be considered, are, 1. Crystalline Form; 2. Elasticity; 3. Resistance to Fracture (including Strength of Materials); 4. Hardness, and 5. Those properties dependent on a permanent change in the arrangement of the molecules—as Ductility, Malleability, Temper, &c.

150. **Structure of solids.**—In solids the particles of matter are held in fixed relation to each other by the molecular forces (146). The relative disposition of the molecules, or of their groups, constitutes what is called *structure* in solids. This structure may be either *symmetrical* or *regular*, as in living beings and crystals, or *amorphous*, as in most rocks and many other substances.

There exists in nature a plan, which cannot be mistaken, to combine matter in complete and symmetrical wholes. The bodies of animals consist, usually, of two equal, (or nearly equal), and similar sets of limbs and organs, one on the right and one on the left side. The organs of most flowering plants are similarly and regularly arranged in whorles of *three* members, as in the lily, or of *five*, as in the rose, or in some other simple numbers, and the same law is beautifully exemplified in the arrangement of the leaves and branches of all plants and trees (*phyllotaxy*). In the animal and vegetable world, the laws which direct the aggregation of matter are those of VITALITY, and it is observed that most of the forms thus produced are bounded by curved lines and surfaces.

In the inorganic or lifeless world different laws are in force, and in the production of solids the atoms, under favorable circumstances, arrange themselves in forms which are angular and bounded by plane surfaces. The geometrical forms thus produced are analogous to the more complicated results of vitality as seen in animal and vegetable life. These forms are called CRYSTALS, and the laws governing the aggregation of matter into such forms, are called *the laws of crystallization.*

When solids are formed in a manner unfavorable to the regular action of the molecular forces, the regular forms of crystals are not produced, but a mass which has perhaps some traces of crystalline structure (as in marble), or which is entirely amorphous(152), according as the act of solidification has been more or less disturbed. Thus, in granite we easily detect the crystalline structure of some of the constituents closely aggregated, while in slates and many mechanical rocks no traces of crystalline structure can be seen.

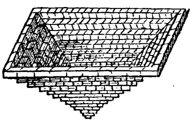
§ 2. Crystallography.

151. **Conditions of crystallization.**—In order that crystals may form, it is a necessary condition that the molecules of the body to be crystallized should have freedom of motion among themselves, and ample time to arrange themselves in accordance with the force of crystallogenic attraction. These conditions may be met in either of the following methods: 1. By solution; 2. By fusion; 3. By sublimation or evaporation; or, 4. By electrical or chemical decomposition.

(*a*) *By solution.*—Many solids dissolve in water; thus most salts, as common salt, Epsom salts, saltpetre, borax, alum, &c., form a clear solution in water, in which all crystalline attractions are subordinated, until by gradual evaporation, or by cooling from a saturated hot solution, the several salts reappear, each in its own appropriate form. Sulphur and phosphorus also, dissolved by heat in bisulphid of carbon, crystallize by the cooling of the solution. Some substances are equally soluble in cold or in hot water, and crystals are obtained from their solutions only by evaporation, with or without the aid of heat.

Common salt is an example of such a substance; when evaporated very gently, as by solar heat, perfect cubes are formed; if rapidly, as by fire, a confused mass of irregular crystalline grains result. Sometimes the floating crystals, as they grow in weight continually, but slowly, sink, giving rise to the curious hopper-shaped forms seen in fig. 93.

93



Alumina dissolves in melted boracic acid, and the solution, exposed for a time to the highest heat of a porcelain furnace, loses the boracic acid slowly by evaporation, and the alumina crystallizes as rubies or sapphires. Many other gems have thus been obtained, of microscopic size, by M. Ebleman, by solution in boracic or phosphoric acids, or their salts, at a high heat.

(*b*) *By fusion.*—By melting sulphur, bismuth, and many other substances, in crucibles, and allowing them to cool very slowly; when a crust has formed on the surface it is pierced, and the contents remaining fluid are turned out, the interior cavity is found lined with crystals of the substance experimented on.

(*c*) *By sublimation.*—By heat many substances rise in vapor, and on cooling again, in a proper receptacle, assume their appropriate crystalline forms—camphor, sulphur, arsenious acid, iodine, arsenic, sal-ammoniac, &c., can be thus crystallized.

(*d*) *By electrical or chemical decomposition.*—By adding to a solution of some substances some other dissolved body, which causes the first to become insoluble, a crystalline powder often falls (this is true in most cases of *precipitation*), due to the formation of a new compound, of a less solubility than either of the substances employed. The crystals of metals, *e. g.* of copper, gold, silver, &c., are easily formed by the processes of electro-metallurgy.

152. **Amorphism.**—Amorphism is that state of a solid in which there is no trace of a crystalline structure; examples of such a state are seen in common glass, gun-flint, wax, obsidian, sugar-candy, &c. An amorphous body, having no planes of cleavage, is broken in one direction as easily as in another. Bodies are generally more soluble, less hard and dense, in the amorphous than in the crystalline state.

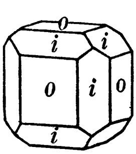
An amorphous body may be produced in a number of ways; for example, by fusion, as in the case of glass; by evaporation of solutions, as those of the gums and glue in water; and by precipitation from their solutions, as is the case with alumina and phosphate of lime.

The property of toughness, seen, as for example, in emery (amorphous corundum), and horn-stone (amorphous quartz), is much more highly developed in the amorphous than in the crystalline varieties of these minerals.

153. **Crystalline forms. Definitions.**—The crystalline forms assumed by the same substance are, with certain limitations, always the same; depending on the nature of the substance, and are therefore *essential* forms. The study of these forms and the laws of crystallogeny, reveal to us all that we know of the ultimate forms of matter.

A crystal is a polyhedron, and the terms of solid geometry are used in crystallography without change.

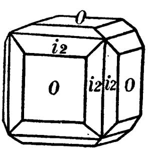
94



Replacement.—An edge or angle is replaced when cut off by one or more secondary planes. Fig. 94, *i i*.

Truncation.—An edge or angle is truncated when the replacing plane is equally inclined to the adjacent faces. Fig. 94.

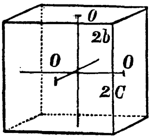
94^b



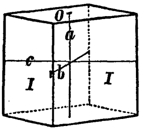
Bevelment.—An edge is beveled when replaced by two planes which are respectively inclined at equal angles to the adjacent faces *i* 2 *i* 2, fig. 94^b. Truncation and bevelment can only occur on edges formed by the meeting of equal planes.

The axes of a crystal are imaginary lines passing through its centre, and about which two or more faces are symmetrically arranged. They connect either the centres of opposite faces, fig. 95, or edges, fig. 96, or the apices of opposite solid angles, fig. 97, or of both edges and angles, fig. 98.

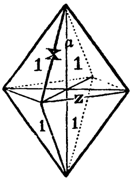
95



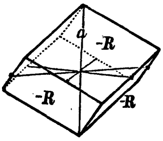
96



97



98



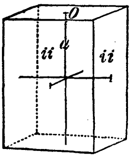
Three axes are employed for the different systems in crystallography (excepting the sixth, fig. 114), whose length may be equal, or only two alike, or all unequal; they may also be at right angles to each other, or oblique.

A prism is a column having any number of sides. In crystallography we have four and six-sided prisms, which may be either right prisms, that is, erect; or oblique prisms, that is, inclined.

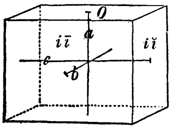
Four-sided prisms occur of a number of kinds; their bases may be either square, rectangular, rhombic, or rhomboidal. If the base is a square, or a rectangle, and the prism erect, *the eight solid angles are equal* and *rectangular;* the edges are twelve, and may vary; for example,

A cube is bounded by six equal sides (the lateral sides being equal to the bases), and the twelve edges are all equal, fig. 95.

99



100



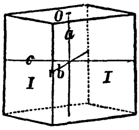
A right square prism, fig. 99, has a square base and a height which may be either greater or less than its breadth; its sides are equal rectangles, the eight basal edges (four at each base) are equal to each other, but differ from the four lateral edges.

A right rectangular prism, fig. 100, has a rectangular base, and sides also rectangular, the opposites only equal; two edges at each base differ from the other two, while the lateral edges are also different; hence there are three sets of edges, four in each set.

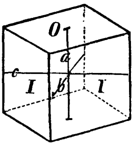
The base may also be a rhomb or rhomboid.

A right rhombic prism, fig. 101, has a varying height and a rhombic base. Its plane angles are two obtuse and two acute, with corresponding solid angles and lateral edges; the four lateral faces are rectangles, and, like the basal edges, are equal.

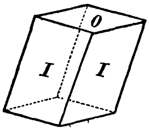
101



102



103



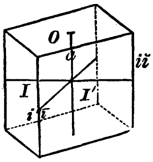
An oblique rhombic prism, figs. 102, 103 (fig. 102 a front view, and fig. 103 a side view); has a rhombic base and a varying height, the lateral faces are rhomboids. The lateral edges, like the basal edges, have two acute and two obtuse angles. When the height is equal to the breadth the form is:—

A rhombohedron, fig. 98, composed of six equal rhombic faces.

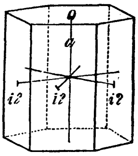
A right rhomboidal prism has a rhomboidal base and a varying height, only the two opposite sides and angles are equal, the lateral rectangular faces correspond to the basal edges; the opposites only are equal. This form is similar to fig. 101.

An oblique rhomboidal prism, fig. 104, has a rhomboidal base and a varying height. The lateral faces are rhomboids. The edges of each base are of four kinds; for two opposite are longer than the other two, and of each pair, one is obtuse and the other acute. In this solid, therefore, only diagonally opposite edges are *similar*, and only opposite solid angles are equal.

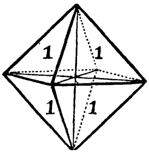
104



105



106



An hexagonal prism, fig. 105, is an erect six-sided prism.

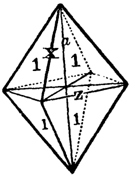
An octahedron has eight triangular faces; its form is like two four sided pyramids united base to base. Three octahedrons are described

The regular octahedron, fig. 106, has a square base and eight faces, equilateral triangles; its solid angles are six, and equal, as also are its twelve edges. The plane angles are 60°, the interfacial angles are 109° 28′ 16″; this solid is symmetrical, like the cube.

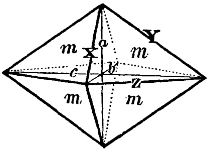
The right square octahedron, fig. 107, has a square base, but a vertical height, greater or less than in the regular octahedron. Its faces are equal isosceles triangles. Its basal edges are equal and similar, but they differ in length from the eight equal pyramidal edges. The vertical solid angles differ from the basal

The right rhombic octahedron, fig. 108, has a rhombic base and a varying height; its faces are equal triangles; the basal edges are equal; the plane angles of the base and the pyramidal edges are of two kinds, two obtuse and two acute.

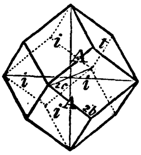
107



108



109



The rhombic dodecahedron, fig. 109, is bounded by twelve equal rhombs; it has twenty-four similar edges, and fourteen solid angles; they are of two kinds: Eight obtuse, formed by the meeting of three obtuse plane angles, and six acute, formed by the meeting of four acute plane angles.

154. **Systems of crystals.**—There are six systems of axes, producing the same number of systems of crystalline forms, by the symmetrical arrangement of planes about these axes. They are called the monometric, dimetric, trimetric, monoclinic, triclinic, and hexagonal systems.

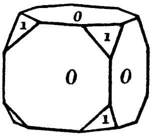
(*a*) *The monometric system* (from *monos*, one, and *metron*, measure), includes the cube, fig. 95, the regular octahedron, fig. 106, and rhombic dodecahedron, fig. 109. Each of these forms is perfectly symmetrical,

---

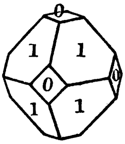
NOTE.—In studying this subject, the pupil will find it of the greatest assistance to his easy comprehension of the forms mentioned, to produce them with a knife, from some soft substance like a turnip or a potato, which are more easily managed than chalk or wood, and neater than clay. Sets of crystalline forms and cards, with the outlines of the various forms prepared for cutting up, are furnished cheaply by the German chemical dealers, for the use of schools.

being equal in height, length, and breadth. Their axes are three in number, of equal length, and at right angles to each other. In the cube, the axes connect the centres of opposite faces, in the octahedron, the apices of opposite solid angles, and in the dodecahedron, the apices of opposite acute solid angles. The relation of the axes in these solids to each other, may be understood by deriving one form from the other. If in the cube (its faces are indicated by *o*) we truncate each of its eight solid angles, fig. 110 is first produced, and as the truncation proceeds, fig. 111, and finally a perfect octahedron. It will be noticed that the centres of *o*, the ends of the axes in the cube, correspond to the apices of the solid angles in the octahedron, which are also the ends of axes.

110

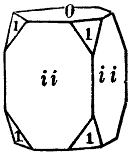


111



(*b*) *The dimetric system* (from *dis*, two-fold, and *metron*, measure), includes the square prism, fig. 99, and square octahedron, fig. 107, bearing the same relation to each other as the cube does to the regular octahedron. In this system there are three axes, all at right angles to each other, but only the two lateral are equal, the third, or vertical axis, being of varying length. In the prism, the axes connect the centres of opposite faces, in the octahedron the apices of opposite solid angles. If a square prism has each of its solid angles truncated, we shall have first, fig. 112, and, finally, the square octahedron is produced.

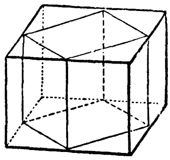
112



(*c*) *The trimetric system* (from *tris*, three-fold, and *metron*, measure), includes the rectangular prism. fig. 100, the rhombic prism, fig. 101, and the rhombic octahedron, fig. 108. Each of these forms has its three axes at right angles to each other, and all are unequal in length. In a rectangular prism (the base a rectangle), the axes connect the centres of opposite faces. In the rhombic prism (base a rhomb), the vertical axis connects the centres of the bases, the two lateral axes connect the centres of opposite edges. In the rhombic octahedron (base a rhomb) the axes connect the apices of opposite solid angles.

(*d*) *The monoclinic system* (from *monos*, one, and *klino*, to incline), includes the right rhomboidal prism, fig. 101, and the oblique rhombic prism, fig. 102. In this system the three axes are unequal, the two lateral axes are at right angles with one another, the vertical is inclined to one of the lateral axes and at right angles with the other. In the right rhomboidal prism the axes connect the centres of opposite faces. In the oblique rhombic prism the vertical axis connects the centres of the bases, and the two lateral axes, the centres of opposite lateral edges. The truncation of the lateral edges of one prism finally produces the other. The relation of these prisms to each other is seen in fig. 103.

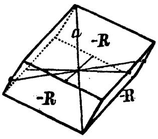
113



(*e*) *The triclinic system* (from *tris*, three, and *klino*, to incline), includes the oblique rhomboidal prism, fig. 104. All the axes are unequal and oblique, the vertical axis connects the centres of the bases; the lateral axes connect the centres of the lateral edges.

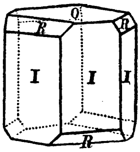
(*f*) *The hexagonal system* includes the hexagonal prism, fig. 105, and rhombohedron, fig. 114. In the hexagonal prism, fig. 105, the vertical axis connects the centres of the bases, the three lateral axes connect the centres of opposite lateral faces or edges, and cross each other at an angle of 60°, at right angles to the vertical axis.

114



In the rhombohedron, two diagonally opposite solid angles consist of three equal obtuse or three equal acute plane angles; the diagonal connecting these solid angles is called the vertical axis; placed with this axis in a vertical position, the rhombohedron is said to be in position, and looking from above, it will be noticed that the lateral edges are at an equal distance from the vertical axis; the three lateral axes connect the centres of the lateral edges intersecting each other, as do the lateral axes of the hexagonal prism, at an angle of 60°. Placing the rhombohedron in position, if we remove the six lateral edges, replacing them by planes parallel to the vertical axis, there is produced a regular hexagonal prism, terminated by three-sided pyramids. If their vertical solid angles are also removed the regular hexagonal prism results. If we remove from an hexagonal prism three alternate basal edges, and at the other extremity also, three edges, alternating with the first, as shown in fig. 115, and continue the removal until the original form is obliterated, a rhombohedron is produced; it also results by removing, in a corresponding manner, the alternate solid angles from the hexagonal prism. When the plane angles forming the vertical solid angles are obtuse, the rhombohedron is called obtuse, and if acute, the solid is called an acute rhombohedron.

115



155. **Modified forms.**—If bodies in crystallizing assumed only the fundamental forms, there would be but comparatively little variety and beauty in crystalline solids; it is to the modification of the fundamental forms that we owe that endless variety of crystalline figures

which we observe in nature, and that are produced in the laboratory. These modified forms are called secondary or derivative forms, and are produced by the replacing of the edges and angles of the fundamental forms by planes, which are called secondary planes. The modifications of crystals take place according to two simple laws.

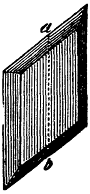
1st. *All the similar parts of a crystal may be simultaneously and similarly modified.* The forms thus resulting are called *holohedral* forms (from *holos*, whole, and *edra*, face).

2d. *Half the similar parts of a crystal may be simultaneously and similarly modified.* The forms thus resulting are called *hemihedral* forms (from *hemisa*, half, and *edra*, face).

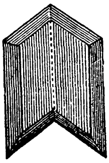
[It is beyond the design of this elementary work to enter into more detail concerning the different systems of crystallography, and of modified forms. For further information the student is referred to Dana's Mineralogy, from which, by permission, this chapter has been condensed.]

156. **Compound crystals.**—Sometimes we find two or more crystals united regularly and symmetrically together. The form, if composed of two individuals, is called a twin crystal. Fig. 116 is a simple crystal

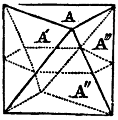
116



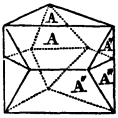
117



118

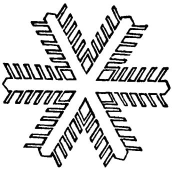


119

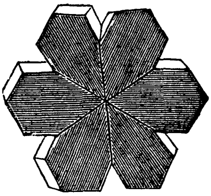


of gypsum; if it be bisected along the imaginary line *a b*, and the right half be inverted and applied to the other half, it will form fig. 117. If an octahedron, as fig. 118, be bisected through the dotted line,

120



121



and the upper half revolved half way around be then united to the lower, it produces fig. 119. Both figs. 117 and 119 are twin crystals.

The imaginary axis, on which the revolution of half the crystal is made, is termed the *axis of revolution* and the imaginary section, the *plane of revolution.* Compound crystals, composed of more than two individuals, are frequently observed, as in the case of the snow-flake, a not unusual form of which is represented by fig. 120, composed of six crystals meeting at a point, or of three crossing each other at an angle of 60°. Fig. 121 represents a compound crystal of chrysoberyl.

157. **Cleavage.**—By the application of mechanical force to crystals we observe that they often split in certain directions, leaving even and polished surfaces. The production of such surfaces, in causing the separation of the particles of the crystals, is called their *cleavage;* the planes along which the separation takes place are called *cleavage joints.* Cleavage is often obtained with great ease, as, for example, with mica, which may be separated by means of the fingers into thin leaves. Galena, also, cleaves easily, and as the three cleavage planes are at right angles to each other, a cube results. Calc spar splits in three oblique directions, and thus a rhombohedron is obtained; while in fluor spar a cleavage of its solid angles produces an octahedron. The cleavage of many crystals is obtained with great difficulty, as, for example, in quartz and tourmaline; in others no cleavage can be produced, owing to the strong cohesion among the laminæ. In some crystals but one cleavage is visible, as with mica; several have two; others three, as galena and calc spar; fluor spar has four, blende has six, while others have even more. We obtain, by the cleavage of a crystal, some one of the thirteen fundamental forms. Varieties of the same mineral have the same cleavage. Cleavage occurs parallel to the faces of the fundamental form, or along the diagonals; the facility of cleavage and lustre of the surfaces is always the same, parallel to similar faces.

158. **Determination of crystalline forms.**—In order to determine a crystal, it is essential to refer it to the system to which it belongs, and to determine the simple forms of which it consists, with the relative lengths and inclination of the axis.

§ 3. Elasticity.

159. **Elasticity of solids.**—Elasticity, already mentioned as one of the properties of matter, has a peculiar importance in solids, because it is itself a moving force, and serves to measure the intensity of other forces. All bodies offer a resistance to compression and extension, which is due to elasticity. It is shown in the effort of a compressed spring, or a bent bow, to recover from its forced state of flexion.

Tension, flexure, and *torsion* are also at once evidence and measures of the force of elasticity in solids; while in fluids *compression* is the only evidence of its presence, and hence compressibility alone is a general property of matter.

Every body has a *limit of elasticity* beyond which it cannot be carried without a permanent derangement of its particles, or fracture. A perfectly elastic body is one which returns completely to its original form when pressure is removed; and every body does this, each within its own limit of elasticity. Hence every body may, in a restricted sense, be said to be perfectly elastic. The return of an elastic body to its primitive position is usually made with several oscillations, called *oscillations of elasticity*. This is familiarly seen in the recoil of a bent blade or spring of steel.

It is evident that in bending the steel its molecules are deranged from their position of equilibrium by compression on one side and extension on the other, and that it is the force with which they tend to replace themselves which produces the elasticity of the blade.

There is a similar, although less perceptible, change of figure in an ivory ball, which, dropped upon a hard surface, will rebound nearly to the height from which it fell. It does not immediately recover its spherical shape, but is for several times, alternately, an oblate and prolate spheroid.

160. **Elasticity of tension and compression.**—By tension is to be understood the action of a force exerted in the direction of the length, of a wire, for example. The laws of elasticity of tension have been experimentally deduced, by suspending weights from the lower end of a rod or wire, sustained at top by a firm support. The elongation occasioned by each addition of weight is measured by a telescope mounted on a graduated bar, parallel to the wire (the apparatus is called a *cathetometer*). If the limit of elasticity is not passed, the rod or wire returns to its original length on removing the weights; but if the strain is continued too long, or too great a tension is brought to bear, a permanent change of length results. When the limits of elasticity are not passed, the following laws are developed by this mode of experiment.

1. *For the same substance the elongation caused by each unit of tension is the same, whatever may have been the original tension.* Thus, with a wire loaded with ten or twenty pounds, the elongation for each successive pound is the same as for the first pound.

2. *The elongation is proportional to the tension employed.* This follows from the first law, and signifies that if the rod or wire is elongated one unit by one pound, it will be elongated ten units by ten pounds, &c.

3. *The elongation with a given tension is proportional to the length of the rod.* That is to say, if a rod of a given length is elongated a unit of length by a given tension, a rod two units long is elongated twice as much by the same tension.

4. *The elongation is inversely proportional to the area of the section of the rod.* That is, if of two rods of the same substance, of equal length, and subject to the same tension, one has twice the area of the other, it will be elongated only half as much.

Experiment has shown that in the case of the compression of a metallic bar, or rod, in the direction of its length, by an endwise force, the bar is shortened just as much as it would be lengthened if the same force had been used to stretch it. Hence the laws for the elasticity of compression are quite the same with those for tension.

These laws may be demonstrated mathematically as well as experimentally, but it is not requisite here that we should do more than enunciate and illustrate them.

161. **Coefficient of elasticity.**—From the laws of elasticity, of tension, and compression, just enunciated, it follows that the elongation (l) of a given rod is proportional to a constant quantity, C, depending on the nature of the substance; secondly, to the weight, W, by which it is stretched; thirdly, to its length, L; and, fourthly, that it is inversely proportional to the area of its section, S: *i. e.*

$$l = C \, . \, W \, . \, L \, . \, \frac{1}{S};$$

hence,

$$l = C\frac{WL}{S}, \quad \text{or} \quad C = \frac{lS}{LW}.$$

Putting $K = \frac{1}{C}$ in these equations, they become

$$l = \frac{1}{K}\,\frac{WL}{S}, \text{ or } K = \frac{LW}{lS}.$$

If in the last equation we make $l = L$, and $S = 1$, it becomes $K = W$.

The quantity K is called the *coefficient of elasticity.* In other words, the coefficient of elasticity, in any homogeneous substance, is equal to the weight required to double the length of a bar of that substance having a given area, assuming such an elongation physically possible, which it is not, unless in the case of caoutchouc.

We are indebted to Wertheim for most of our experimental knowledge of this subject. The following table shows the mean coefficient of elasticity of a number of metals, as deduced by him, with various weights, at different temperatures, from 1° to 392° Fahrenheit.

METALS.*	Value of K at the temperature of—				
	1°–14° F.	50° F.	59°–68° F.	212° F.	392° F.
Gold hammered,	9,351	8,603			
" annealed,			5,585	5,408	5,482
Silver hammered,	7,800	7,411			
" annealed,			7,140	7,274	6,374
Palladium hammered,	10,659	10,289			
Platinum hammered,	16,224	15,647			
" annealed,			15,518	14,178	12,964
Copper hammered,	13,052	12,200			
" annealed,			10,519	9,827	7,862
Iron-wire (ordinary),	17,743	18,613		19,995	
" annealed,			20,794	21,877	17,700
Steel-wire annealed blue,	17,690	18,045		18,977	
English steel-wire annealed,			17,278	21,292	19,278
Cast-steel annealed,			19,561	19,014	17,926
Berlin-brass hammered,	9,782	9,005			

An inspection of this table shows that the coefficients of elasticity of the metals, generally diminish as the temperature rises from 1° to 392° F. But for iron and steel, the coefficients augment up to 212° F., and then diminish, until at 392° F., they have the same tenacity as at common temperatures.

Wertheim has also determined by experiment that the coefficient of elasticity in the metals is increased by all means which produce an increase of density, and decreased by the contrary means. He found, also, that the passage of an electric current in a conductor momentarily diminishes this coefficient, independent of the alteration of temperature produced by the electricity. In alloys the coefficient is nearly the mean of the coefficients of the several metals compounded, even when there is a difference of bulk between the mass of the alloy and the sum of its ingredients.

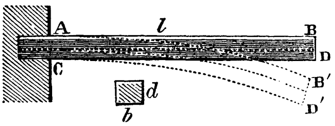
162. **Elasticity of flexure.**—Let A B, fig. 122, be a rectangular beam, fixed horizontally by one of its ends. If the free end of such a beam is acted on by any force tending to bend it in the direction B D′, the bar will, in virtue of its elasticity, return again to its horizontal position when the flexing force ceases to act, after performing a certain number of oscillations.

The elasticity of flexure is due chiefly to the united effect of the elasticity of tension and compression. For the molecules on the upper side of the curve are extended, while those on its under side are compressed, and the united effort of these two forces—which are equal—is to restore the beam to its first position. The conversion of a straight line into a curve, as in this case, is also accompanied by a disturbance in the equilibrium of the molecules, independent of the change due to their separation and compression; and such a change develops elasticity.

---

* The rods employed in these experiments had each a section of one square millimetre, and the values of K are the weights, in kilogrammes, that would be required to stretch the rods to double their original length. These values, in general, greatly exceed the limit of elasticity.

122



The laws of elasticity of flexure are comprised in the following formulæ:

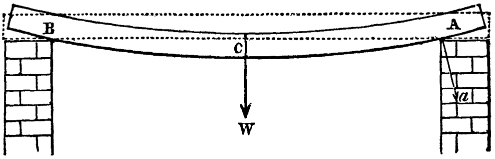
$$(1.)\quad a = \frac{Wl^3}{Dbd^3} \text{ or } W = \frac{Dabd^3}{l^3}.$$

In which a is the arc $B\,B'$, described by the flexure; W, is the flexing weight acting in a perpendicular; b, the horizontal breadth of the bar; d, its thickness; l, the length of the bar, and D, a constant quantity, depending on the nature of the substance used. If in the above we make each of the quantities a, b, d, and l, equal unity, it follows that $D = W$, or D, is a weight which will bend a given bar one unit long, and of a given diameter (say one centimetre), through a unit of arc (say one degree). This quantity, D, is called the *coefficient of elasticity of flexure*, and in any case the value of a, b, d, and l, being known experimentally, the value of D is readily ascertained by calculation.

If a beam is supported at its two extremities, A, B, fig. 123, and the weight is applied in the middle, the formula becomes

$$(2.)\quad W = \frac{16Dabd^3}{l^3},$$ where a is the flexure and l the distance from the supports.

123



The following laws are deduced from the first formula:

1. *The displacement of the free end of the bar is proportional to the load.*

This is equally evident from the experiments of Coulomb and from an analysis of the isochronism of the oscillations accompanying the effort to restore the equilibrium.

2. *The load requisite to produce a given flexure is proportional to the breadth of the beam or bar.*

This is evident if we consider a beam two or three times as broad, composed of two or three separate beams, each requiring the same load as the first bar to flex it through the same arc.

3. *The load is also proportional to the cube of the depth or thickness of the bar.*

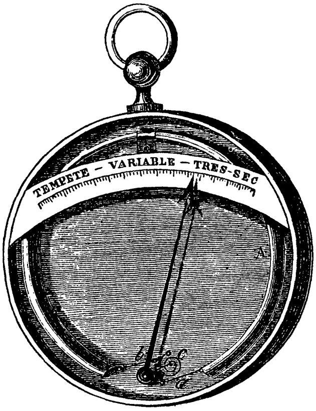
4. *The load is in the inverse ratio of the cube of the length of the bar.*

If the section of the beam is not a rectangle, with one side perpendicular to the direction of the flexing force, these laws cannot be directly applied. It is assumed in all the cases that the bar returns to its first position when left to itself; or, in other words, that the pressure has not exceeded the limit of elasticity.

Applications.—Constant use is made of the elasticity of flexure. The dynamometer of Reynier has already been named (37). Springs of all kinds, for balances, carriages, time-pieces, bows, &c., employ this agency. The aneroid barometer of Vidi, and the metallic manometer and thermometer of Bourdon are familiar and most useful applications of this force.

163. **M. Bourdon's metallic barometer.**—M. Bourdon, of Paris, has applied the principle of elasticity of flexure to the construction of a metallic barometer, which, with great simplicity of construction, has all the advantages of the aneroid. The essential part of the instrument, fig. 124, consists of a very thin and elastic brass tube, A, bent into the form of an arc of a circle, whose cross section is a flattened ellipse, with its longer diameter perpendicular to the plane of curvature. This tube, exhausted of air, and hermetically closed, is attached only at its centre, so that the ends are free to move. With a diminished atmospheric pressure, the ends separate from each other. If the atmospheric pressure increases, the ends come nearer together. By means of the metallic wires, *a*, *b*, and the spring, *c*, these movements of the ends of the tube are communicated to a needle moving over a graduated plate.

124



The same principle Bourdon has applied to the construction of manometers for locomotives and other steam-boilers, which are now extensively used in all countries.

164. **The aneroid barometer.***—The construction of this instrument, invented by Vidi, of Paris, depends upon the elasticity of flexure. Being of small size, and containing no mercury, it is very portable, and it gives results sufficiently accurate for all ordinary purposes. It consists of a circular copper box, the cover of which is very thin, and hermetically sealed, after the air is partly exhausted from its interior. This chest is contained in an outer case, fig. 125, about four inches in diameter, and which has a dial-plate like that of a watch. Variations in the pressure of the atmosphere will cause the cover of the exhausted box to move with the change of tension. By means of a combination of levers and springs, the movements of the centre of this cover are communicated to a pointer which moves over the graduated plate.

125

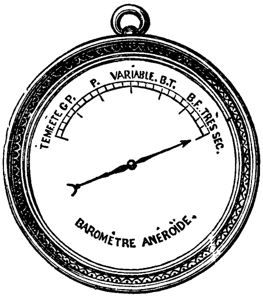
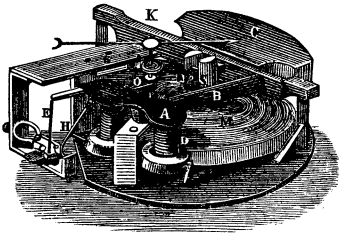


Fig. 126 shows the interior construction of this instrument. To the cover M of the exhausted box, are attached two uprights, S, which act upon a lever, P, by means of a pin uniting them. This lever, P, is attached to a bar, moving freely on two pivots placed at its extremities. A lever, B, unites the bar, K, to the plate, A, pressing on two springs, D. By means of a spring, represented on the side of the figure, the rod E, in connection with A, communicates movement to the bent lever, H, causing a metallic wire to uncoil itself from the axis, O, of the pointer, thus transmitting to it the movement.

126



Excellent aneroid barometers are now made at Lebanon Spa, N. Y., by E. Kendall, at a moderate cost.

The theory of the barometer and the mode of observing atmospheric pressure with the aneroid barometer, is explained in the chapter on gases.

165. **Elasticity of torsion.**—When a metallic rod or wire is twisted by a force applied at one extremity, while the other remains fixed, it has a constant tendency to return to its first position, and if the force

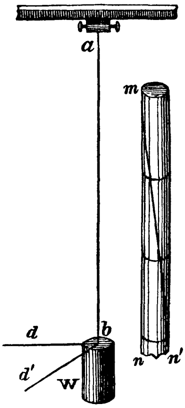
---

* Aneroid is derived from the Greek alpha (*α*), privative, and *ἐρέω*, to flow (a barometer without a fluid), in allusion to the absence of quicksilver.

is withdrawn, when left to itself, the wire makes a number of oscillations before it comes to a state of rest.

We can easily see how torsion is developed from elasticity. Let *a b*, fig. 127, be a metallic wire, made tense by a weight, W, and twisted by a force applied at *d*, acting in a circle of which *d b* is the radius. Let *m n* represent an enlarged view of a row of molecules on the surface of the wire parallel to the axis. If the length of the wire remains unchanged during torsion, and the line *m n* takes the position of the spiral *m n'*, it is evident that the distances between the molecules in this line must be increased. The elasticity of torsion, therefore, depends upon the force with which the particles tend to preserve their respective distances from each other. By the same force with which the molecules on the surface of the wire tend to resist separation, the molecules in the axis of the wire are compressed, and there is a tendency to diminish the length of the wire. Torsion, therefore, tends to separate the molecules on the surface of the wire, and to compress those situated in the axis.

127



The *angle of torsion* is the angular distance, *d b d'*, through which the movable end of the wire is rotated about its axis. The *force of torsion* is the power applied at the extremity of a lever whose length is unity, placed perpendicular to the axis of the wire, to produce the deviation indicated by the angle of torsion; this force is called the *coefficient of torsion*.

166. **Coulomb's laws of torsion.**—For our knowledge of the laws of torsion we are indebted to Coulomb, who has reduced these laws to the following formula:

$$(1)\quad t = \pi a\sqrt{\frac{W}{2gf}};\quad \text{or, (2)}\quad f = \frac{\pi^2 a^2 W}{2gt^2}.$$

When a cylindrical weight, W, is suspended to a wire, as shown in fig. 127, so that its axis corresponds with the axis of the wire, W is the suspended weight, *a* its radius, *g* the accelerating force of gravity (71), *f* the coefficient of torsion for the extended wire, and *t* the time of an oscillation when the force of torsion is removed, and the wire is left free to vibrate. The following laws were deduced by Coulomb from the preceding formula:

(1.) *The force of torsion is proportional to the angle of torsion.*

To prove this law, Coulomb caused the weight to oscillate around its axis by the torsion of the wire, and found that the times of oscillation were the same whatever their amplitude. This result corresponds with the formula in which the time of oscillation is independent of the amplitude.

Based upon this law, Coulomb invented a very delicate *torsion balance* which bears his name. This instrument will be described when speaking of its use in electrical experiments (820).

(2.) *The force of torsion remains the same whatever may be the tension of the wire.*

Experiments prove that the squares of the times of oscillation are proportional to the weights employed, whence it follows that $\frac{W}{t^2}$, in formula (2), is constant whatever may be the value of W, therefore it is evident from the formula that the coefficient of torsion, f, is constant, and that the force of torsion agrees with the preceding law.

(3.) *The coefficient of torsion is inversely proportional to the length of the wire.*

Experiments prove that the square of the time of oscillation is proportional to the length of the wire, and the formula shows that f is inversely proportional to t^2, therefore it must also be inversely proportional to the length of the wire.

(4.) *The coefficient of torsion is proportional to the fourth power of the diameter of the wire.*

According to experiment, the time of oscillation is inversely proportional to the square of the diameter, and the formula shows that the coefficient of torsion is inversely proportional to the square of the time of oscillation, hence the coefficient of torsion is proportional to the fourth power of the diameter of the wire.

167. **Torsion of rigid bars.**—Savart found by experiment that the laws of Coulomb, which had been previously determined for flexible wires of cylindrical form, were equally applicable to rigid bars of brass, copper, glass, or wood, whether the sections were circular, square, rectangular, or triangular, provided that comparisons were made only between bars of the same form.

More recently Poisson has demonstrated these laws, in case of cylindrical rods, by means of the calculus, and M. Cauchy has obtained the same result by the calculus for bars having a rectangular section.

168. **Limit of elasticity.**—When a wire or rod has been stretched by a weight which is very great in proportion to the diameter of the wire or rod, the elongation and the diminution of diameter do not entirely disappear when the tension is removed. The bar is then said to have been *forced*, or to have been stretched beyond its *limit of elasticity.* Similar effects are seen when elasticity has been developed by compression, flexion, or torsion.

These results are explained by supposing that the molecules composing the wire or rod have been forced into new relations with each other, so that elasticity no longer acts on all the particles in the same direction as before, and therefore a permanent change of form is developed. It follows, therefore, that after a rod or wire has thus been forced, there should be a new state of elasticity similar to the first; and such experiment shows to be the case.

If a degree of tension, sufficient to produce permanent elongation, acts for a long time, a rod will be gradually drawn out into wire.

M. Vicat has observed a wire, placed where it was free from any sudden shock, extended by a weight exceeding its limit of elasticity (equal to about one-third what would be required to produce instantaneous rupture), and which continued to be elongated for years without attaining its limit of extension.

Thin plates of glass or steel placed obliquely, or supported only at the ends, will, after a time, contract a permanent curvature.

The limit of elasticity is rapidly diminished by heat.

At temperatures of 59° F., 212° F., and 392° F., Wertheim found that the limits of elasticity for copper varied as the numbers 3, 2, 1; and for platinum as 14½, 13, 11⅓. Annealing diminishes the limits of elasticity, but Wertheim found that a temperature of 392° F. made no sensible difference in the elasticity of those metals which had been previously annealed.

169. **Change of density produced by tension.**—In general, metals that are forced by excessive tension increase in density by a lateral approach of their molecules, but the contrary effect is produced by tension in bars of iron or lead. Annealing restores the density of metals which have been forced by tension.

§ 4. Strength of Materials.

170. **Laws of tenacity.**—The absolute strength or tenacity of a body is its power of resisting a force applied in the direction of its length, and tending to draw it asunder. The following are the laws of tenacity:

1st. *The tenacity of a bar, or rod, or the resistance it is able to sustain, is proportional to the area of its transverse section.*

2d. *The tenacity is independent of the length of the bars.*

The resistance which a rod can sustain is evidently proportional to the transverse section of the body, for the cohesion of two, three, or four times as many particles must be destroyed, if the area of the section is increased two, three, or four times. If a wire supports a certain weight, two such wires, or one of double size of the same quality, will support a double weight. Tenacity is not modified by length, except that the probability of casual defects increases with the length. Tenacity is measured experimentally by securing one end of the body to a fixed point, and hanging gradually increasing weights to the other, until it is broken. The breaking weight measures the absolute strength. To compare the strength of different bodies, we must assume a *unit of area;* the one usually chosen is one square inch.

The following table gives the absolute strength of some of the more important bodies, expressed in pounds, for one square inch area of the transverse section.

1st. Metals:—

Steel untempered,	110,690—127,094
" tempered,	114,794—153,741
" cast,	134,256
Iron, bar,	53,182— 84,611
" wire,	58,730—112,905
" cast,	16,243— 19,464
Silver, cast,	40,997
Copper, "	20,320— 37,380
Brass, "	17,947— 19,472
" wire,	47,114— 58,931
Gold,	20,490— 65,237
Tin, cast,	4,736
Zinc,	2,820
Lead,	887— 1,824

2d. Woods:—

Sycamore,	9,630
Birch,	12,225
Elm,	9,720—15,040
Larch,	12,240
Oak,	10,367—25,851
Box,	14,210—24,043
Ash,	13,480—23,455
Pine,	10,038—14,965
Fir,	6,991—12,876

3d. Cords:—

Hemp twisted, $\frac{1}{4}$ to 1 inch,	8,746
" " 1—3 "	6,800
" " 3—5 "	5,345
" " 5—7 "	4,860

Wrought metals are more tenacious than cast, and alloys are sometimes stronger than either of their constituents. The strength of metals, as a rule, diminishes as they are heated; and sudden, frequent, and extreme changes of temperature always impair tenacity.

Johnson's results.—From an extensive series of carefully conducted experiments, the late Professor Walter R. Johnson ascertained that, if either bars or plates of malleable iron are subjected to a high degree of tension, whilst heated to 550° or 600° F., and are then gradually cooled, the maximum tenacity of the iron is sensibly increased over fifteen ($15\frac{17}{100}$) per cent. From the maximum thus obtained, the tenacity gradually diminishes by heating, but the tenacity will remain greater than before the first heating, unless the temperature is raised above 700° F. (Report to Franklin Institute on strength of materials for steam-boilers, 1837.)

The strength of cords is in proportion to the fineness of the strands, and also to the fineness of the flax or hemp fibres of which the strands consist. They are weakened by overtwisting. Damp hempen cords are stronger than dry ones, twisted than spun, tarred than untarred, and unbleached than bleached. Silk cords are three times stronger than those of flax.

Tenacity of vegetable and animal substances.—Woods are subject to great inequalities. Trees grown on mountains are much stronger than those of the same kind from the plains.

Animal and vegetable substances, converted from the liquid to the solid state, as gums, varnish, glue, &c., possess extraordinary strength. Rumford found that a solid cylinder of paper, glued together, whose sectional area was one square inch, would support 30,000 lbs.; and a similar cylinder of hempen strings, glued together lengthwise, supported 92,000 lbs.—a tenacity greater than that observed in iron.

171. **Resistance to pressure in columns.**—The resistance of a

column to a vertical force which tends to crush it, depends on its form, its sectional area, and its height. Of two columns of the same material, having the same form and equal heights, the one which has the larger sectional area will be the stronger, but the exact ratio of increase in strength is unknown.

According to Euler: When the base remains the same, the strength of a column diminishes as the square of the height; that is, when the height is trebled, the strength is diminished nine times.

The resistance of a right prism is in the inverse ratio of the square of the height, and directly as the width, and the square of the thickness.

A prism, whose base is a parallelogram, has less strength than one of the same height and volume, whose base is a square; and the latter less than a cylinder of equal height and volume.

A solid cylinder resists less than a hollow one of equal height and mass; and lastly, a solid cylinder less than an equivalent cone. A column of one piece is stronger than one composed of several.

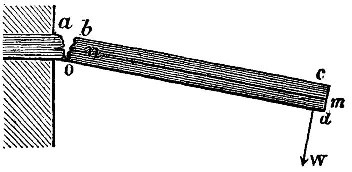
Solids do not offer the same resistance in all positions: stones in the position of their natural bed are stronger than when placed otherwise; and wood is stronger in the direction of its fibres than across them.

The strength of rectangular columns is directly as the product of the longer side of the section into the cube of the shorter side, and inversely as the square of the height.

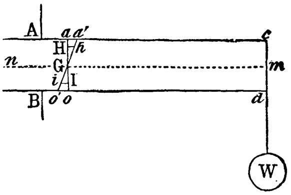
172. **The lateral or transverse strength of materials** *is their power to resist a breaking force applied at right angles to their length.*

Let *a b c*, fig. 128, be a beam secured at one end, and supporting at the other extremity a weight, W, acting at right angles with its length. It is evident that while the suspended weight tends to produce extension and rupture at the upper surface, *a*, the particles at the opposite or under surface, *o*, will be compressed. Between these two points there will be a certain plane, *m n*, called the *neutral axis*, where there is neither extension nor compression.

128



Suppose the power of the beam to resist compression is the same as its power of resisting extension, then the neutral axis will divide the transverse section into two equal parts, an area of compression and an area of extension. When the fibres on the surface *a* are extended to their limit of tenacity, the fibres at *o* will be compressed with an equal force, while no force is exerted on the neutral axis, therefore the entire force required to be overcome to produce rupture is equal to one-half the longitudinal tenacity of the beam. If *t* represents the absolute tenacity of a unit of area (See Table, § 170), *b* the breadth of the beam, 128 *a*



and d its depth, then the resistance to be overcome, $R, = \frac{1}{2} tbd$. The effect of the breaking force tends to turn the section $a\,o$ about the neutral axis. The sum of all the extending forces, H h, fig. 128 *a*, will be represented by the area of the triangle $a\,\mathrm{G}\,a'$. These forces act at different distances from the neutral axis, G, but their entire effect will be the same as if they were all concentrated at the centre of gravity of $a\,\mathrm{G}\,a'$, which is at a distance from G equal to $\frac{2}{3}\,a\,\mathrm{G} = \frac{1}{3}\,a\,o = \frac{1}{3}\,d$. The statical moment of the extending force will therefore be

$$\tfrac{1}{4} tbd \times \tfrac{1}{3} d = \tfrac{1}{12} tbd^2.$$

The statical moment of the compressing force is also $\frac{1}{12} tbd^2$. Hence the sum of the moments of the statical forces opposed to fracture is $\frac{1}{6} tbd^2$. To overcome this moment of resistance, the weight, W, acts at the end of the beam, the length of which we represent by l; then, since at the moment of fracture the statical moment of the weight must equal the statical moment of the resistance, we shall have

$$Wl = \tfrac{1}{6} tbd^2, \quad \text{or,} \quad W = \frac{tbd^2}{6l}.$$

The lateral cohesion of the beam prevents the different laminæ from sliding on each other, and thus tends to prevent fracture, but this element of strength is neglected in the preceding analysis.

To find the weight required to break a beam supported at one end, we have the following—

RULE: *Multiply the absolute tenacity of a beam of the same dimensions, by its depth, and divide the product by six times the length; the quotient is the weight suspended at the extremity required to break the beam.*

Practical applications.—To apply this rule to practical purposes, it is necessary to take into consideration the weight of the beam itself. This weight may be considered as acting at its centre of gravity, consequently the strain produced by it will be only half as much as if it acted at the extremity of the beam, we must therefore subtract from the breaking weight one-half the weight of the beam. Calling the weight of the beam w, the formula becomes,

$$W = \frac{tbd^2}{6l} - \frac{w}{2}.$$

If we would estimate the load which a beam can sustain without danger of breaking, we must consider that beams, of whatever material they may be constructed, are liable to be more or less imperfect. To afford security from accident, it is customary to estimate the working load as only $\frac{1}{3}$, $\frac{1}{4}$, or, in some cases, only $\frac{1}{10}$ part what would be required to produce fracture.

For a cylindrical beam supported at one end, the breaking weight, diminished by the weight of the beam itself, is

$$W = \frac{t\pi r^3}{4l} - \frac{w}{2}, \quad r \text{ being the radius of the beam.}$$

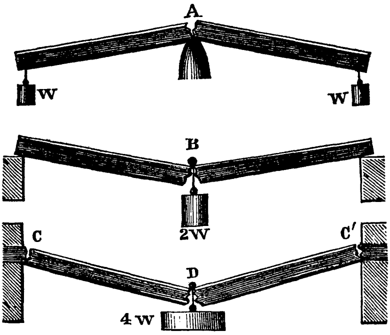
For a tube whose external and internal radii are r and r', the breaking weight will be

$$W = \left(\frac{t\pi r^3}{4l} - \frac{w}{2}\right) - \left(\frac{t\pi r'^3}{4l} - \frac{w'}{2}\right) = \frac{t\pi}{4l}\left(r^3 - r'^3\right) - \frac{w - w'}{2}.$$

For a rectangular tube supported at one end,

$$W = \frac{t}{6l}\left(bd^2 - b'd'^2\right) - \frac{w - w'}{2}.$$

129



If a rectangular beam is supported at the centre, and the weight is divided into two parts, resting upon opposite ends of the beam, as shown A, fig. 129, we must replace W, w, and l in the preceding formula by ½W, ½w, and ½l. Making these substitutions, and reducing:

$$W + \frac{w}{2} = \frac{4tbd^2}{6l}; \text{ or,}$$

The weight which a beam can support when it rests upon its centre, and the weight is equally divided at its extremities, is four times as great as if the beam were supported only at one end, and the weight was suspended from the other end.

One-half of the beam is included as a part of the breaking weight.

If the beam is supported at its two extremities, and the weight is suspended at the centre, as shown at B, fig. 129, the breaking weight will obviously be the same as the sum of the two weights in the preceding case.

If the beam is secured at both extremities, as shown at D, fig. 129, and the weight is placed at the centre, three fractures are to be produced simultaneously. To produce the fracture at the centre separately will require the same weight as in the preceding case, and the fractures at C and C′ will each require one-half as much more force, while one-half of the weight of the beam is to be included Therefore,

$$W + \frac{w}{2} = \frac{8tbd^2}{6l}.$$

General estimate of the strength of beams.—If the weight is evenly distributed over the whole beam, it will support twice as much as if the whole pressure were placed in the centre. If a rectangular beam has two or three times the breadth of another, the depth and length being the same, it will have two or three times the lateral strength; and if the length is increased two or three times, other things being equal, the power of suspension will become one-half or one-third respectively. When the length and breadth remain the same, the strength increases with the depth, but in a higher proportion. A beam having the same length and breadth as another, but twice or three times the depth, will bear a four or nine times greater weight. A thin board or beam is therefore much stronger when placed on its edge than on its side; on this principle, the rafters and floor timbers of buildings are made.

In round timber, the power of suspension is in proportion to the cubes of the diameters, and inversely as the length; a cylinder whose diameter is two or three times greater than that of another, will carry a weight 8 or 27 times heavier. The lateral strength of square timber is to that of the tree whence it is hewn as 10 : 17 nearly.

The strongest rectangular beam that can be sawn from a piece of round timber is one whose breadth is equal to the square root of one-third multiplied by the diameter, and its depth is equal to the square root of two-thirds multiplied by the diameter.

A tube is the form which combines the least weight of materials with the greatest lateral strength. Galileo was the first who remarked that the bones of animals, the quills of birds, the stalks of plants which bear a heavy weight of seed at their summit, and other similar hollow cylinders, offer a much greater resistance than solid cylinders of the some length, and constructed of the same quantity of matter.

A round tube whose external and internal diameters are to each other as 10 : 7 has (according to Tredgold) twice the lateral strength of a solid cylinder containing the same amount of material.

A rectangular tube, whose height is considerably greater than the breadth, will sustain a greater amount of lateral pressure than a hollow cylinder of the same thickness, and containing the same amount of material, because a much greater amount of material is placed at a remote distance from the neutral *axis*. Hollow rectangular beams of iron are used in architecture, and the same form of construction was selected by Stephenson for the Britannia and Victoria Bridges.

The Britannia Tubular Bridge, across the Menai Straits, is an immense rectangular iron tube, or corridor, 17 feet wide, 22 feet high at the ends, 30 feet at the centre, and 1834 feet long, through which the English Western Railroad passes from Wales to the island of Anglesea, 104 feet above high water mark. Two of the openings between the piers are each 460 feet, and the other two each 230 feet. The ordinary pressure of the railroad trains which pass this bridge produces a depression of merely one-eighth of an inch, or less; discernible only by the aid of instruments.

The Victoria Tubular Bridge, for the passage of the Grand Trunk Railway across the St. Lawrence, at Montreal, is 6600 feet, or a mile and a quarter in length. It has 24 openings of 242 feet each, and a centre span of 330 feet. The abutments are 36 feet, and the central piers 60 feet above summer water. The breadth of these immense iron tubes is 16 feet, the height at the ends of the bridge 19 feet, and at the centre 21 feet 8 inches. The tubes are constructed of boiler iron, varying from $\frac{1}{4}$ to $\frac{1}{2}$ an inch in thickness, strongly braced with lateral irons placed at distances of from 3 to 6 feet. The cost of this stupendous bridge was $7,000,000.

173. **Limits of magnitude.**—The materials of all structures must support their own weight, and therefore their available strength is the *excess* only of their absolute strength above what is necessary to support themselves.

When *all* the dimensions of materials are increased, the absolute strength augments as the square of the ratio of increase, but at the same time, the weight of the materials augments as the cube of the increase.

If the dimensions of a beam are doubled, it is four times stronger, and eight times heavier; or if its magnitude is multiplied 4 times, its strength will be multiplied 16 times, and its weight 64 times.

In consequence of unequal ratio in increase, the strength of a structure of any kind cannot be estimated from its model alone, which is always much stronger in proportion to its size than the structure. In enlarging a structure, a limit is soon reached at which it has no available strength, its total absolute strength being required to support itself; and if this limit is passed, it will fall to pieces by its own weight.

All works, natural and artificial, have such limits of magnitude which they cannot surpass while their materials remain the same.

In conformity to this principle, small animals are stronger than large ones, and insects and animalculæ are capable of feats of strength and agility, which seem miraculous when translated into the proportions of man. The operation of the same law may be seen by comparing the unwieldy movements of the elephant with the lithe and active tiger, or the easy motion of song-birds, and the arrowy swoop of the hawk, with the laborious and measured flight of the swan, and the condor of the Andes. For the same reason, the gigantic saurians, whose bones are mentioned by geologists, had their home in the ocean, where, in modern times, are found sea-weeds of interminable length, and animals, whose ponderous mass would be incapable of motion, if they were not floated buoyantly by the element they inhabit.

§ 5. Properties of Solids depending on a permanent displacement of their Molecules.

174. **Malleability**, or the property of being wrought under the hammer, belongs to many of the metals in an eminent degree, and upon it their utility in a great measure depends.

The following is the order of malleability of the principal metals, when extended under the hammer; viz., lead, tin, gold, zinc, silver, copper, platinum, iron. This order represents the facility of extension depending on relative hardness, but gold may be beaten thinner than any other metal (compare § 19).

The property of extending into plates in the rolling-mill is somewhat different from the facility of extending under the hammer, and is possessed by the metals in the following order; viz., gold, silver, copper, tin, lead, zinc, platinum, iron. Malleability varies with the temperature.

Iron is most malleable when it first attains a white heat, and in that state huge masses of it are taken from the furnace to be forged, the metal yielding like wax to the pressure of the rolling-mill, or the blows of the hammer. Zinc is most malleable at 300° or 400°, and lead and copper when they are cold. Glass, which is very brittle when cold, becomes malleable at a high temperature. Gold may be hammered into leaves so thin that a million of them are less than an inch thick. Metals lose their malleability by constant hammering, but recover it again by being heated and slowly cooled—a process called *annealing*.

175. **Ductility**, or the property of being drawn into wire, must not be confounded with malleability, for the same metals are not always both ductile and malleable, or do not possess these properties to an equal extent. In general, ductility increases with the temperature.

The following is the order of ductility in the principal metals; viz., platinum, silver, iron, copper, gold, zinc, tin, lead.

Iron may be drawn into the finest wire, but it cannot be rolled into plates of proportional thinness. Tin and lead possess these qualities in the reverse order.

176. **Hardness** has no relation to density, or the number of particles within a given space, but depends only on the nature of the particles, their mutual arrangement, and cohesion.

The metals may be scratched by glass, which is far lighter than most of them, and among metals, density is not connected with relative hardness. Alloys are often harder than either of their constituents, and some metals, as steel, may have their hardness modified by heat at pleasure.

The following table gives the scale of hardness used by mineralogists, commencing with *talc*, the softest crystalline solid, and ending with *diamond*, which is esteemed the hardest, since it cuts all other bodies, but cannot be cut by any but itself.

Deg.	Substance.	Deg.	Substance.
1	Talc.	6	Feldspar.
2	Gypsum.	7	Quartz.
3	Calc spar.	8	Topaz.
4	Fluor spar.	9	Sapphire.
5	Apatite.	10	Diamond.

The terms *hard* and *soft* are seen by an inspection of this table to be entirely relative, since each succeeding body is harder than the one preceding, and vice versa, the extremes only being respectively softer and harder than all others.

We employ hardened steel to cut wood, and even iron; emery (the rough sapphire) is required to cut and polish steel and glass. The diamond, set in a staff of metal, is an efficient tool for cutting plates of glass into any required size. Even the hardest rocks, as porphyry and jasper, are readily turned into any required form in the lathe, by the use of a diamond properly set as a turning tool.

177. **Brittleness.**—Bodies which are easily broken in pieces and pulverized are said to be *brittle.* Such are hard bodies generally, and also many highly elastic substances.

178. **Hardening; Temper; Annealing.**—When certain metals are heated to redness or to a higher temperature, and then are suddenly cooled, by plunging into cold water, oil, or mercury, they become hard, brittle, and more elastic than before. This process is called *hardening.*

The effects of this method of hardening are most important in the case of steel, since it is in virtue of this quality that its application to a great variety of purposes depends.

When steel is raised to a high heat and slowly cooled, it becomes soft, ductile, flexible, and much less elastic than before. This process is called *softening* or *annealing,* although the latter term is more frequently employed to denote a similar process of removing the hardness produced by hammering, and other mechanical means.

Softened steel may be hammered, rolled into sheets, drawn into wire, or wrought into any form required in the arts. If hardness or great elasticity is required, the steel is then heated to redness and plunged into cold water, oil, mercury, or some other fluid, by which it is rapidly cooled. If, as is generally the case, it is then too hard for the use to which it is to be applied, a portion of the hardness is removed by what is called *drawing the temper*, by heating to a lower temperature, and allowing the article to cool gradually. The proportion of hardness removed depends on the temperature to which the articles are heated. This process of reheating and cooling, by which the degree of hardness is modified to suit any special purpose, is called *tempering.*

Colors of tempered steel.—The workmen carefully observe the color of the steel, as it is reheated, and determine, from the tint, when a degree of heat is obtained sufficient to produce the temper desired. The tints which correspond approximately to the different temperatures are as follows:—

Light straw,	428° F.	Violet yellow,	509°	Blue,	560°
Golden yellow,	446°	Purple violet,	530°	Deep blue,	603°
Orange yellow,	464°	Feeble blue,	550°	Sea-green,	626°

Dies used in coining require to be made of the hardest steel. Files have but a very little of the hardness removed by tempering. Razors and fine cutlery are reheated to a pale yellow; penknife blades to a light straw color; table cutlery, where flexibility is required more than hardness, is reheated to violet; watch-springs to a full blue; coach-springs to a deep blue.

Tempering by a bath.—In many manufactories the requisite heat for tempering is obtained by immersing the hardened steel articles in a bath of metallic alloys, mercury, or oil, the temperature of which can be exactly regulated by a thermometer. The articles to be tempered are placed in the bath, which is then heated to the required temperature, and then allowed to cool slowly. In this way a great number of articles of the same kind can be made to assume a uniform temper at small expense.

It is a remarkable property of steel that when it is heated to a temperature that allows it to begin to harden, by rapid cooling it receives its full degree of hardness. It cannot be partially hardened at any lower temperature. A bar of steel, heated at one end while the other remains cold, will be found, after rapid cooling, hardened at one end, with a sharp line of demarkation between the hard and soft parts. Where it has been heated to a temperature insufficient for hardening, and is then rapidly cooled, it is sensibly softened.

Temper of glass.—Glass undergoes the same changes by tempering as steel. It becomes, by rapid cooling, more brittle, harder, and less dense. A specimen of glass, examined by Chevandier and Wertheim, having a density of 2·513, acquired by annealing a density of 2·523. For hardening glass it is sufficient to allow it to cool rapidly in the open air, by moving it about. Glass-ware is annealed by passing it slowly through a very long oven, called a "*leer,*" the end where it enters being nearly a red heat, and the other extremity nearly cold. Without the process of annealing, glass utensils would be almost worthless.

Prince Rupert's Drops.—A beautiful illustration of the properties of unannealed glass may be seen in the scientific toy called Prince Rupert's Drops, or Dutch Tears. They are made by dropping melted glass into water, by which means it is suddenly cooled and becomes very hard and brittle. They have a long oval form tapering to a point at one end, fig. 130. The body of these glass drops will bear a smart stroke from a hammer without breaking, but if a portion of the smaller end is broken off the whole mass will be broken into an almost impalpable powder, with a violent shock. The Bologna vial is another similar example.

130

Tempering copper and bronze.—Most metals are acted on by heat and cold in the same manner as steel, but to a less extent. Copper is a remarkable exception, since its properties in this respect are exactly the reverse of those manifested by steel. When copper is rapidly cooled it becomes soft and malleable, but when it is slowly cooled it becomes hard and brittle.

Bronze, which is an alloy of copper and tin, undergoes, by change of temperature, the same changes as copper, but in a more remarkable degree.

A recent fracture of bronze, which has been rapidly cooled, presents a yellow color; but after it has been slowly cooled the color is a brilliant white, like pure tin. It is thus evident that hardening and annealing cause different arrangements of the particles of copper and tin of which the bronze is composed.

179. **Hammering.**—By hammering, the molecules of many bodies are brought nearer to each other, so that their density is increased. By rolling, wire-drawing, extension, compression, bending, twisting, or any mechanical means by which the limits of elasticity are passed, changes are effected similar to those produced by hammering.

Lead yields under the hammer or rolling-mill without increasing its density, but its density may be increased by compressing in dies, or in any situation where it has no room to spread out under the action of the compressing force.

By such mechanical means the physical properties of solid bodies undergo changes analogous to those produced by tempering. They become dense, tenacious, hard, brittle, and their limit of elasticity (168) is increased, although their elastic force is unchanged, as was shown by Coulomb, from the fact that their vibrations in either condition are accomplished in the same time.

Annealing restores the metals to the same condition as before they were submitted to mechanical force. Iron and platinum require to be frequently annealed during the process of drawing into wire.

The ancients gave to the bronze plates of their armor the necessary hardness by hammering.

When zinc and iron have been hardened by rolling, the tenacity and elasticity are not the same in both directions of the plates, and rolling does not increase the tenacity in so great a degree as drawing into wire. M. Navier found that a rod of forged iron, having a section of one square millimetre, required to break it a weight of 40 kilogrammes. When it had been rolled into thin plates, a strip cut in the direction of its length, having the same sectional area, required a weight of 41 kilogrammes to break it; but only 36 kilogrammes if cut in a transverse direction.

Fine wire twisted into cables is used to support suspension bridges, because such cables are much stronger than the same amount of iron in the form of rods or coarse wire.

180. **Changes of structure, affecting the mechanical properties of metals**, take place spontaneously by the lapse of time, or more rapidly by the influence of heat or vibrations.

These changes have been principally observed in iron. When this metal is recently forged it is very strong and flexible, and its fracture is fibrous and of a dull color; but when it becomes old, or has been subjected to frequent vibrations, or changes of temperature, it becomes hard and brittle, and its fracture is coarsely granular, presenting many brilliant facets. This change, often seen in the axles of railway carriages, is produced by vibration combined with the heat of friction, especially in the severe weather of northern winters. These facts, well known to engineers upon railroads, explain the necessity of running trains at but moderate speed when the thermometer indicates a temperature near to zero.

Similar changes, diminishing the tenacity, take place in chains and anchors, which have been for a long time imbedded in ice. Gay Lussac has observed bars of iron, which became almost as brittle as glass by remaining for a long time at a high temperature in an oven.

Some curious results produced by vibration were discovered by Savart, who, having caused strips of glass and bars of metal, drawn into wire, to vibrate in the direction of their length, found that they gave out at first very indistinct and confused sounds, but on continuing the experiments for a long time this confusion gave place to clear and distinct tones, which were obtained with ease.

Brass wire strained like the strings of a piano, where it is exposed to atmospheric changes and constant vibration from currents of air, in a few months loses its tenacity and becomes completely friable, showing in its cross fracture a radiated structure.

Annealing produces the same effect, in this respect, as vibration. Substances cast in the form of plates do not resound well until after the lapse of several days. Sulphur, cooled in the form of a disc, does not at first give out a clear sound, but after a time it vibrates readily, and after the lapse of some months the sound emitted is very much changed. This proves that there has been some modification of structure.

§ 6. Collision of Solid Bodies.

181. **Motion communicated by collision.**—*When two solid bodies come into collision, the motion is redistributed through them, whether one or both bodies are in motion.*

1. If the bodies are soft and tenacious, or if the shock is not so

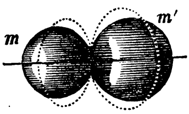
great as to overcome the tenacity of one or both bodies, they acquire a uniform motion, and move in contact as one body, as stated under impact, § 112.

2. If the bodies are hard and inelastic, and the collision takes place at only a small portion of the surfaces, and if the shock is so great as to overcome the tenacity of one or both bodies before motion is uniformly distributed through the masses, they will be broken in pieces, and the fragments scattered in different directions.

3. If the bodies are so elastic that motion may be distributed through the mass before the limit of elasticity is passed, a reaction will take place, and the elasticity will modify the distribution of motion.

182. **Direct impact of elastic bodies.**—When two elastic spherical bodies m and m', fig. 131, come into collision, while moving in the same straight line, they will undergo compression or flattening, as shown in the figure, and their centres will continue to approach each other until they acquire a common velocity (112). This velocity is represented by $x = \frac{mu + m'u'}{m + m'}$, u and u' being the velocities before impact, and x the common velocity of their centres of gravity at the instant of greatest compression. If the limit of elasticity has not been passed, the elasticity of the two bodies will now act as an internal force, causing their centres of gravity to recede from each other, until the bodies recover their original forms, when they separate, and the shock terminates.

131



183. **Modulus of elasticity.**—When two elastic bodies meet, the force with which they are urged towards each other is called the *force of compression;* the force of elasticity causing their centres to recede during the second interval of the shock is called the *force of restitution;* the ratio of these two forces is called the *modulus of elasticity.* When this ratio is unity, or the force of restitution is equal to the force of compression, the bodies are said to be *perfectly elastic.* When this ratio is zero, the bodies are said to be *inelastic.* For any value of this ratio, between these extremes, the bodies are said to be *imperfectly elastic.*

There are no bodies known that are perfectly elastic, or perfectly inelastic; hence, in considering the collision of solid bodies, it is necessary to take into account the degree of elasticity which they possess.

The following table exhibits the degree of elasticity of several common substances, perfect elasticity being taken as unity.

Substances.	Degree of elasticity.	Substances.	Degree of elasticity.
Glass, ...	0·94	Bell metal, . . .	0·67
Hard-baked clay,	0·89	Cork, ...	0·65
Ivory, ...	0·81	Brass, ...	0·41
Limestone, . . .	0·79	Lead, ...	0·20
Steel, hardened, .	0·79	Clay just yielding to the hand,	0·17
Cast-iron, . . .	0·73		
Steel, soft, . . .	0·67		

184. **Velocity of elastic bodies after direct impact.**—If two imperfectly elastic bodies, m and m', move in the same straight line with velocities u and u', u being greater than u', the first body will overtake and strike against the second with the same force as if the second body were at rest, and the first body were moving with a velocity equal to $u - u'$. The two bodies, being elastic, will suffer compression until they acquire a common velocity,

$$x = \frac{mu + m'u'}{m + m'}.$$

The velocity lost by m at this instant will be $u - x$, and the velocity gained by m' will be $x - u'$

Let e be the modulus of elasticity, and v and v' the velocities of the two bodies after they recover their original forms at the close of the second period of the shock.

Since the forces of compression and restitution are in proportion to the velocities they generate or destroy, the velocity destroyed in m by the force of restitution will be $e\ (u - x)$, and the velocity gained by m', by the force of restitution, will be $e\ (x - u')$.

Hence the whole velocity lost by m will be $(1 + e)\ (u - x)$, and the whole velocity gained by m' will be $(1 + e)\ (x - u')$. The velocity of m after impact will be

$$v = u - (1 + e)\ (u - x) = x - e(\ u - x). \qquad (a)$$

The velocity of m' after impact will be

$$v' = u' + (1 + e)\ (x - u') = x + e(x - u'). \qquad (b)$$

Substituting the value of x in the formula (a) and (b), and reducing, we obtain

$$v = \frac{mu + m'u'}{m + m'} - \frac{m'e(u - u')}{m + m'}. \qquad (c)$$

$$v' = \frac{mu + m'u'}{m + m'} + \frac{me(u - u')}{m + m'}. \qquad (d)$$

If the two bodies move in opposite directions before impact, we must make either u or u' negative in the preceding formulæ. If one of the bodies is at rest before impact, u' becomes zero. If $e = 1$, the formulæ represent the conditions of perfectly elastic bodies, and if $e = 0$, the formulæ will apply to inelastic bodies.

From the preceding formulæ we may deduce the following general conclusions:

1. *If two bodies are perfectly elastic, their relative velocities before and after impact are the same.*

Making $e = 1$ in (a) and (b), and subtracting the latter from the former, we have,—

$$v - v' = u' - u.$$

2. *If the bodies are perfectly elastic and equal, they will interchange velocities by impact.*

Making $m = m'$, and $e = 1$ in (c) and (d),

$$v = \tfrac{1}{2}(u + u') - \tfrac{1}{2}(u - u') = u'.$$

$$v' = \tfrac{1}{2}(u + u') + \tfrac{1}{2}(u - u') = u.$$

3. *By the impact of bodies, whether elastic or otherwise, no motion is lost.*

Multiplying (c) by m, and (d) by m', and adding and cancelling similar terms, we have

$$mv + m'v' = mu + m'u',$$

in which the first member of the equation is the sum of the momenta of the two bodies after impact, and the second member the sum of their momenta before impact.

4. *The velocity which one body communicates to another at rest, when perfectly elastic, is equal to twice the velocity of the former, divided by one plus the ratio of the masses of the two bodies.*

In formula (d) let $m' = rm$, $u' = 0$, and $e = 1$, we shall then obtain

$$v' = \frac{2u}{1 + r}.$$

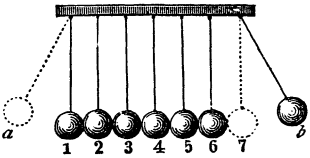
5. *When a body in motion strikes another equal body at rest, both bodies being perfectly elastic, the first body comes to a state of rest, and the second flies off with the previous velocity of the first.*

If the two masses are equal, $r = 1$ in the last formula, and $v' = u$.

Scholium.—If a series of perfectly elastic balls are arranged in a line, and all are in contact before impact, except the first, which is made to strike against the second, all the balls except the last will remain in contact, and at rest, after impact, and the last will move off with the velocity of the first.

185. **Transmission of shock through a series of elastic balls Experimental illustration of elasticity.**—If several equal ivory balls are suspended, as shown in fig. 132, when 1 is drawn back to *a*, and let fall against 2, only the last in the series, 7, will move; this will start with the velocity which 1 had at the instant of striking against 2, and it will fly off to the position *b*, at a distance equal to the limit to which the first had been drawn back; it will then return striking against 6, which will again be set in motion, all the others remaining at rest. This alternate movement of the extreme balls of the series will continue until friction, and the resistance of the air conspiring with the slightly imperfect elasticity of the balls, at length causes the action to cease.

132



Problems.—Elasticity of Tension.

74. A rectangular bar of iron whose transverse section is one square centimetre, and whose length is three feet, is suspended with its upper extremity attached to a firm support: what weight must be suspended at its lower extremity to cause a temporary elongation of one-fourth of an inch when the temperature is 14° F., 50° F., 212° F., 392° F.?

This problem may be solved for rods of any of the metals mentioned in the table (161).

75. A rectangular bar of hammered brass $2\frac{1}{2}$ feet in length, suspended by its upper extremity, supports a weight of 75 kilogrammes: how much will it be temporarily elongated when the temperature is raised to 50° F.?

Elasticity of Flexure.

76. If a beam 10 feet long, 3 inches wide, and 6 inches in depth, suffers a certain amount of flexure when one end is firmly supported in a wall, what weight will be required to give an equal flexure to another beam of the same material, 15 feet long, 6 inches wide, and 9 inches in depth?

77. What weight suspended at the middle of the last-mentioned beam will be required to produce an equal amount of flexure when the beam is supported at both ends?

Tenacity.

78. How many pounds weight, suspended by a steel wire hanging vertically, will be required to break it when the wire is one-fourth of an inch in diameter?

Calculate for both tempered and untempered steel.

79. In a pendulum experiment, it is required to suspend a weight of 64 lbs. by a copper wire. What must be the diameter of the wire, when, for the sake of security, $\frac{1}{5}$ is deducted from its strength as given by the table?

Transverse Strength.

80. If a beam of oak, 6 inches wide by 9 inches in depth, projects 20 feet

from a wall in which it is secured, what weight suspended at the end, as in fig. 128, will be required to break it, no account being taken of the weight of the beam itself?

81. What weight will be required to break a cylinder of cast iron 25 feet long, having an external diameter of 2 feet, and an internal diameter of 20 inches, the weight being supported at the extremity?

82. What weight, placed at the centre of the greater span of the Britannia Tubular Bridge, would be required to break it, calling the breadth of the tube 17 feet, height 30 feet, calculating for a single thickness, ¾ of an inch of plate iron, and estimating the tenacity equal to a force of 60,000 lbs. to the square inch?

83. Deducting from the weight found in the last problem, the weight of one-half the length of tube spanning the greater opening, and estimating the working load as ⅕ the breaking weight, how heavy a train might safely cross the Britannia Bridge?

84. How heavy a train may safely cross the Victoria Bridge (172), if the thickness of the tube at the centre is ½ an inch, and if, disregarding the weight of the tube itself, the train is limited to ⅙ the absolute tenacity of the structure?

85. What weight can be sustained at the middle of the strongest rectangular beam that can be sawn from a birch log 2 feet in diameter, and 30 feet long, reckoning the weight of the timber nine-tenths that of water?

Impact of Elastic Bodies.

86. Two glass spheres weighing 12 oz. and 7 oz. respectively, move in the same direction with velocities of 8 feet and 5 feet in a second. Find the respective velocities of the two balls after impact, and their common velocity at the instant of greatest condensation.

This problem may be varied by substituting for glass, balls made of each substance whose degree of elasticity is given in the table (183).

87. A perfectly elastic body m, moving with a velocity of 12, impinges on another perfectly elastic body, m', moving in the opposite direction with a velocity of 5; by impact m loses one-third of its momentum. What are the relative weights of the masses m and m'?

88. A body, m $(= 3m')$, impinges on m' at rest. The velocity of m after impact is ⅗ of its velocity before impact. Required the value of e, the modulus of elasticity.

89. Two bodies m and m', whose elasticity is ⅔, moving in opposite directions with velocities of 25 and 26 feet per second respectively, impinge directly upon each other. Find the distance between them 4½ seconds after impact.

90. A number of perfectly elastic balls are placed in a right line. The first is made to start with a given velocity; determine the ratio of the balls so that its momentum may be equally divided among the remainder.

91. An elastic ball falls from a height of 40 feet. How high will it rebound, supposing that one-fifth of the final velocity is lost at the impact in consequence of imperfect elasticity?

92. An ivory ball falls from an elevation of 100 feet. With what velocity must another similar ball be projected upward in the same vertical line, that after the two balls meet, the first ball may return to the same elevation from which it fell?

CHAPTER III.

OF FLUIDS.

HYDRODYNAMICS.

§ 1. Hydrostatics.

I. DISTINGUISHING PROPERTIES OF LIQUIDS.

186. **Definitions.—Fluids—Hydrodynamics.**—*Fluids* are bodies in which the attractive and repulsive forces (146) are, 1st, either in perfect equilibrium, producing *liquids* or *inelastic* fluids; or 2d, in which the repulsive force holds sway, producing *gases* or *elastic* fluids.

Hydrodynamics treats of the peculiarities of state and motion among fluid bodies, both liquids and gases (15). It is subdivided into *hydrostatics*, or fluids at rest, and *hydraulics*, or fluids in motion.

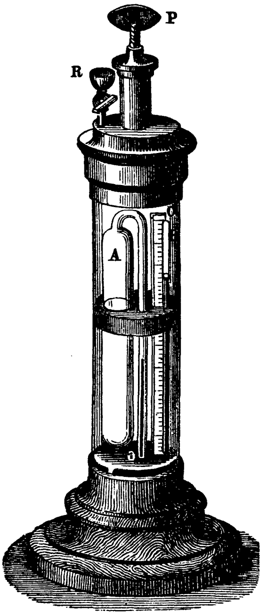
187. **Mechanical condition of liquids.**—Liquids, owing to the slight cohesive attraction among their particles, possess no definite form, but adapt themselves to the shape of the containing vessel. This is a necessary consequence of the perfect freedom of motion among the particles of a liquid. Liquids vary very much, however, in the degree of their fluidity, as between thin mobile liquids like alcohol or water, and thick, *viscous*, bodies like oils and tar. In viscous bodies, the imperfect fluidity is a consequence of the only partial ascendency of the repulsive force, leaving still a notable amount of cohesion among the particles. Heat serves to increase the repulsive force, and so converts viscous into thin liquids.

Liquids and gases are conveniently distinguished as non-elastic and elastic fluids, but this distinction is not absolute, since all liquids possess some elasticity, more, usually, than belongs to the solid state of the same bodies.

188. **Elasticity of liquids.—Compressibility.**—We have already, in sec. 21, illustrated the compressibility of water. The researches of Canton, in 1761, Oersted, in 1823, and others of later dates, have proved that all liquids are slightly compressible. The *piezometer* (πιεξω, to press, and *μετρον*, a measure) is an instrument designed to measure the compressibility of liquids. Oersted's apparatus, fig. 133, consists of a strong glass cylinder; twenty-four or twenty-five inches

in height, mounted on a stand: the upper part is accurately closed by a brass cap, through which passes the funnel tube, R, to supply the vessel with water, and a cylinder, furnished with a piston, moving by the screw P. In the interior is a vessel A, containing the liquid to be compressed, having a capillary tube at its upper part, which bends and descends to the mercury O, contained in the lower part of the vessel This capillary tube is subdivided into equal parts, and the number of these parts the vessel A can contain, is accurately determined. There is also in the interior of the cylinder, a tube of glass, B, furnished with a graduated scale, C; this tube is closed at its upper end, and has its lower end immersed in the mercury, O. This tube is called a manometer (280), and is, when at rest, nearly filled with air.

133



In order to experiment with this apparatus, fill the vessel, A, with the liquid to be compressed, and by means of the funnel, R, fill the cylinder with water, having previously placed mercury in its lower part. Turning the screw, P, the piston descends; in consequence, the air in the tube, B, is compressed, and the mercury is elevated; the degree of elevation shows the amount of pressure; at the same time, the mercury rises in the capillary tube, and gives the measure of the compression of the liquid in A.

Supposing each division of the capillary tube held but a millionth part as much as the vessel A, then if the liquid to be compressed was water (at the pressure of one atmosphere), we should observe the mercury to rise between 49 and 50 divisions.

There is one correction to be made in the observations obtained by this instrument; it might be supposed that the capacity of A would be invariable, the exterior and interior walls being compressed equally by the liquid, but it is not so; the interior capacity of the vase undergoes the same diminution as would a body of glass of the same form and volume, submitted to the same pressure. This diminution amounts to about 33 ten millionths ($\frac{33}{10,000,000}$), of the primitive volume, for each atmosphere of pressure.

This quantity can be calculated in any case, as well for glass as for copper and brass (the three materials used in the improved piezometer of Regnault), from the known elongation of rods of these substances at the same tension, according to the laws of Wertheim (160). M. Grassi (*Ann. de Chimie et Phys.*, 3e *Série. Tom.* XXXI., p. 437) has lately revised the results of Oersted, Canton, and of Messrs. Colladon and Sturm, on a great number of liquids, and at different temperatures The compressibility of water diminishes with increasing temperatures. On the other hand, heat increases the compressibility of alcohol, ether, chloroform, and wood spirit.

Salt or sulphuric acid diminishes the compressibility of water in proportion to the quantity dissolved, but for a given density these solutions obey the same order as pure water.

Grassi's principal results are given in the following table, the compressibility being parts in a million at a pressure of one atmosphere:

Liquids used.	Temperature.	Compressibility.	Pressure in atmospheres employed.
Mercury, ...	32° F.	0·000,002·95	
Water, ...	32°	0·000,050·3	
Do.	77°	0·000,045·6	
Do.	128°	0·000,044·1	
Ether, ...	32°	0·000,111·0	3·408
Do.	57°	0·000,140·0	1·580
Alcohol, ...	45°	0·000,082·8	2·302
Do.	55½°	0·000,090·4	1·570
Wood spirit, . . .	56°	0·000,091.3	
Chloroform, ...	47°	0·000,062·5	
Do.	54°	0·000,064·8	1·309

It appears, that of all liquids tried, mercury has the least, and ether the greatest, ratio of compressibility. The pressures were carried to the great extent of 220 atmospheres.

Elasticity.—It is hardly requisite to remark that the return of compressed liquids to their original bulk, on removal of pressure, is proof of an elastic force in them equal to their ratio of compressibility

Consequences.—It follows from what has been said,—

First. That the molecules of liquids, owing to the entire freedom of motion among themselves, are in equilibrium.

Second. That liquids possess perfect elasticity, and a slight degree of compressibility.

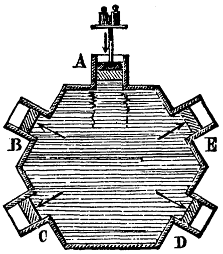
Let us now farther consider the necessary consequences of these mechanical conditions of liquids, both independent of gravity, and also with reference to that force.

II. TRANSMISSION OF PRESSURE IN LIQUIDS.

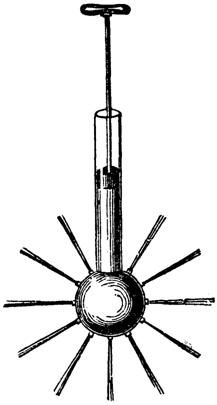
189. Liquids transmit pressure equally in all directions.—*Liquids transmit in all directions, and with the same intensity, the pressure exerted on any point of their mass.*

This important theorem was first clearly announced by B. Pascal. Let fig 134 be a vessel filled with a liquid, and furnished with a number of equal cylinders, in each of which is a well-fitting piston. The vessel and liquid are both assumed to be without weight, consequently none of the pistons have any tendency to move. If pressure is applied to the piston A, it will be forced inwards, and the other pistons B, C, D, and E, of equal area, will each be forced outwards with the same pressure, so that if the piston A was pressed inwards with a force of one pound, it would be found necessary to apply a force of one pound to each of the other pistons, in order to keep them in their place. If the area of B and C was two or three times that of A, then the pressure upon them would be two or three times as great. We cannot perfectly demonstrate, that liquids transmit pressure *equally* in all directions (because we cannot obtain for experiment, as would be necessary, liquids without weight, and pistons working without friction), but that this pressure is exerted in all directions, is shown by the simple apparatus, fig. 135, consisting of a cylinder, furnished with a piston and terminated by a sphere; on this sphere are placed small tubes, jutting out in all directions; upon filling the sphere and cylinder with water, and pressing upon the piston, the water is forced out from each of the jets with equal energy. This is a necessary consequence of the mechanical constitution of liquids (187).

134



135



Let A represent the area of any portion of the inner surface of a vessel, and A' that of any other portion of the same vessel, while P and P' are the pressures exerted on these surfaces respectively by any force of compression on the liquid, and we have

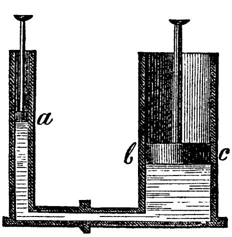
$$P : P' = A : A'.$$

It also follows that this expression represents correctly the pressure exerted on any solid body plunged in the liquid, as well as for any part of the sectional area of the liquid itself. Hence:—

The entire pressure sustained by any surface is proportional to its area, "and thus," says Pascal in his *Treatise on the Equilibrium of Fluids,* it appears that a vessel full of water is a new Principle in Mechanics, and a new Machine which will multiply force to any degree we choose." Pascal also referred the equilibrium of fluids to the principle of virtual velocities which regulates the equilibrium of other machines (105).

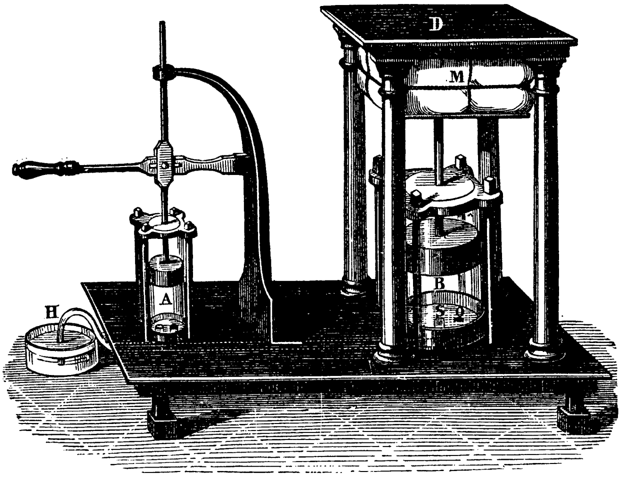
190. **The Bramah Hydrostatic Press.**—This powerful apparatus depends upon the principle just announced. Want of good workmanship alone prevented Pascal from realizing his conception of this machine (in 1653), as was long afterwards (A.D. 1796), done by Bramah, at London.

136



As the form of the vessel has no influence on the equal transmission of pressures, and the point of application of force may be situated at any convenient distance from the press, it is plain that the mechanician can use this principle as circumstances demand. Thus, in fig. 136, the piston *a* may be to the larger one *b c* as 1 : 20, and hence a pressure of one pound exerted on *a* will raise *b c* with a force of twenty pounds, and conversely any pressure exerted on *b c* will be diminished twenty fold at *a*.

137



The main parts of the Bramah hydrostatic press, fig. 157, consist of a small forcing-pump, A, in which is a piston worked by a lever. This pump communicates with a large and strong cylindrical reservoir, B, by a tube indicated by the dotted lines in the figure. In this cylinder a water-tight piston moves, bearing at its upper end a flat metallic plate, between which and the top of the frame, D, the substance, M, to be compressed, is placed.

The cylinders are filled by means of the curved tube H, one end of which rests in a vessel containing water, or oil, the other terminates in the barrel A, and has a valve at its end opening upwards. This valve opens when the piston is raised, thus drawing in water, and closes when the piston descends. By working the piston, the barrels A and B are completely filled with water. The orifice O is in connection with a stop-cock, by which the water can be drawn off when the pressure is to be reduced.

If the cylinder B has an area of 200 square inches, and the small cylinder an area of half a square inch, the pressure of the water on the piston above B, will be 400 times that applied at the lever. But let the arms of the lever be to each other as one to fifty, then when a force of fifty pounds is applied at the long arm, the piston will descend with a force of 2500 pounds ($50 \times 50 = 2500$), and there will be exerted, theoretically, a force of 1,000,000 pounds upon the piston in B ($50 \times 50 \times 400 = 1{,}000{,}000$), or, deducting one-fourth for the loss occasioned by the different impediments to motion, a man would still be able to exert a force of 750,000 pounds.

This enormous result is gained, of course, very slowly, in accordance with the well-known relation of power to weight (109).

Uses in the arts.—The hydrostatic (often called *hydraulic*) press is of extensive use in the industrial arts. It is employed for compressing cloth, oil-cake, paper, hay, gunpowder, candles, vermicelli, and for numerous other articles, to which the proper form or condition is imparted by severe pressure; also for testing steam-boilers and chain cables. The tubes of the famous Britannia tubular bridge over the straits of Menai (172) were raised to their place by means of powerful hydraulic presses.

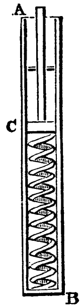
191. **Pressure of a liquid on the bottom of a vessel.**—*The pressure exerted by a liquid on the horizontal base of a containing vessel, is* 1*st, independent of the shape of the vessel, and* 2*d, is equal to the weight of a column of this liquid, whose base is that of the vessel, and whose height equals the depth of the liquid.*

In a conical vessel, standing on its base and filled with liquid, conceive any number of horizontal planes, dividing the contents of the vessel into a series of frustums, so thin that each frustum may be considered a cylinder. It is evident that the pressure exerted by each cylindrical mass on its own base is equal to its own weight. But by the principle of Pascal each succeeding section will have to support a pressure, as much greater than the weight of the superincumbent masses as the area of its base is greater than the area of the base of that preceding it. Hence, the base of the conical vessel will support a pressure equal to the weight of a column of water whose base and height are respectively those of the vessel.

Evidently, from this reasoning, if the conical vessel is inverted, or its form is in any way modified, the same law holds good.

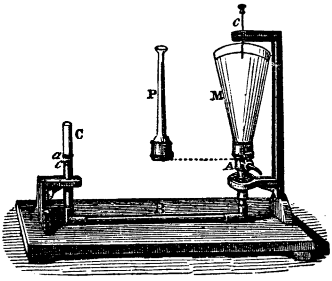
The truth of this principle may be experimentally demonstrated by means of the apparatus in fig. 138. If this instrument is placed in a liquid, the piston C is forced in with a pressure equal to the weight of a column of the liquid, whose base has the area of the piston, and whose height is equal to the depth of the liquid above the surface of the piston.

138



To demonstrate that the pressure is independent of the form of the vessel, M. Haldat has contrived the apparatus, fig. 139. It consists of a tube, A B *c*, bent twice at right angles. On A, may be placed the vessels M and P, of equal height, but of different forms. The tube A B *c* is filled with mercury, which rises to an equal height in A and *c*; M is then placed on A, and filled with water; the mercury immediately rises in *c*, to a certain point, as *a*. We then replace M by P, and fill with water to the same height as before. The mercury again rises to the point *a*, as it did with the vessel M; it is evident that the pressure transmitted to the mercury in the direction A B, was the same in both cases, proving, most conclusively, that the pressure does not depend upon the quantity of liquid, for the vessels M and P differ greatly in capacity. The area of the base formed by the surface of the mercury, and the vertical height formed by the column of water, were, however, the same in both cases, and upon these, as before stated, the pressure depends. In the case of a vessel having vertical walls, the pressure would be equal to the weight of the liquid the vessel contained.

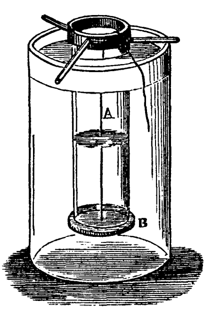
139



192. **Upward pressure.**—Having shown that pressure in liquids is exerted from above, downwards; it follows, from the law of equality of pressure, that a corresponding force is exerted from below, upwards. This pressure is made very manifest by the buoyancy experienced when we plunge the hand into a liquid of great density, as into mercury In

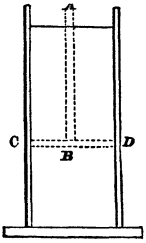
order to demonstrate this upward pressure experimentally, a tube of glass is taken, open at both ends, fig. 140, having at the lower end a disk of glass, B, which is supported by means of a thread from its centre: the whole is then placed in a vessel of water and abandoned to itself; the disk remains attached to the end of the cylinder, owing to the upward pressure of the water. If now the interior tube be carefully filled, the disk will not fall until the level of the water within the tube is nearly the same as that in the outer vessel, proving that the upward pressure is equal to the weight of the interior column, and therefore that:—*The upward pressure, in any vessel, is equal to the weight of a column of liquid having the same base as the cylinder,* A, *and whose height equals the depth of the section below the surface of the liquid.*

140



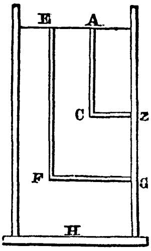
193. **Pressure on the sides of a vessel.**—*The pressure of a liquid on any portion of a lateral wall, is equal to the weight of a column of liquid, which has for its base this portion of the wall, and for its height the vertical distance from its centre of gravity to the surface of the liquid.* Thus, in fig. 141, the pressure at the height, C D, of the wall is, by § 191, equal to the weight of the column A B, since the pressure of this is communicated laterally to all the particles lying on the same horizontal plane.

141



This lateral pressure increases, of course, with the depth of liquid in the vessel. Thus, in fig. 142, the column of liquid A C pressing with a certain force on Z, the column E F will press on G, with a force as much greater, as E F is deeper than A C. This may be further illustrated by plunging the apparatus, fig. 138, at various depths and in a horizontal position, the piston will be forced in with a pressure corresponding to the depth; also, if it is placed in any position intermediate between the horizontal and vertical, the piston will be similarly pressed in, thus showing that pressure is exerted equally in all directions.

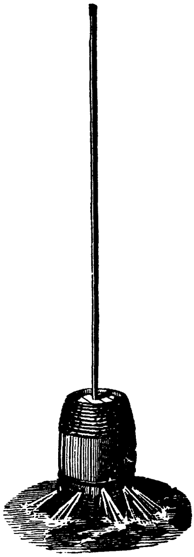
142



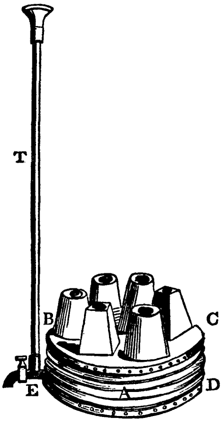
194. **Pascal's experiment with a cask.**—Pascal made a striking experiment at Rouen, in 1647, to illustrate the enormous pressure

exerted by a lofty column of water contained in a small tube. A strong cask, filled with water and arranged as in fig. 143, was fitted with a small tube about forty feet high. When this tube was filled with water, the effect of the pressure transmitted to all parts of the cask was sufficient to burst the vessel.

143



144



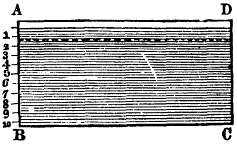
195. The water bellows, or hydrostatic paradox.—This familar experiment is only a modification (in form) of Pascal's cask.

The hydrostatic bellows, fig. 144, consists of two boards, B C, and E D, connected with leather or India-rubber cloth, A, in such a manner that the upper board can rise and fall, like the common air bellows. The tube T E communicates with the interior of the apparatus. Supposing the tube to have a cross section of one square inch, and the top of the bellows to have a surface of 100 square inches, one pound of water in the tube would lift 100 pounds on the bellows, the weight of the water acting with a pressure equal to one pound on each square inch of the surface. The pressure is proportioned to the height of the column of water; for if we use a smaller tube, for the same bulk of fluid, the height of the column of water will be greater, and will raise a greater weight; if the tube be larger, the column will not be so high, and will not raise so large a weight.

196. Total pressure on the walls.—In the vessel A B C D fig.

145, divide the side A B into 10 equal parts. Supposing the pressure at 1 to be one pound, then the pressure at 2 would be two pounds, at 3 three pounds, &c., as the intensity of the pressure increases directly with the depth. The average intensity of pressure would be found at the 5th division (or a point midway between the 1st and 10th), and the total pressure on the walls would be the same as if it sustained the average intensity over the whole lateral surface, and therefore *the total pressure upon a wall of such a vessel, is equal to the weight of a column of the liquid whose base is equal to the area of the side, and whose height is equal to one-half of the depth of the liquid in the vessel.* This is true, whether the vessel be vertical or inclined in any direction. In the case of a cubical vessel, this pressure on one side would be equal to one-half the weight of the liquid contained in the vessel.

145



Total pressure on the bottom and sides of a vessel.—The total pressure exercised on the bottom and sides of a vessel, is much greater than the weight of the liquid contained in the vessel. In the case of a cubical vessel, the pressure exerted on the bottom is equal to the whole weight of the liquid (191), the pressure exerted on each side being equal to half the weight of the liquid on the four sides, it is equal to twice its weight, consequently, *in a cubical vessel the entire pressure exerted on the bottom and sides is equal to three times the weight of the contained liquid.*

Table, showing the pressure in pounds, per square inch, and square foot, produced by water at various depths.

Depth in feet.	Pressure per square inch.	Pressure per square foot.
1	0·4328	62·3232
2	0·8656	124·6464
3	1·2984	186·9696
4	1·7312	249·2928
5	2·1640	311·6160
6	2·5968	373·9392
7	3·0296	436·2624
8	3·4624	498·5856
9	3·8952	560·9088
10	4·3280	623·2320

By aid of the above table, the pressure of water on any surface of a vessel containing it, can be determined. As, for example, the pressure of water on a square foot, at the bottom of a vessel twenty-three feet in depth; at two feet, the pressure is 124·6464; at twenty feet, ten times as much; = 1246·464; at three feet, 186·9696, and $1246{\cdot}464 + 186{\cdot}9696 = 1433{\cdot}4336$, the pressure of water on a square foot of surface, at a depth of twenty-three feet.

That the pressure produced at great depths is really immense, can be shown by confining a piece of wood at great depths in the sea. The pressure forces the water into the pores, so that it will not be capable of floating afterwards. A bottle, the body of which is square, if tightly corked and lowered into the sea, will be broken by the pressure. If the body of the bottle is strong and cylindrical, the cork will be forced in. Below a certain depth, divers cannot penetrate, and the same may, perhaps, be true of fishes.

197. **The centre of pressure** upon any surface immersed in a fluid is the point of application of the resultant of all the pressures acting upon it.

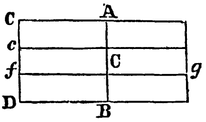
If the pressure of a fluid upon an immersed surface were the same at all depths, the centre of pressure would be at the centre of gravity of the surface. But as the pressure increases with the depth, the centre of pressure will always be below the centre of gravity.

The centre of pressure in an immersed surface, or in the side of a vessel containing a fluid, is a point to which a force equal and opposite to the resultant of all the pressures must be applied to keep the surface at rest.

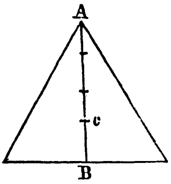
The position of this point, for various regular surfaces, has been determined by the calculus.

The centre of pressure of a rectangular surface, vertically or obliquely immersed, so as to have one side in the surface of the liquid, is in a line joining the centres of the superior and inferior bases, and at a distance from the inferior base, equal to one-third the height of the rectangle. In fig. 146 the point C, in the line A B, distant from B one-third of A B, is the centre of pressure.

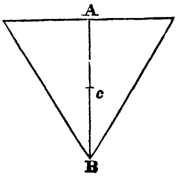
146



147



148



When the immersed surface is a triangle having one side horizontal, and the apex in the surface of the fluid, the centre of pressure is in a line joining the apex and the centre of the horizontal base at a distance from the centre of the base equal to one-fourth the bisecting line. The centre of pressure in the triangle, fig. 147, is at *c*, distant from B one-fourth of the line A B.

When the base of the triangle lies in the surface of the fluid, the centre of pressure is midway between the apex and the centre of the base, as at *c*, fig. 148, which is equidistant between A and B.

198. **Pressures vary as the specific gravities of liquids.**—*Two liquids press on the same area and at the same depth, directly in the ratio of their specific gravities.* We have seen (191) that the pressure exerted on the base of a vessel having vertical walls is equal to the weight of the liquid the vessel contains. Plainly, therefore, the pressures exerted on the base of two equal vessels filled with equal volumes of liquid of unlike density will vary directly with their specific gravities; or representing the pressures in the two cases by P and P′, and the specific gravities by (Sp. Gr.) and (Sp. Gr.)′, we have

$$P : P' = (Sp.\ Gr.) : (Sp.\ Gr.)'$$

III. EQUILIBRIUM OF LIQUIDS.

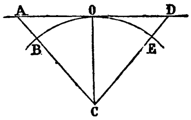
199. **The conditions of equilibrium in liquids.**—The joint effect of gravitation, and of the perfect mobility of the particles of a liquid, is:—

1. *That the surface of a liquid at every point must be perpendicular to the direction of gravity,* i. e., *it must be horizontal or level.*

This principle, first distinctly enunciated by Archimedes, follows from the nature of gravitation, which acting on a body free to move, causes its centre of gravity to descend as low as possible. It is only when the surface is horizontal that all the particles of the fluid mass are equally solicited by the force of gravity.

The inequalities of the solid surface of the earth exist, because cohesion is opposed to gravitation. Otherwise the mountains would sink, and the valleys rise, until the whole mass had a uniform level.

149



By this principle a surface of water is perfectly horizontal only when its area is so limited that the direction of the forces of gravity can be regarded as parallel at each point. If an observer is stationed at O, fig. 149, and O A is one mile, the subtense of curvature (A B or D E) is eight inches. But O C and B C are lines perpendicular to the points O and B, and are therefore *plumb* lines (60), and hence the surface E O B is a spherical surface. In other words, we reach the more general principle:—

That the resultant of all the forces acting at any point on the surface of a liquid mass, when in equilibrium, must be normal to the surface at that point.

It follows from this again:—

2. *That every liquid mass, when in equilibrium, can be considered as made up of an infinite number of very thin layers, sustaining at all points the same pressure, and, at each point of surface, normal to all the forces there acting.*

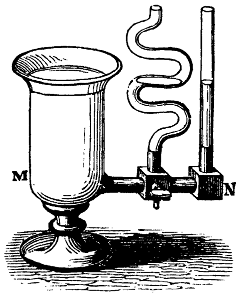
200. **Equilibrium of liquids when freed from the influence of gravity.**—It follows, as a consequence of the last principle, if a mass of liquid is freed from the influence of gravity, and abandoned undisturbed to its own molecular attractions, that it will assume a spherical figure; since then the sphere is the only form which can satisfy the conditions of equilibrium. This theory is most beautifully demonstrated by a celebrated experiment called—

The experiment of Plateau, who conceived that the influence of gravity might be avoided by suspending a mass of oil in alcohol, diluted to exactly the density of the oil. This conception is perfectly realized by experiment. By care and certain precautions to secure clearness in the liquids, a considerable sphere of oil may be suspended in any part of the alcoholic mixture, and by a wire arranged to rotate as an axis, and about which the sphere of oil readily arranges itself, the oblate figure of the earth, the appearance of satellites, or even the rings of Saturn, may be imitated in a most instructive and striking manner.

The spherical form of drops of rain or dew, and the globular drops of mercury, are referable to the conditions of fluid equilibrium.

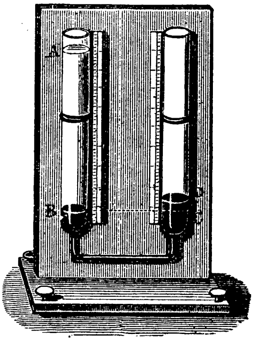
201. **Equilibrium of a liquid in communicating vessels.**—If two or more vessels communicate with each other, the liquids in both or all the vessels stand at the same level. This law rests upon the fact, that the pressure of liquids at equal depths, is equal in all directions. If the fluid stands at a higher level in one vessel than the other, the particles of the former exert a greater lateral pressure on the channel of communication than the other can; these particles are, therefore, continually pushed upwards, until they exert an equal and opposite pressure, which obtains when the columns are at an equal height. The effect is the same, whatever may be the size and number of the vessels. Fig. 150 represents a number of vessels of different shapes and capacities, connected with a common reservoir; if we pour water into one of them, it will rise to the same height in the other vessels.

150



202. **Equilibrium of liquids of different densities in communicating vessels.**—When two liquids of different densities are placed in communicating vessels, their surfaces will not rest at the same point or level; *for in communicating vessels, the heights of the liquid columns are in the inverse ratio of the specific gravities of the liquids.*

151



If mercury is first poured into the lower part of the apparatus, fig. 151, and the tube A B is then filled with water, this liquid will exert a pressure on the mercury, causing it to be depressed in A B, and to rise in the other tube. Measuring the height of the columns of mercury, C D, and water, A B, which are in equilibrium, they will be found to be as 1 to 13·59. These numbers represent the densities of water and mercury.

Demonstration.—Let S represent the surface of the mercury at B, and H be the height of the column of water, B A, and *Sp. Gr.* the specific gravity of water; then, by § 198, the pressure on the surface is $P = S\,H\,(Sp.\ Gr.)$ for the column of water, and for the mercury, C D, it is $P' = S'\,H'\,(Sp.\ Gr.)'$ But by § 189 equilibrium can obtain only when the pressures exerted on B and C are proportional to the area of those surfaces, or where $P : P' = S : S'$. Substituting the value of P and P', it follows that

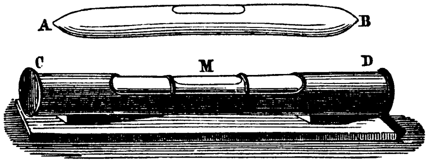
$$H\,(Sp.\ Gr.) = H'\,(Sp.\ Gr.)'$$

$$H : H' = (Sp.\ Gr.)' : (Sp.\ Gr.)$$

In other words, the columns are in equilibrium when their heights are inversely as their specific gravities, which was to be proved.

203. **The spirit level.**—Since by § 199 the surface of a liquid at rest is always horizontal, we have thereby a ready means for determining the horizontal line by use of the *spirit level.* This instrument is a glass tube, A B, fig. 152, very slightly curved upwards, nearly filled with alcohol, hermetically sealed and sheathed in brass, C D. The small bubble of air, M, always rises to occupy the highest point of the apparatus. The base is carefully adjusted, so that only when the instrument is placed horizontally, does the bubble remain in the centre, at a fixed mark.

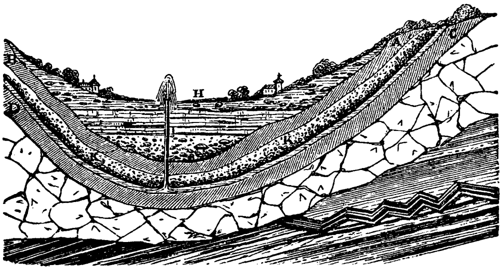
152



204. **Artesian wells.**—All springs and fountains are examples of the laws of equilibrium of liquids in communicating vessels. Among similar phenomena, artesian wells are the most remarkable examples. These are wells (named artesian from the ancient province of Artois in France, where they were early made, although known long before in China), bored into the earth's crust, often to a great depth. The crust of the earth consists often of various beds or strata, some pervious to water like sandstones, and others, like clay, impervious.

Fig. 153 presents an imaginary section of a portion of the earth's crust, containing two impervious strata, A B, C D, and one pervious stratum, K K. Let these strata reach the surface in elevated land, and we have thus a basin into which the meteoric waters filter and from which they cannot escape, being confined by the impervious strata already named; now an artesian boring in the valley H, will reach the imprisoned water after passing A B, and the water will be thrown up in a jet, the height of which will depend on the elevation of the edges of the basin, which may come to the surface in lofty hills hundreds of miles away from the well.

153



A well of this kind was sunk at Louisville, Ky., in 1857-8, to the great depth of 2086 feet (Dupont's well), which delivers, through a bore of 13 inches, over three hundred thousand gallons of sulphuretted mineral water in 24 hours, at 170 feet above the surface, with a constant temperature of 76½° F. (82½° at the bottom). (Am. Journ. Sci. [2] xxvii. 174.) Belcher's well in St. Louis is 2199 feet deep, and yields also sulphuretted water; while the famous Grenelle well in Paris is 1806 feet deep, and yields daily 600,000 gallons of soft water, warm enough to answer the purposes of the great slaughter-houses surrounding it. Artesian wells have lately been successfully bored in the African desert on the great caravan route.

IV. BUOYANCY OF LIQUIDS.

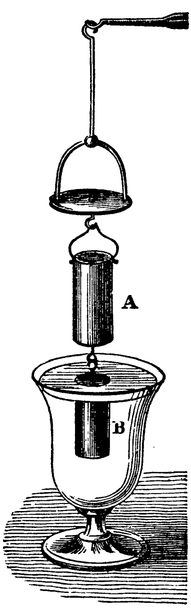
205. **Theorem of Archimedes.**—*Solids immersed in liquids are buoyed up by a force equal to the weight of the liquid displaced.*

This very important principle was discovered by Archimedes, about 230 years B. C., and is called after him, the *Principle of Archimedes.* Its correctness is proved by means of the hydrostatic balance, from one of the arms of which (fig. 154) is hung a hollow cylinder, or bucket, A, having a cylindrical mass of copper, B, exactly fitting into it, and suspended from it by means of a hook. Having exactly counterpoised the beam by weights on the other arm of the balance, fill up the glass vessel with water, until the cylinder B is wholly immersed. The cylinder will then appear to have lost weight, the other arm going down. If the bucket, A, is now exactly filled with water, the equilibrium will be restored; proving that the weight lost by the immersed body is equal to its own bulk of water.

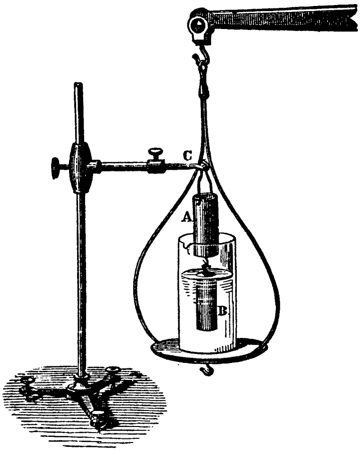
The same is true of any liquid whatever. It is also true, however, that the weight lost in this case by the cylinder must, as a necessary result of the law of action and reaction, be gained by the water in the vase.

This fact is illustrated by arranging the apparatus as seen in fig. 155. After first balancing the vase of water, the cylinder B is suspended in it from a separate support C. The vase then appears to have gained in weight, and it will be found requisite in order to restore the equilibrium to remove therefrom enough water exactly to fill the cup A.

154



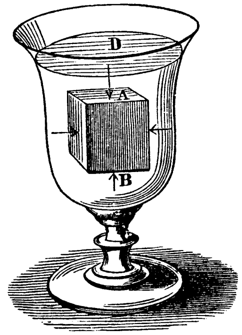
155



The theorem of Archimedes is a necessary consequence of the mechanical condition and laws of equilibrium of liquids, and of the impenetrability of matter. The whole immersed body is buoyed up by a force equal to the resultant of all the forces normal to each point of its surface, that is by a force equal to the weight of the liquid which it displaces.

206. **Another demonstration of Archimedes' principle.**—Conceive a cube A B, fig. 156, of water for example, say one cubic inch or a cubic centimetre in bulk, isolated and sustained in its position by the pressure of the surrounding particles—such being the condition of equilibrium existing among the particles of liquids at rest (199). Hence it is evident that the weight of the ideal cube A B is sustained in its position of equilibrium by a buoyant force exactly equal to its own weight. If A B is now solidified by any cause which does not change its volume, it is evident that the conditions of its equilibrium also remain unchanged. We may therefore replace it by any other substance of whatever weight, having the same dimensions, and the new solid will still be buoyed up by a force equal to the weight of the ideal cube of water, or of any other liquid in which it is immersed.

156



The form of the body is evidently immaterial, and therefore it follows, as before, that a body plunged in a liquid is sustained by a power equal to the weight of the liquid displaced.

Floating bodies.—Accepting the Theorem of Archimedes, it follows, that if the immersed solid be of the same weight as the displaced fluid, the former will remain at rest in the fluid, in any position in which it may be placed, the upward pressure exerted upon the solid being equal to its own weight.

Since the specific gravities of any two substances are to each other as the weights of equal volumes of these substances (99), it follows that any homogeneous solid will float when its specific gravity is less than that of the liquid, and that it will sink when these conditions are reversed.

Hence, iron sinks in water, but floats on mercury; some woods which float on water will sink in oil or alcohol; while oak, which floats on salt water, will sink in fresh water. But if the iron is fashioned into a thin-walled vessel, and the dense woods into hollow boxes, they will then float on the same liquids in which they before sank, because their volumes have been increased, respectively, without increasing their weight, and they float because each displaces a volume of water greater in weight than the weight of the floating body.

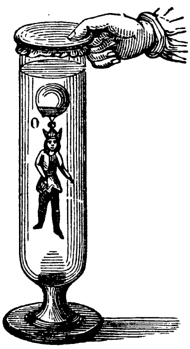
Examples illustrating this principle are of familiar occurrence. *Iron ships—e. g.,* the Great Eastern—float as buoyantly as ships of wood, and have besides a vast capacity for floating their heavy machinery, coal, and cargo. The problem of weighing a ship and cargo resolves itself into a question of mensuration of the volume of water displaced by her.

Camels are tanks of iron or wood, which are first filled with water, and after being secured to the sides of loaded vessels the water is pumped out, when their buoyancy aids the vessel in floating over a bar, or in shallow water.

Floating docks, so much in use in the seaports of the United States, are similar contrivances by aid of which the heaviest and largest ships are safely raised entirely out of water for repairs. The elevating force is solely the buoyancy of large sectional tanks previously sunk beneath the vessel, and then pumped out by steam-engines.

Cartesian devil.—This hydrostatic toy, known also as the *ludeon,* exhibits the principle just stated. It consists of a small glass or enamel figure, fig. 157, at whose head is fixed a bulb of glass having a small opening, O, beneath. It is filled with water to such an extent, that when placed in the cylinder of water as represented, it just floats. Over the mouth of the vessel is tightly fixed a piece of caoutchouc. Pressure exerted by the thumb on the caoutchouc will be conveyed through the water to the air contained in the bulb O. Sufficient water will thus enter O to render the specific gravity of the apparatus heavier than that of water, when it sinks. On removing the pressure, expansion of the air in O expels the water which was previously forced into it, and the apparatus rises. By a contrivance similar to this, the beautiful nautilus shell rises, to float upon the surface of the sea, or sinks again at pleasure, by a voluntary contraction or expansion of an internal cavity.

157

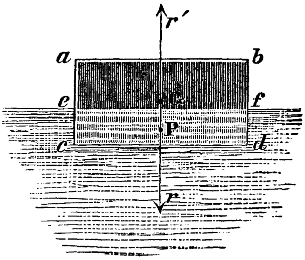


Fishes are bodies floating in a state of equilibrium, when immersed in their own element. But in order to preserve this state at different depths, they have an *air bladder,* by contracting or expanding which, their bodies acquire the mean density of the water in which they are.

207. **Equilibrium of floating bodies.**—In order that a floating body may be in equilibrium it is necessary: First, That the weight of the fluid displaced should be equal to the weight of the floating body: Second, That the resultant of all the upward pressures of the liquid should act in the vertical line, passing through the centre of gravity of the body.

As the weight of a body may be considered as acting at a single point called the centre of gravity, so the upward pressure of a liquid, acting upon a body immersed in it, may be considered as acting in a single point which will be the centre of gravity of the fluid displaced. This point is evidently different from the centre of gravity of the body, and may therefore appropriately be called the *centre of buoyancy.* In a homogeneous solid this point is always below the centre of gravity when the body floats, and coincides with it when the body sinks. Let *a b c d*, fig. 158, be a homogeneous solid, G will represent the centre of gravity of the body, and P the centre of buoyancy, or upward pressure, situated at the centre of gravity of the liquid displaced.

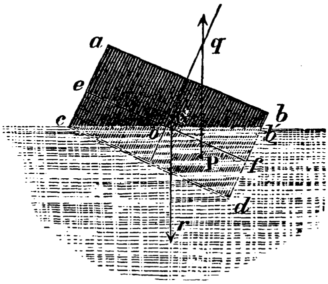
158



When the floating body is not homogeneous the centre of gravity may be below the centre of buoyancy, as in the case of a ship having ballast or heavy cargo stowed in the hold.

Let the floating body take the position shown in fig. 159, the force of gravity will act at G in the direction G *r*, but the upward pressure will act from a new centre of buoyancy, P′, at the centre of gravity of the displaced fluid, and in the direction P′ *q*. This force being equal to the force of gravity and parallel to it, but acting in an opposite direction, the two forces form a couple (48), and tend to rotate the body till the two forces again act in the same vertical line.

159



When the centre of gravity and centre of buoyancy are in the same vertical line, the floating body will be in equilibrium.

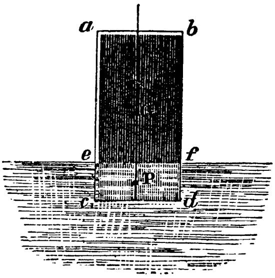
This equilibrium may be *neutral*, or the same in any position of the floating body ; *unstable* when by any movement of the body the centre of gravity descends :—or *stable* equilibrium when movement of the body in any direction causes the centre of gravity to ascend.

Neutral equilibrium.—A sphere of uniform density floating in a liquid is an example of neutral equilibrium, because, whatever position it may assume, the part immersed is a segment of a sphere of the same magnitude and form, and no alteration can be effected in the relative positions of the centre of gravity and the centre of buoyancy.

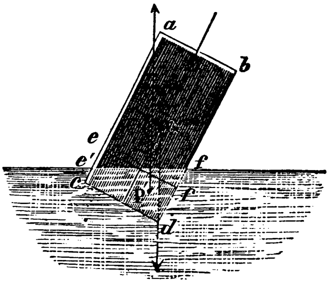
Unstable equilibrium.—Let *a b c d*, fig. 160, represent a rectangular prism of uniform density, floating on one end, the centre of gravity being at G, and the centre of buoyancy or upward pressure being at P.

Although G and P are in the same vertical line, it is evident that the equilibrium will be unstable, because when the body moves to any new position, as at fig. 161, the centre of gravity descends.

160



161



Stable Equilibrium.—The centre of buoyancy, or centre of upward pressure, may be considered as the centre of support of a floating body. When this centre is above the centre of gravity, the body will evidently be in a position of stable equilibrium. It will also be in a position of stable equilibrium when the centre of gravity occupies a lower position than it would acquire in any other position of the floating body. But in such cases the stability of the equilibrium of the floating body is more readily understood by reference to another point called the *metacentre.*

208. **The metacentre** *of a floating body is the point where the vertical passing through the centre of buoyancy, in the position of equilibrium, meets the vertical drawn through the new centre of buoyancy, when the body has been slightly displaced from this position.*

By reference to figs. 158 and 159 it will be seen that G r' or G q is the vertical which passes through the centre of buoyancy in the position of stable equilibrium, and P′ q the vertical passing through the centre of buoyancy when the body is moved a little from the position of equilibrium; hence, q is the metacentre related to the position of stable equilibrium, and in this case it is above the centre of gravity.

Referring to figs. 160 and 161, we see that the metacentre is at q', fig. 161, or at a point below the centre of gravity.

The metacentre may also be found by taking the point of intersection of verticals passing through the centres of buoyancy in any two positions near each other.

A floating body will be in stable equilibrium whenever the metacentre is above the centre of gravity, and the degree of stability will be in proportion to the distance of the metacentre above the centre of gravity. This depends on the form of the floating body.

When the centre of gravity is below the centre of buoyancy, the metacentre must evidently always be above the centre of gravity, and this condition is always stable. It is also evident that the stability of a floating body increases with the breadth of the part submerged. These principles are of great importance in the construction and loading of ships. The metacentre may be regarded as a sort of fulcrum above which is the pressure of the sails, and below the weight of the ship.

Vessels designed for transporting passengers and light cargo require heavy ballast of iron or stone placed near the keel, to preserve the equilibrium. On the other hand, vessels loaded with iron have the centre of gravity so low as to cause injurious strain upon the ship, unless the cargo is elevated by cross piling or other supports to raise the centre of gravity so as to allow the ship to roll easily in a heavy sea. The equilibrium of small boats is from the same cause often disturbed by the unguarded movements of the passengers. The rolling of a vessel in a storm may so shift the position of the cargo, and thus remove the centre of gravity, that the vessel may be thrown upon her beam-ends, and be lost.

V. DETERMINATION OF SPECIFIC GRAVITY.

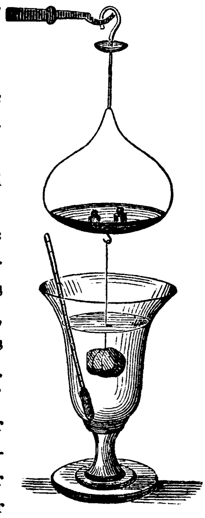
209. **The problem stated.—Methods.**—We have already considered the relations of density and specific weight to mass and weight (96–99). Most of the methods in use to determine specific gravity depend on the principles of hydrostatics just considered, and serve as illustrations of them. The problem is:—

To compare the weight of any body whose specific gravity is sought with the weight of an equal volume of water taken as unity. The specific gravity is found by dividing the first weight by the second.

Methods.—This operation is performed first, by the *hydrostatic balance;* second, by the *specific gravity bottle;* third, by various floating instruments called *hydrometers* or *areometers.*

All these methods resolve themselves into special cases of the Theorem of Archimedes, § 205.

162



210. **Specific gravity by the hydrostatic balance.**—The solid (heavier than water) is suspended beneath the pan of a balance by means of a fine filament of raw silk, and then weighed, hanging in air. It is then immersed in water as in fig. 162, and the weight it loses determined. This loss is equal (according to the principle of Archimedes) to the weight of a volume of water of the same bulk as the immersed body. Subtracting the weight of the substance in water from its weight in air, and dividing the latter by the difference, the product will be the specific gravity required

Example.—A piece of iron weighed in air 460 grains, in water 401·16 grs. Then 460—401·16 = 58·84 grs., which equals the weight of a volume of water equal to the iron, and 460 ÷ 58·84 = 7·8 = specific gravity of the iron.

To make the case general, let W be the weight of the body, and W' the loss of weight in water, then by the definition

$$(Sp.\ Gr.) = \frac{W}{W'}.$$

The result thus obtained is always to be reduced to a standard temperature.

For solids lighter than water.—If the body whose specific gravity is to be determined is lighter than water, it must be attached to some solid (whose weight in air and in water is known) sufficiently dense to sink it in water. The compound mass is weighed first in air, and then in water, and the loss determined, the weight lost by weighing the heavy body alone in water being known, the weight of the light body in air, divided by the difference between these losses, gives the specific gravity.

Example.—A substance weighed in air 200 grains, attached to a piece of copper it weighed in air 2247 grs., in water 1620 grs., suffering a loss of 627 grs. The copper itself loses, when weighed in water, 230 grs., 627 — 230 = 397, then

$$Sp.\ Gr. = \frac{W}{W'} = 200 \div 397 = \cdot 504.$$

163

For liquids.—The hydrostatic balance also applies to liquids as well as to solids—whether the liquids are denser or lighter than water.

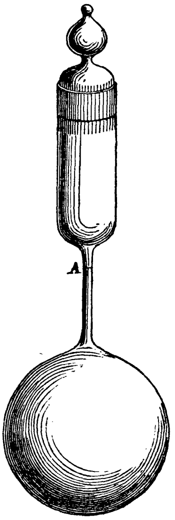
For this purpose a small glass tube is prepared, including enough mercury to sink it in any liquid not heavier than mercury. It is hermetically sealed, the end bent into a hook, and the whole suspended by a very thin platinum wire from the pan of a balance. Fig. 163 shows this apparatus of full size.

The weight of the volume of water which this system displaces at 60° F. (or at 4° C.) is first determined by the mode described for solids. This is a constant quantity, and may be called C. If the tube is now immersed in another liquid, as in alcohol for example, it will require a certain weight to restore the equilibrium (the weight of the tube and mercury is supposed to be counterpoised in each case by a constant weight prepared for the purpose). The amount of this weight, W (required to restore the equilibrium), is the weight of a volume of the liquid displaced by the tube. But the weight of the same volume of water is known (C.) Hence the specific gravity of the liquid is $\frac{W}{C}$.

Example.—A glass tube, like fig. 163, lost in water 2·9910 grains $= C$, in alcohol it lost 2·4081 $= W$. $\frac{W}{C} = {\cdot}80511 =$ *Sp. Gr.* of the alcohol.

211. **Specific gravity bottle.**—*For liquids.*—When it is required to determine the specific gravity of a liquid, the specific gravity bottle offers the easiest and most simple method. Such a bottle is shown in fig. 164. It is closed by a ground glass stopper, and the neck is drawn out to a fine tube (the upper portion of which serves for a funnel in filling the bottle), upon which, at A, is traced a fine line to which the bottle is to be filled at each experiment. The tare of the bottle is accurately determined and noted once for all. It is then filled to A with pure water and weighed again. This weight less the tare gives its capacity of water at a fixed temperature. To determine the specific gravity of any other liquid, the bottle is filled with it and weighed as before. Deducting the tare of the bottle, we now know the weight of a volume of the liquid equal to the same volume of water. Representing these two weights by W' and W, we have (*Sp. Gr.*) $= \frac{W}{W'}$. In all cases, the result must be reduced to a standard temperature as described in the Chapter on Heat.

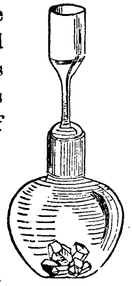
164



For solids, when broken in small fragments, we may also use the specific gravity bottle. In this case the weight of the bottle when empty, and also when filled with pure water, being known, a known weight of the solid in fragments is introduced, as in fig. 165. Calling the weight of the bottle and water $= Wa$, and the weight of the solid added W, and the weight of the bottle solid and water Wb, it is plain that the weight of water displaced by the solid is $W' = Wa + W - Wb$, and that the specific gravity of the solid is

$$(Sp.\ Gr.) = \frac{W}{Wa + W - Wb}.$$

165



This value must be corrected for temperature as before.

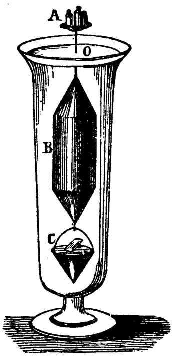
For solids soluble in water we must employ some liquid in which the substance is insoluble, as alcohol, oil of turpentine, &c. The specific gravity thus determined is reduced to the standard of water by multiplying it by the known density of the liquid employed; thus, for

Example.—A substance soluble in water was weighed in oil, and its specific gravity, compared with the oil, was 2·6, the specific gravity of the oil was ·87; then 2·6 × ·87 = 2·262 the specific gravity of the substance.

212. **Specific gravities by hydrometers or areometers.**—In this mode the balance is replaced by floating bodies called *hydrometers* or areometers. There are two classes of these instruments, namely, first, *hydrometers with a constant volume;* and, second, *hydrometers with a constant weight.*

1. **Nicholson's hydrometer or areometer** is an instrument of the first class, used for determining the specific gravity of solids. It consists of a hollow cylinder of metal or glass, B, fig. 166, having attached at its lower end a cone, C, loaded with lead, which causes the apparatus to assume an upright position when placed in water. The upper part of the cylinder is terminated by a slender rod, on the end of which is a small pan, A, for holding weights. The whole apparatus must have a less specific gravity than water, so that a certain weight, represented by *C*, must be put in the pan to sink the areometer to the water mark, O. If we wish to determine the specific gravity of a solid (whose weight must be less than *C*), we place it in the pan A, and add weights until O is brought to the level of the water. The weight *C*, minus the weights last added, will be the weight of the body in air. It is now taken from A, and placed in C; the additional weight now required to sink the cylinder to the index, O, will be the weight lost in water. We have now the data for determining the specific gravity of the solid.

166



For example, if the counterpoise weighed 250 grs., and a mass of lead whose specific gravity we wish to ascertain, requires, when placed in A, 50 grs. to be added in order to bring the hydrometer to the point O, then (250—50) 200 is the weight of the lead in air; placing now the lead on C, we find that it requires the addition of 17·47 grs. on A, in order to counterbalance the instrument; consequently the specific gravity of the lead is 11·45. $\frac{200}{17\cdot47} = 11\cdot45$. If the substance is lighter than water, it is confined under a perforated cover or wire cage placed on C, which prevents its rising.

If we represent these successive weights by C, W, and W', then in any case

$$(Sp.\ Gr.) = \frac{C - W}{W'}.$$

a. Fahrenheit's hydrometer is the same instrument (omitting the lower pan) constructed of glass and designed to measure the specific gravity of liquids. Knowing (by the balance) the constant weight (C) of the instrument, and also the weight (c) required to sink it to a fixed point on the stem—the sum of which weights (by 210) is equal to the weight of the water displaced. We have only to float it in any liquid whose specific gravity we would ascertain, and note the weight, W, required to sink it to the fixed point on the stem. The weight of the liquid displaced is then $C + W$, and since $C + c$ and $C + W$ are the weights of equal volumes of water and of the liquid, the specific gravity of the liquid is found by dividing the latter by the former, or $(Sp.\ Gr.) = (C + W) \div (C + c.)$

b. Rousseau's hydrometer is a form of this instrument adapted to determining the specific gravities of liquids of which we possess too small a portion to float a common hydrometer. For this end, a cup of glass replaces the pan A, which holds say one cubic centimetre. Thus loaded, the instrument sinks to a point marked 20° near the middle of the stem. The stem is divided between this point and zero into twenty equal parts, each of which consequently measures one-twentieth of a gramme or 0·05 gramme. The specific gravity of a liquid is then found by this instrument by multiplying 0·05 by the number of the division to which it sinks when loaded with one cubic centimetre of the liquid used.

2. **Gay Lussac's and Beaume's hydrometers** are instruments having a constant weight, and by which we determine the specific gravity of a liquid by measuring the volume of fluid displaced by the floating instrument—which weight, as we have seen, is the same as the weight of the instrument itself. But we have shown (99), that for equal absolute weights the specific weight is inversely as the volume or $(Sp.\ Gr.) = V' \div V$, where V' equals the volume of water displaced by the instrument, and V the volume of any other liquid displaced by it. In other words, we can find the specific gravity of any *liquid by dividing the volume of a given weight of water by the volume of the same weight of the liquid whose specific gravity is required.*

Instruments of this class are very common and in constant use for determining the specific gravity of alcohol, acids, alkaline solutions, urine, milk, and many other liquids. Figs. 167 and 168 show the form of Gay Lussac's *densimeter*, as it is often called. It is a glass tube containing enough mercury in the lower end to cause the tube to float in pure water at the hundredth division of a scale of equal parts traced on paper and sealed up inside the tube. If it displaces 100 measures of water, floated in sulphuric acid it displaces only 54 measures, and therefore the specific gravity of sulphuric acid is 100 ÷ 54 = 1·85.

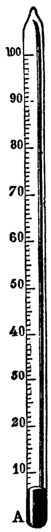
For fluids lighter than water, the graduation is carried up say to 166 (as we know of no liquid of a less specific gravity than 0.60). Then placed in pure alcohol it rises say to 125 degrees, or the specific gravity of alcohol is 100 ÷ 125 = 0·80.

By giving the instrument the form shown in fig. 168, much needless length is saved on the stem, since the ball is so placed as to be always immersed, and its buoyancy is equal to that of a much greater length of tube. The scales are also usually divided among the instruments—one for liquids lighter than water, one for specific gravities from 1· to 1·33 (corresponding to 100 to 75), and another reading from 75 (corresponding to 1·33) at the top down to 50 (corresponding to 2·06) near its middle. These instruments are not of scientific accuracy, but are ready modes of determining off-hand the approximate specific gravity of a given liquid.

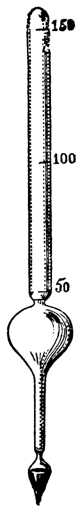
The scales of Beaumé (that most in use), as well as those of Cartier and Beck, are purely arbitrary. Table V. at the end this volume shows the correspondence of their degrees to real specific gravities.

Table VI. gives the specific gravity of some of the more frequently occurring liquids and solids.

167



168



§ 2. Hydraulics.

I. MOTION OF LIQUIDS.

213. **Definition.**—*Hydraulics* (from the Greek ὕδωρ, water, and αὐλος, a pipe), is that part of hydro-dynamics which treats of the flow and elevation of liquids, especially water, and the construction of all kinds of instruments and machines for moving them, or to be moved by them. Hero of Alexandria (about 130 B. C.) appears to have been the earliest author on this subject.

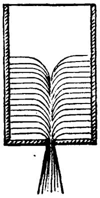
214. **Pressure of liquids upon the containing vessel.**—A vessel filled with water, or any other liquid, and closed, is subject to two pressures acting in opposite directions: namely, 1. The atmospheric pressure, acting from without inwards; and 2. The pressure of the column of contained liquid acting against the walls. If a vessel so situated is pierced, and the pressure from within outwards is stronger than the external pressure, the liquid will flow out; but if the external pressure is the stronger, the liquid will not escape.

This statement may be illustrated by filling a glass vessel, as a wine glass, with water, placing a piece of paper over its top, and supporting the paper with the hand, at the same time inverting the glass; then removing the hand from the paper and holding the glass inverted, the fluid will not escape, the external (atmospheric) pressure against the paper being greater than the weight of the column of water pressing downwards.

The mass of liquid escaping from an orifice in a vessel, is called a *vein.*

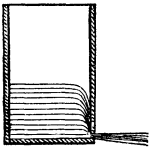
215. **Appearance of the surface during a discharge.**—The surface of a liquid, discharging itself from an orifice in a containing vessel, does not usually remain horizontal.

169



When the vein issues from an opening in the bottom of the vessel, and the level of the liquid is near the orifice, a funnel-shaped depression is found in the liquid, fig. 169. If the liquid has a rotatory movement, the funnel is formed sooner than if it is at rest. If the orifice is at the side of the vessel, there is a depression of the surface upon that side, above the orifice, fig. 173. These movements depend upon the form of the vessel, the height of the liquid in it, and the dimensions and form of the orifice.

170



216. **Theoretical and actual flow.**—The actual flow from an orifice, is the volume of liquid which escapes from it in a given time. The theoretical flow is a volume equal to that of a cylinder which has for its base the orifice, and for its height the velocity, furnished by the theorem of Torricelli. That is, the theoretical flow is the product of the area of the orifice multiplied by the theoretical velocity.

It is, however, observed that the vein escaping from an orifice, contracts quite rapidly, so that its diameter is soon only about two-thirds of the diameter of the orifice. If there was no contraction of the vein after leaving the orifice, and its velocity was the theoretical velocity, the actual flow would be the same as that indicated by theory. But the section of the vein is soon much less than at the orifice, and its velocity is not so great as the theoretical velocity, so that the actual is much less than the theoretical flow; and, in order to reduce this to the first, it is necessary to multiply it by a fraction which is named "the co-efficient of contraction."

From comparative experiments, made by a great number of observers, the actual flow has been determined to be only about two-thirds of the theoretical flow.

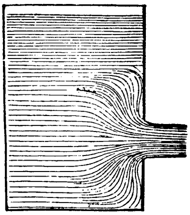
Practically, the flow, F, in a unit of time, is calculated by the formula $F = m\,v\,s$, where m is a constant, representing the ratio between the actual and theoretical velocity or flow; in other words, between the area of the orifice and the area of the section of greatest contraction in the vein. This coefficient of contraction, $m = 0{\cdot}62$; and thus the above formula becomes

$$F = 0{\cdot}62.\ s\sqrt{2gH} = 2{\cdot}75\,s\sqrt{H}.$$

$=$ the sectional area of the orifice.

The contraction of the vein is most noticeable in downward flowing jets. If the jet is thrown upwards, at an angle of 25° to 45°, the vein preserves its own diameter; but if it surpasses 45°, its section increases.

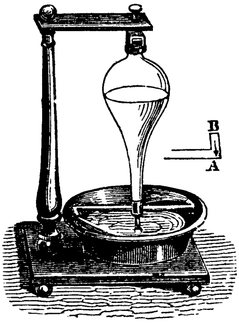
171



By suspending solid particles in the water, the currents that are formed by an escaping vein are made visible. The solid particles direct themselves, in curved lines, towards and into the orifice, as a centre of attraction, fig. 171. The particles in immediate contact with the orifice, owing to friction, not moving so easily as those near the axis, contraction must result; we can see also, that gravity, by accelerating the velocity, must cause continual decrease in the section of the jet.

217. **Reaction of the escaping vein.—Barker's mill.**—When a jet of liquid escapes from an orifice in a containing vessel, the pressure of the liquid upon the walls at the point of escape finding no counteracting force, the horizontal component of the column is not destroyed as when the opening is closed; and this force reacts to thrust the vessel in a direction opposite to that of the escaping vein.

172



This reaction is made sensible by suspending the containing vessel on a free vertical axis, as in the apparatus known as Barker's Mill, fig. 172. The orifices of escape for the vein are here in the ends of a horizontal pipe bent at right angles, and in opposite directions, formed as seen at AB, where the arrow shows the point of reaction of the escaping vein upon the end-wall of the tube.

It might be supposed, as was assumed by Newton, that the moving force in this case was only the horizontal component of a force equal to a column of liquid whose base was equal to the area of the orifice, and whose height was the distance of its centre of gravity from the level. But the effects from pressure are not the same for a liquid in motion as when in equilibrium; and D. Bernoulli has demonstrated—

That it is, in this case, requisite to estimate the force of reaction as double the height of the liquid above the centre of gravity of the orifice.

This principle is applied in the construction of reaction water-wheels.

218. **Flow.**—The volume of liquid escaping in a given time from an orifice is called its *flow.* This depends on the size of the opening and the velocity of the jet. Assuming the motion of the jet to be uniform for a given time, say one second, the distance passed over by an escaping molecule in this time is called its *velocity.* The velocity depends chiefly on the height of the liquid above the centre of gravity of the orifice; this height is called the *head* or column.

The velocity of flow is modified among other causes also by the friction of the liquid, both at the opening and against the walls. When the aperture is made in a very thin wall of a large vessel, so as to reduce as much as possible the causes tending to modify the motion of the escaping fluid, the laws of the escape are comprised in the following theorem, announced by Torricelli, in 1643, as a consequence of the law of falling bodies discovered by Galileo.

219. **Theorem of Torricelli.**—*Liquid molecules, flowing from an orifice, have the same velocity as if they fell freely in vacuo, from a height equal to the vertical distance from the surface of the liquid to the centre of gravity of the orifice.*

If H represents the height of the head above the centre of gravity of the orifice, then the velocity is expressed by the formula

$$V = \sqrt{2gH}.$$

Deductions from the Torricellian Theorem.—1. *The velocity depends on the depth of the orifice from the surface, and is independent of the density of the liquid.*

Water and mercury in vacuo would fall from the same height in the same time; and so escaping from an orifice at the same depth, below the surface, would pass out with equal velocity; but mercury being 13·5 times as heavy as water, the pressure exerted at the aperture of a vessel filled with mercury, will be 13·5 times as great as the pressure exerted at the aperture of a vessel filled with water.

2. *The velocity of flow of liquids from an orifice is as the square roots of the head.*

Thus, stating the velocity of a liquid escaping from an orifice one foot below the surface, to be *one;* from a similar orifice, four feet below the surface, it will be *two;* and at nine feet, *three;* at sixteen feet, *four;* and so on.

Let H represent the height of the liquid above the orifice, g the accelerating force of gravity, and v the velocity of discharge; we shall have $v = \sqrt{2gH}$.

220. Demonstration of the theorem of Torricelli.—The theorem of Torricelli may be demonstrated by means of the apparatus shown at fig. 173.

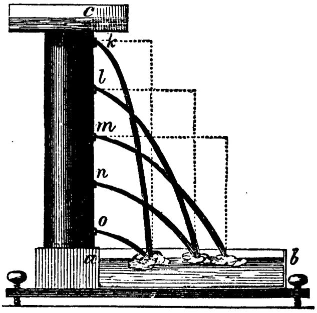
A cylindrical vase, $a\ c$, enlarged into a reservoir at the top, is filled with water. In the side of the vase are orifices, k, l, m, n, o, so situated that m is at the centre of $a\ c$, and k and o are equidistant from m, as are also l and n. Let x represent the horizontal range of a spouting jet, and y the height of the orifice above the horizontal line $a\ b$, let H be the height of the water above the orifice, and a the angle of elevation of the direction of the jet as it issues from the orifice: then by the laws of falling bodies (71), combined with the laws of projectiles (103) we shall have

$x = vt \cos. a$, and $y = \frac{1}{2}gt^2 - vt \sin. a$.

Eliminating t from these equations, we obtain

$$(1.) \quad y = \frac{gx^2}{2v^2 \cos.^2 a} - x \text{ tang. } a.$$

173



When the water issues horizontally from the orifice, a becomes zero and $y = \frac{gx^2}{2v^2}$, $x = v\sqrt{\frac{2y}{g}}$, and

$$(2.) \quad v = x\sqrt{\frac{g}{2y}}$$

The values of x and y being determined by observation in any case, the value of v, or the velocity with which the jet issues from the orifice, is readily calculated by formula (2). This velocity is found to accord very nearly with the velocity which a body would acquire in falling freely from a height equal to the head of the fluid above the orifice.

Bossuet found by using mercury that the variation from this value of v was less than one hundredth part of the velocity.

There is a remarkable consequence of this law which may easily be verified by experiment. In the formula for the value of x replacing v by its value $\sqrt{2gH}$ we have $x^2 = 4Hy$.

(a). The value of x is the greatest possible when $H = y = \frac{1}{2}ac$, as is shown in the figure, where the jet issuing from the centre of the cylinder has a greater range than any jet either above or below the centre.

(b). Since $y = ac - H$, the values of y and H may be interchanged without altering the value of x; that is, two jets issuing from orifices at equal distances above and below m meet the horizontal line ab at the same point as is shown in the figure where the jets issuing from k and o have the same range, and also the jets l and n.

The value of x and y being determined by observation, the value of v, or the velocity of the jet, becomes known by the formula (2).

221. **The inch of water** named by hydraulic engineers as the unit of measurement in the scale of water, is the volume of water which escapes in a given time, say one minute, through an orifice of one inch diameter whose centre is one and one-twelfth inches below a constant surface.

Prony has harmonized this unit with the French metrical system by employing a pipe of two centimetres internal diameter and 17 millimetres long, under a head of two centimetres. He preserves the term *inch of water*, restricting it to the quantity of water escaping in one minute from such an opening, equal to 13·333 litres, or 11·766 quarts. In 24 hours this orifice will furnish 20 cubic metres, equal to 4,402 gallons English measure.

222. **Constitution of liquid veins.**—The form and constitution of liquid veins have been studied by a great number of experimenters The results of F. Savart, and more lately of G. Magnus (Poggendorff *Annalen*, cvi., p. 1), are those here given.

It is determined, 1. That if a liquid vein issues quite calmly and vertically downwards, from a circular orifice in a plane and thin horizontal wall, no movement of rotation existing in the mass of the liquid, such a stream forms a continuous perfectly smooth cylindrical mass, the diameter of which diminishes with the distance from the orifice to the point where disintegration commences. From this point the vein assumes a turbid appearance, enlarges in diameter, and commences to spirt off small drops laterally.

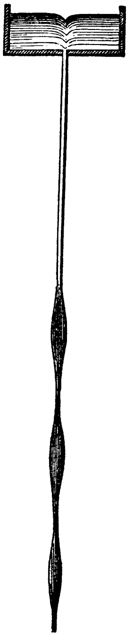
2. If the mass of liquid is in rotation in the vase, or any cause of vibration exists, as from the sounding of a musical note, then the vein is separated into two distinct parts, fig. 174. The portion nearest the orifice is calm and transparent, like a rod of glass, gradually decreasing in diameter. The second, on the contrary, is constantly agitated, and takes an irregular form, in which are distributed, at regular distances, elongated swellings, called "ventral segments," whose maximum diameter is greater than that of the orifice: while the position of the first swelling is always much nearer the orifice than the point where the jet without swellings commences to become turbid.

Magnus found that the best means to produce these "ventral segments" were a large tuning fork sounding C below the line—and the monotonous hum of the magnetic hammer or break-piece used in electro-magnetic apparatus.

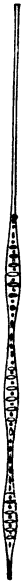
3. The swellings consist of separate isolated masses of water, as shown in fig. 175. However regular their external form may be, they are still formed of separate masses, as may be readily distinguished by

holding a piece of wire in the hand so that one of its ends penetrates a little way into the jet. A uniform pressure is felt when the wire is struck by the smooth part of the stream, but when struck by a swelling a strong vibratory and intermitting motion is felt. The separate masses of water forming the swelling, clearly communicate this motion to the air, and thus disturb the flame of a gas jet brought near them, which the smooth part of the stream does not do.

174



175



Savart found that the swellings are formed of disseminated globules, elongated in the transverse direction of the vein, and that the contractions or knots are formed of globules, elongated in the longitudinal way, fig. 175: also that the limpid part of the vein is formed of annular swellings which originate very near the orifice, propagating themselves at unequal intervals to the troubled part, where they separate, of the same form at the instant of their separation, but changing periodically.

4. The "ventral segments" are produced by the vibration of the orifice through which the water flows, and they change with the strength of the note producing the vibration, as well as with the diameter of the orifice.

The vein itself occasions a tone, partly because its single separate masses of water set the adjacent air in motion, but especially by the impact of these masses upon some sonorous or elastic substance. Where the orifice is made of caoutchouc, and this is carefully insulated by woolen pads from the bottom of the vase, not even the loud tone of a heavy tuning fork on a sounding box (377) sufficed in Magnus's experiments to cause the production of ventral segments. Without such precautions they are often set up spontaneously by vibrations communicated from the falling stream through the solid parts of the apparatus.

To observe the constitution of the swellings, Magnus used a revolving card perforated by a narrow radial slit, like the toy known as the anorthoscope, illuminating the stream by a lamp, but the details of his results exceed our space.

5. If the vein flows from a very small orifice (less than a millimetre), the small drops into which the stream breaks up move quite irregularly. But on sounding a note the drops arrange themselves in groups with great regularity—a certain number always follow each other imme-

diately—a somewhat greater interval succeeds, and then the former grouping of drops occurs again.

So in a stronger stream, under the power of a harmonious note, the swellings and knots assume more regularity, and usurp the transparent part, which almost entirely disappears—the flow of the liquid from the orifice remaining the same as at first.

6. The constitution of veins thrown out in any direction is essentially the same; but the number of pulsations is diminished in proportion as the vein is projected more vertically upwards.

223. **Escape of liquids through short tubes.**—Short tubes (called *adjutages*) are often placed in an orifice to increase the flow. They are either cylindrical or conical. If the vein pass through the tube without adhering to it, the flow is not modified; if the vein adhere (the liquid wetting the interior walls), the contracted part is dilated, and the flow increased. In the last case, and with a cylindrical adjutage, its length not being more than four times its diameter, the flow is augmented about one-third.

Conical *adjutages* converging towards the exterior of the reservoir, increase the flow still more than the preceding, the flow and velocity of the vein varying with the angle of convergence. Conical adjutages diverging towards the exterior, give the greatest flow. They may give a flow 2—4 times as great as that which an orifice of the same diameter in a thin wall furnishes, and 1·46 times greater than the theoretical flow.

Practically, the flow during a second from cylindrical adjutages of a length three and a quarter times the diameter, is found by the formula,

$F = 0{\cdot}82\, s\sqrt{2gH} = 3{\cdot}62\, s\sqrt{H}$; s being the area of the tube and H the head.

224. **Escape of liquids through long tubes.**—When a liquid passes through a long straight tube, the velocity of the flow soon diminishes greatly owing to the friction between the liquid particles and the walls. Bends or curves in the tube increase the loss in velocity, for the same reason. The discharge thus becomes very much less than it would be from an orifice in a thin wall, and to obviate this evil the tube is generally inclined; the liquid then passes down an inclined plane, or it is forced through by pressure, applied at the opposite end.

Formulæ.—The discharge, D, per second through straight tubes of uniform diameter entirely open at the end may be determined by the formula,

$$D = 20{\cdot}8\sqrt{\frac{Hd^5}{l + 54d}};$$

in which H is the height of the water above the orifice of discharge, d the diameter, and l the length, of the tube. All these quantities are to be taken in metres. The formula gives the value of D in cubic metres, which may be reduced to "*inches of water*" (221) by multiplying the result by 75. This formula was deduced from the experiments of Eytelwein. When the tube is very long, we may neglect $54d$ as very small in comparison with l, and the formula to determine the diameter required to discharge a given volume of liquid is,

$$d = 0{\cdot}298\sqrt[5]{\frac{lD^2}{H}}.$$

The velocity of the discharge is given by the formula,

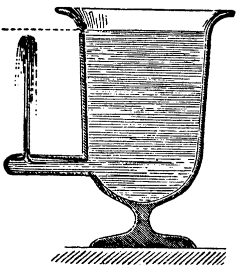
$$v = \frac{4D}{\pi d^2} = 26{\cdot}44\sqrt{\frac{Hd}{l + 54d}}.$$

For long tubes the term $54d$ may be neglected, and modifying the coefficient to correspond with the results obtained by Prony with tubes 2280 metres long.

$$v = 26{\cdot}79\sqrt{\frac{Hd}{l}}.$$

225. **Jets of water.**—As the velocity of a liquid escaping from an orifice is the same as that which a body acquires falling from a height equal to the distance from the surface of the liquid to the orifice, a jet of water spouting upwards, should rise to the level of the liquid in the reservoir. But this never quite takes place, fig. 176, because of—1st, the friction in the conducting tubes destroying the velocity—2d, the resistance of the air—3d, the returning water falling upon that which is rising. The height of the jet is increased by having the orifices very small, in comparison with the conducting tube; piercing them in a very thin wall, and inclining the jet a little, thus avoiding the effect of the returning water.

176



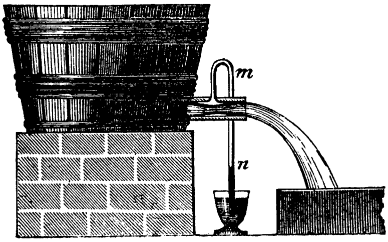
It has been determined that the differences between the height of vertical jets and that of the reservoirs are approximately as the squares of the height of the jets. Experiment has assigned the number 0·01 as the coefficient, and the formula which gives the height, h, of a jet under a head represented by H, is $H - h = 0{\cdot}01h^2$:—the unit of measure being the French metre.

If air is mingled in the water, the mixture being lighter than water, the jet can be made to rise higher than its source.

226. **Pressure exerted by liquids in motion.**—When a liquid is in motion, either in a conduit tube or an adjutage, the pressure it exerts

on the walls is not the same as it is in equilibrium, and generally it is less, as the velocity of flow is greater.

177



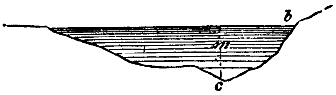
If the effective velocity is equal to theory, the interior pressure upon the walls of the adjutage will be equal to the statical pressure in a state of equilibrium. As the effective velocity increases, the interior pressure upon the walls of the adjutage becomes less than the pressure in a state of equilibrium, and it may even become less than the external atmospheric pressure, but it can never become null.

This principle may be demonstrated by the apparatus shown in fig. 177, where a bent tube, *m n*, is inserted into a cylindrical adjutage, and when the lower end is placed in a vessel of water, as shown in the figure, the fluid will mount up in the tube to a certain point *n*.

If the tube *m n* is not too long, the water will mount up and enter the adjutage, and flow out with the jet. But the fact that the water will not mount over in the tube *m n*, unless it is very short, proves that the external atmospheric pressure is always opposed by a certain amount of internal pressure. It may also be shown that the interior pressure never becomes null, but that there is merely a diminution of pressure, by placing the apparatus in a vacuum, when the water will flow out in the direction *m n*.

227. **Velocity of rivers and streams.**—The velocity of streams varies very much. The slower class of rivers have a velocity of less than three feet per second, and the more rapid as much as six feet per second, which gives respectively about two and four miles per hour

178



The velocities vary in different parts of the same transverse section of a stream, for the air upon the surface of the water, as well also as the solid bottom of the stream, has a certain effect in retarding the current. The velocity is found to be greatest in the middle, where the water is deepest, fig. 178, somewhere in *m*, below the surface; then it decreases with the depth, towards the sides, being least at *a* and *b*.

Stream measurers.—To measure the velocity of streams, various means are employed. The most simple is a glass bottle filled with water, sunk just below the level of the current, and provided at the cork with a small flag, that stands above the surface.

A wheel may also be used, furnished with float-boards, placed in the stream and immersed, so that the whole surface of the boards is covered with water. The friction in this case is very small, so that the wheel revolves with very nearly the velocity of the stream. By observing the number of revolutions of the wheel in a given time, the rapidity of the current is measured.

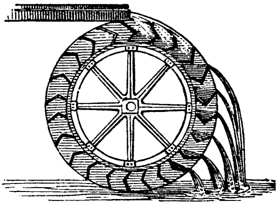
To ascertain the velocity at different depths, the simplest instrument is Pictot's tube. It consists of a tube bent nearly at right angles, terminated by a funnel-shaped mouth : the upper part of the tube, above water, is of glass. To observe with this instrument, it is sunk with its funnel up stream at the depth where its velocity is required. If the water was still, the height of the column within and without the tube would be equal; but as it is in motion, the water will rise in the tube to counterbalance the force with which the water is impelled (the impulse of the stream), the column of water in the tube rising higher as the velocity of the stream is greater.

II. WATER-POWER AND WATER-WHEELS.

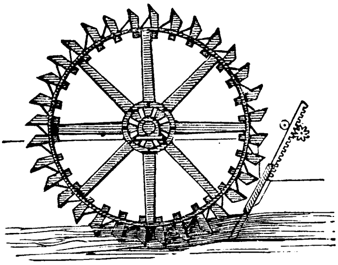
228. **Water-wheels.**—The motive power of water is of extensive practical importance, from the number of machines driven by water-wheels.

229. **The overshot wheel.**—Fig. 179 is used when the supply of water is moderate and variable. The water is delivered at the top of the wheel, which may move with the hands of a watch, as in the figure, or the reverse. It is furnished with buckets of such a shape as to retain as much of the water as possible, until they reach the lowest practicable point on the wheel, and none after that point. In this wheel the effect is produced both by impact, and by the weight of the water. The water is received as near the summit as possible, and the buckets are so shaped as to retain the water to the lowest practicable point in its descent, corresponding to about five on the face of the watch.

179



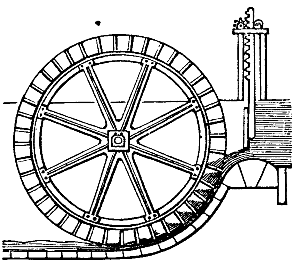
180



230. **The undershot wheel.**—Fig. 180 receives its impulse at the bottom; it is furnished with float-boards instead of buckets. If they are placed at right angles to the rim of the wheel, they may turn either way. When the wheel is required to turn only in one direction, the float-boards are placed as in the figure, so as to represent an acute angle towards the current: the water acts then partly by its weight.

The breast-wheel.—Fig. 181 is moved both by the weight and momentum of the water. It is furnished with buckets, formed to retain the water as long as possible. The breast-wheel is the form most generally adopted, as it allows of a larger diameter for a given fall than the overshot-wheel, with more economy of power than the undershot-wheel.

181



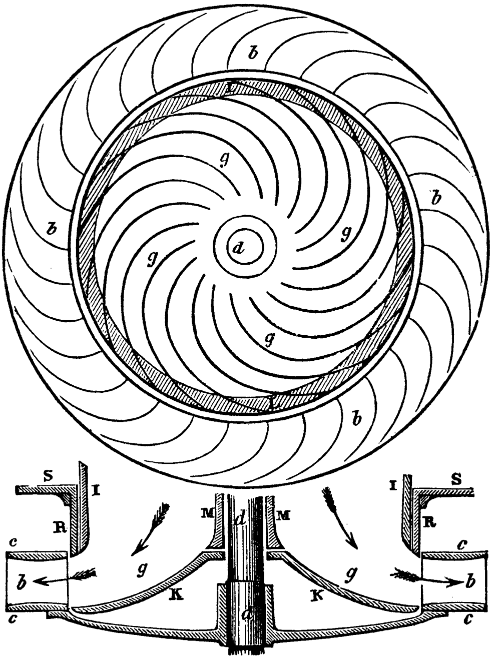
A more distinct idea of these different water-wheels may, perhaps, be gained by illustration from the face of a watch. In the breast-wheel, the water may be received (according to the desired motion of the wheel) between eight and eleven o'clock, or between one and four o'clock. According as the water is received above half-past nine or below half-past three on the watch, the wheel is called a high or low breast-wheel.

231. **Boyden's American Turbine.**—The turbine is a horizontal water-wheel, revolving entirely submerged, and is, of all forms of water-wheel, the most energetic and economical of power. This machine was first constructed in an efficient form by M. Fourneyron in 1827 as the result of experiments commenced in 1823; but the honor of perfecting the turbine and establishing the mathematical principles by which it may be adapted to every variety of water-power, whether with high or low fall of water, in both small and large streams, is due to U. A. Boyden, Esq., of Massachusetts, under whose direction turbines have been extensively introduced in the cotton manufactories of Lowell and elsewhere. Two of the turbines constructed under the superintendence of Mr. Boyden have been found to give a useful effect to eighty-eight per cent. of the power of the water employed.

The water enters the centre of the wheel, descending in its vertical axis, and is delivered through a great number of curved guides so arranged that the water enters the buckets in directions nearly tangent to the circumference of the wheel. The water is received by the curved buckets in the direction of greatest efficiency, and having expended its force, it escapes from the wheel in a direction corresponding very nearly with the radii.

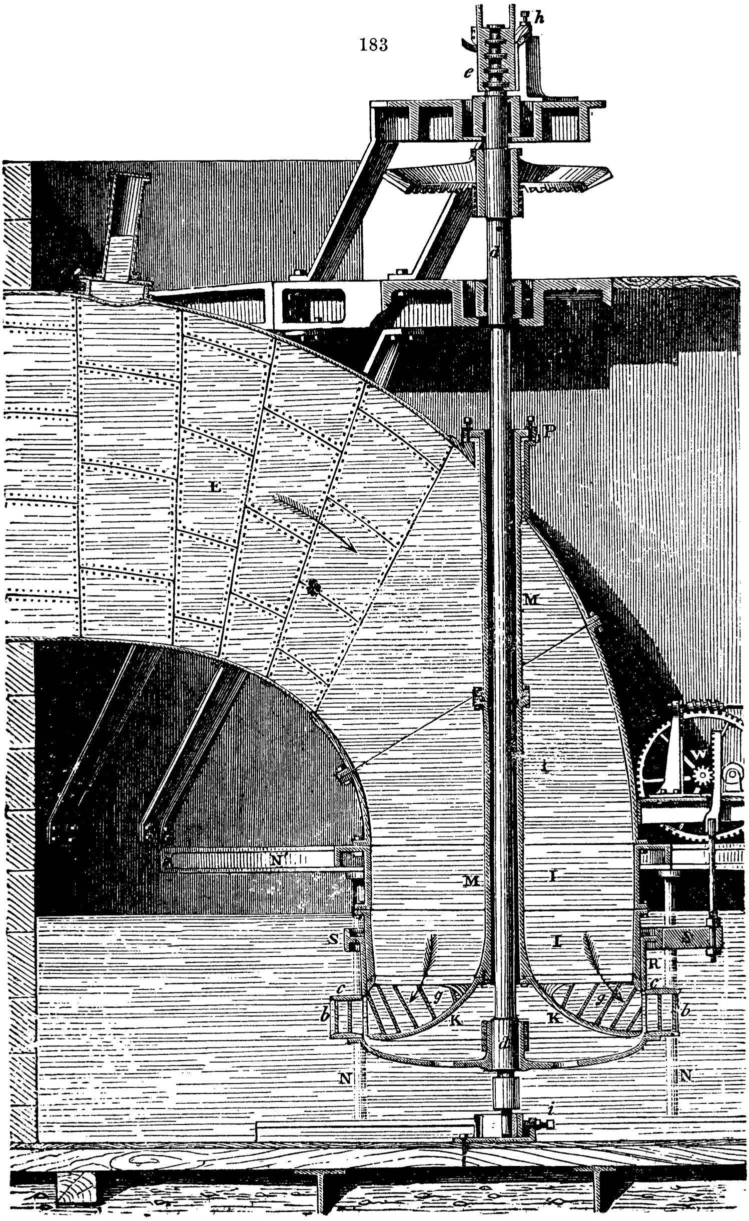
The upper part of fig. 182 shows a horizontal section of the turbine, and a perpendicular section is shown in the lower part of the same figure. Fig. 183 shows a section of the turbine with the iron sluice and other attachments as they stand in the wheel-pit. The letters refer to the same parts in both figures. K K is a stationary disc of cast iron supported by the disc tube M M made fast to the upper curb at P. The curved guides, *g g g g*, made of plate iron, are secured to the disc K, and to the rim L L above, in such a manner as to give the least possible obstruction to the water as it flows through the guides into the revolving wheel. The arrrows show the course of the water through the iron sluice E, and on the disc through the guides into the wheel. The wheel itself consists of a central plate of cast iron, and of two crowns *c c c c*, of the same material, between which are the curved buckets *b b b b*. The lower crown is firmly secured to the central plate. The buckets are let into curved grooves in the crowns, and have tenons, passing through mortices in the crowns, riveted above and below.

182



The vertical shaft, *dd*, is made of cast iron, and is accurately turned in every part. The entire weight of the wheel is supported by a series of collars attached to the shaft and moving in the suspension box *e*. The box *e* is hung upon gimbals at *h* (like a mariner's compass), supported by framework resting in the masonry of the wheel-pit. The lower end of the shaft is steadied by a pin passing into the step *i*, which is adjusted by a screw. *RR* is a cylindrical gate which drops down between the guides and the movable part of the wheel, to regulate the flow of water according to the amount of power required. Attached to the gate are the brackets *SS*, connected with the rackwork and endless screw shown at *W*, by which the gate is raised or lowered. A self-regulating adjustment of the gate is secured by a governor not shown in the figure. Ordinary gearing,

183



attached to the upper part of the shaft, communicates the power of the wheel to the machinery to be driven. The curved iron sluice E rests upon beams N', secured in the masonry of the wheel-pit and by stancheons NN.

Turbines may be divided into high and low pressure machines. High pressure turbines are adapted to hilly countries and deep mines where high falls of water may be commanded; in these cases the height of the column of water will compensate for the smallness of its volume, reservoirs being provided to keep up a constant supply.

The low pressure turbines produce great effect with a head of water of only nine inches, and are suitable for situations in which a large volume of water flows with a small fall.

The results of an investigation by Arago, Prony, and others, who were appointed by the French Academy of Sciences to report upon turbines, are as follows:—

(1). That these wheels are applicable equally to great and small falls of water.

(2). That they transmit a useful effect equal to from 70 to 78 per cent. of the total moving force of the water employed (88 per cent. has been secured by Boyden's wheel).

(3). That they will work at very different velocities above or below that corresponding to the maximum effect, without the useful effect varying materially from that maximum.

(4). That they will work from one to two yards deep under water, without the proportion which the useful effect bears to the total force being sensibly diminished.

(5). In consequence of the last-mentioned property, they utilize at all times the greatest possible proportion of power, as they may be placed below the lowest levels to which the water surface sinks.

The mathematical formulæ for adapting turbines to every variety of water-power, and much other valuable information, will be found in a treatise on the Hydraulic Experiments at Lowell, by Mr. J. B. Francis, from whose work the above condensed description has been principally obtained.

*MOLECULAR FORCES ACTING BETWEEN PARTICLES OF UNLIKE KINDS.

I. CAPILLARITY.

232 **Observation.—Definition.**—The complete discussion of the action of Molecular Forces between particles of unlike kinds belongs appropriately to Chemical Physics. We have already noticed some of the phenomena of adhesion properly referable to this section (147 and following). It now remains to consider briefly those special cases of this general subject which affect the laws of fluid equilibrium. We refer especially to the *Phenomena of Capillarity and Endosmose.*

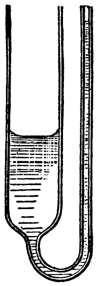
The laws of fluid equilibrium which we have already considered apply only to vessels of considerable diameter, in which the effects of adhesion between liquids and solids (148) may be safely neglected.

In very narrow vessels, and particularly in tubes of small bore, the effects of this kind of molecular attraction become very sensible. Such tubes are called *capillary tubes*, from *capillus* a hair, in allusion to the hair-like fineness of their bore. The effects of such tubes on liquids are distinguished by the general term *capillarity.*

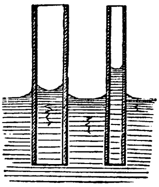
233. **General facts in capillarity.**—If tubes of small bore, open at both ends, are placed vertically in water, the liquid is seen to mount both in the tubes and on the outside, fig. 184, rising higher within as the tubes are smaller. If the bore is over half an inch in diameter, this effect is not very sensible. The experiment becomes more satisfactory if made in communicating vessels (202), of which one branch is much narrower than the other, as in fig. 185. Two slips of glass plunged in water, and brought near each other, also exhibit the effect of capillarity. In narrow communicating vessels, then, the laws of equality of level do not hold good.

If the experiment is tried with mercury (which does not wet the glass) there is a depression of the surface of the liquid both within and without the tube, fig. 186, and this becomes greater as the tubes are smaller, as seen in the two branches of the communicating vessels, fig. 187. In a greased tube water is similarly depressed.

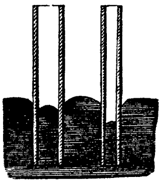
185



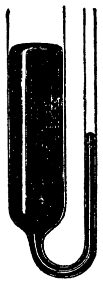
184



186



187



These phenomena are independent of atmospheric pressure—taking place equally in a vacuum or in compressed air. They are also independent of the thickness of the walls of the tube (148), but they vary with the material of the tube, and with the nature of the liquid.

Thus, in tubes of the same internal diameter, placed in liquids capable of wetting the surface of the glass, the elevation is different for each liquid. In tubes of 0·0472 inch diameter of bore, water rises 0·905 inches (or about 4 inches in tubes of $\frac{1}{100}$ inch bore), essence of turpentine 0·385, pure alcohol 0·278, whale oil about the same, while ether rises still less. In some liquids the elevation is scarcely sensible, while, as we have seen in mercury and other liquids not wetting the surface of the tube, there is a depression.

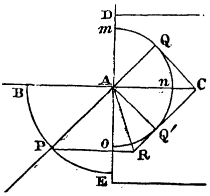
Form of the surface.—These changes of level are accompanied by a change of form in the surface of the liquid in the capillary column. It is *concave* if there is elevation—*plane* if there is no change of level, and *convex* if there is depression. The first case is called the *concave meniscus*, and the last the *convex meniscus.*

The cause of these phenomena is to be sought in the mutual action of molecular forces (146) and of gravity.

A needle covered with grease, gently placed upon the water, floats, because, not being moistened by the liquid, there is produced a depression in which it is supported. Thus, many insects walk and skim on the surface of water without plunging in. Oil and other burning-fluids in lamps, and the melted tallow and wax of candles, are supplied to their flames by means of the capillarity of their wicks; so there is an absorption of liquids in wood, in sponge, in cloth, and in all bodies that possess sensible pores.

234. **Cause of the curve of liquid surfaces by the contact of solids.**—The form of the surface of a liquid in contact with a solid, depends upon the relation which exists between the attraction of the solid for the liquid, and the liquid particles for each other.

188



Let A B, fig. 188, represent a fluid surface, and D E the surface of a solid immersed vertically in the fluid. Any liquid particle, as A, is submitted to the action of three forces, viz.: 1st. Gravity, which, as it acts equally upon all the particles of the fluid, may be omitted from the present discussion. 2d. The cohesive attraction of the fluid acting through the quadrant B A E, and having its resultant in A P. 3d. The adhesive attraction of the solid for the particle A. This latter force may be considered as divided into two parts; the attraction of that part of the solid above the surface of the fluid, whose resultant will be A Q; and the attraction of that part of the solid below the surface of the fluid, which will have a resultant in A Q′. Let B P E be drawn at the limit of sensible cohesive attraction of the fluid for the particle A, and let A P, or P, represent the intensity of the resultant of all the cohesive attraction of the liquid for the particle A; also let *m n o* have the same relation to the adhesive attraction of the solid, and Q and Q′ will represent the intensity of this force above and below the surface of the liquid.

Completing the parallelogram A Q C Q′, A C = 2 Q cos. 45° will represent the resultant of all the attraction of the solid for the particle A. On A C and A P, construct the parallelogram A P R C, and A R will be the resultant of A P and A C.

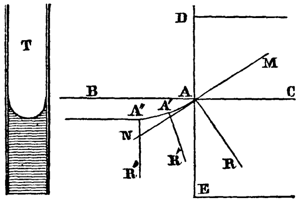
The direction of the resultant A R will be determined by the value of $2Q \cos. 45° - P \cos. 45°$, or $2Q - P$. There will evidently be three cases:

$$2Q - P > 0, \quad 2Q - P = 0, \quad \text{and } 2Q - P < 0.$$

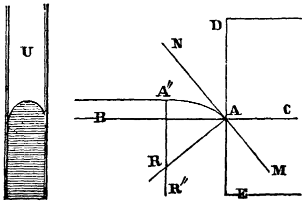
In the first case the resultant will lie in the angle C A E, and the fluid will wet the solid. In the second case the resultant will lie in A E, in the plane which separates the solid and the fluid. In the third case the resultant will lie in the angle B A E, and the fluid will not wet the solid. In this case there is no necessity to suppose any repulsion between the solid and the fluid, but only that the cohesive attraction of the fluid is more than twice as great as the attraction of the solid for the fluid.

As the surface of a liquid is always perpendicular to the direction of the forces which solicit its molecules (199) in the first case, the surface of the fluid at A will be tangent to the plane M N, fig. 189, which is perpendicular to the general resultant A R. At A′, where the attraction is more feeble, the resultant A′ R′ will be more nearly perpendicular, and at a point A″, where the sensible attraction is zero, the resultant A″ R″ will be vertical, and the curve A A′ A″ will become tangent to a horizontal plane.

189



190



In the case of a small tube, T, the concave surface of the fluid will be sensibly spherical. When $2Q - P = 0$, the resultant A R lies in the line A E, and the surface of the liquid in contact with the solid is horizontal, because the attractive force of the solid and fluid combined is the same as if the surface of the fluid was indefinitely extended.

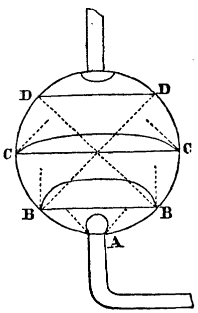
When $2Q - P$ is less than zero, or negative, the resultant A R will be found in the angle B A E, and the surface will be tangent to the plane M N at the point A; it will therefore be convex, as shown at A A″, fig. 190. In a capillary tube, U, the surface will be convex and sensibly spherical.

235. **Experimental illustrations.**—Capillary phenomena are easily explained, when we know that under the double influence of the attraction of a solid and a liquid, the surface of the liquid may be either concave, plane, or convex, as the relative intensities of these forces vary; we can also see that the ascent or depression of the liquid in capillary tubes is a direct consequence of the terminal form of the liquid. This explanation is easily verified by experiment.

a. Take a bent tube, similar to fig. 185, but let the capillary branch be shorter than the other, and pour water, drop by drop, into the larger branch, the liquid, as it rises to the top of the tube, in the short capillary branch will present successively a concave, then a plane, and at last a convex surface. As the water stands in a convex form above the end of the tube, the concave surface in the larger branch will rise above it, showing that the rise of the liquid above the level due to hydrostatic pressure depends upon the form of the surface.

b. Let a capillary tube, with a small sphere blown in it, as shown in fig. 191, be soldered into the bottom of a small glass vessel or tube, into which mercury is slowly dropped. As the mercury rises into the sphere, it will take, at A, the form of a very convex button. As it rises to B B and C C, the convexity of the surface will gradually diminish, although it will make a constant angle (about 45°) with the walls of the glass sphere, as shown by the dotted lines. When it arrives at D D, where the surface of the sphere is inclined 45°, the surface of the mercury will be horizontal, and still higher it will be concave. These successive stages of curvature are seen in filling a mercurial thermometer.

191



This experiment, depending upon the constant angle made by the surface of the mercury with the walls of the tube, enables us to show that the level of the mercury in the capillary tube is higher or lower than in the vessel with which it communicates, according as the surface is concave or convex.

The level at which a liquid may be maintained in a capillary tube depends on the diameter of the canal at the upper level of the liquid.

c. An impressive verification of this fact is obtained by soldering a capillary tube to the top of a glass vase or low air-bell, like a cupping-glass or beaker. If the diameter of the capillary tube is not more than the one-hundredth of an inch, a column of water of the diameter of the vase will be sustained by the capillary force at the height of nearly four inches—the height of the column requisite to restore equilibrium being independent of the diameter of the vase. The same apparatus being reversed and plunged in a bath of mercury will evince a corresponding depression of the level of the mercury in the capillary tube, the vase remaining void of mercury.

It is evident from these experiments that capillarity is a very energetic force, and when we remember that the capillary canals in vegetables are usually smaller than the one-hundredth of an inch, and those in the animal body are very much smaller, it is easy to understand the ascensional force of sap in plants, and the functions of the capillaries in animals.

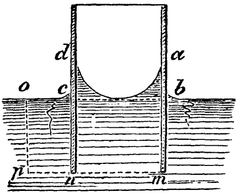
d. By this power it is that the soil in dry seasons receives moisture from below to supply the waste of evaporation,—and conversely that the benefits of rain descend to the lower strata. Hence in dry climates the surface soil is covered with saline efflorescences left by the evaporation of water holding salts in solution.

e. Rocks are split by the swelling of wooden plugs driven forcibly in the dry state into holes drilled for the purpose, and afterwards wet with water.

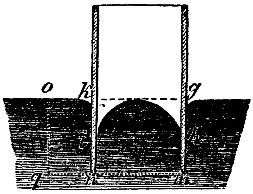
236. **Influence of the curve on capillary phenomena.**—The ascent or depression of liquids in capillary spaces, is owing to the

form of the surfaces. Let *a b c d*, fig. 192, represent a concave meniscus, the particles of which are sustained in equilibrium by the forces before mentioned; these particles do not exercise any pressure on those below them, and therefore at *c b* there is a line above which the particles of fluid are sustained by upward attraction, and below which it is sustained by equal external pressure of the column *o p*.

192



193



The sensible attraction of the walls of the tube does not extend so far as the perpendicular height, *d c*, of the curve. It is only a small portion of the wall of the tube just above the extremity, *d*, of the curve that supports the fluid. The action of every part below *d*, while it tends to elevate the fluid below, also tends to depress the portion of fluid above it, and these two influences neutralize each other. The particles of fluid about *d*, and within the limit of sensible attraction, are drawn upward by the attraction of the tube, and these particles, by their cohesive attraction, support those below, until the weight of the capillary column becomes equal to the adhesive attraction of the solid for the particles within the limit of attraction about the point *d*.

In determining the height of liquids in capillary tubes, the height of the column supported by capillary attraction must be added to the elevation produced by external pressure.

When, as in fig. 193, the meniscus is convex, the equilibrium still exists, for the liquid molecules being attracted obliquely inwards and downwards, the downward pressure is greater than on the exterior of the tube, and therefore the surface of the liquid within the tube descends until the pressure on the base, *m n*, is the same as on any exterior point, *g*, of the same layer.

237. **Law of the elevation and depression of liquids in capillary tubes.**—It has been demonstrated by Laplace, that the attraction of the meniscus is equal to a constant coefficient, depending on the nature of the liquid and that of the tube. In a cylindrical tube with a circular base, experience has demonstrated, that the concave surface is sensibly a hemisphere, with a radius equal to half the diameter of the tube. The attraction of the meniscus is, therefore, in inverse ratio

with the radius, or the diameter of the tube, and in consequence, the liquid column will be raised by this force to a height which varies according to its intensity. The length of the liquid column contained in the tube is a little less than calculation would indicate, according to the above rule, because of the weight of the meniscus, but this error is very small, less as the capillarity of the tube is less, the influence of the weight of the meniscus decreasing rapidly as the diameter of the bore diminishes. The height of the liquid in the tube is, therefore, never absolutely in inverse ratio to the diameter, but the law is nearly exact when we add to the height one-sixth of the diameter of the tube, which is the correction required for the weight of the meniscus.

Corrections for this error being thus made, the law would be correct, had the meniscus an accurately spherical surface, but this obtains only when the diameter is very small (2 or 3 *m. m.*, ·07874, or ·11811 inches) the surface in general ceases to be truly spherical, and the ascent or depression depends on the curve of the surface, which varies much more rapidly than the diameter of the tube.

238. **Depression of mercury in capillary tubes.**—The rapidity with which capillarity diminishes, in tubes of great diameter, is seen in the following table:—

TABLE OF DEPRESSIONS OF MERCURY IN CAPILLARY TUBES.

Diameter of tube.	Depressions in m. m. according to Laplace.	According to Young.	According to Jacoby.	According to Cavendish.
20· m. m.	0·038	0·031	0·031	
15· "	0·137	0·111	0·118	0·131
10· "	0·445	0·402	0·406	0·406
8· "	0·712	0·669	0·673	0·820
6· "	1·171	1·139	1·134	1·377
5· "	1·534	1·510	1·513	1·735
4· "	2·068	2·063	2·066	2·187
3· "	2·918	2·986	2·988	3·054
2·5 "	3·566			
2· "	4·454	4·887	4·888	4·472

The numbers contained in the first column have been calculated by M. Bouvard, according to the formula of Laplace; those of the last two columns have been obtained directly by experiment.

239. **Ascent of liquids in capillary tubes.**—For all liquids, the ascent or depression in capillary tubes, decreases according to analogous laws. If the tubes are very small, the heights augmented with one-sixth of their diameter, are inversely as the diameters. If the tubes are very large, we may ascertain very accurately the heights to

which liquids would rise by very complicated calculations, or we may obtain, approximately, their capillary effects, in supposing them proportional to the depression mercury undergoes in tubes of the same diameter. For the same tube, and for the same liquid, the capillarity depends much on the temperature, decreasing more rapidly than the density.

According to Gay Lussac, the elevation of water in a capillary tube of 1 m. m., (·03937 in.) is 30 m. m., (·11811 in.) and different liquids elevate themselves, in the same tubes, to heights, which are in the following relation :—

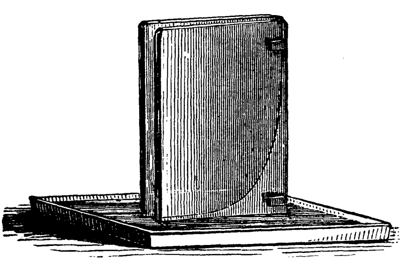
Water,	100·
Saturated solution of chloride of ammonium,	102·7
“ “ sulphate of potash,	95·7
“ “ “ copper,	84·
Nitric acid,	75·
Hydrochloric acid,	70·1
Alcohol,	40·8
Oil of lavender,	37·5

240. **Laws of the equilibrium of liquids between parallel or inclined laminæ.**—Phenomena analogous to those presented in capillary tubes, may be observed when two laminæ, plunged in a liquid, are brought near to each other. If the laminæ are made wet, the liquid elevated between them, is terminated by a cylindrical surface; if not moistened, the liquid is depressed, and is terminated by a convex surface; and it is observed that:—

1st. *A liquid is regularly elevated or depressed between two laminæ, inversely as the interval which separates them.*

2d. *That the height of the ascension or depression for a given interval, is half that which would take place in a tube having a diameter equal to that of the interval.*

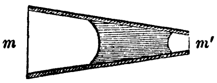
194



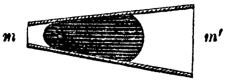
When we plunge two inclined laminæ (with their line of contact in a vertical position) in a liquid which wets them, a concave surface may be observed between them, fig. 194, the liquid rising toward the upper point of their line of contact. The surface of the liquid takes the form of the curve known in geometry, under the name of the *equilateral hyperbola;* this curve is produced by capillarity.

241. **Movement of drops of liquid in conical tubes or between laminæ.**—When a drop of liquid is contained in a conical tube, or between two laminæ having their lines of contact horizontal, the liquid, if it wets the tube or laminæ, is terminated by two concave surfaces, and the liquid is drawn towards the smaller end of the tube, or towards the angle of the laminæ, *i. e.* in the direction from *m* to *m′*, fig. 195, because the liquid being terminated by concave surfaces, the pressure from without inwards decreases as the radius of the concave surface diminishes, so that the resultant pressure is directed from *m* towards *m′*. If the liquid is mercury, or any fluid that does not wet the surrounding body, the two surfaces will be convex, and the pressure from without inwards will be greater in proportion as the radius of curvature becomes less, hence the resultant pressure will be from *m* towards *m′*, fig. 196, and the drop will be driven towards the larger end of the tube, or towards the more open parts of the laminæ.

195

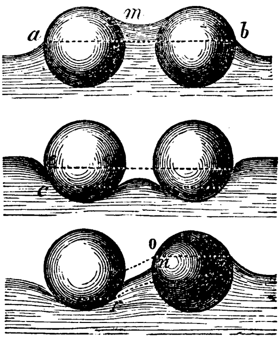


196



242. **Attraction and repulsion of light floating bodies.**—The attraction and repulsion which we observe between light bodies floating on the surface of liquids, is due to capillarity. Two floating bodies are drawn near to each other either when both are, or both are not moistened, and repelled if the liquid wets only one of them.

197



Let *a* and *b*, fig. 197, be two floating bodies whose surfaces are wet by the liquid. Between the bodies a small mass of fluid is elevated by capillary attraction, of which the point *m* is higher than the level of *a b*, the highest points of the exterior curves.

The weight of the column *m* tends to draw the two bodies together, acting like a loaded cord suspended between the two bodies. The mutual cohesion of the molecules of the liquid surface, *m*, causes it to serve as a cord, and the adhesion of the liquid to the floating bodies at the highest points, serves as the attachments of the extremities. When the two bodies are not wet by the liquid, the liquid is depressed between the bodies, and the external pressure upon the two bodies drives them together, as shown in the middle section of the figure.

When one body is wet by the liquid, and the other is not, the result of capillary attraction is to cause the two bodies mutually to repel each other.

If the body which is wet is removed beyond the influence of the other body, the concave meniscus will rise on both sides of the body to the dotted line *o*, in the lower part of the figure. In the same manner, if the body not wet were alone, the meniscus of depression would extend on both sides to the dotted line, *n*

If now we suppose the two bodies brought so near each other that the concave meniscus of fluid attached to one body will come in contact with the convex depression of the other, the surface of the liquid will take a form, *n k*, intermediate between the two curves which would have been formed when the bodies were entirely separated; that is, the more elevated point *n* will be below *o*, and the point of greatest depression at *k* will be above the point *r*. The body wet will therefore be drawn outward by the weight of that part of the external meniscus which is more elevated than the internal, as represented by the distance *o n*; and the body not wet will be driven away from the first by the excess of hydrostatic presssure due to the difference of level, *k r*, on the two sides of the second body.

Escape of Liquids from Capillary Tubes.

243. **Flow of liquids from capillary tubes.**—Fluids escaping from capillary tubes are subject to the following laws:—

1. *For the same tube the flow is proportioned to the pressure.*
2. *With tubes having an equal pressure and length, the flow is proportional to the 4th power of their diameters.*
3. *For the same pressure and the same diameter, the flow is in inverse ratio to their length.*
4. *The flow increases with the temperature.*

The inequalities in the flow of different liquids under the same circumstances does not seem to depend on their viscosity or their density; for alcohol flows slower, and oil of turpentine, or sugar solution, faster than water. So also nitrate of potash solution flows faster, and serum flows less swiftly, than pure water; alcohol added to serum retards its movement, while if nitrate of potash solution be added to the mixture, the serum recovers its usual velocity.

These experiments made with glass tubes, were repeated on the bodies of animals recently killed, by injecting the various fluids into the principal arteries. The results were found to accord, tending to prove that the circulation of blood and other fluids in the arteries and veins of living bodies, is subject to the same laws as the flow of liquids in capillary tubes of glass.

II. OSMOSE OR OSMOTIC FORCE.

244. **Osmose, Exosmose, Endosmose.**—Osmose is the transmission of liquids into each other through the pores of an interposed medium which ordinarily offers more resistance to the passage of one of the liquids than of the other.

When a membranous sac, or a vessel filled with a fluid, and closed by a membrane, is plunged into another liquid capable of mixing with the first, two currents are established through the membranous partition. The current from within outwards is called *exosmose* (from ἔξω outwards, and ὠσμος impulsion), and the current from without inwards is called *endosmose* (from ἔνδον inwards, and ὠσμος).

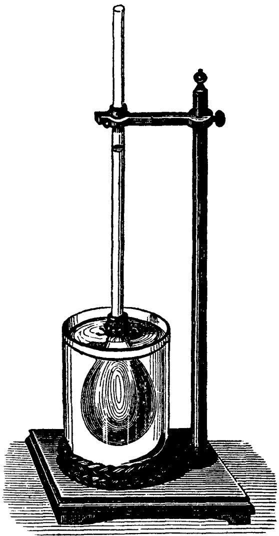
The more general term *osmose* has been substituted by Graham for the two correlative terms just defined, as being a better expression of all the phenomena concerned.

The phenomena of osmose are closely allied to those of capillarity. They have been very accurately studied, particularly by Dutrochet, who brought forward his researches in 1827, and more recently by Prof. Graham.

245. **Endosmometer.**—The existence and rapidity of these currents is ascertained by the *endosmometer*, an instrument which may be thus constructed. To a membranous pouch or bladder is fitted, hermetically, a glass tube as in fig. 198.

The jar or bladder, and part of the tube, is filled with a dense liquid, as a strong solution of gum or sugar, and placed in a tall cylindrical jar, which is then filled with distilled water, until it stands exactly at the level of the fluid in the tube. For very exact experiments, this level is constantly maintained by the addition to, or the removal of the water in the outer jar. After a time, gum will be found in the outer vessel, a current from without, inwards, also taking place.

198



If we wish to determine more accurately the actual as well as the comparative flow of different liquids, we may use an apparatus constructed as follows:—Over the open mouth of a bell jar, of a few ounces capacity, is placed a plate of perforated zinc, to support firmly a piece of fresh ox-bladder, which is securely tied over it. To the upper aperture is attached a graduated tube, open at both ends, the capacity of whose interior bears a certain definite relation, as $\frac{1}{100}$th, to that of the lower opening of the bell jar; so that a rise or fall in the tube as of 100 m. m. (3·937 in.) indicates the entrance or removal of a stratum of liquid, 1 m. m. (0·03937 in.) thickness over the whole surface.

246. **Necessary conditions.**—According to M. Dutrochet, in order successfully to produce the phenomena of endosmose, it is necessary:—

1st. *That the liquids be susceptible of mixing.*

2d. *That they are of different densities.*

3d. *That the membrane or wall* (*septum*) *which separates them is permeable to one or both liquids.*

Materials for septum.—All thin animal and vegetable membranes thin plates of burnt clay, slate, marble, pipe-clay, &c., produce endosmotic effects in a more or less notable degree. Of inorganic materials, those which contain most silicic acid are less permeable. A chemical action on the materials of the septum, invariably takes place (excepting with alcohol and cane-sugar solutions), whether it is formed of bladder or of earthenware. Where the partition is not susceptible of being acted upon, the endosmotic action is very slight.

247. **Direction of the current.**—The endosmotic current is in general directed towards the more dense liquid, but alcohol and ether are exceptions; they acting as denser liquids, although lighter than water; so also as acids are more or less diluted, there is endosmose towards the acid or towards the water. The excess in the quantity of the liquid which passes into the endosmometer, is proportional to the surface of the membrane, and to the different heights to which the liquids mount in capillary spaces, the elevation taking place from that side of the liquid which has least capillary action.

248. **Organic solutions.**—Neutral organic substances, such as gum-arabic, urea, and gelatine, produce but little endosmotic action. Of all vegetable substances, sugar solution; and albumen among animal bodies; are those which, with equal density, possess the greatest power of endosmose. The figures attached to the following substances, indicate the proportional height to which the liquids rose when the endosmometer, being filled successively with solutions of them, of the same density, was placed in pure water: gelatine 3, gum 5, sugar 11, albumen 12.

249. **Inorganic solutions.**—Neutral salts do not possess any peculiar power of endosmose, but diffuse themselves with nearly the same rapidity as if no porous partition was used. Alkaline solutions greatly accelerate endosmose. This may be observed, even in solutions which contain but 1 part of the alkaline salt in 1000 of water. In moderately dilute solutions (containing not more than 2 per cent. of the salt) the action is most rapid.

The soluble salts in the soil are taken up by the rootlets of plants by the combined action of capillarity and endosmose; the salts entering the plant more rapidly than the water which holds them in solution.

250. **Endosmose of gases.**—There is endosmose between gases, as between liquids; if we connect two vessels containing different gases, having a dry membrane between them, the gases will gradually mix, equal currents being established in both; but if the membrane is moist, unequal currents (that is, endosmose) are formed. Thus, a soap bubble placed in a jar of carbonic acid, will, in a little time, burst, owing to the increase of volume caused by endosmose.

251. **Theories of endosmose.**—Many theories have been proposed to

account for these phenomena: such as that endosmose was due to an unequal viscosity of the two liquids; to currents of electricity passing in the direction of the endosmose; to the unequal permeability of the membrane for the two liquids, or, that the phenomenon was due to capillary action, joined to the affinity of the two liquids. Very probably endosmose depends on the same forces that produce capillarity, but obviously they are not the only forces which exert influence, for we find that heat, which always diminishes capillarity, augments the strength of the endosmose.

Problems on Hydrodynamics.

Elasticity of Liquids.

93. A cubic foot of water at the freezing point is submitted to a pressure of 20 atmospheres. How great is the condensation? and what is the specific gravity of the condensed liquid?

94. What is the specific gravity of sea water at the depth of three miles, reckoning the specific gravity at the surface 1·026, and the compressibility 0·0000436, and allowing a column of fresh water 33 feet high to equal the pressure of one atmosphere?

95. How much would the volume of a cubic foot of alcohol, at 45° Fahrenheit, be diminished by a pressure of four atmospheres?

Hydrostatic Pressure.

96. In the hydrostatic press, given the diameters of the two cylinders A and B, and the force applied to the pump P: determine the pressure produced.

97. In the hydrostatic press, suppose the diameter of the smaller cylinder to be 1 inch, and the diameter of the larger cylinder to be 15 inches, the length of the pump-handle to be 3 feet from the fulcrum, and the distance of the piston 2 inches from the fulcrum, the lever being one of the second order: what is the relation of the pressure exerted to the power employed?

98. A cubical vessel is filled with fluid: compare the pressures upon the sides and bottom.

99. A slender rod is immersed vertically in a fluid: divide it into three portions which shall be equally pressed.

100. Compare the pressures on two equal isosceles triangles just immersed in the same fluid, one with its base upwards, the other with the base downwards.

101. A cylindrical vessel is filled with a heavy fluid: what proportion does the pressure on the cylindrical surface bear to the entire weight of the fluid?

102. If the tube, T, of the water-bellows, fig. 144, is 10 feet high, and the surface of the bellows, B C, is 18 inches in diameter, what weight will be sustained when the tube is filled with water? and what when the tube is filled with mercury?

103. The sides of a hollow pyramid are isosceles triangles, the base is a rectangle having sides a and b, and the height of the pyramid is c. If the pyramid be placed with its base on a horizontal plane, and filled with water, how does the whole amount of pressure on the four sides compare with the pressure upon the bottom?

104. A hemispherical vessel, 6 inches in diameter, without a bottom, stands on a horizontal plane. When just filled with water, the liquid begins to run out at the bottom. Determine the weight of the vessel.

105. What height must a column of mercury have to balance a column of water 25 feet high in a communicating vessel?

106. If a vessel of water communicates with a vessel of ether, standing at a height of 20 inches, at what elevation will the water stand?

Buoyancy of Liquids.

107. A man exerting all his force can raise a weight of 300 lbs.: what would be the weight of a stone (*Sp. Gr.* = 2·5) which he could just raise under water?

108. If a given piece of silver be balanced by its weight of iron in air, what addition must be made to the iron so that the iron and silver may be in equilibrium when immersed in water?

109. How much will 12 ounces of gold weigh when immersed in alcohol (*Sp. Gr.* = 0·798)?

110. If an alloy of gold and silver, weighing 22 ounces in air, loses 1½ ounces when weighed in water, how much of the alloy is gold, and how much silver?

111. To avoid imposition in the purchase of lead (due to the fraudulent introduction of pigs of iron encased in lead), the Russian government are in the habit of weighing the lead offered for sale with weights of lead, on a balance so arranged that both pans can be immersed in water after they are brought to equilibrium. If the equilibrium remains undisturbed, the lead is pure. Otherwise its degree of adulteration can be calculated. Suppose, by the above test, that 1500 lbs. of commercial lead is found to be adulterated to such a degree as to require the addition of 10 lbs. of lead weights under water to produce equilibrium, what is the amount of iron encased in the commercial lead (*Sp. Gr.* of lead = 11·45; of cast iron = 7·64)?

Floating Bodies.

112. Supposing the specific gravity of a man, of water, and of cork, to be 1·12, 1, and 0·24 respectively, what quantity of cork must be attached to a man weighing 150 lbs., that he may just float in water?

113. A cylinder, whose length is greater than its diameter, having a specific gravity of 0·63, floats in water, what portion of the diameter of the cylinder is immersed?

114. What is the bulk of a hollow vessel of copper, weighing 5 lbs., which just floats in water?

115. How much bulk must a hollow vessel of iron occupy, weighing one ton, that it may float with only one-half its bulk immersed in water?

116. A ship entering a river from the ocean sinks 2 inches, and after discharging 12,000 lbs. of cargo rises one inch; what is the weight of the ship and cargo, reckoning the specific gravity of sea water 1·026?

117. A life-boat contains 100 cubic yards of wood (*Sp. Gr.* = 0·8), and 50 cubic yards of air (*Sp. Gr.* = 0·0012). When filled with fresh water, what weight of iron ballast (*Sp. Gr.* = 7·645) must be thrown into it before it will sink?

118. A parallelopiped of ice whose dimensions are 15·75 yards, 20·45 yards, and 10·5 yards, is floating in sea water on its broadest face; the specific gravity of sea water is 1·026, and that of ice 0·93. Required the height of the ice above the surface of the water.

119. When two persons, A and B, descend together to the bottom of a lake in a cylindrical diving-bell, it is observed that the water stands 1 inch lower within the bell than when A descends alone; the pressure of the atmosphere is equal to a column of water 33 feet high, the diameter of the bell is 4 feet, and the sur-

face of the water within it, at the bottom of the lake, is 20 feet below the surface of the lake; find the volume of B.

120. A piece of flint glass weighs in air 4320 grains, and in water it weighs 3195 grains: what is its specific gravity?

121. Determine the specific gravity of granulated tin from the following data:

Weight of bottle filled with water at 60° F.,	44·378 grammes.
" " tin,	9·431 "
" " bottle tin and water,	52·515

122. The same specific gravity bottle used in the last example is supplied with 7·432 grammes of powdered glass. The weight of bottle water and powdered glass is 49·859 grammes, what is the specific gravity of the powdered glass?

123. A body weighs 14 lbs. in air, and 9 lbs. in water; another body weighs 81 lbs. in air, and 7 lbs. in water: what are their respective specific gravities, and how do they compare with each other?

124. The counterpoise of a Nicholson's hydrometer requisite to sink it to zero weighs 25 grammes; with a piece of brass on the upper pan it requires 8·171 grammes to sink it to zero, and 10·241 grammes when the same piece of brass is on the lower pan: what is the specific gravity of the brass?

125. A Fahrenheit's hydrometer weighs 700 grains—to sink it in water 300 grains are requisite (volume of water = to volume of hydrometer = 1000 grains), placed in alcohol 132 grains are required to bring it to zero (832 grains = volume of alcohol = volume of hydrometer). What is the specific gravity of the alcohol?

Motion of Liquids.

126. What volume of water will flow from an orifice 2½ inches in diameter, in 7 seconds, if the centre of the orifice is 10 feet below the surface of the fluid?

127. A vessel 20 feet deep is raised 5 feet above a plane: how far will a jet reach issuing 5 feet from the bottom?

128. A jet of water issuing from a vessel, 3 feet below the surface, and an equal distance above the horizontal plane on which it falls, is seen to have a horizontal range of 2·3 feet; how does the velocity of discharge compare with the theoretical velocity?

129. A jet of water issues from a cylindrical adjutage 2 inches in diameter, 6½ inches long, with a head of 10 feet. What amount of water is discharged per hour?

130. What quantity of water will be discharged per day through a tube one inch in diameter and 19 feet long, under the pressure of 21 feet head?

131. If a fire-engine discharges 16·8 cubic feet of water through a ¾ inch pipe in one minute, how high will the water be projected, the pipe being directed vertically?

Capillarity.

132 What will be the difference of level in a glass tube $\frac{1}{20}$ inch diameter, bent in the form of the letter U, when one branch is filled with mercury and the other branch with alcohol?

133. A block of marble 10 feet long and 2 feet thick (the tenacity of marble being 9000 lbs. to the square inch), is burst asunder by the capillary attraction of a series of wooden plugs. The series of plugs being 8 inches apart, 1 inch in diameter, and 5 inches long, the plugs are driven dry, and afterwards allowed to absorb water by capillarity. What height would be required for a series of columns of water, acting upon the orifices where the plugs are inserted, to produce a similar rupture of the marble?

CHAPTER IV.

OF ELASTIC FLUIDS, OR GASES.

Pneumatics.

I. DISTINGUISHING PROPERTIES OF GASES.

252. **Definitions.—Pneumatics — Gases, vapors,—tension.—** *Pneumatics* (from *Πνεῦμα,* a spirit or breath,) is a subdivision of the general subject of Hydrodynamics (186), and is devoted to the consideration of the properties of *elastic fluids.*

Gases are elastic fluids, aëriform, transparent, and usually colorless and invisible. The blue color of the air is due to watery vapor in the atmosphere. In gases, the molecular force of repulsion (146) prevails over the force of attraction—and in the permanent gases this force has never been overcome.

Vapors differ from gases chiefly in that they are produced by the action of heat upon liquids—as steam is produced from water; and by their returning again to the liquid state at ordinary temperatures by the loss of heat.

Tension is an expression for the tendency of a gas to expand; the degree of expansive force in each gas being specific and varied by temperature, and mechanical means.

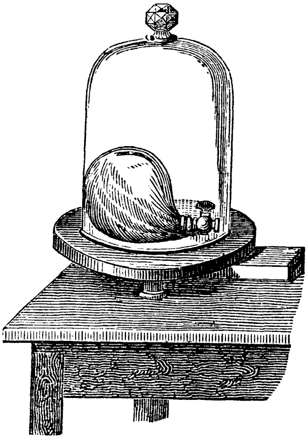
Gases, simple or compound.—Of the thirty-four gaseous bodies known in chemistry, four only are simple or elementary, viz.: oxygen, nitrogen, hydrogen, and chlorine. The three first named of these gases, together with the compound gases, oxyd of carbon (CO) and the binoxyd of nitrogen (NO_2), are the only aëriform bodies which have thus far resisted the united effects of cold and pressure, and permanently retained their gaseous state. Hence they are called permanent or incoercible gases.

All other gases, whether simple or compound, have, by the means named, been coerced into the liquid or solid state, and are hence called *non-permanent* or coercible gases.

253. **Expansion of gases.**—Expansion is the most characteristic property of gases. This molecular force, for all that appears, would separate the particles of a gas indefinitely through all space, were there no counteracting causes.

Under normal conditions, the atmosphere is in a state of equilibrium between the earth's attraction and its own expansive force. If we disturb this condition of equilibrium, we see evidence of the exercise of the power of expansion. In fig. 199, a moist bladder, partly filled with air, is subjected to a partial vacuum under the air-bell. As the pressure in the bell is diminished by working the air-pump, the portion of confined air expands, and distends the flaccid bladder until it fills the jar. As soon as the equilibrium of pressure is restored by opening a communication with the external air, it contracts again to its original dimensions.

199



It appears, therefore, that gases, like liquids, are in a state of equilibrium, the only difference in the conditions of equilibrium being that, in liquids, this state results from the opposite effects of the two molecular forces; while in gases, the repulsive force is held in control by gravity, or some extraneous force.

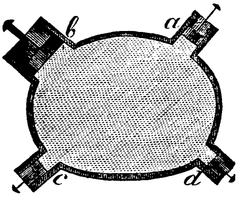
254. **Mechanical condition of gases.**—Perfect freedom of motion among their particles, as a consequence of equilibrium, brings gases under the general definition of fluids. Being also elastic, ponderable, and impenetrable (14), it follows that all the characteristic properties of liquids already discussed, apply also to gases. Atmospheric air is the type of permanent gases. For its chemical constitution, reference is made to chemistry.

II. PROPERTIES COMMON TO BOTH LIQUIDS AND GASES.

255. **Gases transmit pressure equally in all directions.**—The theorem of Pascal, already demonstrated with respect to liquids (189), is also true of gases.

Suppose the vessel, fig. 200, to be filled with air in the usual state of tension. By its elasticity it exerts an equal pressure in all directions; and by the reasoning in § 189, the pressure it exerts on the pistons *a*, *b*, *c*, *d*, is in proportion to their areas. The same is true of any part of the inner surface of the vessel, or of any section of its interior.

200



If the air within and without the vessel has the same tension, then the pistons have no tendency to move, the inner and outer pressures exactly balancing each other. Any pressure applied upon either of the pistons develops an increase of elastic force in the gaseous contents of the vessel, proportioned to the amount of compression. This pressure reacts on every portion of the inner surface of the vessel, and moves each of the other pistons outwards with a force proportioned to its area.

The chief difference between the transmission of pressure in gases and liquids is, that in gases, owing to their elasticity, the effects of pressure are not felt at long distances, so instantaneously as in liquids.

The distribution of illuminating gas in cities through many miles of pipes illustrates both the law and the exception.

The reaction due to elasticity prevents, as is well known, the driving of a blast of air in an effective manner through small and tortuous passages.

The laws and illustrations regarding the pressure and equilibrium of liquids. contained in §§ 191, 192, 193, and 199, are also true of gases; and it is therefore needless to repeat them in this connection.

256. **The atmosphere.**—*Its general phenomena.*—A vast aërial ocean rests upon the surface of the earth, penetrates even its solid crust, and is dissolved, to a certain extent, in its waters. It is composed of the two incoercible gases, nitrogen and oxygen, in the proportion of nearly four parts of the first, to one part of the second, by measure. It is held in its place by the force of gravitation, which, counteracting the molecular force of repulsion, brings it to equilibrium at about forty-five miles above the earth. This height of the atmosphere has been determined chiefly from the phenomena of refraction, as observed in its effect on the rising and setting of the heavenly bodies.

It is the opinion of Bunsen and others that the atmosphere extends to a distance of about 200 miles, although its density, above 45 miles, is too small to refract light to such a degree as to enable us to observe it. Many pneumatic experiments are thought also to indicate an altitude of the atmosphere exceeding 45 miles.

The atmosphere partakes of the motion of the earth, but its state of rest, with respect to bodies on the earth's surface, is disturbed by winds and currents, caused by agencies to be considered hereafter.

Like the ocean, the upper surface of the atmosphere must, theoretically, have a definite surface, since each particle is influenced by gravity in a similar manner, and the resultant direction of these actions at any point must be a radius of the earth (199).

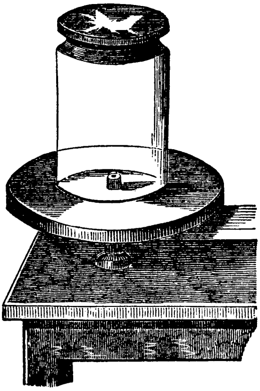
It follows from § 191, that each molecule of air exerts, at a given level, the pressure due to the weight of a continuous line of molecules, extending vertically from the point chosen to the outer limits of the atmosphere. Therefore its upward pressure (192), its pressure on the sides of any vessel (193), and its buoyancy (205), are the same, and governed by the same laws as those already enunciated for liquids;

provided always that there is a communication, however small, between the outer air and the interior of any given vessel.

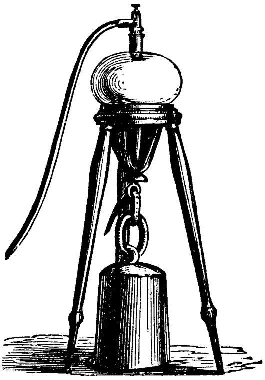
257. **Atmospheric pressure.**—The great weight, and consequent pressure of the atmosphere upon bodies near the surface of the earth, was unsuspected by mankind in general, until Torricelli, in 1643, first announced it.

As it is exerted, in obedience to the laws of fluid equilibrium (199), alike above, below, and on the sides of all bodies, a man of usual size moves about unconscious that he sustains a constant load of over 30,000 pounds, or more than fifteen tons. If this pressure is partially removed from one surface of a body, its existence then becomes very manifest.

201



202



In fig. 201, the upper end of an air-jar is hermetically sealed by a bladder skin tied on when wet and dried. Its lower edge rests upon the well-ground plate of an air-pump. As the air in the jar is gradually exhausted by working the pump, the surface of the bladder becomes more and more depressed, until, finally, the membrane bursts, with a sharp report, owing to the pressure of the atmosphere resting upon it.

This experiment demonstrates the downward pressure of the atmosphere only.

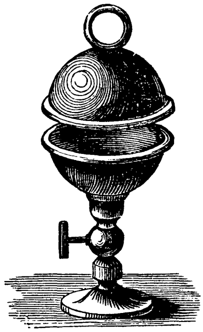
Its upward pressure is illustrated by the apparatus seen in fig. 202.

A glass jar having an open bottom, and sustained on a tripod, is covered by an impervious caoutchouc bag. When a partial vacuum is produced in the jar through the upper opening, the yielding bag rises and carries with it the weight which is hung below. This heavy mass is sustained in mid air as on an elastic spring by the upward pressure.

The upward pressure of the air may also be illustrated by a familiar experiment with a tumbler. Fill a tumbler with water, and lay over it a piece of paper,—hold the paper in its place by laying upon it a board or the palm of the hand,—turn the tumbler bottom upwards, and remove the hand or board, the upward pressure of the atmosphere will then retain the paper in its place, closing the tumbler, and preventing the discharge of the water.

The pressure of the air from all sides is shown by the well-known Magdeburg hemispheres.

203



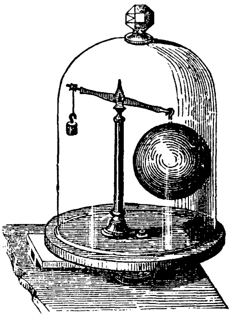
This apparatus is composed of two hollow hemispheres of brass, fig. 203, whose accurately fitting edges are well greased. One of the hemispheres is furnished with a stop-cock, by which connection is made with an air-pump. Placing this apparatus upon the air-pump, and exhausting the air, it will be found that the hemispheres can no longer be separated, no matter in what position they may be held; proving that the atmospheric pressure which alone keeps the hemispheres together, is exerted in all directions. Rings adapted to each hemisphere, enable two persons to test their strength against the atmospheric pressure. Otto V. Guerick, who invented them, employed a pair which held all the power of a strong team of horses.

These illustrations, easily multiplied by the ingenuity of the teacher, give evidence of the fact of atmospheric pressure in all directions, but do not indicate its amount.

258. **Buoyancy of air.**—*Bodies weighed in air are sustained or buoyed up by a force equal to the weight of the volume of air displaced,* in accordance with the Archimedean principle (205).

This law is well illustrated by the apparatus seen in fig. 204. A hollow globe of brass is counterpoised on one arm of a balance by a brass weight at the other end. Placed on the plate of an air-pump, and covered by a bell-glass, the air may be removed from contact with the two masses previously in equilibrium; and, in proportion as the vacuum is produced, the globe begins to preponderate by a force as much greater than the action of gravity upon the counterpoise as the weight of its own volume of air is greater than that of the counterpoise.

204



The brass and platinum weights used in delicate determinations of weight are standards only when in vacuum. Let us then represent the various values as follows :—

W' = weight of the body in air as estimated by standard weights, and also the weight of the standard weights themselves in a vacuum.

V' = volume of the standard weights in cubic inches.

V = volume of the body in cubic inches.

w = weight of one cubic inch of air at the time of the weighing.

W = weight of the body in a vacuum—which we wish to find.

We can now easily deduce the following values:—

$V'w$ = buoyancy of air on the weights.
Vw = buoyancy of air on the body.
$W' - V'w$ = actual weight of standard weights in air.
$W - Vw$ = actual weight of body in air.

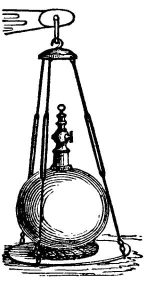
Since these weights just balance each other, we have,

$$W - Vw = W' - V'w, \quad \text{or} \quad W = W' + w\,(V - V').$$

The correction $w\,(V - V')$, which must be made to the weight determined by the balance in air in order to obtain the weight in a vacuum, is evidently additive when the volume of the body is greater than the weights, and subtractive when those conditions are reversed. When the volumes are equal, the correction becomes zero, and the balance yields the same results in air as in a vacuum.*

If a vessel, whose capacity is 100 cubic inches, is exhausted of air and weighed, fig. 205, and after filling it with dry air at the ordinary temperature and pressure, it is weighed again, it will be found that its weight is 31·074 grains more than at first; that is, 100 cubic inches of air weigh 31·074 grains. Air is the standard of comparison in density for all gases and vapors.

205



259. **Impenetrability of air.**—Air is impenetrable. This may be shown by inverting a hollow vessel, as a tumbler, upon the surface of water; when pressed downward the water will not rise and fill the tumbler, because of the impenetrability of the air. The *diving-bell* depends on this quality of air: it consists of a large bell-shaped vessel, sunk by means of weights into the sea, with its mouth downwards. Notwithstanding the open mouth, and enormous pressure of the sea, the water is excluded from the bell, because of the air contained within.

260. **Inertia of air.**—Wind is only air in motion. If the air had no inertia, it would require no force to impart motion to it, nor could it acquire momentum. We know that the force encountered by a body moving through the air (that is, displacing the air), is in proportion to the surface exposed, and the velocity with which it is moving (143).

The sailing of ships, the direction of balloons, the wind-mill, and the frightful ravages of the tornado, are all familiar examples of the power of moving air, and consequently proofs of its inertia.

III. BAROMETERS AND BALLOONS.

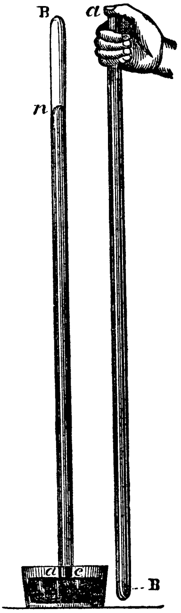
261. **Torricellian vacuum.—Measure of atmospheric pressure.**—The amount of pressure exerted by the atmosphere was first determined by Torricelli, a disciple of Galileo, in 1643.

---

* Cooke's Chem. Physics, p. 269.

If a glass tube, *a* B, fig. 206, about 32 inches in length, is filled with mercury, and then inverted in a vessel of the same fluid, the liquid column will fall some distance, and after several oscillations will come to rest at *n*, a height at the level of the sea, of about thirty inches above the level, *a c* of the mercury in the vessel. The space *n* B, above the mercury, is the most complete vacuum attainable by mechanical means, and is called the Torricellian vacuum. If, after having closed the mouth of the tube, we lift it out of the dish, we shall find that the weight of the column of mercury pressing against the finger is very considerable. When we place the tube in the vessel of mercury, we have this same force exerted, the column of mercury tending to flow out of the tube, and another force, the weight and pressure of the atmosphere, tending to push the mercury up in the tube. The length of the mercurial column, it is evident, is in proportion to the atmospheric pressure, which under ordinary circumstances, is equivalent to a column of mercury thirty inches in height.

206



We may now easily estimate the pressure on any given surface, as, for example, a square inch. If we should take a tube whose base is a square inch, and repeat the above experiment, the column *a n* would, as before, be sustained at a height of thirty inches; but the weight of a column of mercury thirty inches in height and one inch square, is very nearly fifteen lbs.; therefore the atmospheric pressure on a square inch is fifteen lbs. (accurately 14·7225 lbs).

If the tube were filled with a liquid lighter than mercury, a proportionally longer column would be sustained by the pressure of the atmosphere: the length of the column being inversely as the densities of the two fluids. If water, which is about 13·5 times lighter than mercury, was used, the column of water sustained would be 13·5 times as long as the mercurial column, or about thirty-four feet.

262. **Pascal's experiments.**—The experiment of Torricelli excited the greatest sensation throughout the scientific world, and the explanation he gave of it was generally rejected.

Pascal, who flourished at that time, perceived its truth, and proposed to subject the experiment to a test which must put an end to all further dispute. "If," said Pascal, "it be really the weight of the atmosphere under which we live, that supports the column of mercury in Torricelli's tube, we shall find by transporting this tube to a loftier point in the atmosphere, that in proportion as we leave below more and more of the air, there will be a less column of mercury sustained in the tube." Pascal therefore carried a Torricellian tube to the top of a lofty mountain, called the Puy-de-Dome, in Auvergne (central France). It was found that the column gradually diminished in height as the elevation to which the instrument was carried increased. He repeated this experiment at Rouen (France), in 1646, with a tube of water, and found that the column was sustained at a height of about thirty-four feet, or 13·59 times greater than the height of the column of mercury.

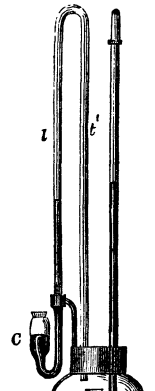
263. **Construction of barometers.**—Barometer is the name given to Torricelli's tube. This instrument has different forms, according to the use for which it is designed. There are, however, certain conditions to be fulfilled in the construction of barometers, whatever may be their form.

1st. It is necessary that the mercury be perfectly pure and free from oxyd, otherwise it adheres to the glass; again, by impurities, its density is changed, and the height of the column in the barometer is greater or less than it should be.

2d. It is necessary that there be a perfect vacuum above the surface of the mercury in the tube; for if there be a little air, or vapor, as of water, the elasticity of these will continually depress the mercurial column, preventing its rising to the true height.

To obtain a perfect vacuum, a small portion of pure mercury is boiled in the barometer tube, and when cooled, another portion of mercury is added, and again boiled, and so on, until the tube is full; by this means the air and moisture which adhered to the walls of the tube are driven out completely. The boiling must not be too long continued, otherwise a portion of oxyd will be formed, which will dissolve in the mercury and alter its density. The tube being filled, we invert it in a vessel of pure mercury. In order to determine whether there is not some air or moisture in the tube, we incline the tube quickly; if the mercury gives a dry metallic sound when striking the summit of the tube, it is a proof of their absence, while, if they be present, the sound is deadened.

207



264. **Apparatus illustrating the principle of the barometer.**—By means of the air-pump, and the apparatus fig. 207, the principle of the barometer is beautifully shown.

The apparatus consists of a large bell-glass, R, with two syphon barometer tubes attached. One of them, B, has its cistern within the bell. The other barometer, whose cistern is without the bell, communicates with its interior by the curved tube *l t'*. When this apparatus is placed on the air-pump, and exhausted of air, the mercury in B falls in proportion to the vacuum produced, and rises in the tube C *l*, in the same proportion. In B we see the effect of diminished pressure, as on a mountain or in a balloon; in C the pressure of the external air causes the mercury in it to rise, forming a gauge of the exhaustion. When the air is allowed to enter, the mercury in the tubes resumes its former position, 265 **Height of the barometric column at different elevations.**—The following table gives a comparative view of the height of mercury in the barometer at different elevations above the sea.

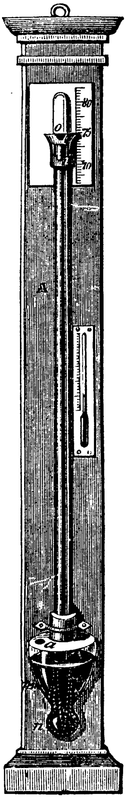
At the level of the sea, the mercury stands at	30	inches.
5,000 feet above " " "	24·773	"
10,000 " [height of Mt. Ætna,] "	20·459	"
15,000 " [height of Mt. Blanc,] "	16·896	"
3 miles	16·361	"
6 " [above the top of the loftiest mountain,]	8·923	"
9 "	4·866	"
15 "	1·448	"

266. **Cistern barometer.**—The cistern barometer is the most simple form of this useful instrument. It consists of a Torricelli's tube of glass, filled with mercury and plunged into a vessel containing the same metal; this vessel or cistern is of various forms.

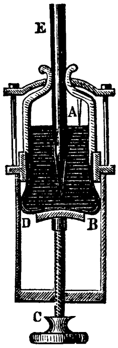
That it may be transported easily, the cistern is divided into two compartments, *m, n,* fig. 208; the upper division is cemented to the tube, communicating with the atmosphere by the small hole *a*. The two compartments are united by the narrow neck into which the lower part of the barometer tube enters, fitting closely, although not touching the walls; leaving only so small a space, that capillarity will not allow the mercury to escape from the lower compartment when we incline the barometer. So that in whatever position we place it, no air can enter the lower end of the tube.

This barometer is always fixed on a wooden support, at the upper part of which is a graduated scale, whose zero is the level of the mercury in the cistern. The sliding scale *i* indicates the level of the mercury in the tube. There is attached to barometers also a slider, moving by the hand upon which is a *vernier*, by means of which we can distinguish very small variations. But the level of the mercury in the cistern varies as the column of mercury in the tube ascends or descends, for then a certain quantity of mercury passes from the cistern into the tube, or the reverse, so that the zero (the level) changing the graduation on the scale, does not indicate the true height of the barometer.

208



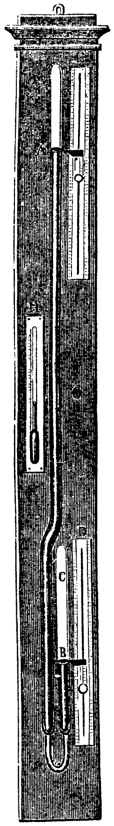
209



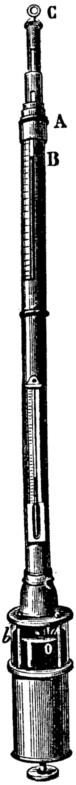
Fortin's barometer.—This error is avoided in the barometer of Fortin, fig. 211, by means of a cistern of peculiar construction, shown in fig. 209.

The lower part is of deer-skin, and is elevated or depressed by means of the screw C, pressing the plate D B. At the upper wall of the cistern is fixed a small ivory needle, A, whose point corresponds exactly to the zero of the scale, graduated on the case. At each observation with this instrument, care is taken to make the level of the mercury in the cistern, correspond with this point, which is accomplished by turning the screw up or down.

212



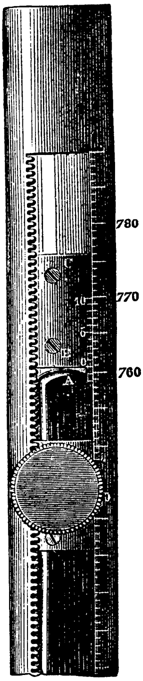
211



Vernier.—To secure great accuracy in measuring the height of the mercurial column, the barometer is furnished with a vernier, B C, fig. 210. Ten divisions on the vernier correspond with nine divisions of the graduated scale. The vernier is moved by a rack and pinion until its lower extremity corresponds very accurately with the surface of the mercury in the barometer tube. In the figure the mercurial column is seen to stand a little above the division marked 760. Counting upward we see that the seventh division of the vernier is exactly opposite one of the divisions on the graduated scale. This gives the small portion of the column above 760 equal to seven-tenths of a division of the scale, and the height of the column is read 760·7.

This form of barometer has been adopted by the Smithsonian Institution, and is made by Mr. Green, of N. Y.

210

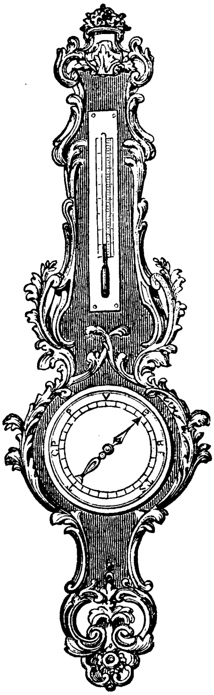


267. **The syphon barometer**, invented by Gay Lussac, consists of two tubes, fig. 212, of the same internal dimensions, united by a very capillary neck, both closed at their upper extremities, the air entering the cistern through a small hole at C. The large tubes being of the same interior diameter, the capillary action is mutually destroyed. The capillary tube is made small, so that when we turn the instrument over, it remains full, because of its capillarity. For measuring the height of the mercury, there are two scales, E and D, graduated in different directions, having their common zero at O, on a line intermediate between the two mercurial surfaces; so that by adding the indications of these two scales, we have the difference in the level of the mercury in the two tubes.

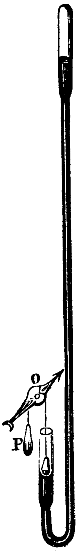
But a quick movement, transportation in a carriage or on horseback, may divide the mercurial column in the capillary tube, and thus allow the air to pass into the long arm, whereby the accuracy of the instrument would be destroyed.

In order to obviate this inconvenience, M. Bunten has modified the instrument as represented in fig. 213. The long arm A drawn out to a point, enters into and is soldered to a larger tube, K, which is attached to the capillary tube. With this arrangement, should bubbles of air even pass through the capillary tube, they cannot enter the long arm, but are retained in the top of K. These bubbles of air have no influence on the observations, and may be driven out by simply heating the tube.

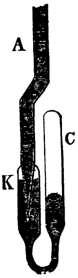
215



214



213



268. **Wheel barometer.**—The wheel barometer is an instrument of no scientific value, but has a certain popular interest as it purports to declare the state of the weather. The apparatus consists, figs. 214 and 215, of a dial plate attached to a syphon barometer having a small cylindrical cistern, upon whose surface rests a float; this is attached to a silk string, which winds around a pulley, O, and is terminated by the counterpoise P; the axis of the pulley carries a needle, which rests upon the face of the dial plate. When the pressure of the atmosphere changes, the column rises or falls in the tube accordingly, and carries along with it the float. The pulley turns and moves the needle to the words rain—fair—changeable, &c., which are designed to correspond to certain heights of the mercurial column.

269. **Causes of error.**—In order to obtain the true height of the mercury in a barometer, we must, after making the observation, determine by calculation the error caused by *capillarity*, and by the variations of density, caused by *changes of temperature.*

Correction for capillarity.—When the barometer tube is of capillary diameter, the surface of the mercury in it becomes convex (233) and the depression is greater by as much as the tube is more capillary.

For correcting this error, it is necessary to know the diameter of the tube, and then by means of the table (238), ascertain the depression which must always be added to the observed height.

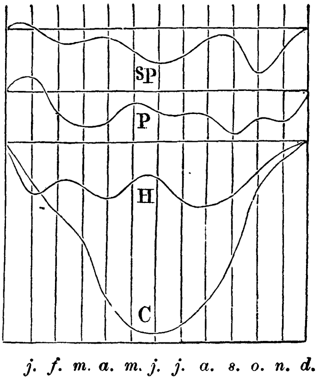
Correction for temperature.—In all mercurial barometers we must have regard to the temperature, for as heat expands mercury, it diminishes its density, and in consequence, under the same atmospheric pressure the mercury would rise as much higher as the temperature was more elevated. Consequently barometric observations cannot be compared, unless they were taken at the same temperature, or are brought by calculation to the same standard.

As it is entirely arbitrary what temperature shall be chosen; that of melting ice has generally been taken. A table showing the expansion and contraction of mercury at different temperatures may be found in the chapter upon heat. The metallic and aneroid barometers have already been described (163, 164).

270. **Variations of the barometric height.**—When we observe a barometer during many days, we notice that not only does its height vary from day to day, but also in the same day. The amount of these variations increases from the equator towards the poles. The greatest variations (excepting extraordinary cases) are 6 m. m. (·2362 in.) at the equator; 30 m. m. (1·181 in.) at the tropic of cancer; 40 m. m. (1·5748 in.) in France, and 60 m. m. (2·3622 in.) 25° from the poles. The greatest variations take place in winter.

The mean diurnal height is the average of twenty-four successive observations taken from hour to hour. M. Ramond has found the height of the barometer at noon to be the mean of the day. *The mean monthly height* is the average of the thirty mean daily heights of a month. *The mean annual height* is the average of the three hundred and sixty-five mean daily heights of a year.

216



At the equator the mean annual height is 758 m. m. (29·842 in.) It increases, passing from the equator, and attains its maximum of 763 m. m. (30·04 in.) between the latitudes of 30° and 40°; it decreases in more elevated latitudes. The mean monthly height is greater in winter than in summer, because of the cooling, and consequent increased density of the atmosphere.

The scale, fig. 216, shows the barometric variations of the different months. Equal distances, taken on the lower horizontal line, *j*—*d*, represent the duration of the different months, and the curved lines at the commencement of each interval, the mean barometric heights corresponding to the successive months. We have then curves, whose inflexions make known the variations of the mean from one month to another. The four curves represent the monthly means as observed at Calcutta, C, at Havana, H; Paris, P, and at St. Petersburg, S. P. The differences of the curves represent, with great distinctness, the differences of the mean barometric heights. Calcutta and Havana, on the same latitude, have, it will be seen, very different monthly means.

Variations observed in Barometers are of two kinds.

1st. *Accidental variations*, which do not offer any regularity in their movements, and which depend on seasons, the direction of the wind, and geographical position.

2d. *Diurnal variations.*—It was about the year 1722, that the hourly variations of the barometer were proved to take place in a regular manner. From that time, many observers have labored to determine the extent and the periods for the different parts of the earth. Alex. Von Humboldt, with others, has demonstrated by a long series of very accurate observations at the equator, that the maximum of height corresponds to 9 o'clock in the morning; the barometer then falls to its minimum at four, or half past four o'clock in the afternoon; it then rises, attaining a second maximum about ten o'clock at night. These movements are so regular, they almost serve to mark the hours like a clock, but they are very small. M. Humboldt found that the distance between the highest point in the morning, and the lowest point in the afternoon, was but two m. m. In the temperate zones, these diurnal variations also take place, but are very difficult to ascertain, because of the accidental variations, so that it requires extended and very accurate observations in order to determine them. The hours of the maximum and minimum of the diurnal variations, appear to be nearly the same in all climates, varying a little with the season. Thus, in winter (in France), the maximum is at nine o'clock in the morning, the minimum at three o'clock in the afternoon, and the second maximum at nine o'clock in the evening. In summer, the maximum takes place before eight o'clock in the morning, the minimum at four o'clock in the afternoon, and the second maximum at eight o'clock at night. In spring and in autumn, the critical hours are intermediate.

271. **Relation between barometric changes and the weather.**—Those variations of the barometer which are not periodic, are generally supposed to be indications of changes in the weather. For it has been noticed that those days in which the column of mercury was 29·72 inches in height, there was very changeable weather; that in a majority of those days when the mercury rose above this point, there was fine weather; when it fell below this point, stormy weather, snow, or rain, prevailed. It is from these coincidences between the height of the barometer and the state of the weather, that there is marked on

the scale or dial plate of barometers, at certain heights, the words stormy, rain or snow, variable, fine weather, &c., and it is supposed that when the mercury stands at the height indicated respectively by these words, we should have corresponding weather. Now, although this may be true to a certain extent, yet a little reflection will show the fallacy of such indications. The height of the mercurial column varies with the position of the barometer, and consequently two barometers, in different places, not upon the same level, would indicate different coming changes. The changes of weather are indicated in the barometer, not by the actual height of the mercurial column, but by its changes of height.

Rules by which coming changes are indicated.—The following rules may, to some extent, be relied upon but, for reasons already stated, must be taken with a considerable degree of allowance.

1. The sudden fall of the mercury is usually followed by high winds and storms.
2. The rising of the mercury indicates generally the approach of fair weather; the falling of it shows the approach of foul weather.
3. In sultry weather, the falling of the mercury indicates coming thunder. In winter, the rise of the mercury indicates frost. In frosty weather, its fall indicates thaw, and its rise indicates snow.
4. Whatever change of weather follows a sudden change in the barometer, may be expected to last but a short time.
5. When the barometer alters slowly, a long continuation of foul weather will succeed if the column falls, or of fair weather if the column rises.
6. A fluctuating and unsettled state in the mercurial column, indicates changeable weather.

272. **Measure of heights by the barometer.**—Since the level of the mercury in the barometer falls, as we ascend above the earth, we see that it is possible to determine by barometric observations, the elevation of a mountain, or of any other place above or below the level of the sea. If the atmosphere had a uniform density, we could ascertain, by a very simple calculation, the height to which the barometer was raised, from the amount of the fall of the mercurial column; for, mercury being 10,466 times heavier than air, a fall of one m. m. (·03937 in.) of the barometric column, would indicate that the column of air had diminished 10,466 m. m. (412·054 in.), and therefore the height measured would be 10,466 m. m. But as the atmospheric pressure diminishes very rapidly as we ascend, such calculations are of no value except for small elevations, and it is necessary to deter-

mine the rate of diminution in density of the air, in proportion as it is further removed from the earth. Tables have also been constructed by which we can easily calculate the level between any two places, when we know the height of the barometer, and the temperature of the atmosphere.*

The altitude of any place above the level of the sea may also be calculated by the following formula, given by Prof. Guyot, in the tables referred to below:—

If we call

$$\left.\begin{array}{l} h = \text{the observed height of the barometer,} \\ T = \text{the temperature of the barometer,} \\ t = \text{the temperature of the air,} \end{array}\right\} \text{at the lower station.}$$

$$\left.\begin{array}{l} h' = \text{the observed height of the barometer,} \\ T' = \text{the temperature of the barometer,} \\ t' = \text{the temperature of the air,} \end{array}\right\} \text{at the upper station.}$$

If we make, further,

Z = the difference of level between the two barometers;

L = the mean latitude between the two stations;

H = the height of the barometer at the upper station reduced to the temperature of the barometer at the lower station; or,

$H = h' \left\{ 1 + 0\cdot00008967\ (T - T') \right\}$;

The expansion of the mercurial column, measured by a brass scale, for 1° Fahrenheit = 0·00008967;

The increase of gravity from the equator to the poles = 0·00520048, or 0·00260 to the 45th degree of latitude;

The earth's mean radius = 20·886,860 English feet;

Then Laplace's formula, reduced to English measures, reads as follows:—

$$Z = \log.\frac{h}{H} \times 60158\cdot6 \text{ Eng. feet,} \left\{ \begin{array}{l} \left(1 + \dfrac{t + t' - 64}{900}\right). \\ \left(1 + 0.00260 \cos. 2\,L\right). \\ \left(1 + \dfrac{z + 52252}{20886860} + \dfrac{h}{10443430}\right). \end{array}\right.$$

z, in this formula, is the approximate value of Z, as given by that part of the formula preceding the parenthesis in which z is introduced.

Heights may be calculated by the above formula, but the calculation is much facilitated by the use of the Smithsonian Tables.

273. **Balloons.**—Bodies in air (like solids plunged in liquids) lose a part of their weight, equal to the weight of the air displaced From this it follows, that if a body weighs less than an equal volume of air it will rise in the atmosphere until it meets with air of its own density: hence, heated air, smoke, &c., rise, because they are less dense than cold air.

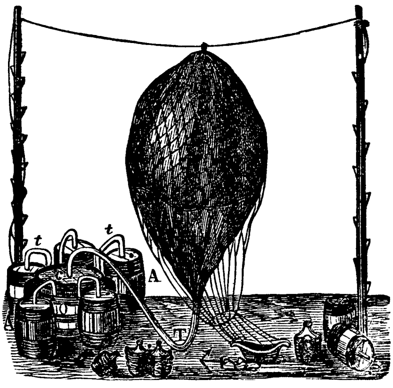
Dr. Black, of Edinburgh, announced in 1767, that a light vessel filled with hydrogen gas, would rise in the air; and Cavallo, in 1782, communicated to the Royal Society in London the fact, that soap-bubbles, filled with hydrogen, would ascend in the atmosphere. The brothers Montgolfier, in 1782, first constructed balloons. These consisted of globes of cloth, lined with paper. The one that they first exhibited publicly, was a globe about thirty feet in diameter, open at the lower part, below which was placed a fire. This, expanding the air within the globe, diminished its density, and the balloon rose to a height of nearly a mile. Hot-air balloons are, therefore (in allusion to their inventors), usually called Montgolfiers. Balloons filled with hydrogen were first introduced by Mr. Charles, professor of physics in Paris, in 1782; and in November of the same year, Pilatre de Rosier made the first aërial voyage, in a balloon filled with hot air. The ascension took place from Boulogne. Soon after, Messrs. Charles and Robert, in the garden of the Tuilleries, repeated the same experiment in a balloon filled with hydrogen gas. At this epoch, aërial voyages multiplied. In January, 1784, seven persons rose from Lyons, three from Milan, &c.; and soon, so familiarized were the public with this method of navigating, that it was not uncommon for people to ascend in a balloon which was restrained from going too far by means of a cord; when the adventurers had attained a certain height, the balloon was drawn down by means of the cord, and other voyagers took their place.

---

* Guyot's Meteorological and Physical Tables, Smithsonian Collections.

Gay Lussac, September 16, 1804, made an ascent remarkable for the facts with which it enriched science, and for the height which was attained, namely 7016 metres, or about 23,019 feet. In those elevated regions, Gay Lussac found respiration and the circulation of the blood much accelerated, because of the rarefaction of the atmosphere; his heart making 120 pulsations in a minute, while 66 was its normal rate. He also collected there specimens of air for chemical analysis, and determined the cold of space.

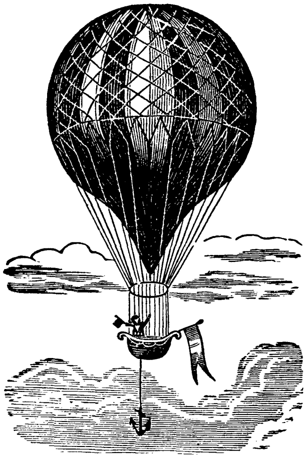
217



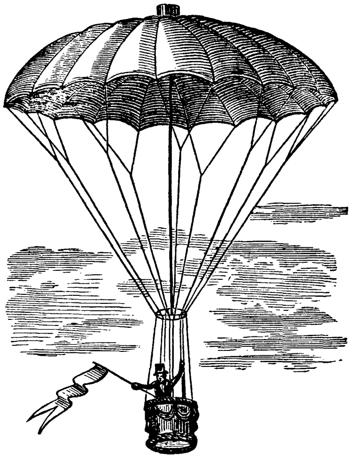
Construction and filling of balloons.—Generally the balloon is pear-shaped. It is made of a material impervious to hydrogen gas, often of strips of taffeta sewed together, and covered with a varnish, composed of linseed oil and caoutchouc, dissolved in essence of turpentine, or of a tissue formed of a layer of caoutchouc interposed between two layers of taffeta, and called *mackintosh*.

To fill the balloon, A, fig. 217, an aperture at its lower end is placed in communication, by means of a tube, T, with vessels, *t t*, generating hydrogen (from the action of dilute sulphuric acid on iron). When the balloon is sufficiently filled, the aperture is closed. Suspended by means of a net-work of ropes covering the whole apparatus, is a boat formed of wicker-work, for the reception of the aeronauts, fig. 218. At the upper part of the balloon is a valve, which the manager may open or shut at pleasure by means of a cord. As illuminating gas can usually be procured more easily than hydrogen, it is frequently used by aëronauts; but being at least seven times more dense than pure hydrogen, the balloon requires to be of a correspondingly larger size, in order to obtain the same ascensional force.

218



219



The balloon must not be completely filled, for the atmospheric pressure diminishing upwards, the gas in the interior will expand in a like ratio, and tend to burst the balloon. A number of fatal accidents have taken place from this cause.

When the aëronaut wishes to descend, he pulls the cord which opens the valve in the upper part of the balloon, and thus the hydrogen escapes, and the balloon comes down. If he wishes to ascend, he throws out bags of sand which he has taken up with him, and the balloon, becoming thus lighter, rises to a correspondingly greater height.

Parachute.—Aëronauts often abandon their balloons, and descend in a *parachute*. This apparatus is composed of strong cloth, and when extended, has the appearance of an umbrella, fig. 219, with this difference, that the whale-bones are replaced by cords, sustaining a small boat, in which the aëronaut places himself. There is a small chimney, or hole, in the top of the parachute, in order to allow the air, which would accumulate, to escape regularly, otherwise it would escape fitfully by the sides, throwing the apparatus violently around, to the imminent peril of its occupants.

IV. COMPRESSIBILITY OF GASES.

274. **Mariotte's law.**—Boyle and Mariotte discovered the law of the compression of gases, which is as follows:—

At the same temperature, the volume occupied by the same bulk of air, is in inverse ratio to the pressure which it supports. From which it follows, that the density and tension of a gas are proportional to the pressure.

Let V and V' represent the volume of a gas at different pressures P and P', and D and D' the different densities. Then

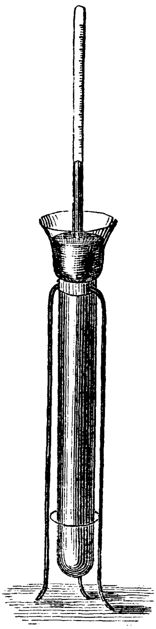
$$V : V' = P' : P, \quad \text{whence } V' = V\frac{P}{P'} \text{ and } P' = P\frac{V}{V'}.$$

$$D : D' = P : P', \quad \text{whence } D' = D\frac{P'}{P} \text{ and } P' = P\frac{D'}{D}.$$

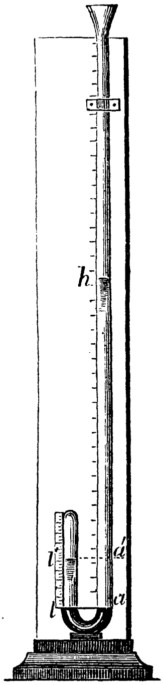
Experimental verification of Mariotte's Law.

In order to verify this law, the apparatus called Mariotte's tube is employed. To an upright support of wood is attached a bent tube, fig. 220, whose two vertical branches are unequal in length. The longer limb is open at the top, and furnished with a scale which indicates heights; the shorter is closed at the top, and is divided into parts of equal capacity. Mercury is poured into the tube so that the level of the liquid in the two branches is found on the same horizontal line, $l a$. The air in the shorter limb then occupies a definite volume, indicated by the graduation. If more mercury is added, until the measured volume of air is reduced one-half, as from ten to five, occupying only the space above l', and we now measure the difference of level between the two surfaces of mercury, viz.: $a' h$, we shall find that it is the same as the height of the barometric column. That is, the pressure of the column of mercury in the Mariotte's tube is equivalent to one atmosphere; adding this pressure to that which the atmosphere exerts on the mercury, we have the air subjected to double of its usual pressure, and it is, conse

219



220

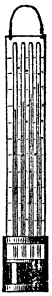


quently, reduced in volume one-half. If we subject it to a pressure of three atmospheres it will be reduced to one-third; of four atmospheres, to one-fourth of its original bulk, &c. By this law, at a pressure of 814 atmospheres air would become as dense as water.

The law of Mariotte may also be verified for pressures less than one atmosphere, by using a barometer tube, about two-thirds filled with mercury, and inverted in the deep cistern, fig. 221, filled with mercury. Sinking the tube to such a depth that the level of the mercury within and without is the same; the contained air is under the pressure of one atmosphere, and occupies a known volume. If the tube is now raised until by a diminution of pressure the given volume of air is doubled, it will be found that the length of the mercurial column in the tube is half that in the barometer: that is, the air under a pressure of one-half an atmosphere has doubled its volume. The volume here, as in the other case, is in inverse ratio to the pressure.

275. **Experiments of Despretz.**—Mariotte's law was generally received as correct, until Despretz, not doubting its correctness, so far as air was concerned, undertook to test this law in its application to other gases. For this purpose he filled several tubes of the same height with different gases, and inverted them in a vessel of mercury, placing behind them a graduated scale, as shown in fig. 222. This apparatus was then introduced into a glass cylinder filled with water, and subjected to pressure by means of a forcing-pump.

222



As the pressure increased, the height of the mercury in the different tubes varied, as shown in the figure, and this variation increased with the pressure. Carbonic acid, sulphuretted hydrogen, ammonia, and cyanogen were compressed more, and hydrogen less, than common air. These experiments, in which the probability of error is extremely small, show that each gas has a special law of compressibility, differing more or less from the law announced by Mariotte.

A series of very accurate experiments, subsequently conducted by Pouillet, by a different method, confirmed the results of Despretz, who had announced the unequal compressibility of different gases.

276. **Experiments of Regnault.***—Mariotte's law has been subjected to the test of very careful and repeated experiments by Dulong, Arago, and others. But the most complete and reliable experiments are those conducted by Regnault.

Regnault kept the column of gas upon which he was experimenting at a uniform temperature by a stream of cold water flowing through a cylinder which surrounded the tube of condensed air or other gas. The utmost precaution was taken to remove every trace of moisture from the gases employed. The temperature and atmospheric pressure were carefully noted at every experiment, and due allowance made for their changes. The temperature of the column of mercury employed to measure the pressure was noted, and the height of the column corrected accordingly. Finally the condensation of the mercurial column due to its own weight was also considered, and every possible precaution was observed to secure the utmost accuracy in the experiments.

---

* Mémoires de l'Académie des Sciences, Tom. XXI., p. 329.

Results obtained by Regnault.

The following table gives some of the principal results obtained by Regnault. Let P represent the pressure, when any gas occupies a volume V, and P' the pressure when the volume of the same gas is V'. If Mariotte's law were strictly correct, we should have $P\,V$ equal to $P'\,V'$, or $\frac{P\,V}{P'\,V'}$ ought to be equal to unity.

Air.		Nitrogen.		Carbonic Acid.		Hydrogen.	
P	$\frac{PV}{P'V'}$	P	$\frac{PV}{P'V'}$	P	$\frac{PV}{P'V'}$	P	$\frac{PV}{P'V'}$
m. m.		m. m.		m. m.		m. m.	
738·72	1·001414	753·46	1·000088	764·03	1·007597	“	“
4209·48	1·002765	4953·92	1·002952	3186·13	1·028698	2211·18	0·998584
8177·48	1·003253	8628·54	1·004768	9351·72	1·045625	2845·18	0·996121
9336·41	1·006366	10981·42	1·006456	9619·97	1·155865	9176·50	0·992933

We here see that in the four gases examined, the ratio $\frac{P\,V}{P'\,V'}$ was found very nearly equal to unity, showing that though Mariotte's law is not absolutely true, it is sufficiently accurate for most purposes.

In the case of air, nitrogen, and carbonic acid, the compressibility augments more rapidly than the increase of pressure, while in the case of hydrogen the compressibility diminishes. It has also been ascertained that the rate of compressibility for any gas varies with the temperature. For example, carbonic acid at 212° F. agrees almost exactly with Mariotte's law.

277. **General conclusions on the compressibility of gases.**— From a careful consideration of all the experiments upon the condensation of gases, it seems reasonable to conclude that:—

1st. There is some temperature, differing for different gases, at which the compressibility of gases corresponds with Mariotte's law. That at higher temperatures the compressibility diminishes, and at lower temperatures the compressibility increases.

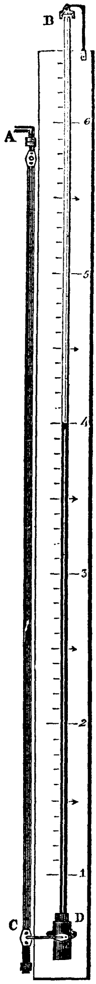
2d. Almost all gases, by a certain amount of pressure, are liquefied, and it is found that their compressibility increases very rapidly near the point of liquefaction.

Although these conclusions are based upon the analogies of science, and apparently indicated by experiments, yet further observations are required for their confirmation

V. INSTRUMENTS DEPENDING ON THE PROPERTIES OF GASES.

278. **Manometers.**—Manometers are instruments designed to measure the tension of gases or vapors above the atmospheric pressure. The unit of measurement which has been chosen for these instruments is the pressure of the atmosphere, which, at the level of the sea, is (261) equal to about 15 lbs. to the square inch, and therefore a pressure of two or of three atmospheres signifies a pressure of 30 lbs. or of 45 lbs. Manometers are of very various construction, two of which will be mentioned, namely:—

223

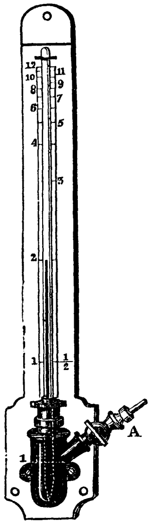


279. 1st. **Manometer with free air.**—This consists of a glass tube, B D, fig. 223, open at both ends, placed in a cistern of mercury, to which it is cemented. The cistern is connected with an iron tube A C. By this tube the pressure of the fluid is transmitted to the mercury. The gases whose tension we wish to find, being often of a temperature sufficiently high to melt the cement attached to the apparatus, the tube A C is filled with water, which receives the pressure, direct, and transmits it to the mercury.

In order to graduate this instrument, A being open to the atmosphere, that point where the mercury rests in the tube is marked 1 (one atmosphere). At distances of thirty inches, the numbers 2, 3, &c., are marked, which indicate the number of atmospheres, for it will be remembered, that a column of mercury thirty inches in height represents the atmospheric pressure. The apparatus being placed in connection with a steam-boiler, we ascertain the pressure to which it is subjected by the height to which the mercury rises in B D; if to 2·5, the pressure is 2·5 atmospheres, or 37½ lbs. to the square inch.

280. 2d. **Manometer with compressed air.**—This form of the instrument consists of a glass tube filled with dry air, placed in a cistern of mercury, to which it is cemented. This by a lateral tube, A, fig. 224, communicates with the vessel containing the elastic fluid to be gauged.

224



In order to graduate the manometer, such a quantity of air is placed in the tube, that when A communicates with the atmosphere, the level of the mercury is the same in the tube as in the cistern. At this point, therefore, 1 is marked upon the scale. Following Mariotte's law, it might be supposed that we should mark for two atmospheres, at a point in the middle of the tube, but when the column of air is reduced half, the tension of two atmospheres is increased by the weight of the column of mercury raised in the tube, and therefore the middle point of the tube would represent a pressure greater than two atmospheres. The true position for the second mark is at a point a little below the middle of the tube, where the elastic force of the compressed air, added to the weight of the column of mercury, is equal to two atmospheres.

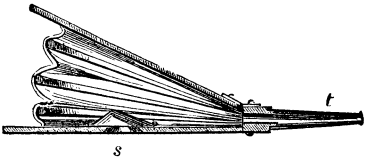
The true position of the points, indicating 3, 4, &c., atmospheres, is determined on the scale of the manometer by calculation. This is not a very desirable form of manometer, because the volume of air growing smaller, the divisions must continually diminish in size, and therefore, even considerable variations of pressure are not easily observed in the upper portion.

Bourdon's metallic barometer, described in § 163, is much used as a manometer or gauge for steam-boilers. It is sometimes called Ashcroft's gauge, and is the best instrument in use for the purpose.

[For the diffusion, effusion, and transmission of gases, the mixture of gases and liquids, and the absorption of gases, see the Author's "*Chemistry.*"]

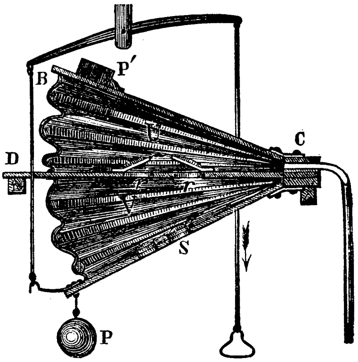
281. **Bellows.**—The most common instrument for producing a current of air is the ordinary bellows, fig. 225, consisting of two leaves of wood united by leather, and terminating in a metallic tube *t*. A valve *s* is placed in the lower leaf, opening upwards.

225



When the leaves are pressed together, the valve *s* closes, and the contained air escapes through *t*. But when the leaves are separated, air rushes in through the valve and also through the tube, through which last it is again ejected upon pressing the leaves together.

226



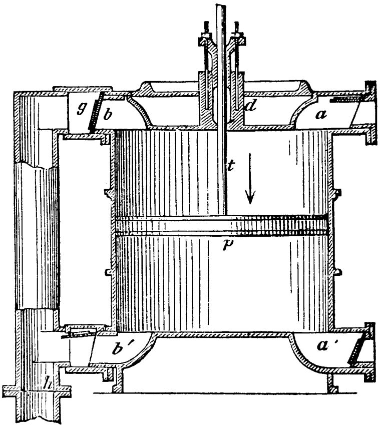
Bellows with a continuous blast.—In the ordinary bellows, the blast of air is intermittent. Where a continuous jet is wanted, as at a smith's forge, a double blast bellows is used, fig. 226.

This consists of three pieces of wood, of which one, D, is immovable, the others are connected with this by means of leather. The apparatus is divided into two compartments, U V The blast-pipe communicates with the one above; in the lower one, air is introduced through the lower valve, S. When the lever is drawn down, as shown by the arrow, the valve S closes, and the air being compressed, passes into U through the valves *r r*, raising C B, and partially escaping through the tube. With the reverse motion (accelerated by the weight P), the valves *r r* close, and the exterior air enters V by the valve S. During this time, the upper weight, P′, causes C B to descend, and thus there is continually an escape of air by the blast-pipe. The weight may be replaced by a spring.

282. **Furnace blowers.**—In blast, or high furnaces, blowing machines are employed, by means of which a large volume of air is forced into the fire; these machines are of very various construction.

Fig. 227 represents one of them; it consists of a cast iron cylinder, containing a piston, *p*, of which the rod, *t*, passes, air tight, through a packing-box, *d*; there are four valves, two of which *a a′* opening inwards, draw in air; the air passes out through the valves *b b′* which open outwards. The piston is set in motion by a steam-engine or water-wheel; during its *descent* the valves *a* and *b′* only are opened; through the first, air is drawn in, through the second, it is expelled; during the *ascent* of the piston, the other valves *a′* and *b*, act in the same manner. The expired and compressed air is received into the tube *g h*, through which it is conveyed to the furnace. In the great blowing machines at Scranton, Pa. the blast is used under a pressure of five pounds to the square inch, driven by two engines of 1200 horse power each.

227



283. **Escape of compressed gases.**—When a compressed gas escapes from an opening in a thin wall, the velocity of its escape depends on the difference of the interior and exterior pressures, and on the density of the gas passing out. It has been proved,

1. *That with the same gas at the same temperature, the velocity of flow into a vacuum is the same at any pressure.*

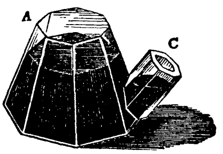
That is, if we had a vessel filled with air, compressed at 1, 2, 3, or 1000 atmospheres, and allowed it to escape by a small orifice, the velocity of its flow would be the same during the whole time of its discharge. But the quantity of the gas that could escape in the same time would vary, being evidently proportional to the density of the gas, that is, to the pressure. If the escape took place in a gas, as air, instead of in a vacuum, the velocity is then proportional to the difference between the elastic force of the interior and exterior air.

2. *The velocity of the escape of gases into a vacuum is in inverse ratio to the square root of their densities.*

Where the gas escapes through long tubes instead of through orifices in a thin wall, the velocity is very much diminished, because of the friction, and is less in proportion as the tube is longer and its diameter smaller.

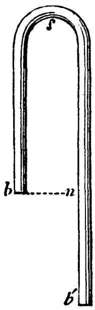
284. **Pneumatic ink-bottle.**—In the pneumatic ink-bottle, fig. 228, the ink in the tube *c* is kept constantly at nearly the same level. By inclining the bottle it may be filled as seen in A. The ink in A tends to force itself in the tube C, but is opposed by the atmospheric pressure, which is much greater than the pressure of the column of ink in A. As the ink in C is consumed, its surface, falling, will allow a small bubble of air to enter A, where it will exert an elastic pressure, and cause the ink in C to rise a little higher. This effect will be continually repeated until the bottle is emptied of ink. Bird-cage fountains are constructed on a similar principle.

228

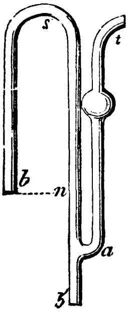


285. **The syphon.**—The syphon used for decanting liquids, depends for its operation on the principle of atmospheric pressure. It consists of a bent tube, *b b′*, fig. 229, having one of its arms longer than the other. It may be filled by turning it over, and pouring the liquid in, or by immersing the shorter arm in a vesssel of water, and applying the mouth at *b′*; upon exhausting the air, the water will be forced up by atmospheric pressure, to supply the place of the air withdrawn, and there will then be a continual discharge until the vessel is emptied.

229



230

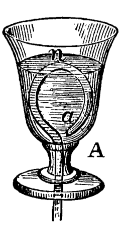


The two branches being filled with liquid, the pressures exerted at the points *b* and *n* will be equal, for they are on the same level; but the pressure exerted at *b′* will be greater, because of the column *n b′*, and the liquid will escape from this long branch because of this excess of pressure, and will draw after it the liquid in the shorter branch; if the end of this be immersed, there will be a continual discharge as long as *b* is below the surface of the liquid, for the atmospheric pressure will cause the liquid to ascend, to supply the place of that which is passing out; otherwise there would be a vacuum produced.

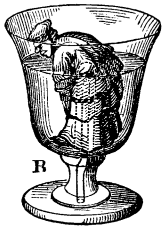
It is evident that water could not be raised by means of a syphon more than thirty-four feet: for a column of water of that height is in equilibrium with the pressure of the atmosphere (261). The velocity of the flow from a syphon will be the same as if the liquid fell freely from a height equal to the distance between the level of the liquid in the vessel and the end of the long arm. To avoid the necessity of filling a syphon by pouring, the form represented in fig. 230 is employed. To use this instrument, the open end, *b′*, of the longer limb is closed by the finger, while a partial vacuum, created by sucking at the small ascending tube, *t a*, occasions the liquid to pass over as in the ordinary syphon.

Intermittent syphon. Tantalus' vase.—Fig. 231 consists of a vessel, A, containing a syphon, of which one of the branches opens below the bottom of the vessel; the other is curved. When water is poured into the vessel A, it will rise to the same height in the interior of the tube as it attains outside. The tube will not act as a syphon until the vessel is filled to the height *n*, but when it reaches that point, the water will flow through *a* into the long branch, filling it completely, and the syphon being now supplied, will discharge water until the vessel is emptied. The syphon may be concealed in a little image, fig. 232, B, representing Tantalus, so that just before the water touches his lips the syphon is filled, and the vessel is emptied.

231

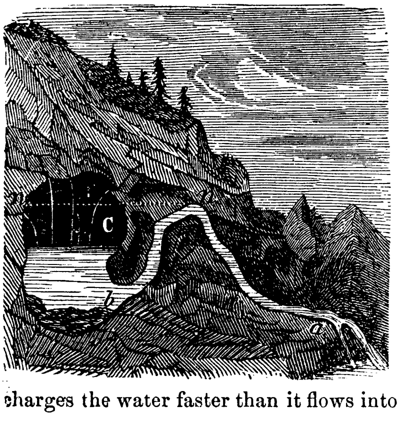


232

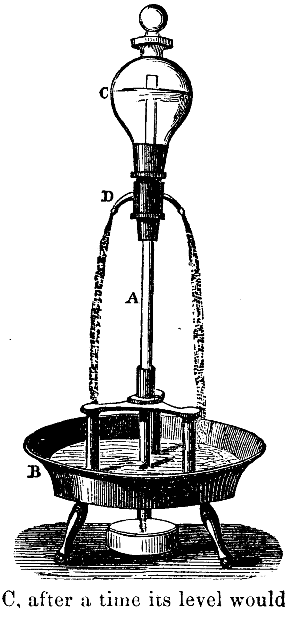


286. **Intermittent springs.**—There exist in nature intermittent springs, the water flowing regularly for a time, and then suddenly ceasing. In these springs the opening, as at *a*, fig. 233, communicates with a subterranean cavity, C, by means of a channel, *a n b*, which has the form of a syphon. This cavity is gradually filled, until at last the water attains the level *n n*, when the syphon is filled, and the water escapes. If the syphon discharges the water faster than it flows into C, after a time its level would

233



234



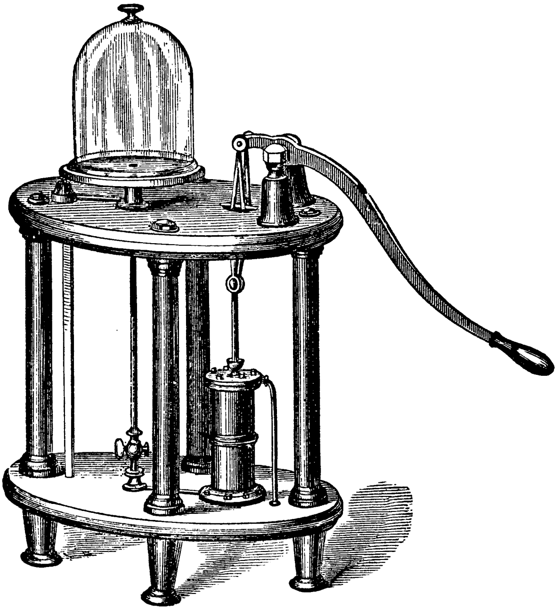
be lowered to *b*; air would then rush in by the syphon, the flow of water would cease, and would not recommence until it had again attained the level *n n*.

Intermittent fountain.—The intermittent fountain consists of a vessel of glass, C, fig. 234, whose aperture for the admission of water is hermetically sealed by an accurately-ground stopper.

A glass tube A, passes through the vessel C, its upper end terminating above the surface of the liquid; its lower end rests in a copper cistern, B, which has a small aperture for the escape of water. The globe being partially filled, the water escapes through the capillary orifices of the tube at D, in consequence of the atmospheric pressure transmitted through the lower end of the tube A. When the end of this tube becomes covered with water, which after a time happens (because the orifice in the cistern B does not allow so great a flow of water as can escape from the tubes at D), the exterior air cannot enter the globe, and in consequence the flow ceases. The water continuing to escape from D, in a little time the surface is so much lowered, that the end of the tube, A, is out of water; the air then entering the globe, the escape recommences, and so continues at intervals until C is emptied of water.

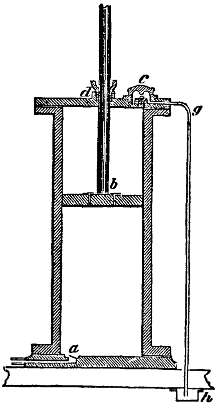
287. **Air-pump.**—The air-pump, designed to produce a vacuum in any confined space, was invented by Otto v. Guericke, burgomaster of Magdeburg, in 1650. Fig. 235 exhibits a very excellent form of the

235



air-pump, manufactured by Ritchie of Boston, usually called the Leslie form of air-pump. The essential part of the air-pump is the cylinder shown in section in fig. 236. This cylinder communicates with the bell glass, by means of a tube, shown in fig. 235, and with the external air by means of the tube *g h* (fig. 236). There are three valves, *a*, *b*, and *c*, all opening upward. The piston-rod passes through a packing-box, *d*, in which it moves air-tight, and the power is applied by means of a lever, as shown in fig. 235.

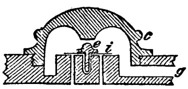
236



Suppose the piston to be standing at the bottom of the cylinder, when we depress the lever, the air from the receiver expands, rushing through the valve, *a*, into the empty space formed in the bottom of the cylinder, while the air above the piston is forced out through the valve *c* and the tube *g h*. With the reverse motion the valves *a* and *c* close, excluding the external air from the cylinder, and preventing the return of air from the cylinder to the receiver. At the same time the piston-valve, *b*, opens and allows the air below the piston to pass through into the upper part of the cylinder. When the piston rises again, this new volume of air which has passed above the piston is forced out through the valve *c*, into the external atmosphere, while another portion of rarefied air from the receiver expands into the cylinder below the piston, to pass upward and be forced out through the valve *c* at the next stroke of the piston; and so on continuously, as long as the rarefied air in the receiver and cylinder has sufficient tension to open the valves. At each stroke of the piston the air undergoes renewed rarefaction until the amount remaining in a good instrument is about one-thousandth of the original quantity, and the space within the receiver may be regarded as a vacuum. The pump here figured is furnished with a barometric manometer, seen in the left of fig. 235, by which the degree of exhaustion is directly indicated. The efficiency of the air-pump depends in a great measure upon the valves, which are best made of oiled silk.

The construction of the upper valve, *c*, as made by Ritchie, is shown in fig. 237. The disk of oiled silk, *i*, is kept in place by the pin *e*, and the whole is protected by the dome-shaped covering *c*. The tube *g h* (fig. 236) discharges the air, and the oil which escapes with it is collected in a reservoir placed below the pump.

237



An air-pump with two cylinders is commonly used in France, the pistons of which are alternately raised and depressed by a rack and pinion motion.

Degree of Exhaustion.—It is plain, on a moment's reflection, that by mechanical means alone, it is impossible to produce a perfect vacuum. There must always remain a certain volume of air, inferior in tension to the gravity and friction of the pump valves. By employing an atmosphere of dry hydrogen to rinse out the residue of common air from an exhausted receiver, an approach to a perfect vacuum is made, inversely as the density of the two gases. Also by using carbonic acid for the same end, and absorbing the residue of this gas by dry quick-lime previously placed on the pump plate, a perfect vacuum may be produced; but by chemical and not by mechanical means.

288. **Compressing machine.**—This machine is used to compress the air or any other gas; it is constructed like the air-pump, the only difference being that its valves open in a contrary direction, viz.: downwards.

238

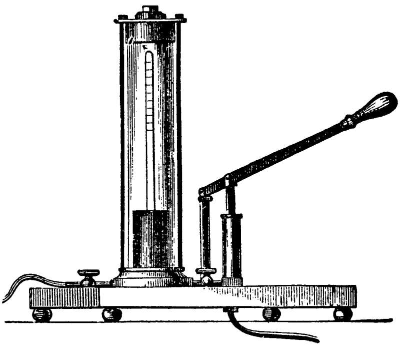


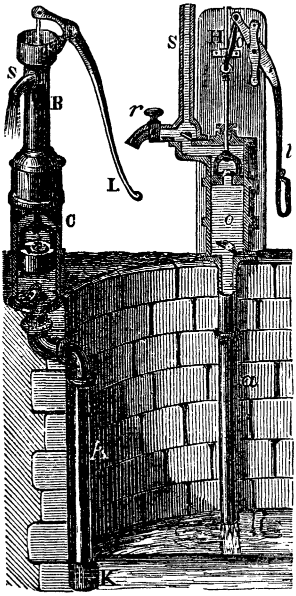
Fig. 238 shows a very neat form of the condensing pump as constructed by Ritchie, to illustrate the Mariottian law (275) and to liquefy gases.

289. **Water-pumps.**—Pumps are machines designed to elevate liquids above their former level. They are of two classes: 1st, those acting by atmospheric pressure; 2d, those which act independent of such pressure. They are commonly called either suction, or forcing pumps, or both united.

290. **Suction-pumps.**—The suction-pump, fig. 239, is composed of a tube, A, whose lower end is immersed in the water to be elevated This is attached to the body of the pump, C, which contains a piston furnished with a valve, *r*, opening upward. The upper extremity of the tube A, also contains a valve, *o*, opening in the same direction.

When the piston is elevated from the lower part of the pump by working the handle, L, the valve *r* closes, and a partial vacuum is produced, but the elastic force of the air in A causes the valve *o* to open, and part of the air thus passes into C. The air in the tube is thus rarefied, and the water rushes up to such a height, that the weight of the column of water raised, added to the elasticity of the interior air, keep it in equilibrium with the atmospheric pressure. When the piston descends, the valve *o* closes by its weight, and prevents the return of the air from the body of the pump, C, into the tube A. The compressed air opens the valve *r*, and thus escapes into the atmosphere through B. After a number of strokes of the piston, fewer as the capacity of the tube *a* is less, the water will be elevated above the lower valve; now when the piston is lowered, the valve *r* will open and the water pass above it. Upon elevating the piston, *r* closes, and the water is raised in B, and escapes through the spout S.

239

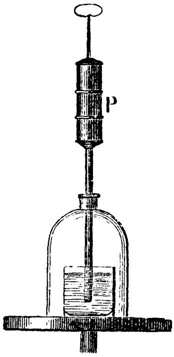


As in this and the following pump, the water is elevated to the top of the tube by means of atmospheric pressure, it is evident, that even in the most perfectly constructed pumps, the distance from the level of the water to the top of the pump must not exceed thirty-four feet (261), but those of ordinary construction contain defects, so that generally we do not gain a greater height than twenty-six or twenty-eight feet. But after the water has passed above the piston, the height to which we may elevate it, is limited only by the power applied at the piston; for it is the ascensional force of this which elevates the water.

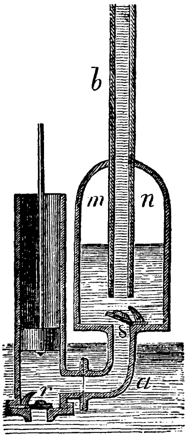
291. **Suction and lifting pump.**—Sometimes the water raised above the piston, instead of passing upwards in the tube in which the piston works, rises by a lateral ascensional tube, S, furnished with a valve which prevents the return of the water, as is shown in *a*, *r*, S, fig. 239.

That the rising of the water in the tube is due to the atmospheric pressure, may be demonstrated by the apparatus, fig. 240. After forming a vacuum in the reservoir which contains the vessel of water, the liquid will not rise in the tube when the piston in the pump, P, is raised, but upon admitting the air it is rapidly elevated, as usual.

240



241



292. **Forcing-pump.**—In the forcing-pump, the piston has no valve. The lower part of the cylinder in which it works is

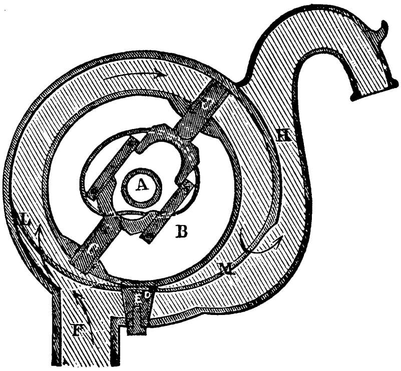
placed in the water to be elevated, so that the valve *r*, fig. 241, which opens upward, is always immersed. The ascending tube *a b* contains a valve, S, also opening upwards, and an air chamber, *m n*.

When the piston is raised, S is closed, and water is introduced by the open valve *r;* upon the descent of the piston, *r* closes, and the water is forced into the ascending tube, *a b*. The reservoir, *m n*, filled with air, is designed to render the jet of water continuous. When the water is forced by the piston into the tube, the air is compressed in *m n;* reacting afterwards by its elasticity, it continues to drive the water into the upper part of the tube, after S is closed, and while the piston is rising.

It is found necessary to have the air-chamber twenty-three times the capacity of the body of the pump, in order to render the jet continuous.

293. **Rotary pump.**—The rotary pump is a mechanical contrivance for raising water by a continuous rotary movement. Fig. 242 represents one of the most successful of these pumps (Cary's). Within a fixed cylinder is included a movable drum, B, attached to the axis, A, and moving with it. The heart-shaped cam surrounding A, is immovable. The revolution of B causes the plates or pistons *C C* to move in and out, in obedience to the form of the cam. The water enters and is removed from the chamber through the ports or valves, L and M; the directions are indicated by the arrows.

242



The cam is so placed that each valve is in succession forced back into its seat when opposite E, while at the same time the other valve is driven fully into the cavity of the chamber; thus forcing before it the water already there, into the exit pipe H, and drawing after it, through the suction pipe F, the stream of supply. When the pump is set in action, the suction-pipe is gradually exhausted of air, in which, consequently, the water ascends, and being thrown into the cylinder, it is there carried around by the plates *C C*, in the manner just described.

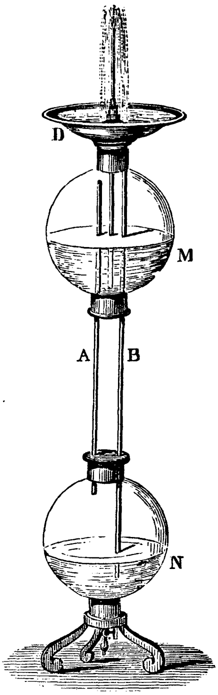
This is a form of pump often employed in the steam fire-engines now coming into general use.

294. **Fire-engine.**—In order to obtain a continuous and powerful jet of water from fire-engines, they are usually constructed with two forcing-pumps, which are alternately discharging water into a common air-chamber. The pistons are moved by brakes, having an oscillating motion. The water from both pumps, forced into the air-chamber,

escapes through a long leathern hose, terminated by a metal tube, which serves to direct the jet.

295. **Hiero's fountain.**—In this apparatus we also obtain a jet of water by means of air, compressed in this case by a column of water A common form of this apparatus is represented by fig. 243.

243

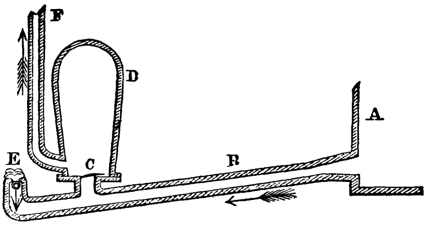


It consists of a metallic cistern and two globes of glass. The cistern, D, communicates with the lower part of the globe N, by the tube B; a second tube, A, joins the globes, ending in the upper part of both; M is partially filled with water; and lastly, a third tube passes through the cistern, and terminates at the bottom of M. The upper extremity of this tube has a small orifice, from which the jet of water issues.

Upon pouring water into the cistern D, the liquid descends to N, by the tube B, consequently the water in the lower globe, N, supports, besides the atmospheric pressure, the pressure of the column of water in the tube. This pressure is transmitted to the air in the globe, M, which, reacting on the water, forces it out through the jet, as seen in the figure. If there was no friction, and no resistance from the air, the water would spout to a height equal to the difference in level of the water in the two globes.

296. **Hydraulic ram.**—In the hydraulic ram, the momentum of a part of the fluid in motion, is effective in raising another portion. A simple form of this apparatus is seen in fig. 244. The water descends from the spring or brook, A, through the pipe B, near the end of which is an air-chamber, D, and rising main, F. The orifice at the extreme end of B, is opened and closed by a valve, E, opening downwards.

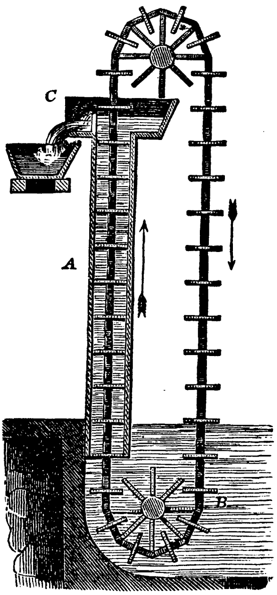
244



When the valve E is open, the water flows through B, until the current becomes sufficiently rapid to raise the valve E, and thus to close the orifice. The water in B having its motion thus suddenly checked, exerts a great pressure, and having raised the valve C, will rush into the air-vessel D, where it compresses the air. The compressed air in D, because of its elasticity, causes the water to rise in the pipe F, until the water in A B is brought to rest. When this takes place, the pressure is again insufficient to sustain the weight of the valve E, which opens (descends), the water in B is again put in motion, and the same series of effects ensue as have already been described.

The hydraulic ram, when well constructed, is capable of utilizing about 60 per cent. of the moving power.

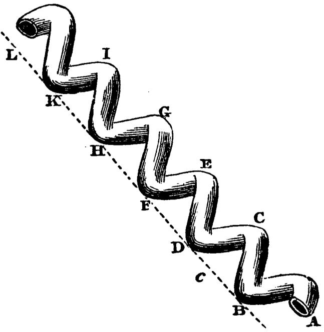
245



297. **Chain-pump.**—The chain-pump acts independent of atmospheric pressure. It consists of a cylinder, fig. 245, whose lower end is immersed in the water of the reservoir B, and whose upper part enters into the bottom of a cistern, C, into which the water is to be raised. An endless chain is carried around the wheels above and below, and is furnished, at equal distances, with circular plates, which fit closely into the cylinder. As the wheel is revolved by means of power applied usually by a winch, the circular plates successively enter the cylinder and carry the water up before them into the cistern, from which it passes out by a spout.

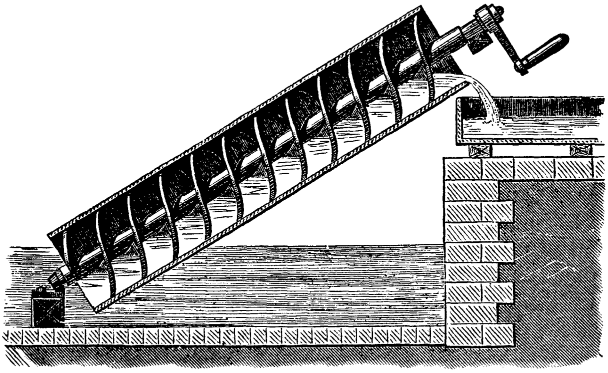
298. **Archimedes' screw.**—This machine is said to have been invented by Archimedes in Egypt, to aid the inhabitants in clearing the land from the periodical overflowings of the Nile. The instrument varies in its form, according to the manner and purposes of its application. To render the principle upon which it works intelligible, let us suppose a tube bent in the form of a corkscrew, and inclined in the manner shown in fig. 246. If a ball be placed in A, it will fall to B, and there remain at rest; if the screw be now turned so that the mouth A is placed in its lowest position, the point B, during such a motion, will ascend, and will assume the

246



highest position it can have. The ball will then fall to C; by continuing the revolution of the screw, the ball will ascend in the tube, and finally will be discharged from the upper mouth. The same would happen with a portion of liquid. If the lower extremity of the screw was immersed in a reservoir of liquid, it would gradually be carried along the spiral as the screw was turned, to any height to which the screw might extend. In practice, the screw is more commonly formed of a cylinder, to the walls of which is attached a spiral thread, as shown in fig. 247. Besides liquids, these machines are used for elevating ores in mines, or grain in breweries, &c. They are commonly used at an inclination of about 45°, but may be used at 60°; revolving 100 to 200 times a minute.

247



Problems on Pneumatics.

Atmospheric Pressure.

134. What weight could be lifted by the apparatus shown in fig. 202, if the mouth of the jar is 5 inches in diameter, and the air within the jar is exhausted, so as to leave it but one hundredth part its normal density?

135. What force would be required to separate two Magdeburg hemispheres, having an internal diameter of ten inches, if a perfect vacuum were formed within?

136. A mass of metal, whose specific gravity is 11·35, weighs in a vacuum 735 grains; how much will its weight be diminished if weighed in the open air?*

---

* In these problems the barometer is supposed to stand at 30 inches, and the thermometer at the freezing point.

137. A mass of iron (Sp. Gr. 7·8) weighed in air with brass weights (Sp. Gr. 8·3) 460 grains, what would it weigh in a vacuum?

138. A glass globe, from which the air has been exhausted, weighs 254·735 grammes; when full of air, it weighs 5422·737 grammes; when full of another gas, 651·175 grammes; what is the capacity of the globe, and what is the specific gravity of the gas?

Barometer and Balloons.

139. To what height will sea water (Sp. Gr. = 1·026) rise in a Torricellian tube, when the barometer stands at 28·75 inches?

140. When the mercury barometer stands at 30 inches, what must be the length of a water barometer inclined to the horizon at an angle of 30°?

141. What would be the height of a sulphuric acid barometer (Sp. Gr. sulphuric acid, 1·85) when the mercurial barometer stands 29·35 inches?

142. Measurement of the height of the highest peak of the Smoky Mountain, (Lat. 36° N.) in North Carolina, September 8, 1859, by Prof. A. Guyot. By observation at 8½ A. M.

	Barometer. Eng. inches.	Temperature of Barometer.	Temperature of air.
Lower station, R. Collins' house, 4 ft. above ground, ...	$h = 27{\cdot}862$,	$T = 66^{\circ}{\cdot}4$,	$t = 65^{\circ}{\cdot}1$.
Upper Station, Smoky Dome, 4 feet below summit, ...	$h' = 23{\cdot}963$,	$T' = 51^{\circ}{\cdot}8$,	$t' = 51^{\circ}{\cdot}4$.

Mr. Collins' house being 2500·2 feet above the ocean.

Calculate from these data the height of Smoky Dome above the ocean.

Altitude calculated by Prof. Guyot, 6655·85 feet.

143. What is the ascensional force of a spherical balloon, 30 feet in diameter, filled with common illuminating gas (Sp. Gr. ·485), the weight of the balloon and car attached being 200 lbs.? What if it were two-thirds filled with hydrogen (Sp. Gr. of hydrogen, 0·069)?

144. A balloon entirely filled with illuminating gas (Sp. Gr. ·500), is so ballasted that it rises to an elevation where the mercury stands at 15 inches. Suppose one-half the gas is now liberated, will the balloon rise or fall? and what amount of ballast should be put in, or thrown out, to cause the balloon to remain stationary, at the same elevation as before any gas was liberated?

Mariotte's Law. (*Regarded as invariable.*)

145. What proportion of a tube, 34 feet high, can be filled with water, the contained air being assumed to be compressed at the bottom of the tube?

146. A faulty barometer (containing air) indicated 29·2 and 30 inches, when the indications of a correct instrument were 29·4 and 30·3 inches respectively; find the length of tube which the air in the column would fill under the pressure of 30 inches?

147. A glass globe, 10 c. m. in diameter, hermetically sealed, weighs 45·120 gram. when the barometer stands at 74·5 c. m. What would it weigh if the barometer stood at 76 c. m.?

148. A glass globe hermetically sealed, 30 c. m. in diameter, suspended to one arm of the balance, is poised by 320·422 gram. in brass weights, when the barometer stands at 76·21 c. m. After a time, it is found to have lost in weight 0·022 gram. What is now the height of the barometer, supposing the temperature not to have changed?

CHAPTER V.

OF UNDULATIONS.

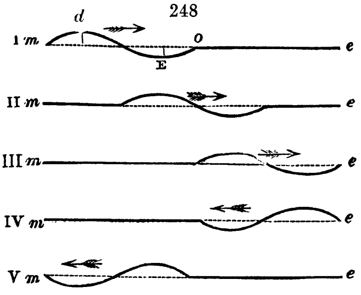
§ 1. Theory of Undulations.

299. **Origin of undulations.**—By the operation of certain forces, the different parts of all bodies are, ordinarily, held in a state of equilibrium or rest. If the molecules of a body are disturbed by any extraneous force, they will, after a certain interval, return to the state of repose. This return is effected by the particles approaching the position of equilibrium, and receding from it, alternately, until at length the body, by the resistance of the medium in which it is placed, and by other causes, is gradually brought to rest. The alternate movements thus produced, are variously expressed by the terms vibrations, oscillations, waves, or undulations, according to the state or form of the body in which such movements occur, and the character of the motions which are produced.

300. **Progressive undulations.**—Undulatory movements are of two kinds, progressive and stationary. In progressive undulations, the particles which have been immediately excited by the disturbing cause, communicate their motion to the particles next them, and as this movement of the particles is successive, the position they assume at any particular moment during their motion, appears to advance from one place to another.

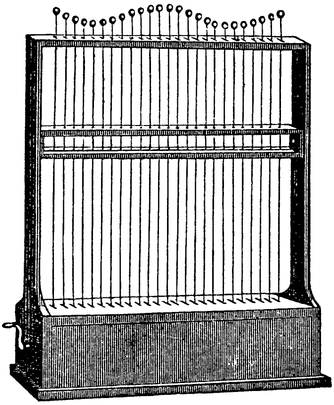
This kind of undulation is observed in a cord made fast at one end, while the other is smartly shaken up and down; the portion of the cord nearest the hand will assume the position in fig. 248, I, *m d* E O. Such a wave does not continue stationary; the moment it is formed, it advances toward the other extremity of the cord, II, on reaching which, III, an inverted curve is produced, IV, and the wave returns, V, to the position from which it started, the relative position of the elevation and depression being reversed. This alternate movement may be repeated a number of times before the cord comes to rest. These are sometimes called waves of translation.

248



301. **Mechanical illustration of undulations.**—In fig. 249 is shown *Powell's apparatus* for illustrating progressive undulations. A series of balls are so mounted upon metallic rods that they have freedom of motion only in a vertical direction. On a shaft turned by a crank, shown in the lower part of the figure, are placed a series of eccentric wheels (one under each rod) so arranged as to raise the rods one after the other. When one rod is rising another is falling, and the wave appears to travel from one end of the series to the other. As soon as one wave disappears another is formed, and these waves succeed each other like the undulations of a cord.

249



302. **Stationary undulations.**—Undulations are termed stationary when all parts of the body assume and complete their motion at the same time.

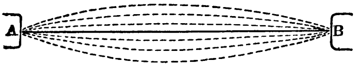
Thus, when a cord stretched between A B, fig. 250, is drawn at the middle from its rectilinear position, it ultimately recovers its original position, after performing a series of vibrations, in which all parts of the cord participate.

303. **Isochronous vibrations.**—Those vibrations that perform their journey on either side of their normal position in equal times, are termed isochronous (from ισος, equal, and χρονος, time).

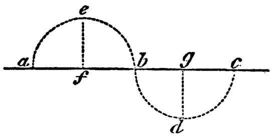
The movements of a pendulum furnish a perfect illustration of such vibrations (78).

304. **Phases of undulations.**—In every complete oscillation, or perfect wave, the following parts may be recognised. The curve *a e b d c*, fig. 251, is called a wave. The part *a e b*, which rises above the position of equilibrium, is called the phase of elevation of the wave, *e* being the point of greatest elevation; the curve *b d c* is called the phase of depression of the wave, the point *d* being that of greatest

250



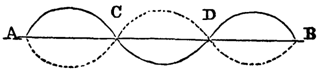
251



depression. The distance *e f*, of the highest point above the position of equilibrium, is called the height of the wave, and in like manner the distance *g d*, of the lowest point below the position of equilibrium, is called the depth of the wave. The distance *a c*, between the beginning of the elevation and end of the depression, is called the length of the wave, the distance *a b* the length of the elevation, and *b c* that of depression.

305. **Nodal points.**—When a body, as a string, is made to assume a series of stationary vibrations, the points where the phases of elevation and depression intersect, are always at rest.

252



Let the cord stretched between A B, fig. 252, be temporarily fixed at the points C and D, and the three parts be drawn at the same moment equally in contrary directions, so that the cord will assume the undulating form represented in the figure; if now the fixed points at C and D be removed, no change will take place in the vibratory motion of the cord; but as it continues to vibrate, the points C and D, although free, will be in a state of rest.

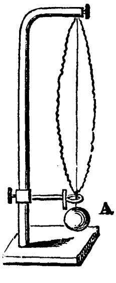
Pieces of paper resting upon these points will be undisturbed, while, if placed on the intermediate positions, they would be thrown off immediately. These are called nodal points (Latin, *nodus*, a knot).

§ 2. Undulations of Solids.

306. **Solid bodies.**—All solid bodies exhibit the phenomena of vibration in various forms and degrees, varying in an infinite variety of ways, according to the form of the body, and the manner in which the force producing the vibration is applied.

307. **Forms of vibration.**—Bodies of a linear form, as tense strings, fine wire, &c., are susceptible of three kinds of vibration, which are called (1st) the transverse, (2d) the longitudinal, and (3d) the torsional vibrations. A simple apparatus to exhibit these effects experimentally, contrived by Prof. August, is represented in fig. 253. It consists of a spirally twisted wire, stretched from a frame by a weight. If the weight be raised to A, and then let fall, it will advance and recede from its normal position, the wire performing a series of *longitudinal vibrations.*

253



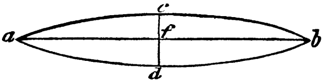
Transverse vibrations are produced by confining the lower end of the wire by a clamp. The wire is then drawn from its position of equilibrium and suddenly let go. The vibrations which it then makes, shown by the dotted lines, are transverse to the axis of the wire. *Torsional vibrations* are produced by turning the weight around its vertical axis;

upon letting it go, the torsion, or twist of the wire causes it to turn back, its inertia carrying it beyond its position of equilibrium, until arrested by the resistance of the wire, and these alternate twistings will continue with a constantly decreasing energy, until gravity, and the molecular forces of the solid, restore the equilibrium.

308. **Vibration of cords.**—Cords and wires, as is familiarly seen in stringed instruments, have their elasticity developed by tension. The transverse vibrations of a body are well illustrated by the simple apparatus annexed.

Thus if the cord *a f b*, fig. 254, be drawn out in the middle to *a c b*, upon being let go, its elasticity causes it to return to its former position. This movement is effected with an accelerated velocity, and is at its maximum when the cord has reached the line of equilibrium *a f b*, consequently it passes with a constantly decreasing velocity to *a d b*, where its motion is nothing; it then returns to *a f b*, and so continues.

254



One complete movement, (as from *a c b* to *a d b*,) is termed an oscillation or vibration, and the time occupied in performing it is called the time of oscillation. The vibrations of tense strings are isochronous.

309. **Laws of the vibration of cords.**—Calculation and experiment have demonstrated, that the vibration of cords is in accordance with the four following laws.

1. *The tension being the same, the number of vibrations of a cord is in inverse ratio to its length.*

That is, if an extended cord, as of a violin, makes in a certain time a number of vibrations, represented by 1, then, in order to make a number of vibrations, represented respectively by 2, 3, 4, the cord must be ½, ⅓, ¼ as long.

2. *The tension being the same, the number of vibrations in cords of the same material, is in the inverse ratio of their thickness or diameter.*

That is, if we take two cords or wires of the same length, of copper or steel, as those of a piano, one of which is twice the diameter of the other, and which vibrate equal lengths, the small one will make, in the same time, twice as many vibrations as the larger.

3. *The number of vibrations of a cord is proportional to the square root of the stretching weight.*

That is, if we represent by 1 the number of vibrations made by a cord, extended by a weight of 1, then the number of vibrations made by a similar cord of the same length, in the same time, becomes respectively 2, 3, 4, &c., when the weight is increased to 4, 9, 16, &c. Thus, if we would cause a given cord, as of a violin, to vibrate with a four-fold velocity, it is necessary to strain it to sixteen-fold the original tension.

4. *All other things being equal, the number of vibrations of a cord is inversely proportional to the square root of its density.*

Thus, if we take a cord of copper which has a density of 9, and one of cat-gut, whose density is about 1, the number of vibrations of the last in the same time will be three times that of the former.

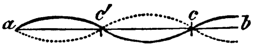
It is evident that these laws apply only to homogeneous cords, and not to those cords which are covered with another material, as a harp string of cat-gut, covered with metallic wire.

310. **Vibrations of rods.**—Rods, like cords, vibrate both in longitudinal and transverse directions. If they are fixed firmly by one of their extremities, as in a vice, they will give, when set in motion, a series of isochronous vibrations.

Elastic rods may, like strings, be divided by stationary undulations into several vibrating parts. The nodal points may be ascertained by placing upon the rods light rings of paper; these will be thrown off as long as they rest upon any point except a node, but when they reach a node, they will remain there unmoved.

The space between the free extremity and the first nodal point is equal to half the length contained between two nodal points, but it vibrates with the same velocity. Thus *a*, fig. 255, being the fixed, and *b* the free end, the part between *b* and *c* is half the distance *c c'*. The nodal points may be rendered sensible by sand strewn upon the horizontal surface of the vibrating rod; the sand is seen to move to certain points, where it remains stationary; these are the nodes.

255



Rods may also, like cords, vibrate longitudinally, and the nodal points are formed in the same manner. It has been observed in elastic rods of the same nature, that the number of longitudinal vibrations is in the inverse ratio of their length, whatever may be their diameter and the form of their transverse section.

A prismatic bar, vibrating longitudinally, undergoes a very considerable increase of length, which, in the state of repose, could not be produced except by a very strong tension, while the vibratory movement is obtained by a very feeble force.

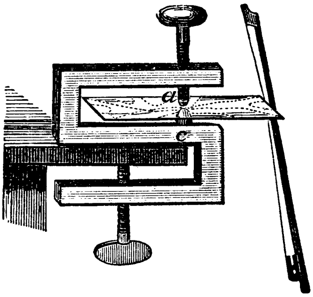
The number of vibrations by torsion in rods, is in the inverse ratio of their length, and is proportional to their thickness, the substance in all cases remaining the same.

311. **Paths of vibration.**—The motion performed by vibrating rods is often very complex. This may be beautifully seen by the contrivance of Prof. Wheatstone, consisting of a polished bead fastened on the extremity of an elastic rod, as of a knitting-needle, firmly fixed in a board or vice.

Upon making the rod vibrate, the bead, by reflection, will produce a continuous line of light. It will be seen that the arc described is not circular, but the rod appears to be impressed at the same time with two vibratory movements, at right angles to each other, and moves in a curve produced by the composition of these forces.

312. **Vibration of elastic plates.**—Vibrations are readily excited in elastic plates by the friction of a violin-bow or by blows. The plate may be confined either at its centre or from one corner, in the vice, fig. 256, resting upon a cone of cork, *c*, and pressed by the screw *a*, tipped with cork.

256

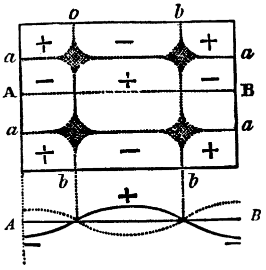


In the vibration of plates, nodal lines will be formed, which do not participate in the movements of the plane, but remain in a state of rest.

313. **Nodal lines.**—These nodal lines answer to the nodal points in linear vibrations, and if we suppose the plane to be made up of a series of rods, these lines will answer to their nodal points. They run in various directions across the vibrating surface, the contiguous ones moving in contrary directions, dividing the planes into numerous portions in opposite phases of vibration.

This is shown in fig. 257, by the signs + and —, A B being the vibrating plane. The dimensions of these internodes (vibrating portions), are regulated in the same manner as those of vibrating rods. The outside ones, *a b*, *a b*, are always half the size of those in the interior. The nodal lines vary in their number and position, according to the form of the plates, their elasticity, the number of vibrations, the mode of vibrating, &c.

257



314. **Determination of the position of the nodal lines.**—The position of the nodal lines may be determined by scattering sand or other fine material over the plate, and vibrating, as by means of a violin-bow drawn across the edge of the plate; the grains of sand will remain upon the points which are at rest, and which are therefore nodal points. Those which are upon vibrating portions, will be thrown aside until, after a time, they will settle quietly down upon the nodal lines.

It is observed that if *lycopodium*, or some other very light powder, is placed upon the plates, it will accumulate on those parts which are in greatest vibration. Mr. Faraday proved that this phenomenon was due to small currents of air produced during the vibration of the plate, and which drew the powder with them; for in a vacuum, the powder of lycopodium is disposed, like sand, upon the nodal lines, and for the same reason; if the plate covered with sand is vibrated under water, the sand collects upon the most agitated portions of the plate, because of the similar currents excited in the water by the vibrations.

315. **Laws of the vibration of plates.**—Observation has determined that the vibration of plates of the same substance, and having the same degree of rigidity, are subject to the following laws:—

1. *That the number of the vibrations is independent of the breadth of the laminæ.*

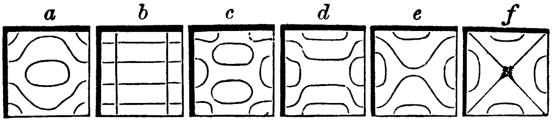
2. *It is proportional to their thickness.*

3. *The thickness being the same, it is in inverse ratio of the square of their length.*

316. **Method of delineating nodal lines.**—As these nodal lines assume various and complicated figures, difficult to delineate with accuracy by common drawing, Savart replaced the sand by powdered litmus, previously mixed with gum water, dried and pulverized to a uniform size. The acoustic figures being produced with this powder, a paper moistened with gum water was then gently pressed upon them, thus giving an exact transfer.

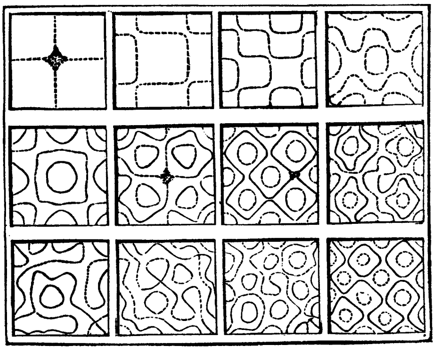
This method gave great facilities for the comparison and study of these fugitive figures, so difficult to produce with perfect identity, and enabled the inventor to determine the exact limits of the nodal lines and areas of unequal vibration.

258



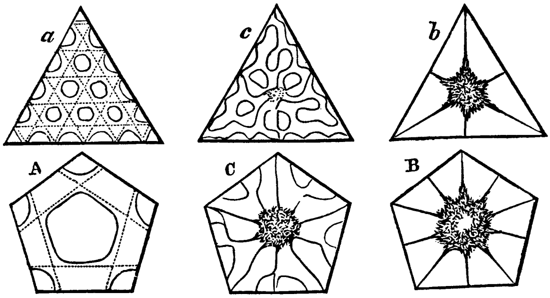
317. **Nodal figures.**—Nodal (or acoustic) figures have always a great symmetry of form, and their lines are generally as much more numerous as the number of vibrations is greater. The same plate may furnish an infinite variety of figures, which pass from one to another in a continuous manner, and not by sudden changes. Thus the figures *a b c d e f*, fig. 258, pass into one another without intermission.

259

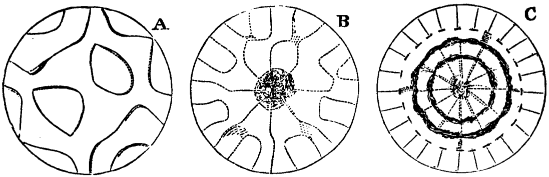


Many hundred forms of nodal figures have been figured. Fig. 259 represents a few of those obtained on square plates. Triangular and polygonal plates all give symmetrical figures, analogous to those obtained with square plates, as is seen in fig. 260. With circular plates it is observed that the nodal lines distribute themselves in the direction of the diameter, dividing the circle into an equal number of parts, or into more or less regular circular forms, having the centre of the plate as their common centre, or in both of these forms combined. Fig. 261 represents these different varieties of form.

260

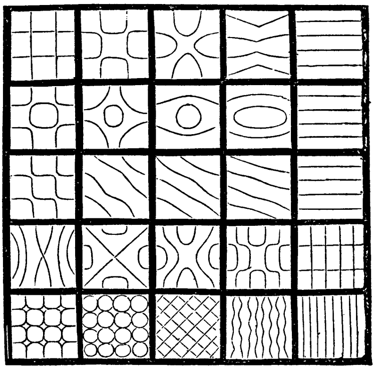


261



318. **Vibration of membranes.**—The flexibility of membranes does not permit us to vibrate them unless they are stretched as in a drum. They present modes of vibration which have much analogy to those of solid plates, vibrating either by concussion, as in the drum, or by the influence of vibrations in the air. If we stretch over the top of a funnel a piece of moistened bladder, and when it is dry, suspend the apparatus by a knotted hair, passed through the centre of the membrane, we can produce symmetrical nodal lines upon its surface, strewed with sand, by passing the fingers, covered with resin over the hair. The same phenomenon may be

262

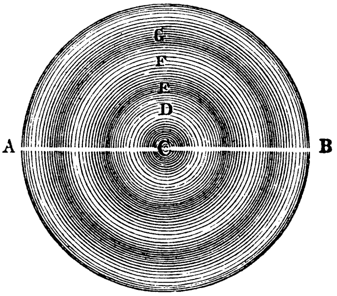


observed, if we bring the membrane near a bell while it is vibrating. The acoustic figures obtained by the vibration of membranes are extremely varied. Savart has observed that square membranes are divided by their nodal lines into the same forms as square plates under the same circumstances, with this difference, that the vibrating parts in the vicinity of the edges are smaller for the last, while they are equal to the others in membranes. Fig. 262 represents a few of the forms produced in the vibration of membranes. It has been found that wood and metals, in very thin laminæ, vibrate like membranes.

§ 3. Undulations of Liquids.

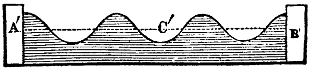
319. **Production of waves.**—Liquids are capable of assuming undulatory movements, similar to the vibrations of solids, differing from them, however, in some respects, in consequence of the different physical arrangement of their atoms. If a depression be made at any point in the surface of a fluid in a state of rest, by the dropping in of a solid, as of a pebble into water or by immersing and then withdrawing the solid, a circular undulation will be produced. Around the point of depression there first rises a circular elevation above the level of the liquid when in equilibrium, and immediately beyond this is a circular depression, and so, alternately, successive elevations and depressions. Thus the initial motion will be gradually propagated in a series of progressively widening circles; wave follows wave, until opposing causes allow the equilibrium to be regained. Thus, in fig. 263, the light circles D and F represent the elevations, and the shaded ones, C, E, G, the depressions of these circular waves.

263



As in the case of the vibrations of solids, an entire undulation consists of a phase of depression, and another of elevation. This may be rendered more intelligible by conceiving a vertical plane, A B, to pass through the point C, whence the waves originate. It is plain that a progressive lineal undulation will arise on it, resembling that of the cord, fig. 248. This section is seen in fig. 264, A′ C′ B′, and the nomenclature used for the cord applies to it, with this difference only, that by the breadth of the wave is meant the periphery of its circle, and by its length, the length of both the elevated and depressed portions. (Peschell.)

264

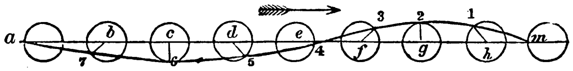


320. **Progressive undulations in liquids.**—In a movement of the kind just indicated, the fluid appears as if its entire mass advanced pro

gressively from the point of excitation; but this is a delusion. Floating bodies, as pieces of wood, are not hurried forward on the surface of the water, but merely rise and fall, alternately, as the waves pass. The true nature of the motion is such, that each particle of the fluid describes a vertical circle about the spot where it may happen to be, revolving in the direction in which the wave is advancing. The particle thus returns to its former position in the same plane, one-half being above, and the other half below the level of the fluid. Each particle of fluid thus set in motion, imparts a similar movement to its contiguous particle, this again to the next, and so on. But as a certain time must elapse for this transmission of motion, the different particles will be describing different portions of their circular movement at the same moment. Some will be at the highest point of their vertical circle, while others are in an intermediate position, and others at the lowest, giving rise to a wave which advances a distance equal to its whole length, while each particle performs one entire revolution.

For the sake of simplicity, we will consider only eight of the many particles which we may conceive as occupying the horizontal surface between *a* and *m*, fig. 265. Imagine the particle *a* to be at rest, when a descending wave strikes it. It will be depressed, and will begin to revolve in a vertical circle in the direction of the arrow. If we consider eight such particles to be situated on the line *a m*, and that each particle begins its motion $\frac{1}{8}$ of a revolution later than its neighbor next on the left hand; then at the instant when *a* has completed one entire revolution, the second will be one-eighth behind it, viz.: at 7; the 3d, two-eighths behind it, viz.: at 6; and the fourth, fifth, sixth, seventh, and eighth, at the points 5, 4, 3, 2, and 1 respectively; whilst the 9th particle is but just beginning to move. Connect the points *a*, 7, 6, 5, 4, 3, 2, 1, *m*, and the line will represent the form of the fluid surface at that precise moment of the undulation.

265



The diameter of the circle which each particle describes, is the amplitude or intensity of the wave, *c*6 its depth, and *g*2 its height, each of which is equal to the radius of a circle which any particle describes during one oscillation. This radius is longer or shorter according to the amplitude of the wave. It is sometimes twenty feet, which makes a very high wave, probably the largest which ever occurs on the ocean in a violent storm, unless it be those waves which have been increased by the accumulation of wave upon wave.

321. **Stationary waves.**—Stationary undulations may be produced by exciting waves in a circular vessel, from its central point. The waves being reflected from the circular wall, will produce another series,

which, combined with the first, will produce the effect of a stationary undulation. So also they may be produced on a surface of a liquid confined in a straight channel by exciting a succession of waves, separated by equal intervals, moving against the side or end of the channel, and reflected from it.

322. **Depth to which waves extend.**—Waves, or undulations, are not only propagated laterally, but also in all other directions. It has been ascertained (by the Messrs. Webber) that the equilibrium of the liquid is not disturbed to a greater depth than about three hundred and fifty times the altitude of the wave.

323. **Reflection of waves.**—If a series of progressive waves are arrested by impinging against any solid surface, they will be reflected again from that surface under the same angle at which they struck it. This reflection of waves is occasioned by elasticity, and obeys, precisely, the laws which regulate the impact of elastic bodies.

Since this law applies to all the rays which constitute the breadth of a wave, the path of a reflected wave may readily be determined by a knowledge of the surface and the angle of incidence. If the wave is linear, (that is, if the line resting upon the highest point of the elevation at right angles to the direction in which it is moving, is a straight line), and it meets a plane surface, it will be reflected, and return in the same path. If it meet the surface at an angle of 20° or 30°, it will be reflected at the same angle, on the other side of the perpendicular to the reflecting surface.

324. **Waves propagated from the foci of an ellipse.**—If the vessel is in the form of an ellipse, and a wave originate at one of the foci, all the rays will converge so as to fall simultaneously, after reflection, on the other focus.

266

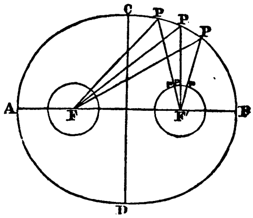


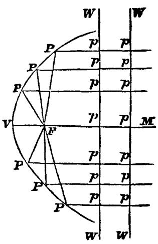
Fig. 266 represents an ellipse, of which F and F′ are the foci, which have the following property. If lines be drawn from the foci to any points, P, P, P, in the ellipse, these lines will form equal angles with the ellipse at P, and their lengths taken together, will be equal to the major axis A B. If we suppose a series of circular progressive waves propagated from the focus F, their rays will strike successively and at equal angles upon the elliptical surface, as at the points P P P; they will be reflected in the direction P F′, towards the other focus. But as all the points of the same wave move with the same velocity, they will all reach the focus F at the same time, for the distances they pass over are equal. Hence it follows, that each circular wave that expands around F, will, after it has been reflected from the surface of the ellipse, form another circular wave around F′ as a centre.

325. **Waves propagated from the focus of a parabola.**—If the

surface be a parabola, and a wave originate at its focus, all the rays will, after reflection, pass in parallel lines. Or, if the rays impinge in parallel lines, they will, after reflection, converge to the focus.

Fig. 267 represents the parabolic curve. The point V is its vertex, the line V M its axis. The point F, upon the axis, is the focus, and has the following property. If lines be drawn from the focus to any points, P, in the curve, and other lines be drawn from the points, P, severally parallel to the axis V M, meeting lines W W, drawn perpendicular to the axis, the lines F P and P p will be inclined at equal angles to the curve at the point P, and the sum of their lengths will be everywhere the same. Hence it may be demonstrated, as in the case of the ellipse, that if a series of progressive waves be propagated from the focus F, these waves, after striking the curve, will be reflected, so as to form a series of parallel straight waves.

267

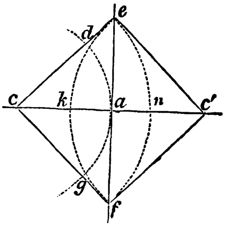


Moreover, it is evident that if two parabolas face each other, so as to have their axes coincident, a system of progressive circular waves, issuing from one focus, will be followed by a corresponding system, having for its centre the other focus. For if a series of parallel straight waves strike a parabolic surface, their reflection would form a series of circular waves, of which the centre would be the focus.

Rays reflected from spherical surfaces, whose extent is small compared with their diameter, will, in their direction, approximate closely to those reflected from a parabolic boundary.

326. **Circular waves reflected from a plane.**—If the diverging rays of a circular wave fall upon a plane surface at right angles to it, their path, after reflection, is the same as it would have been had they originated from a point on the opposite side of the plane, and as far back as the point of origin itself; that is, the form of the reflected wave will be the reverse of the incident wave, for the rays which first strike the surface will be reflected first, and will have returned to the same distance from the surface at the time when the last rays meet it, that these last rays were at the moment when the first were reflected.

268



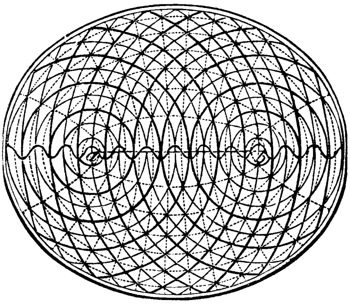
Thus, suppose the wave $g\ a\ d$, proceeding from the centre c, fig. 268, impinges on the plane surface, $e f$. The form of the wave, after reflection, will be the same that it would have been had it proceeded from c', on the other side of $e f$, (at the same distance). It is evident that with a circular wave, all its points cannot impinge at the same time on a plane, therefore, the portions in advance will impinge first, and will first be reflected; and when a impinges, the rays at d and g have still to go through the distances $d\ e$ and $g f$, before they can be reflected; but in the space of time required for this, the ray at a will have returned to the point *k*. In the same way it may be shown that the intermediate rays will return to intermediate positions, and be found in the line *e k f*, symmetrically situated to the line *e n f*, in which they would have been had they not been reflected from the plane. And it further follows, that the centre, *c′*, of the reflected wave *d k g*, is as far from *e f*, as is the centre, *c*, of the incident wave, *e n f*, but on the opposite side of the median plane, *e a f*.

327. **Combination of waves.**—Where two systems of waves, coming from different centres, meet each other, several effects may follow, according to the mode of meeting, which curiously illustrate the principles of undulation in all departments of physics. 1st. If the elevations of the two waves coincide, and, consequently, their depressions also, then a new wave will be formed, whose elevations and depressions will be the sum of those of the two originals. 2d. If the two waves of equal amplitude are so superimposed that the reverse of the last case is true, *i. e.*, that the elevation of one fits the depression of the other, then both waves disappear, and the surface remains horizontal. Or, 3d, if one wave has greater amplitude than the other, and the two waves meet in the same phase, then the resulting wave will have a height equal to the difference between the greater and the less.

Combinations, and *interference* of waves, are of universal occurrence in all media, in which force of any kind is propagated by undulations.

328. **Interference in an ellipse.**—The two systems of waves formed by an elliptical surface, and propagated, one directly around one of the foci, and the second formed by reflection around the other, exhibit not only the phenomena of reflection, but also of interference. These phenomena are represented in fig. 269, where *a* and *b* are the two foci. The strongly marked lines are the elevations, the more lightly traced lines are the depressions, the points where the more strongly marked circles intersect the more faintly marked circles, are points where an elevation coincides with a depression, and are therefore points of interference. The series of these points form lines of interference which are indicated in the diagram by dotted lines, which, as will be seen, arrange themselves regularly in the form of hyperbola and ellipses about these foci.

269



329. **Undulations of the waters of the globe.**—The undulations produced in the oceans, lakes, rivers, and other large collections of

water upon the surface of the globe, are of extreme importance in the economy of nature. Did not water possess, as a consequence of the mobility of its particles among each other, the property of being thus set in motion, the ocean would soon be rendered putrid by the decomposition of the mass of organized matter it contains. The principal physical cause which produces these undulations on a moderate scale, is the motion of the atmosphere. On a large scale they are produced by the combined effects of the attraction of the sun and moon upon the surface of the ocean, which causes the ebb and flow of tides. Differences in temperature and density of the waters of different parts of the ocean, cause currents, by the efforts of these waters to assume a state of equilibrium; and lastly, the rotation of the earth upon its axis, originating the constant easterly current. A full discussion of these interesting questions belongs to Physical Geography.

§ 4. Undulations of Elastic Fluids.

330. **Waves of condensation and rarefaction.**—The undulations of liquids already described (319) are surface waves. Undulations of the same kind may also be produced in elastic fluids. Elastic fluids are also subject to undulations of a totally different kind called *waves of condensation* and *waves of rarefaction.* Such undulations are due to elasticity, and are produced in air and gases by any disturbance of density. If any elastic fluid be compressed, and again suddenly relieved from compression, it will expand, and in its expansion exceed its former volume to a certain extent; after which it will again contract, and thus oscillate alternately on either side of the position of repose. It is obvious that we must regard these undulations, or pulses of air, as extending equally in all directions in the free air, and limited only by the walls of the containing vessel or apartment when the air is confined. Therefore, the effects of the united oscillations extend equally in the course of radii, from the centre to every point of the surface of a sphere.

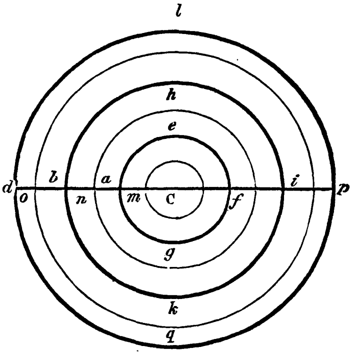
331. **Undulations of a sphere of air.**—The oscillations of air will not be confined to the sphere in which they commence. When air is first contracted, an aërial shell, bounding the sphere of contraction, expands, and becomes thereby less dense than when in equilibrium. Again, upon the expansion of the original sphere, the bounding shell contracts, and becomes more dense, in virtue of its inertia and elasticity. This exterior shell of air thus acts upon another, external to it; this in its turn on another, and so on, and thus the initial force is propagated upon successive concentric portions of air; its effects becoming less marked with each enlargement, until, like the ripple of a wave of water, it becomes too evanescent to be appreciated. Compare § 653.

This alternate condensation and expansion of an elastic fluid, extending spherically around the original centre of disturbance, is perfectly analogous to a series of circular waves formed around a point on the surface of a liquid. The condensation of the elastic fluid being analogous to the elevation of a surface wave, and the phase of rarefaction being analogous to the phase of depression.

The radius of the hollow sphere, or the distance the undulation had traversed when the first particles resumed a position of rest, is called the length of a wave; the entire sphere comprised within these limits constitutes a wave, and the time of vibration is equal to the time in which motion is propagated through the entire length of a wave. If the cause which excited the undulation continues to operate, the first wave will expand, and there will arise a second and third wave, &c., within the first, and concentric with it.

This radial propagation of undulations in air can take place with equal velocity in all directions, only when the atmosphere is of uniform density, so far as the vibrations extend. If this is not the case, such a wave cannot have a spherical form.

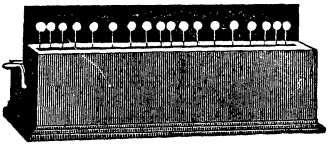
270



Let fig. 270 represent a section of a sphere of air, or other elastic fluid, in which waves of condensation and rarefaction have extended outwards from the centre C; then the heavy lines, *a e f g*, *b h i k*, and *d l p q*, will represent the phases of greatest condensation, the finer intermediate lines will represent the spaces of greatest rarefaction, and the distances *m n*, and *n o*, between circles of greatest condensation, will be the length of the waves.

Mechanical illustration.—Professor Snell has contrived the apparatus, represented in fig. 271, to illustrate these undulations. It consists of a shaft turned by the crank seen at the left, on which are elliptical grooves, inclined to the axis, in which the rods carrying the white balls are made to vibrate from right to left and back again. These grooves are so arranged that each successive ball moves to the left later than the preceding. Thus the balls are seen crowded together at certain parts of the series; and, as the crank is turned, the phases of greatest condensation travel across the field from left to right, while other similar waves are formed and continually succeed them.

271



332. **Velocity and intensity of aerial waves.**—The velocity with which such undulations are propagated through the atmosphere, depends on, and varies with, the elasticity of the fluid. Waves, both large and small, are transmitted with an equal velocity, so long as the elasticity remains the same. The intensity of vibration, *i. e.*, the dimensions of the spaces which the individual particles traverse while in this state of

movement, depends on the energy of the disturbing force, which, it is also evident, is a measure of the degree of compression of the wave.

333. **Interference of waves of air.**—If two series of aërial waves coincide as to their points of greatest and least condensation, a new series of waves will be formed, whose greatest condensation and rarefaction is determined by the sum of these points, as prevailing in the separate undulations. But where the series are so arranged that the point of greatest condensation of one coincides with the point of greatest rarefaction in the other, the resulting series will have condensations and rarefactions equal to the difference between the waves which meet. If they are equal, total interference takes place, as in the case of non-elastic fluids, and silence results, if the waves are those of sound.

Indeed all the effects described in the case of waves formed upon the surface of a liquid are reproduced under analogous conditions in the case of undulations of aëriform bodies. It must, however, be borne in mind, that these aërial waves have always a spherical form.

334. **Intensity of waves of air expanding freely.**—The undulations produced in air form progressively increasing spheres (330), the magnitude of whose surfaces are to each other as the square of their radii, or as the square of their distance from their respective points of impulse. As the intensity of the wave is diminished in proportion to the space over which it is diffused, it follows, that the effect or energy of these waves diminishes as the square of the distances from the centre of propagation increases. So soon, however, as the radial extension of the wave meets with any resistance which reflects the rays in a parallel or concentric direction, this rule ceases to be applicable.

Problems.

On the Laws of Vibrations.

149. If a cord of a given length makes 48 vibrations per second, what must be the respective lengths of similar cords to make 63, 64, 72, 81, and 90 vibrations per second?

150. If a cord, 3 feet long, extended by a weight of 10 lbs., makes 96 vibrations per second, with what force must a similar cord, 2 feet long, be extended that it may make 108 vibrations per second?

151. If an iron wire, one-tenth of an inch in diameter (Sp. Gr. 7·8), makes 72 vibrations per second, what must be the diameter of a platinum wire of the same length (Sp. Gr. 21·23) which will make 45 vibrations per second?

152. An iron rod, vibrating by torsion, makes 30 oscillations per second; how much longer must a rod, having twice the diameter, be to vibrate (with the same force) 15 times per second?

CHAPTER VI.

ACOUSTICS.

§ I. Production and Propagation of Sound.

335. **Acoustics.—Sound.**—Acoustics (derived from the Greek verb, ἀκούω, to hear), teaches the science of sounds, their cause, nature, and phenomena. Sound is the impression produced on the sense of hearing by the vibrations of sonorous bodies. These vibrations are transmitted to the ear by the surrounding medium, which is ordinarily the atmospheric air.

Sound a sensation.—It will be understood, therefore, that all sound, whether unmusical, like mere noise, or musical, like what is technically called a *tone* (a sound of definite and appreciable pitch), is a *sensation;* and the causes which produce this sensation may exist without the sensation itself—that is, without sound. The cause of sound being atmospheric vibration, if there be no delicately constructed organ, like the ear, to receive the impression of this vibration, there is no sound. It would follow, that even at the Falls of Niagara, if there were no ear present to receive the impression, those gigantic vibrations would exist only as such—without sound.

Key-note of nature.—The aggregate sound of nature, as heard in the roar of a distant city, or the waving foliage of a large forest, is said to be a single definite tone, of appreciable pitch. This tone is held to be middle F of the pianoforte—which may therefore be considered the key-note of nature.

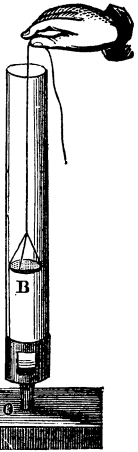
Noise.—The distinctive character of mere noise is determined by the nature and duration of the irregular vibrations causing it. If these vibrations are short and single, the effect is that of a crack, or an abrupt explosion, as in the snapping of a whip, or the explosion of cannon. If they are continuous and prolonged, the effect is that of a rattle, or rumble, like the rolling of thunder, or the noise of carriages over a stony road.

Musical sounds.—Sound, in a musical sense, or tone, is the sensation produced by a series of equal atmospheric vibrations. Noise is the sensation produced by unequal vibrations. If we throw a single stone into the centre of a placid lake, a single wave circles off to the shore: such is the effect upon the air when a *tone* is produced. If a handful of pebbles is thrown into the lake, each separate pebble produces its own circle, these circles intersect each other and become confused to the eye: such is the effect upon the air when a noise is produced. Pulling the string of a harp would correspond to the single stone thrown into the lake: striking a table or chair, where the separate fibres of the wood vibrate unequally, would correspond to the handful of pebbles.

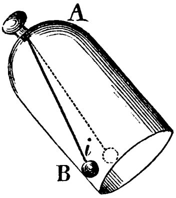
A church-bell, to a cultivated ear, is noisy, or musical, according as the tones in the vibrating metal (for every bell produces more than one distinguishable tone) chance to be at musical or unmusical intervals. If the intervals are (musically considered) dissonances, the bell will be discordant; if they are concords, the bell will be harmonious. Again, if in these concordant tones the intervals be "major," the bell will be cheerful; if they be "minor," the bell will be sad.

336. **All bodies producing sound are in vibration.**—If the sonorous body is solid, and presents a large surface, as a bell-jar, the vibrations may be shown by suspending a small ivory ball, *i*, by a thread in the interior of the jar A B, inclined in the position seen in fig. 272. When the jar resounds with a blow, the ball is thrown from the sides, as shown by the dotted line, and returning, is again thrown off, and so continues bounding, in consequence of the vibrations. A touch from the hand arrests the vibratory movement in the glass: the sound ceases, and the ivory ball remains quiet.

273



272

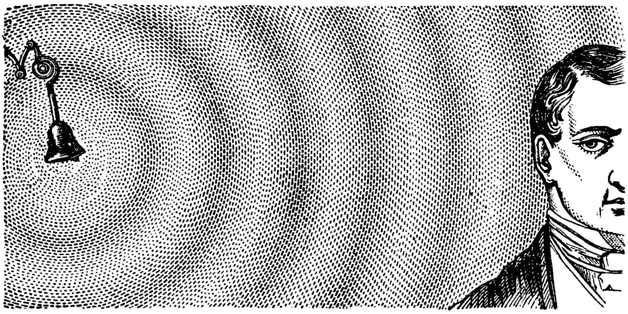


If the sonorous body is a plane surface, its vibrations may be shown by the formation of nodal lines with grains of sand scattered upon it. When the sound is produced by a stretched cord, the vibrations may be felt by touching the cord lightly with the hand, or may be seen by placing bands or rings of paper upon the vibrating cord. In wind instruments, it is the air which they contain, whose vibrations produce the musical sounds.

This may be proved by an organ-pipe, or by a glass tube, fig. 273, when a current of air is passed into it through the foot, O. A small membrane of gold-beater's skin, extended on a circle of pasteboard, B, is placed within the tube, and sustained in a horizontal position by means of a thread. Grains of sand strewed upon the membrane, will be arranged in nodal figures, proving that the membrane obeys the vibratory movement of the air which surrounds it. That the vibrations are not due to an ascending current of air, is proved by the fact, that the membrane does not vibrate when it is placed midway of the length of the tube [a nodal point], but above or below this point it vibrates, and more strongly as it is further removed from the centre.

337. **Sound propagated by waves.**—Sound is propagated by waves of condensation and rarefaction (330), as shown in fig. 274. The vibrations of the sounding bell are communicated to the surrounding medium, and by vibrations alternately forwards and backwards. Motion is communicated to an ever increasing spherical portion of the medium until it reaches the ear, by which the sensation of sound is perceived.

274



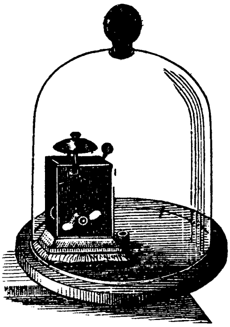
338. **Co-existence of sound-waves.**—Many sounds may be transmitted simultaneously in different directions by the same medium without destroying each other, all the sounds penetrating and crossing in space without modification. In complicated symphonies, a practiced ear readily distinguishes the sound of each instrument. A very intense sound may overpower a feeble sound, as a loud noise renders the human voice inaudible.

339. **Sound is not propagated in a vacuum.**—The vibrations of elastic bodies do not produce an impression on the ear, unless there exists between this organ and the sonorous body an uninterrupted elastic medium, vibrating with it. This medium is ordinarily the atmospheric air, but other gases, vapors, liquids, and solids, transmit sound, and generally with a facility varying with their density.

To prove that sound is not propagated in a vacuum, place under the receiver of an air-pump, a bell, kept in constant vibration by a clock-work movement, fig. 275. The bell apparatus should be placed upon wadding, otherwise the vibrations would be communicated to the plate of the air-pump, and thus to the air. While the receiver is filled with air at the ordinary pressure, the sound is distinct; as the air is gradually exhausted, the sound grows more and more feeble, until finally, when a vacuum is obtained, it ceases to be heard, but is immediately revived by admitting air again.

340. **Sound is propagated in all elastic bodies.**—If, in the experiment just described, the vacuum is supplied with hydrogen gas (density 0.0692), or any gas of less density than atmospheric air, a sound will be transmitted from the bell, very feeble for the hydrogen, and increasing as the gas is more dense. In like manner, a person whose lungs have been filled with hydrogen gas, utters only a shrill piping sound.

275



Vapors, water and other liquids, transmit sounds like gases, but with much more energy. When two bodies are struck under water, the sound is distinct to a person having his ear under water, or communicating with the water by means of some solid substance. The conductibility of sound is so great in solids, that if we apply the ear to one end of a beam of wood, the slightest shock, as the scratch of a pin at the other extremity, may be heard distinctly. The noise of cannon has been heard a distance of more than two hundred and fifty miles, by applying the ear to the solid earth. In several mines in Cornwall, England, there are galleries which extend under the sea, where the sound of the waves is clearly heard when the sea is agitated, rolling the pebbles and boulders over the rocky bottom of the ocean.

The music from a company of musicians, playing in orchestra upon numerous instruments, has been transferred to an apartment in another house, by a cord stretched across the intervening street, connecting at one end with a sounding-board, and at the other extremity with a wooden box. On placing the ear at an opening in the box, the whole musical movement was heard, reproduced in miniature, being transmitted by the vibrations through the cord. A bystander in the same apartment was unconscious of there being any performance.

341. **Hearing** is a sense depending upon the ear, a beautifully constructed instrument, designed to gather in the vibrations of the surrounding air. This vibratory movement is communicated to the acoustic nerve by the aid of organs which will be described in detail at the close of this chapter.

342. **Time is required for the transmission of sound.**—Experience testifies to the truth of this statement. We hear the blows of a hammer at a distance a very sensible interval of time after we see them struck. An appreciable time elapses after we see the flash of a cannon, at a little distance from us, before we hear the explosion. The report of the meteor of 1783 was heard at Windsor Castle ten minutes after its disappearance.

343. **The velocity of all sounds is the same.***—The velocity of sound is the space that it traverses in a second. Theory demonstrates that the velocity of the vibrations of sonorous bodies in the same me

---

* See note in Appendix, p. 668.

dium, is the same for all sounds, grave or sharp, strong or feeble, and whatever may be their pitch. Observation confirms this result, at least for those distances at which experiments have been made. There is no confusion in the effects of music, at whatever distance it may be heard.

If the different notes simultaneously produced by the various instruments of an orchestra, moved with different velocities, they would be heard by a distant auditor at different moments, so that a musical performance, except to those in its immediate vicinity, would produce only discords. M. Biot, in playing an air upon a flute at the extremity of a pipe of the aqueduct of Paris, found that the sounds came to the other end, having exactly the same interval, demonstrating that the different sounds travelled with the same velocity.

344. **Velocity of sound in air.**—Numerous experiments have been made for estimating the velocity of sound; that is, the space that it travels over in a second. The most extensive and accurate system of experiments were those made in 1822, by the Board of Longitude of France, conducted by Messrs. Prony, Arago, Humboldt, Gay Lussac, and others.

Two pieces of cannon were used, one placed at Montlhéry, the other at Montmartre, between which the distance is 18,612 m. (= 61,063·8 feet), or more than ten miles. The discharges were reciprocal, so as to avoid the influence of the wind. At each station were numerous observers, furnished with chronometers, who noticed the time between the appearance of the light, and the arrival of the sound. This time may be called that which the sound requires to pass from one station to another, for the time occupied by the passage of the light between the two points is wholly inappreciable. The mean time required to transmit the sound was 54·6 seconds. By dividing the distance between the two stations by this number, the velocity, per second, is obtained. The velocity of sound at 61° F. (16° C.), that being the temperature of the atmosphere during the experiment, is 1118·3 feet (340·88 m.), (for $61{,}063·8 \div 54·6 = 1118·3 +$); or at the temperature of 32° F. it would be about 1086·1 feet.

Messrs. Bravais and Martin, in 1844, have determined that the velocity of sound between the summit and base of the Faulhorn (a lofty mountain in the Swiss Alps) is the same, whether ascending or descending, and that it is 1090·47 feet per second at 32° F.

It has been determined, 1. That the velocity of sound decreases with the temperature; at 50° F. (10° C.), it is 1106·091 feet (337 m.) So that as the temperature is lowered, sound diminishes in velocity about one foot and a tenth for every degree. 2. That at the same temperature the velocity of sound is not materially affected, whether the sky is bright or cloudy, the air clear or foggy, the barometric pressure great or small, provided the air is tranquil. All of these circumstances, however, exert a great influence on the intensity of the sound as it reaches the ear from a given distance. Fogs, snow, &c., prevent the free propagation of sound, but do not materially affect its velocity. 3. That its velocity varies with the velocity and direction of the wind.

345. **Velocity of sound in different gases and vapors.**—The velocity of sound in the different gases, is in the inverse ratio of the square root of their densities.

Dulong has determined by calculation the velocity of sound in the following gases, at the temperature of 32° F. Carbonic acid 860 feet (262 m.), oxygen 1040 feet (317 m.), olefiant gas, 1030·2 feet (314 m.), air, 1092·54 feet (333 m.), carbonic oxyd, 1105·6 feet (337 m.), and hydrogen, 4163 feet (1269 m.), each in a second. The theoretical velocity of sound in vapor of alcohol at 140° is 862 feet, in vapor of water at 154° is 1347 feet. The observed velocities are generally not very far from those given by calculation.

346. **Calculation of distances by sound.**—The known velocity of sound per second (1118 feet), enables us to obtain a close approximation of the distance of the sonorous body. This follows as a consequence of the very experiments (344) by which the velocity of sound was determined. From the known laws of falling bodies (71), we may also, with the aid of the known velocity of sound, obtain an approximate estimate of the height of a precipice, or the depth of an abyss, from the time occupied by the sound of any projectile, let fall from the hand, in reaching the ear.

347. **Velocity of sounds in liquids.**—Sound is conveyed through liquids as well as through gases. The velocity of sounds in liquids is much greater than in air. In 1827, Messrs. Colladon and Sturm, experimenting upon the velocity of sound in the Lake of Geneva, found it to be 4708 feet (1435 m.) per second, or about four and a half times greater than in air, at the temperature of 46·6° F.

Agitation of the water, liquids, &c., did not affect either the rapidity or intensity of the sound. But the interposition of solid bodies, such as walls, or buildings, between the sounding body and the observer, almost destroyed the transmission of sound in water; an effect which does not take place nearly to the same degree in air (350).

348. **Velocity of sounds in solids.**—Sound is transmitted by solid bodies with much greater rapidity than by air, but by no means with equal velocity, varying much with the elasticity and density of the different solids, as well as their homogeneity and uniformity of structure.

Want of homogeneity in any medium interferes with the propagation of sonorous vibrations. Let a tall glass be half filled with champagne wine: as long as there is effervescence, and the wine contains air bubbles, a stroke on the glass gives only a dead disagreeable sound; as the effervescence subsides the tone becomes clearer, and when the liquid is tranquil the glass rings as usual. The dullness of sound alluded to is owing to the fact that the wine which forms part of the vibrating system lacks homogeneity, and therefore is incapable of regular vibration.

The most exact experiments have been made by M. Biot, with a series of water-pipes in Paris, which had a length of 3120 feet (951 m.). A bell was hung at the centre of a ring of iron, fastened to the mouth of the tube, so that the vibrations of the ring would affect only the metal of the tube, and the vibrations of the bell only the included air. When the ring and bell were struck simultaneously, an observer, placed at the other end, heard two sounds; the first transmitted by the metal, the second by the air. By noticing the interval of time between the arrival of the two sounds, it was ascertained that the velocity of propagation of sound in cast iron is about 10·4 times that observed in air; that is, 11,609 feet (3538·5 m.). Similar experiments were made by Hassenfratz on the velocity of sound in stone, on the walls of the galleries of the catacombs which underlie Paris, by observing the interval of time between the arrival of a sound transmitted by the stones and of that transmitted by the air of the gallery.

Were the earth and sun connected by an iron bar, nearly three years would elapse before the sound of a blow applied at the sun could reach the earth.

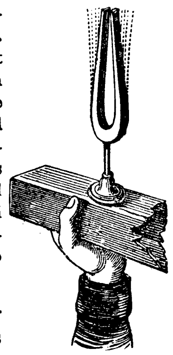
The velocity of the propagation of sound has been determined theoretically by Savart, Chladni, Masson, and Wertheim, from the number of longitudinal and transverse vibrations of the bodies, or their coefficient of elasticity. Chladni found, by the aid of longitudinal vibrations, that in wood, the velocity of sound is from ten to sixteen times greater than in air. In metals, the velocity is more variable, being from four to sixteen times as great as in air.

349. **Interference of sound.**—When two series of sonorous undulations encounter each other in opposite phases of vibration, the phenomena of interference are produced. The undulations will become mutually checked, and if the two sounds are of equal intensity, instead of producing a louder sound, as might be expected, they will altogether destroy each other and produce silence. If, however, *one* of the sounds ceases, the other is heard immediately.

If two sounding bodies were placed in the foci of an ellipse, fig. 269, no sound would be heard, if an ear was placed on any of the lines of interference indicated by the dotted lines, but if one sound was stilled, the other would be heard, or if the ear was placed between the lines of interference, then both sounds would be heard simultaneously, and would be louder than either alone.

The interference of sounds may be shown by means of a common tuning-fork.

276



When in vibration, its branches recede from, and approach each other, as shown by the dotted lines in fig. 276. If the instrument, when vibrating, is placed about a foot from the ear, with the branches equidistant, both sounds will be heard; for the waves of sound combine their effects; but as it is slowly turned around, the sound will grow more and more feeble, until at length a position will be found in which it will be inaudible. For, as the tuning-fork is turned, the waves of sound interfere, and produce partial or total silence. This may also be illustrated by attaching a tuning-fork, lengthwise, to any rotating support. When the fork is vibrating, no sound will be heard so long as it continues to rotate.

350. **Acoustic shadow.**—Persons cut off from observation by a wall, or other obstacle, still hear sounds distinctly, although with a diminished volume. Thus a band of music

in an adjacent house, or neighboring street, is readily followed in the softest melody. Intervening obstacles, therefore, however opaque to light, do not cast perfect shadows to sound. The sound is not entirely cut off, because the obstacle is elastic, and propagates the vibrations it receives in a manner analogous to light passing through a translucent medium.

To a distant observer, the roar of a railway train is instantly hushed on entering a tunnel, and as suddenly renewed on its emergence.

Acoustic shadows are much more distinctly recognised when large masses, as edifices or rocks, intervene; so large as not to enter into vibration.

Although there is not complete silence in the acoustic shadow, still it is analogous to the shadow of light, for there is never complete obscurity in the latter case, even when we take the utmost precaution, for the light spreads behind the obstacles which arrest it.

351. **Distance to which sound may be propagated.**—The distance at which sounds are audible does not admit of precise measurement. In general, it may be stated, that a sound will be heard further, the greater its original intensity, and the denser the medium in which it is propagated. It also depends, greatly, on the delicacy of hearing of different individuals. The intensity of sound, like that of all forces acting in lines, diminishes in the inverse ratio of the squares of the distance of the sounding body. Thus, if the linear dimensions of a theatre be doubled, the volume of the performers' voices at any part of the circumference will be diminished in a fourfold proportion.

That this difference of the agitating impression is the true cause, is shown by confining the air on all sides in a tube. Biot experimented with 2860 feet of the water-pipes of Paris. At this distance the lowest whisper made at one end was accurately heard at the other extremity of the tube.

A powerful human voice in the open air, at the ordinary temperature, is audible at the distance of seven hundred feet. In a frosty air, undisturbed by winds or current, sound is heard at a much greater distance with surprising distinctness. Lieut. Forster, in the third polar expedition of Capt. Parry, held a conversation with a man across the harbor of Port Bowen, a distance of one and a quarter miles. Dr. Young states, on the authority of Derham, that the watchword "all's well" has been distinctly heard from Old to New Gibraltar, a distance of ten miles. The marching of a company of soldiers may be heard, on a still night, at from five hundred and eighty to eight hundred and thirty paces; a squadron of cavalry at foot pace, at seven hundred and fifty paces; trotting, or galloping, one thousand and eighty paces distant. When the air is calm and dry, the report of a musket is audible at eight thousand paces. The sound of the cannonading at Waterloo was heard at Dover.

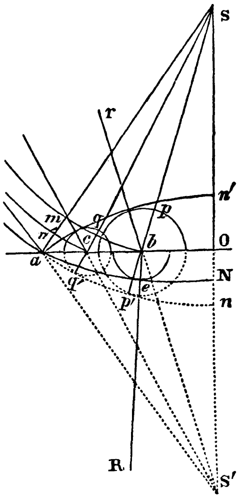
Sounds travel further on the earth's surface than through the atmosphere. Thus it is said, that at the siege of Antwerp in 1832, the cannonading was heard in the mines of Saxony, which are about three hundred and seventy miles distant. The cannonading at the battle of Jena was heard feebly in the open fields near Dresden, 92 miles distant, but in the casemates of the fortifications it was heard with great distinctness. The noise of a sea fight between the English and the Dutch in 1672, was heard at Shrewsbury, a distance of two hundred miles. Sound has been carried by the atmosphere to the distance of three hundred and forty-five miles, as it is asserted, that the very violent explosions of the volcano at St. Vincent's have been heard at Demarara.

Sir Stamford Raffles records however a similar, though much more extraordinary fact. The eruption in Tombers, in Sumbawa, was perhaps the most violent volcanic action recorded; occasional paroxysms were heard, he says, more than nine hundred miles distant.

352. **Reflection of sound.**—When the waves of air on which sound is being borne impinge, in the course of their expansion, on a solid surface, they will be reflected from it, agreeably to the laws regulating the impact of solid bodies (112). Their return is made with equal velocity, and under an equal but opposite angle to that under which they advanced.

Let a spherical wave whose centre is at S, fig. 277, encounter obliquely a plane surface, *a* O, separating two media of different density, a portion of the sound will be reflected as though it emanated from the point S′, as far behind O as S is in front of it, and while *a n* would have been the surface of the wave if the reflecting medium had not intervened, *a n′* will be the wave surface of the reflected sound. Take any portion of the incident wave, as *b m*, let $b p = m a$, when the surface of the incident wave reaches *a*, a wave starting from a new centre of vibration, *b*, will have extended to the circumference described upon *b p*; similarly a new vibration starting from *c*, will describe the circle of which *c q* is the radius. In the same manner, vibrations starting from every point of the line *a* O, will make up the compound wave surface *a n′*, whose centre is at S′, and which is tangent to all the wave surfaces, whose radii are *c q*, *b p*, &c.

277



353. **Echo.**—An echo is the repetition of a sound reflected by a sufficiently distant object, so that the reflected is not confounded with the direct sound.

The ear cannot distinguish one sound from another, unless there is an interval of one-ninth of a second between the arrival of the two sounds. Sounds must, therefore, succeed each other at an interval of one-ninth of a second, in order to be heard distinctly. Now the velocity of sound being eleven hundred and eighteen feet a second, in one-ninth of a second the sound would travel one hundred and twenty-four feet.

To have a perfect echo, therefore, the reflecting surface must be at least sixty-two feet from the sounding body. (62 × 2 = 124.) If we speak a sentence at the distance of sixty-two feet from the reflecting surface, we shall hear the echo of the last syllable only. If twice, thrice, or four times the distance, two, three, or four of the syllables will be echoed, the direct sounds and reflected sound of the other syllables of the sentence, being confounded with each other. If the reflecting surface is at a less distance from the sounding body than sixty-two feet, the direct sound and the reflected sound become confused, so that words and tones cannot be heard distinctly. The original sound will then be prolonged and strengthened; an effect which we express by saying there is resonance. If the distance is comparatively small, as in a common-sized room, the sounds reflected from the walls, the ceiling and the floor, reach the ear at almost exactly the same time as the direct sound, and the apparent power of the voice is strengthened, besides preserving its delicacy. Where, however, the apartment is larger, the direct sound only partially coincides with the reflected sound, and more or less confusion arises. Voices are heard in a remarkably sonorous manner, in large apartments with hard walls, while draperies, hangings, carpets, &c., about a room, smother the sound, because these are bad reflectors. A crowded audience has a similar effect, and increases the difficulty of speaking, by presenting surfaces unfavorable to reflection.

354. **Repeated echoes.**—Repeated or multiplied echoes, are those which repeat the same sound many times. This happens when two obstacles are placed opposite to one another, as parallel walls, for example, which reflect the sound successively.

A striking and beautiful effect of echo is produced, in certain localities, by the Swiss mountaineers, who contrive to sing their *Ranz des Vaches* in such time, that the reflected notes form an agreeable accompaniment to the air itself.

There is a surprising echo between two barns at Belvidere, Allegheny county, N. Y. It repeats eleven times, a word of either one, two, or three syllables; and has been heard to repeat it thirteen times. By placing oneself in the centre, between the two barns, a double echo is heard, one in the direction of each barn, and a monosyllable is thus repeated twenty-two times.

At Ademach, in Bohemia, there is an echo which repeats seven syllables three times; at Woodstock, in England, there is one which repeats a sound seventeen times during the day, and twenty times during the night. An echo in the Villa Smionetta, near Milan, is said to repeat a sharp sound thirty times audibly.

The most celebrated echo among the ancients, was that of the Metelli at Rome, which, according to tradition, was capable of repeating the first line of the Æneid containing fifteen syllables, eight times distinctly.

355. **Change of tone by echo.**—Dr. Chas. G. Page describes an echo in Fairfax county, Virginia, which gives three distinct reflections, the second echo much the most distinct. Twenty notes played upon a flute, are returned with perfect clearness. But the most singular property of this echo is, that some

notes in the scale are not returned in their places, but are supplied with other notes, which are either thirds, fifths, or octaves.

356. **Whispering galleries.**—Whispering galleries are so called, because a low whisper, uttered at one point in them, may be heard distinctly at another and distant point, while it is inaudible in other positions.

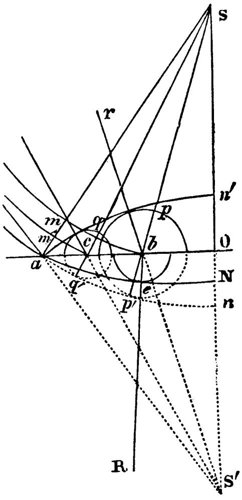
Such galleries are always domed, or of ellipsoidal shape; the best form is that of the ellipsoid of revolution. In such a chamber, whispering in one focus, is very audible to a person at the other focus, because the undulations striking upon the walls, are reflected to the point where the hearer is placed, while in any other position, a feeble sound, or none at all, will be heard, because only a part of the reflected sound will reach the ear at one time. (See fig. 266.)

One of the halls of the museum of antiquities, of the Louvre, at Paris, furnishes an example of such an apartment. In the dome of the Rotunda of the Capitol at Washington, is a fine whispering gallery. The principal room of the Merchants' Exchange, in New York, is of a similar character, and at the same time affords a painful example of confused echoes.

The new Halls of Congress, at Washington, and the Lecture room at the Smithsonian Institution, have been designed with special reference to the best form for public speaking, and to this end an elaborate series of experiments and observations, upon the best proportions and forms of public halls, have been undertaken by Profs. Henry and Bache, by order of the government, the results of which are recorded by the former, in the Proceedings of the American Association for 1856, p. 119, and Smithsonian Report for 1856, p. 221.

357. **Refraction of sound.**—Although sound is reflected by any surface of different density from that in which it originates, the sound also enters the second medium by means of new vibrations originating at the interposed surface.

278

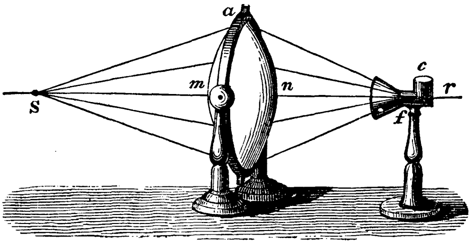


We have seen (345, 347) that the velocity of sound is not the same in different media. Let a sound-wave originating at S, fig. 278, meet with another medium in which sound moves slower than in the first, and let *a* O be the surface of the second medium. Let the difference of velocity be such that while the new vibration, originating at *b*, would advance to *p'*, if the medium were like the first, it can move only to *e* in the new medium: it is evident that if *a n* be the wave surface with the velocity unchanged, *a* N will be the wave surface with the retarded velocity, and the ray S *b* will enter the second medium in the direction *b* R, more nearly perpendicular to the surface *a* O, than its direction in the first medium.

For a fuller discussion of the laws of refraction, see the chapter on Optics.

The phenomena of refraction of sound are in accordance with theory. The researches of Poisson and Green have placed this beyond doubt. Sondhauss has demonstrated the same thing by the following experiment:—A cell, *a m n*, fig. 279, was formed of two films of collodion, united at the edges by a rim of iron. The two films, *m* and *n*, were made spherically convex, and when inflated, the instrument took the form of a lens, like those employed in optical experiments. This apparatus was filled with carbonic acid gas, by means of an opening at *a*. He placed a watch at S, in the axis of the lens that is in the line S *r* which passes through the centres of the two surfaces *m* and *n*. When an observer placed his ear on the opposite side in the axis of the lens, the ticking of the watch was distinctly heard, if at a proper distance from the lens: this distance was less in proportion as the watch was farther removed. If the lens was raised up, the sound ceased to be heard; and the result was the same if the ear was removed from the axis S *r*. Having replaced the watch by an organ-pipe, having an opening like a flute, instead of the ear he employed the bent tube *f c*, having gold-beater's skin extended over the opening *c*, and fine sand placed upon it. When this apparatus occupied the positions in which the ear heard the sound of the watch, the sand was agitated; but on removing the lens the sand remained at rest.

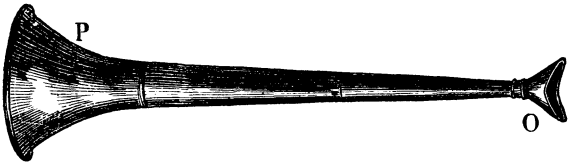
279



To comprehend how these experiments demonstrate the refraction of sound, we must refer to the principles of optics, in which multiplied experiments have rendered the explanation very complete, and easy to be understood.

358. **The speaking-trumpet** is an instrument employed to convey the voice to a great distance. This instrument consists of a conical tube O P, fig. 280, terminated by a bell-shaped extremity, P. At O is a mouth-piece which surrounds the lips without interfering with their movements. The trumpet, it is said by history, was used by Alexander the Great for commanding his army. At the present day it is employed at sea to cause the voice of the commander to be heard above the roar of the winds and waves. On land it is used by firemen.

280

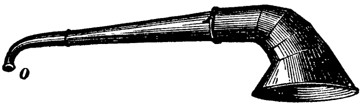


To explain the augmentation of sound by the trumpet, it was formerly supposed that the sound was reflected by the sides of the trumpet, so that the vibrations issued in the direction of the axis of the instrument, and that this effect was also aided by the vibration of the walls of the instrument itself. It was shown by Lambert in 1763 that the vibration of the instrument tended to render articulate sounds confused.

The explanation by reflection of the rays of sound is inadmissible. In reality the form of the extremity has considerable influence, but on the theory of reflection it should be without effect. On the contrary, the conical form should be all important, but Hassenfratz has shown that a cylindrical tube, with a bell-shaped extremity, strengthens the sound as much as a conical tube. In fact, when the interior of the trumpet is lined with cloth, so that the reflection must be very feeble or almost null, the intensity of the sound transmitted remains unchanged We may add that the sound transmitted through a speaking-trumpet is increased, not merely in the direction to which it is pointed, but in every direction, whether the extremity is bell-shaped or otherwise. The efficacy of the trumpet is, therefore, not due to the reflection of sound from its walls, but simply, as stated by Hassenfratz, to the greater intensity of the pulsations produced in the column of confined air which vibrates in unison with the voice at the mouth-piece. A considerable effect is produced by the bell-shaped extremity of the trumpet, but the nature of this influence has not been satisfactorily explained.

359. **Ear-trumpet.**—The hearing-trumpet, fig. 281, intended to assist persons hard of hearing, is in form and application the reverse of the speaking-trumpet, although in principle the same. It consists of a conical tube, turned in any convenient direction, so that the opening, *o*, may enter the ear. The strengthening of the sound by this instrument was formerly attributed to reflection of sonorous waves caused to converge to the ear, and it was sought to obtain the form most favorable to fulfill this condition; thus the cone was replaced by a paraboloid, having its focus at the point *o*. But these different forms have no effect upon the result. Moreover, the nature of the walls, and the condition of the interior surface, whether rough or polished, or lined with cloth, has no effect upon the intensity of the sound. The only essential condition is that the exterior opening should be greater than that which enters the ear. The effect of the ear-trumpet is explained as follows:—The portions of compressed or dilated air, which arrive at the exterior opening, transmit their compression or dilatation to portions of air smaller and smaller, and consequently transmit it with increasing intensity. In this manner the portion of air at *o* receives and transmits to the membrane of the tympanum a compression or dilatation of much greater intensity than in the absence of the instrument. Holding the hand concave behind the ear, as deaf persons are seen to do, concentrates sound in the manner of an ear-trumpet. The form of the external ear in animals favors the collection of sound.

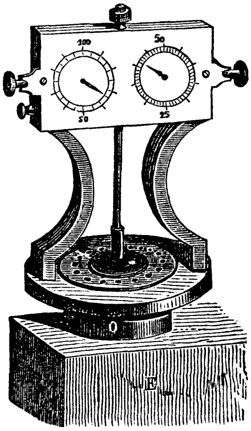
281



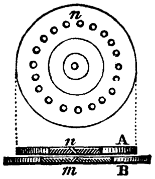
360. **The siren.**—This ingenious instrument was invented by M.

Cagniard de Latour, for the purpose of ascertaining the number of vibrations of a sonorous body, corresponding to any proposed musical sound.

282



283



284

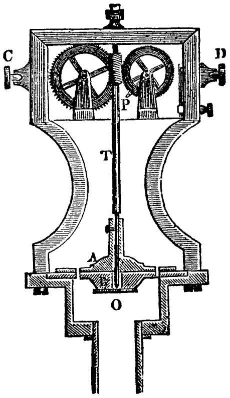


Fig. 282 shows the siren mounted on a wind chest, E, designed to supply a current of air. Figs. 283, 284, show the interior details of the apparatus. The siren is constructed entirely of brass. It consists of a tube, O, about four inches in diameter, terminating in a smooth circular plate, B, fig. 284, which contains, at regular intervals near its circumference, small holes, which are pierced through the plate in an oblique direction. Another plate, A, turning very easily upon its axis, is placed as near as possible to B, without being in contact with it. This plate is pierced with the same number of oblique orifices as those in the plate, B, but inclined in an opposite direction, as shown in fig. 283, *n*, A; *m*, B.

When a current of air arrives from the bellows, it passes through the holes of the first plate, and imparts a rotary movement to the second plate, in the direction *n*, A, fig. 283. As the upper plate revolves, the current of air is alternately cut off, and renewed rapidly by the constantly changing position of the holes. In consequence of this interruption, when the plate A moves with a uniform velocity, a series of puffs of wind will escape at equal intervals of time. These puffs will produce undulations in the air surrounding the instrument, and when the wheel revolves with sufficient rapidity, a musical sound is produced, which increases in acuteness as the velocity of the wheel becomes greater.

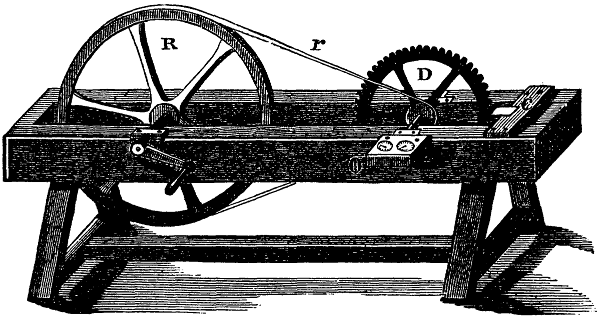
A counter (like that on a gas meter) is connected with the upper plate, by which the number of revolutions is indicated. Pressure upon the buttons, C D, fig. 284, causes the toothed wheels to be set in communication with the endless screw upon the spindle, T. The revolution of these wheels is recorded by the motion of the hands upon the dials in fig. 282. To determine the number of vibrations corresponding to a given sound, a blast of wind is forced from the bellows into the siren, until it gives a corresponding note. The hands on the dials being brought to their respective zeros at the commencement of the experiment, their position, at the end of any known interval, will indicate the number of puffs of air which have escaped from the revolving plate, and will, consequently, determine the number of undulations of the air which correspond to the sound produced.

The siren, with equal velocity, gives the same sound, excepting the timbre, in the different gases, and in water, as it does in air, which proves, that the height of any sound depends on the number of vibrations, and not on the nature of the sonorous body.

361. **Savart's toothed wheel.**—Savart has employed another apparatus to count the number of vibrations corresponding to any proposed pitch.

It consists, fig. 285, of a toothed wheel, D, to be revolved as regularly as possible, by means of the wheel R, and endless band *r*. The toothed wheel, D revolving rapidly, makes the tongue, C, vibrate, producing in the air corresponding undulations, the effect of which is a musical sound. As the tongue is struck on the passage of each tooth, the number of vibrations in a second will correspond with the number of teeth which have struck the tongue in the same time. This is learned from the dial plate, O, which indicates the number of revolutions of the axis, and multiplying this by the number of teeth, we have the whole number of vibrations in a given time. Upon revolving the wheel slowly, we may hear the successive shocks of the teeth against the tongue, and as we increase the velocity, we obtain a more and more elevated sound.

285



362. **Music halls.**—Music halls, theatres, &c., should be so constructed as to convey the sounds that are uttered, throughout the space occupied by the audience, unimpaired by any echo or conflicting sound. On theoretical grounds, the best form for the walls would be that of a parabola. Ornaments, pillars, alcoves, vaulted ceilings, all needless hollow and projecting spaces, break up and destroy the echoes, and resonances. The height of a room for public speaking should be not more than from thirty to thirty-five feet; for at this point, called the limit of perceptibility, the reflection and the voice will blend together well, and thus strengthen the voice of the speaker; if it is higher than

this, the direct sound and the echo will begin to be heard separately, and produce indistinctness.

§ 2. Physical Theory of Music.

363. **Qualities of musical sounds.**—Musical sound is the result of equal atmospheric vibrations, conveying to the ear tones of definite and appreciable pitch. The ear distinguishes three particular qualities in sound. 1. The *tone* or *pitch*, in virtue of which sounds are high or low. 2. The *intensity*, in virtue of which they are loud or soft; and 3d, *quality* or *timbre*, in virtue of which sounds of the same intensity and pitch are relatively distinguishable.

1. **Tone or pitch.**—The tone or pitch of a musical sound is high or low. It depends on the rapidity of the vibratory movement. The more rapid the vibrations are, the more acute will be the sound.

2. **Intensity or loudness.**—The intensity, or force of sound, depends on the amplitude of the oscillations; that is, upon the degree of condensation produced at the middle of the sonorous wave.

A sound may maintain the same pitch, and yet possess greater or less intensity, according as the amplitude of the oscillations varies.

Thus, if we vibrate a tense cord, the intensity or loudness of the tone will vary, as the distance which the vibrating parts pass on each side of the line of rest.

3. **Quality.**—*Quality* is that peculiarity in sound which allows us to distinguish, perfectly, between sounds of the same pitch, and the same intensity.

Thus, the sounds produced by the flute and clarionet are at once distinguishable. The quality of the sound of instruments appears to depend not only on the nature of the sonorous body, and the surrounding bodies set in vibration by it, but also on the form and material of the instrument; and probably, also, on the form of the curve of vibration.

364. **Unison.**—Sounds produced by the same number of vibrations per second, are said to be in unison.

Thus, the siren (360) and Savart's wheel (361) are in unison when we cause them to make the same number of vibrations in the same time.

365. **Melody.—Harmony.**—When the vibrations of a progressive series of single musical sounds bear to each other such simple relations as are readily perceived by the ear, an agreeable impression is produced called *melody*. When two or more sounds, having to each other such simple relations, are produced simultaneously, it is called a *chord*, and a succession of chords, succeeding each other in melodious order, constitutes *harmony*.

If we take a series of sounds, the ratios of whose vibrations are as the following numbers—1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10—we have the notes which will produce a series of chords, which, commencing with the most simple, will gradually become more and more complicated, until the ear can no longer perceive their relations; when this point is reached, they will cease to produce chords and harmony.

In sounds whose vibrations bear to each other the relations of 1 : 2 : 4 : 8 : 16, every vibration expressed by the lower numbers corresponds with similar vibrations in the higher series. The interval between such sounds is very great, and is called an *octave*, because other sounds having simple relations may be so placed between 1 and 2, or 4 and 8, as to form with the two extremes a series of eight sounds having agreeable relations to each other.

286

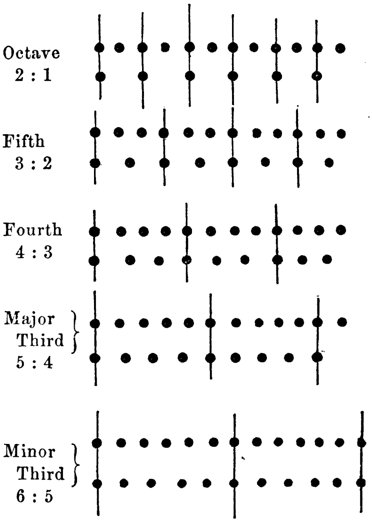


Fig. 286 represents the relations of such tones as produce the most pleasing effects when sounded together. The dots represent vibrations; and those which occur simultaneously, and therefore increase each others' powers, are connected by vertical lines. On the left of the figure are the names applied to these intervals, as explained in section 371.

366. **Musical scale.—Gamut.**—The tones forming a melodious series between any two adjacent sounds which are as one to two, are called the *musical scale* or gamut.

It is generally supposed that the musical scale was invented by Guido of Arezzio, or according to others that it was an improvement upon the Grecian scale, and called the *gamut* from the Greek letter *gamma*, as an acknowledgment of the assistance he derived from that nation.

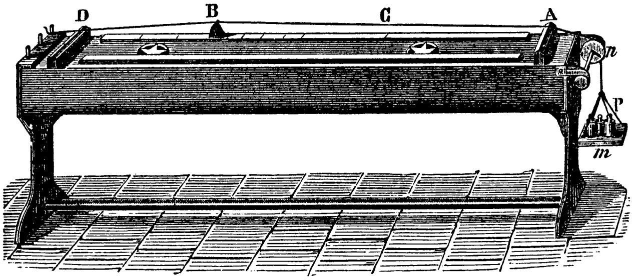
The sounds which compose the musical scale or gamut, are the alphabet of music. They are designated, in English, by the letters C, D, E, F, G, A, B. In French and Italian, by the words ut, or do, re, mi, fa, sol, la, si.

We may also represent the notes of the gamut in numbers. In order to find the relation which exists between the fundamental note, C, or do, and the other notes, the sonometer, or monochord, fig. 287, is employed.

367. **The sonometer or monochord.**—This instrument is used to study the transverse vibrations of cords; and by it we ascertain the relation between the different notes of the musical scale, and, with the aid of the siren (360), the number of vibrations by which they are respectively produced.

Above a case of thin wood, a cord, or metallic wire, A D, is stretched over the pulley *n*, by the weights P, on the pan *m*, fig. 287. The movable bridge, B, can be placed at any desired point; and for convenience of adjustment, the scale is marked off beneath the wire, commencing with C. The string, A D, when vibrating its whole length, produces the note C; in order to produce the note D, the movable bridge, B, must be advanced toward the fixed bridge D, until the length of the cord is but eight-ninths of that which produces the note C. Proceeding in the same manner for the other notes, it will be found, that the length of the cord corresponding to each note is represented by the following fractions:

287



Notes, ...	C	D	E	F	G	A	B	C′
Relative length of cord, . .	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

Continuing to move the bridge on the sonometer, it will be found, that the eighth sound, the octave, is produced by a length of cord half that of the fundamental sound. Upon this note, an octave higher than the fundamental note, we may construct a scale, each note of which is produced by the vibration of a cord half as long as the same note in the preceding gamut. In the same manner we may have also a third and a fourth scale.

368. **Relative number of vibrations corresponding to each note.**—In order to ascertain the relative number of vibrations corresponding to each note in the same time, it is sufficient to invert the fractions of the preceding table. For by the principles already established (309), the number of vibrations is in inverse ratio of the length of the string. Representing, therefore, the number of vibrations corresponding to the fundamental note C, by 1, proceeding as above, we form the following table:

Notes, ...	C	D	E	F	G	A	B	C′
Relative number of vibrations,	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

Which indicates, that in producing the note D, nine vibrations are made in the same time that eight are made by the fundamental note C. So, when the note E is sounded, five vibrations are made for four of C; for B, fifteen to eight of C, &c.

369. **Absolute number of vibrations corresponding to each note.**—By setting the siren, or Savart's wheel, in unison with a given sound, we obtain the absolute number of vibrations corresponding to it. If we set the siren in unison with the fundamental C, in order to obtain the number of vibrations corresponding to the other notes, as D, we have but to multiply it by the fraction $\frac{9}{8}$, &c. But the fundamental C varies with the nature, length, and tension of the cord of the sonometer, and therefore the number of vibrations may be represented by an infinite variety of numbers, corresponding to the different scales. The notes of the scale whose gamut corresponds to the gravest sound of the bass, are indicated by 1. To notes of gamuts more elevated, are affixed the indices 2, 3, &c.; to graver notes are affixed the indices — 1, — 2, &c. The number of simple vibrations corresponding to the note C, is 128, and in order to obtain the number of vibrations corresponding to the other notes, we have but to multiply this number by the fractions indicated in (368), which gives the following table:

Notes, ...	C	D	E	F	G	A	B
Absolute number of simple vibrations, ...	128	144	160	170$\frac{2}{3}$	192	213$\frac{1}{3}$	240

The absolute number of vibrations for the superior gamut, is obtained by multiplying the numbers in the table successively, by 2, by 3, by 4, &c.; for the lower gamut, we divide the same numbers by 2, by 4, &c. Thus, the number of vibrations of A3 is $214 \times 4 = 856$ simple vibrations, or 428 complete vibrations.*

It must, however, be stated, that there is a slight difference in the actual number of vibrations producing a particular note as performed in different cities. Thus, A3 of the pitch adopted at different orchestras, which by the above table should be produced by 426$\frac{2}{3}$ vibrations, varies as follows:

Orchestra of Berlin Opera, ...	437·32
Opera Comique, Paris, ...	427·61
Academie de la Musique, Paris, ...	431·34
Italian Opera, in 1855, ...	449

In piano-fortes, which, for private purposes, are generally tuned below concert pitch, A3 is produced by about 420 vibrations in a second.

There has been a curious progressive elevation of the diapason (pitch) of orchestras, since the time of Louis XIV., when the *la* in the orchestra was (according to Sauveur) 810 simple vibrations (= 405 complete vibrations) per second; the number at the grand opera is now 898, or nearly a tone higher. This rise has taken place mainly in the present century—being a semitone since 1823. The causes of this change (which is still in progress) are doubtless owing the process adopted for preserving and transmitting the true pitch of the undamental note. The final tuning of the diapason is done by the file, and ling a diapason heats it; at the moment it is in tune with the standard it is thus heated, and when it afterwards cools the tone rises. When this second diapason is used for tuning, there is another similar rise, and so it continues. This gradual elevation of pitch becomes quite sensible after the lapse of one or two generations. (Am. Jour. Sci. [2] xx. 262.)

---

* See Appendix, p. 668.

370. **Length of sonorous waves.**—It is easy to ascertain the length of a sonorous vibration, if we know the number of vibrations made in a second. For, as sound travels at the rate of 1118 feet per second, if but one vibration is made in that time, the length of the wave must be 1118 feet; if two vibrations, the length of each must be half of 1118, = 559 feet, &c.

C corresponds, as we have seen, to 128 vibrations per second; the length of its waves is, therefore, (1118 ÷ 128) = 8·73 feet.

The following table indicates the length of the waves corresponding to the C of successive scales :

	Length of waves in feet.	Number of vibrations in a second.
C_{-3}	70·	16
C_{-2}	35·	32
C_{-1}	17·5	64
C_1	8·73	128
C_2	4·375	256
C_3	2·187	512
C_4	1·093	1024

371. **Interval.**—Interval is the numerical relation existing between the number of vibrations made in the same time by two sounds, or it is that which indicates how much one sound is higher than another.

Musical intervals are named from the position of the higher note counting upwards from the lower.

C, D, E, F, G, A, B, C′, D′, E′, F′, G′, A′, B′, C″.
1, $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$ $\frac{15}{8}$, 2, $\frac{18}{8}$, $\frac{10}{4}$, $\frac{8}{3}$, $\frac{6}{2}$, $\frac{10}{3}$, $\frac{30}{8}$, 4.
1st, 2d, 3d, 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, 12th, 13th, 14th, 15th.
$\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{16}{15}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{16}{15}$.

In the above table are given, 1st, the letters by which successive notes are designated; 2d, the relative numbers of their vibrations as compared with the lowest note; 3d, the names of the notes as compared with the first; and lastly, the intervals obtained by dividing each note by that which immediately precedes it. It will be seen that there are but three different intervals between successive notes of the scale; viz. $\frac{9}{8}$, which (being the largest interval found in the scale) is called a *major tone,* $\frac{10}{9}$ called a *minor tone,* and $\frac{16}{15}$ which is called a *semitone,* though it is greater than one-half of the interval of either of the other tones. This last is also called a *diatonic semitone,* to distinguish it from other divisions made for particular purposes. The difference between a major tone and a minor tone is $\frac{1}{81}$ of a major tone, and is called a *comma.*

Comparing the notes of the natural scale two and two, we obtain a variety of intervals named as in the following :—

TABLE OF MUSICAL INTERVALS.

CD = FG = AB = $\frac{9}{8}$; major tone.

DE = GA = $\frac{10}{9} = \frac{80}{81}$ of $\frac{9}{8}$; minor tone.

EF = BC = $\frac{16}{15} = \frac{24}{25}$ of $\frac{80}{81}$ of $\frac{9}{8}$; diatonic semi tone.

CE = FA = GB = $\frac{5}{4}$; major third.

EG = AC′ = BD′ = $\frac{6}{5} = \frac{24}{25}$ of $\frac{5}{4}$; minor third.

DF = $\frac{6}{5}$ of $\frac{80}{81} = \frac{80}{81}$ of $\frac{24}{25}$ of $\frac{5}{4}$; $\frac{80}{81}$ of a minor third.

CF = DG = EA = GC′ = $\frac{4}{3}$; fourth.

AD′ = $\frac{4}{3} = \frac{81}{80}$ of $\frac{4}{3}$; sharp fourth.

FB = $\frac{45}{32} = \frac{25}{24}$ of $\frac{81}{80}$ of $\frac{4}{3}$; $\frac{135}{128}$ of perfect fourth.

CG = EB = FC′ = GD′ = AE′ = $\frac{3}{2}$; fifth.

DA = $\frac{40}{27} = \frac{80}{81}$ of $\frac{3}{2}$; $\frac{80}{81}$ of a perfect fifth.

BF′ = $\frac{64}{45}$; an inharmonious interval.

CA = DB = FD′ = GE′ = $\frac{5}{3}$; sixth.

AF′ = BG′ = $\frac{8}{5} = \frac{24}{25}$ of $\frac{5}{3}$; minor sixth.

FD′ = $\frac{54}{32}$; an inharmonious interval.

CB = FE′ = $\frac{15}{8}$; seventh, an inharmonious interval.

DC′ = GF′ = BA′ = $\frac{16}{9}$; flattened seventh, $\frac{64}{63}$ of $\frac{7}{4}$; decidedly more harmonious than the seventh.*

ED′ = AG′ = $\frac{9}{5} = \frac{24}{25}$ of $\frac{15}{8}$; minor seventh.

CC′ = $\frac{2}{1}$; octave.

372. **Compound chords.—Perfect concord.**—It is evidently easy to take three or four notes whose vibrations have simple relations to each other, and which taken two and two produce a sensation of harmony. Such combinations are called compound chords.

If we take the three notes C, E, G, whose vibrations are to each other as the numbers 4, 5, 6, compared two and two they give the relations $\frac{5}{4}$, $\frac{6}{5}$, $\frac{3}{2}$, which constitute by their union three harmonious intervals called the *perfect major accord.* If we take the three notes E, G, B, which compared two and two give the relations $\frac{6}{5}$, $\frac{5}{4}$, $\frac{3}{2}$, we find it differs from the preceding only in the order of the intervals. This series is called the *perfect minor accord.*

373. **A new musical scale.**—By examining the preceding pages it will be seen that the relative number of vibrations, and the intervals between different notes in the common musical scale, are made up entirely of combinations formed from the three prime numbers 2, 3, 5. and it has been generally stated that relations founded upon the prime

---

* The ratio $\frac{7}{4}$ is claimed to belong to natural music; see section (373).

7 were too complicated to be appreciated by the ear, or to be executed either by the voice or by instruments. But the use of the prime seventh in music has been recently pointed out by H. W. Poole, Esq., of South Danvers, Mass.*

Multiplying the ratios of the ordinary scale by 48, we have the:—

TRIPLE DIATONIC SCALE.

(With Common Chord on C, G, and F.)

C, D, E, F, G, A, B, C′.
48, 54, 60, 64, 72, 80, 90, 96.
$\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{16}{15}$.

By introducing the prime 7, Mr. Poole obtains a series which he calls the:—

DOUBLE DIATONIC SCALE.

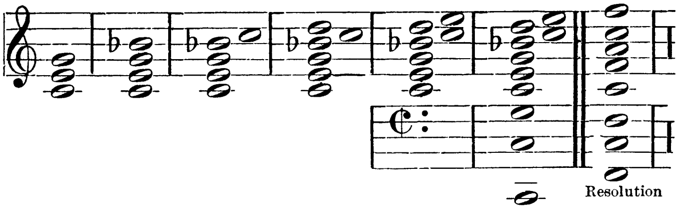
(With Common Chord on C, and Chord of 7 and 9 on G.)

C, D, E, (F), G, (A), B, C′.
48, 54, 60, 63, 72, 81, 90, 96.
$\frac{9}{8}$, $\frac{10}{9}$, $\frac{21}{20}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$.

In defence of this introduction of the prime 7 in musical composition, it is claimed that it is required to fill up the series 1 to 10 (365), as all the other numbers up to ten are universally admitted. It is further claimed that it is of frequent use by such masters as Hayden, Handel, in the "Dead March of Saul," Mozart, in the "O dolce Concento," and that it abounds in the compositions of Rossini, and occurs in all music, especially that which is popular or generally in favor.

It may also be mentioned that the Chinese, in their musical scale, employ the flat sevenths instead of the notes in use with us, which gives G to B as 72 to 84 or 6 to 7, and B to C′ as 84 to 96 or 7 to 8, the very harmony contended for.

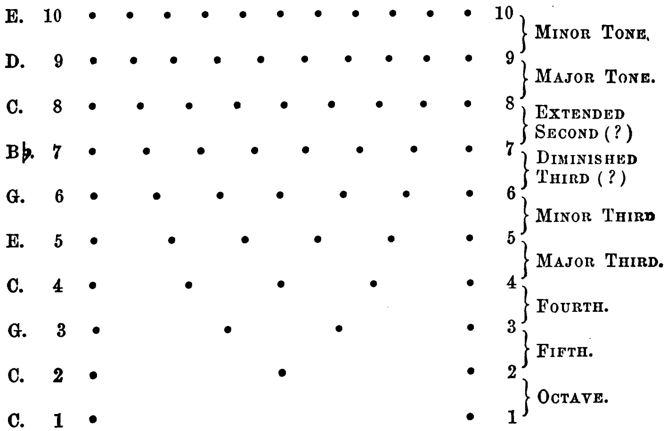
The following example of harmonies, rising through the entire series from 1 to 10, is taken from the essay of Mr. Poole.



Here we have a series of harmonies beginning with the common chord C, E, G, 4, 5, 6, and rising in the last example to the full chord of the 10th, all of which can be appreciated, and are capable of giving a pleasing effect on an instrument of perfect intonation. The full chord of the 10th contains the following series of vibrations and intervals:—

---

* Am. Jour. Sci. [2], IX., p. 68; also Math. Monthly, II., p. 16.



374. **Transposition.**—Any tone of the common scale, or any pitch whatever, may be taken as the basis of another similar scale, provided the same relative intervals are preserved between the successive notes. Such change is called *transposition* of the scale. If such a change of pitch is made on an instrument tuned to play the natural scale, additional notes must be provided in one or more of the intervals already described. Such additional notes are called *sharps* (♯) or *flats* (♭), according as the tone corresponding to any given note is raised or lowered. When new notes are interpolated in every major and minor tone of the natural scale, there are obtained twelve intervals in the octave, and the series thus formed is called the *chromatic scale.*

When the diatonic semitone, $\frac{16}{15}$, is taken from the major tone, $\frac{9}{8}$, the remaining interval is called a *chromatic semitone.* When the diatonic semitone is taken from the minor tone, $\frac{10}{9}$, the interval which remains is called the *grave chromatic semitone.*

375. **Temperament.**—In transposing the scale, so as to commence on any note of the natural gamut, it is supposed, in theory, that every note may be raised or lowered through an interval of a diatonic semitone, $\frac{16}{15}$. This would involve the addition of an inconvenient number of new notes; therefore, in the construction of musical instruments, it is assumed that such notes as C♯ and D♭ are identical, though they are not strictly the same, and are not played alike on a violin or harp, in the hands of a skillful performer.

Temperament is a device by which the multiplication of notes beyond convenient limits is avoided. For practical purposes, organs, pianofortes, and other instruments, are so tuned as to divide the octave into 12 equal intervals, called *chromatic semitones,* of equal temperament.

In this system, all the musical intervals employed, except the octaves, differ more or less from their true value, as given by theory, and as demanded by the cultivated ear.

	True value.	Value in equal temperament.
Minor semitone,	$\frac{25}{24} = 1{\cdot}042$,	$\sqrt[12]{2} = 1.060.$
Major semitone,	$\frac{16}{15} = 1{\cdot}067$,	
Minor third,	$\frac{6}{5} = 1{\cdot}200$,	$\sqrt[12]{2^3} = 1{\cdot}189.$
Major third,	$\frac{5}{4} = 1{\cdot}250$,	$\sqrt[12]{2^4} = 1{\cdot}260$
Fifth,	$\frac{3}{2} = 1{\cdot}500$,	$\sqrt[12]{2^7} = 1{\cdot}498.$

It is here seen that the minor semitones and major thirds are all too sharp, while the major semitone, minor third, and the fifths are all too flat.

Messrs. H. W. Poole and J. Alley have invented an organ, on which every musical interval can be correctly given without tempering; and the perfect musical scale can be performed in as many different keys as may be desired. (Am. Jour. Sci. [2], Vol. IX., p. 68.)

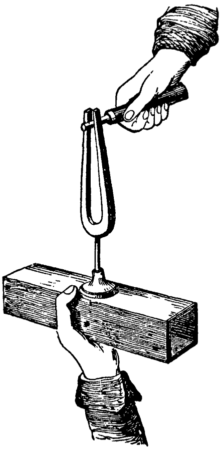
The subject of temperament in all its relations is too extensive to be treated in this work, and we must refer the reader to musical treatises for further information.

376. **Beating.**—When two sounds are produced at the same time, which are not in unison, alternations of strength and feebleness are heard, which succeed each other at regular intervals.

This phenomenon, called beating, discovered by Savart, is easily explained. Supposing that the number of vibrations of the two sounds was 30 and 31; after 30 vibrations of the first, and 31 of the second, there would be coincidence, and in consequence, beating, while at any other moment, the sonorous waves not being superimposed, the effect would be less. If the beatings are near to each other, there is produced a continuous sound, which is graver than the two sounds which compose it, since it comes from a single vibration, while the other sounds are made of 30 and 31 vibrations.

The nearer the vibrations approach to exact unison, the longer is the interval between the beats. When the unison is complete, then no beats are heard; when it is very defective, they produce the effect of an unpleasant rattle.

288



377. **Diapason, tuning-fork.**—The diapason is a familiar instrument, with which we may produce, at will, an invariable note; its use regulates the tone of musical instruments. It is formed from a bar of steel, curved, as seen in fig. 288. It is often sounded by drawing through it

a smooth rod of steel large enough to spring open the limbs, and its vibrations are greatly strengthened to the ear by mounting it, as in the figure, upon a box of thin wood, open at one end. A diapason, giving C 3, or 256 vibrations in a second, produces a sound comparable with that from an organ tube.

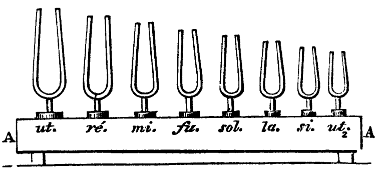
The diapason is ordinarily formed to produce A 3, corresponding to 428 vibrations in a second.

The whole diatonic scale is thus conveniently constructed, by a series of diapasons, arranged as in fig. 289, upon a sounding-box, A A.

378. **Sensibility of the ear.**—According to Savart, the most grave note the ear is capable of appreciating, is produced by from seven to eight complete vibrations per second. When a less number is made, the vibrations are heard as distinct and successive sounds.

The most acute musical sound recognised, was produced by 24,000 complete vibrations per second. Savart maintains, however, that this is not the extreme limit of the sensibility of the ear, which is capable of wonderful training. The same physicist has also demonstrated, that two complete vibrations are sufficient to enable the ear to determine the rapidity of these vibrations; that is, the height of the sound produced. If his wheel made 24,000 vibrations in a second, the two require but $\frac{1}{12,000}$th of a second. The ear may, therefore, compare sounds which act only during this wonderfully brief interval. The limit of perceptible sound depends on the amplitude of the vibrations. By enlarging the dimensions of Savart's toothed wheel, and increasing the distances between the teeth, more rapid vibrations can be heard. Despretz was enabled to recognise sounds made by 36,500 complete vibrations per second.

289



Many insects produce sounds so acute as to baffle the human ear to distinguish them; and naturalists assert, that there are many sounds in nature too acute for human ears, which are yet perfectly appreciated by the animals to which they are notes of warning, or calls of attraction.

The natural *la* is said to be heard by rapidly moving the head; owing to the motion of the small bones of the ear. See § 393.

§ 3. Vibration of Air contained in Tubes.

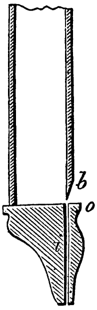
379. **Sonorous tubes.—Mode of vibrating.**—In wind instruments, with walls of suitable thickness, the column of air contained in the tubes alone, enters into vibration.

The material of the tube has no influence upon the pitch, but affects the quality (363) in a striking and important manner. The pitch of the sound produced, depends partly on the size and situation of the embouchure; still more on the manner of imparting the first movement to the air, and partly also on varying the length of the tube containing the column of air. The difference in the quality of the tones produced by pipes of different materials is most probably owing to a very feeble vibration of the materials themselves.

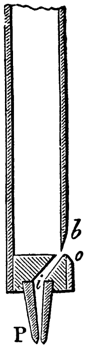
Sonorous vibrations are produced in tubes in a number of ways. 1. By blowing obliquely into the open end of a tube, as in the Pandean pipe. 2. By directing a current of air into an embouchure, or near the closed end of the tube. These tubes are called mouth-pipes. 3. By thin vibrating laminæ of metal, or of wood, called reeds, or by the vibration of the lips, acting as reeds. 4. By a small flame of hydrogen gas.

380. **Mouth-pipes.**—Fig. 290 represents the embouchure of an organ tube, fig. 291, that of a whistle or flageolet. In these two figures the air is introduced by the opening, *i* (called the *lumière*); *b o* is the mouth, of which the upper lip is beveled. The foot, P, fig. 291, connects the pipe with a wind-chest. When a rapid current of air passes through the inlet, it encounters the edge of the upper lip, which partially obstructs it, causing a shock, so that the air passes through *b o* in an intermittent manner. These pulsations are transmitted to the air in the tube, making it vibrate, and producing a sound.

290



291



In order to have a pure sound, there must exist a certain relation between the dimensions of the lips, the opening of the mouth, and the size of the lumière. Again, the length of the tube must bear a certain ratio to its diameter. In those wind instruments, like the flute, flageolet, &c., in which various notes are produced by the opening and closing of holes in their sides by means of fingers or keys, there is a virtual variation in the length of the tube, which determines the pitch of the various notes produced. The number of vibrations depends upon the dimensions of the tube and the velocity of the current of air.

381. **Reed-pipes.**—A reed is an elastic plate of metal, or of wood, attached to an opening in such a manner, that a current of air, passing into the opening, causes the plate to vibrate. This vibration is propagated to the surrounding air. Reeds are found in hautboys, bassoons, clarionets, trumpets, and in the Jews-harp, which is the most simple instrument of this species.

Fig. 292 represents a reed pipe, mounted on the box of a bellows, Q. A glass, E, in one of the walls of the tube, allows the vibrations of the reed to be seen The case, H, serves to strengthen the sound. Fig. 293 represents the reed separated from the tube. It is composed of a rectangular case of wood, closed at its lower end, and open at the top, at a point *o*. A plate of copper, *c c*, contains a longitudinal opening, designed to allow the passage of the air from the tube, M N, through the orifice *o*. An elastic plate, *i*, almost closing the aperture, is confined at its upper end. The sliding rod, *r*, curved at its lower end, permits the regulation of the pitch, by alterations in the length of the vibrating part of the plate.

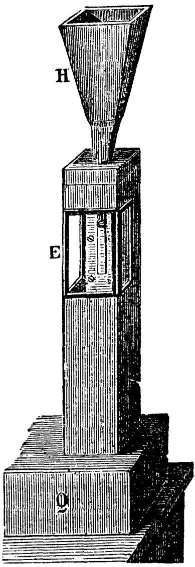
When a current of air passes in through the foot, P, the reed vibrates, alternately opening and closing the aperature. The vibrations being very rapid, the sound produced varies in pitch with the velocity of the current. This sound is transmitted to the exterior air through the opening *o*.

In this kind of reed, the vibrating plate passes through the aperture, without its walls, and the tone is remarkably pure and free from any harshness. The quality of the tone of a reed-pipe depends much on its figure, also upon the construction of the reed, and the material of which it is composed.

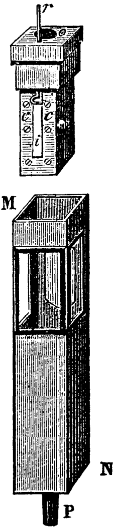
In the French horn, the trumpet, and other similar instruments, the sound is produced by the vibration of the lips of the performer, acting like reeds.

382. Gas jet.—A jet of hydrogen, burned within a tall tube of glass, or other material, occasions, if accurately adjusted, a musical note.

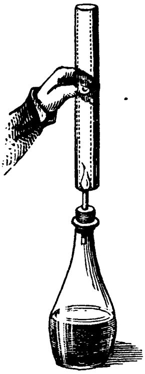
292



293



294



A simple form of this arrangement is seen in fig. 294, where hydrogen, generated by the action of dilute sulphuric acid on zinc in the bottle, is burned from the narrow glass tube, within one of larger dimensions. It is better to take the gas from a reservoir, or gas jet, with a key interposed, to regulate the volume of the flame. The cause of the vibrations and sound in this case is to be found in the successive explosions of small portions of free gas, mingled with common air. The heat occasions a powerful ascending current of air, momentarily extinguishing the flame, and at the same instant permitting the mixture of the atmospheric oxygen with a portion of inflammable gas. The expiring flame kindles this explosive mixture, and relights the jet. As these successive phenomena occur with great rapidity, and at regular intervals, the necessary consequence is a musical note

For a full discussion of the phenomena of singing flames, see a memoir by Prof. W. B. Rogers, Am. Jour. Sci. [2] xxvi. 1.

383. Musical instruments.—The principles already explained in this chapter, will illustrate the peculiar power of the several sorts of musical instruments in common use. It is inconsistent with our limited space to describe, in detail, these several instruments. Such details belong to a special treatise on music. Musical instruments are grouped chiefly, under the heads of wind, and stringed instruments, and those like the drum, in which a membrane is the source of vibration

Wind instruments are sounded, either with an embouchure like a flute, or with reeds. The first division includes the flute, pipe, flageolet, &c., and in the second are found the clarionet, bassoon, horns, trombones, &c. The organ also is a wind instrument, and is, incomparably, the grandest of all musical instruments, as its power and majesty is without parallel in instrumental combinations.

Stringed instruments are all compound instruments. The sounds produced by the vibration of the cords are strengthened by elastic plates of wood, and enclosed portions of air, to which the cords communicate their own vibrations. They are vibrated either by a bow, as in the violin, by twanging, as in the harp, or by percussion, as in the piano.

Drums are of three sorts; the common regimental or snare drum, which is a cylinder of brass, covered with membrane, and beaten on one end only, the bass, or double drum, of much larger dimensions, and beaten on both heads; and thirdly, the kettle drum, a hemispherical vessel of copper, covered with vellum, and supported on a tripod. This drum has an opening in the metallic case, to equalize the vibrations. They all depend, of course, upon the vibration of tense membranes (318).

384. **Vibration of air in tubes.—Laws of Bernoulli.**—The following laws of the vibration of air contained in tubes, were discovered by Daniel Bernoulli, a celebrated geometrician who died in 1782. We may divide tubes into two classes.

a. Tubes of which the extremity opposite the mouth is closed.

b. Tubes open at both extremities.

a. Tubes of which the extremity opposite the mouth is closed.

1st. The same tube may produce different sounds, the number of vibrations in which will be to each other as the odd numbers, 1, 3, 5, 7, &c.

2d. In tubes of unequal length, sounds of the same order correspond to the number of vibrations, which are in inverse ratio of the length of the tubes.

3d. The column of air, vibrating in a tube, is divided into equal parts, which vibrate separately and in unison. The open orifice being always in the middle of a vibrating part, the length of a vibrating part is equal to the length of a wave corresponding to the sound produced.

b. Tubes open at both extremities.

The laws for tubes open at both extremities, are the same as the preceding, excepting that the sounds produced are represented by the series of natural numbers, 1, 2, 3, 4, &c.; and that the extremities of the tubes are in the middle of a vibrating part. Again, the fundamental sound of a tube open at both extremities, is always the acute octave of the same sound in a tube closed at one extremity.

Demonstration.—If the column of air contained in a tube vibrates as a single wave, the number of vibrations will evidently be equal to the velocity of sound, represented by V, divided by the length of the tube, L, or by the quotient $\frac{V}{L}$. If the column of air is divided into a number of segments, each vibrating separately, let l represent the length of one of these segments, and the number of vibrations per second will evidently be $= \frac{V}{l}$.

(a) If the pipe containing the column of air is stopped at both ends, there will be a node at each end. Let n represent the number of nodes including the two ends; the number of vibrating segments will be $n - 1$, the length of each vibrating segment will be $l = \frac{L}{n - 1}$, and the number of vibrations per second will be $\frac{V}{l} = (n - 1).\frac{V}{L}$, in which we may substitute for n any number greater than unity, giving the series of possible vibrations $1.\frac{V}{L}$, $2\frac{V}{L}$, $3\frac{V}{L}$, &c.

(b) If the tube is closed at one end, that end must be regarded as a node, and the open end of the tube as the middle of a vibrating segment. Therefore, $2(n - 1) + 1 = 2n - 1$ will be the number of half segments, and $\frac{2n - 1}{2}$ will be the entire number of vibrating segments contained in the length, L, and the number of vibrations per second will become $\frac{2n - 1}{2}.\frac{V}{L}$.

Substituting for n the integers 1, 2, 3, &c., we obtain the following series of vibrations for the different tones of the pipe:—

$$\tfrac{1}{2}.\frac{V}{L};\ \tfrac{3}{2}.\frac{V}{L};\ \tfrac{5}{2}.\frac{V}{L},\ \&c.$$

If the pipe is open at both ends, as before let n be the number of nodes. The number of complete ventral segments will be $n - 1$, and there will be a half segment at each end, making n segments in the length L. The length of a complete segment will therefore be $\frac{L}{n}$, and the number of vibrations per second is $n\frac{V}{L}$, making $n = 1, 2, 3$, &c. We obtain the following series of vibrations corresponding to different tones of the pipe:—

$$1.\frac{V}{L};\ 2\frac{V}{L};\ 3.\frac{V}{L},\ \&c.$$

The series of vibrations for the tones produced by a pipe will be as follows:—

In a pipe open at both ends, or closed at both ends, . . . 1, 2, 3, 4, 5, &c.
In a pipe open at only one end, ... $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$. "

In the latter case the fundamental note is an octave lower than when both ends of the tube are open, or when both ends are closed. The particular tone that a pipe will produce in either of these series depends on the strength of the blast.

385. **Construction of musical instruments.**—Practically, the end of the tube where sound is formed is never entirely closed, and usually the embouchure is on one side of the tube. The laws of Bernoulli adapted to these conditions give for the construction of musical instruments the following practical rules:—

1. The length of the tube is equal to the quotient of the velocity of

sound divided by the number of vibrations and diminished by twice the breadth of the tube.

2. The number of vibrations is equal to the quotient of the velocity of sound, divided by the length of the tube, increased by twice its breadth.

3. The length of a cylindrical organ tube, flattened at the embochure, is equal to the quotient of the velocity of sound divided by the number of vibrations, and diminished by five-thirds the diameter of the tube. (*Comptes Rendus*, T. L. 1860, p. 176.)

386. **Vibrating dams.**—Illustrations of the vibration of air in tubes are often found at waterfalls. The horizontal column of air enclosed behind the descending sheet of water is rarefied by friction of the water which carries away a portion of the air, and the external air rushing in at the sides is thrown into vibrations, which are often so perceptible as to endanger the safety of persons approaching too near the cataract.

At the falls of Holyoke, Mass., the descending sheet of water has a breadth of 1008 feet, with a fall of 30 feet, and varying from 5 feet to 3 inches thick. The pulsations of the air rushing in behind the waterfall vary from 82 to 258 per minute.

Says Professor Snell, who has described these phenomena at length,* "At one time, when I witnessed the comparatively slow oscillations of 82 per minute, I was surprised by the great strength of the current of air, as it rushed into the opening at the end of the dam. I could not venture within the passage through the pier, lest I should be swept in behind the sheet; nor could I stand at the entrance of the arch without bracing myself, by placing both hands on the corners." These vibrations are shown by Professor Snell to follow the laws of Bernoulli for the vibration of air in tubes.

§ 4. Vocal and Auditory Apparatus.

I. OF VOICE AND SPEECH.

387. **Voice and speech.**—In nearly all the air-breathing vertebrate animals, there are arrangements for the production of sound, or *voice*, in some part of the respiratory apparatus. In many animals the sound admits of being variously modified and altered during and after its production; and in man one of the results of such modification is *speech*.

388. **The vocal apparatus of man** consists of the thorax, the trachea, the larynx, the pharynx, the mouth, and the nose, with their appendages.

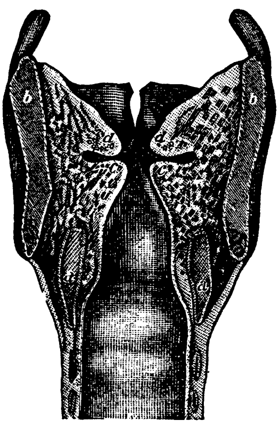
The *thorax*, by the aid of the diaphragm and the intercostal muscles acting on the lungs within, alternately compressing and dilating them, performs the part of the bellows producing a current of air for the production of sound. The *trachea*, or the *windpipe*, extending from the larynx to the lungs, acts as a tube to convey the air from the lungs to the organs more immediately concerned in the production of voice and speech. The *larynx*, which may be considered as the musical organ of the voice, corresponds to the mouth-piece or that part of the organ tube which gives the peculiar character to the sound. The *pharynx* is a large cavity above the larynx, which, by the varying form and tension of its walls, modifies the tones of the voice. The mouth and nasal passages correspond to the upper part of an organ tube from which the vibrations of the column of air are thrown into the atmosphere.

---

* Am. Jour. Sci. [2] Vol. XXVIII. p. 228.

The larynx.—This organ is composed essentially of four cartilages, called respectively the thyroid, cricoid, and the two arytenoid cartilages. The cavity of the larynx is nearly closed by two pairs of membranes, called the vocal cords, the opening between which is called the glottis.

295

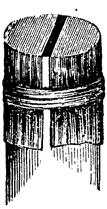


In fig. 295, showing a vertical section through the larynx and glottis, the position of the thyroid and cricoid cartilages is seen in *b b, a a*. The thyroid cartilage consists of two flat plates whose upper edges are curved somewhat like the letter *S,* and forms a prominent projection on the throat of man, visible exteriorly, and vulgarly called Adam's Apple. The cricoid cartilage, *a a*, lies below the thyroid cartilage, and is in fact only an enlargement of one of the cartilaginous rings forming the wind-pipe. The position of the arytenoid cartilages is over the cricoid cartilages. These several cartilages, with the hyoid bone, serve as points of attachment for the muscles forming the proper vocal apparatus. The two chief tongues of the glottis, or proper vocal cords, *c c,* extend from the thyroid cartilage to the arytenoid cartilages, and leave between them a fissure, the *rima vocalis,* or glottis, shown better in fig. 297. This fissure leads on one side into the trachea, which lies below the larynx, and on the other into the cavity of the larynx itself, which communicates with the cavities of the mouth and nose.

Besides the proper vocal cords, there are the ventricular cords, *d d,* situated a small distance above them in the epiglottis; they are less developed than the first. The ventricular cords have no part in the *production* of vocal sounds, which, however, they doubtless serve to modify and strengthen in the same way as the conical case surmounting an organ tube.

Between these two sets of cords are seen the deep depressions, called the ventricles of the glottis.

296



The voice is produced in the larynx; for if an opening is pierced into the trachea, below the larynx, the air escapes by this opening, and it is not possible to produce any vocal sound. If an opening is made above the larynx, it does not prevent the formation of sound. Magendie mentions the case of a man who had a fistulous opening in his trachea, and who could not speak unless he closed it, or wore a tight cravat.

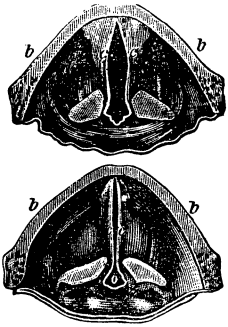
The glottis.—A clear idea may be obtained of the form and action of the glottis, by supposing two pieces of India rubber stretched over the orifice of a tube, so that a small fissure is left between them, fig. 296

By forcing air through such a tube, sounds will be produced, varying with the tension of the membranes and the dimensions of the aperture. The glottis is a fissure-like opening, bounded by similar membranes. By means of a series of small muscles, the vocal cords may be extended or relaxed at pleasure, while other muscles afford the power of altering the width of the vocal fissure.

The vocal ligaments being thicker at their free edges than elsewhere, they vibrate like cords, but as they also extend like plates to the sides of the larynx, their vibrations are very nearly allied to the vibration of reeds. There has been much controversy as to whether the larynx and glottis are to be considered as a reed, or as a stringed instrument. It is probably more correct to say that it acts upon the principles of both combined.

389. **Mechanism of the voice.**—The formation of sound in the larynx, as has been already suggested, is produced by the vibration of the vocal cords, acting as a species of membranous reed, under the influence of air from the lungs. The sound being produced as in ordinary reeds (381) by the intermittent current of air.

297



The glottis is the original seat of the sound, and, although other parts of the respiratory apparatus have a certain influence in modifying the tone, they have no share whatever in the production of the sounds, or in determining their pitch.

When at rest, the lips of the glottis are wrinkled and plicated, so that the air in respiration passing through the fissure fails to put the membranes in vibration. But as the musician tunes his instrument by increasing or diminishing the tension of its vibrating strings, so something like this occurs with the human larynx. The two conditions of the glottis are beautifully shown by the two parts of fig. 297, from Müller. The upper shows the organ at rest, the vocal cords, *c c*, being relaxed, while in the lower, these cords are shown as in the act of vibrating; the small air passage, *o*, opening into the trachea, is never closed. When sounds are to be produced, the fissure is contracted and the membranes receive the degree of tension necessary for vibration. The sound varies according to the tension of the membranes, the magnitude of the fissure, and the form and magnitude of the passages through which the air thus put into vibration, passes before it issues into the atmosphere.

390. **Range of the human voice.**—In speaking, the range of the human voice is subject to but very little variation, being generally limited to half an octave. The entire range of voice in an individual is rarely three octaves, but the male and female voice taken together may be considered as reaching to four.

There are two kinds of male voices, called bass and tenor, and two kinds of female voices, called contralto and soprano, all differing from each other in tone. The strength of the bass voice is in general greater on the low tones than the tenor. The contralto is also stronger on the low tones than the soprano. But bass singers can sometimes go very high, and the contralto frequently sings the high notes like soprano.

The essential difference between the bass and tenor voices, and between the contralto and soprano, consists in the tone or "timbre" which distinguishes them even when they are singing the same note. The barytone is intermediate between bass and tenor, and the mezzo-soprano is, in like manner, found between the contralto and soprano. These two qualities of voice have a middle position as to pitch in the scale of male and female voices.

The different qualities of tenor and bass, and of alto and soprano voices, probably depend on some peculiarities of the ligaments, and the membranous and cartilaginous parietes of the laryngeal cavity which are not at present understood. We may form some idea of these peculiarities, by recollecting that musical instruments made of different materials, *e. g.*, metallic wires and gut-strings, may be tuned to the same note, but that each will have a peculiar quality or "timbre."

The following are the limits of the different classes of voice, as determined by Cagniard de Latour, Savart, and others, the numbers annexed being the number of double vibrations of the glottis produced in a second of time.

Soprano,	1056, 264.	Mezzo-soprano,	930, 220.	Contralto,	704, 176.
Tenor,	528, 132.	Barytone,	352, 110.	Bass,	220, 82·5.

391. **Ventriloquism, stuttering, &c.**—*Ventriloquism* is supposed by many investigators to consist chiefly in the use of inspiratory sounds; this is true only to a certain extent. The art of the ventriloquist depends greatly on the correctness of ear and flexibility of organ, through which common tones are modulated to the position and character in which the imaginary person is supposed to speak; other means often being used to heighten the deception, as concealing the face that the play of organs may not be observed; often in speaking with expiratory notes, the air expelled by one expiration is distributed over a large space of time, and a considerable number of notes.

In stuttering, the several organs of speech do not play in their normal succession, and thus are continually interfered with in convulsive impulses and inefficient adjustments. The cause of this result lies almost wholly in the nervous apparatus which rules over the organs of speech. Important remedial means are, to study carefully the articulation of the difficult letters, to practice their pronunciation repeatedly and slowly, and to speak only when the chest is well filled with air.

In deaf and dumb persons the organs of speech have originally no essential defects. The true cause of their dumbness lies in their inability to perceive sound. The impossibility of appreciating the several sounds, and thus gradually acquiring the power of properly adjusting the organs of speech, is the chief reason why the second infirmity is associated with the first.

392. **Production of sounds by inferior animals.**—The sounds which the different animals produce are peculiar to the class to which they belong; thus the horse neighs, the dog barks, the cat mews, &c. These various modifications depend on the peculiar structure of the larynx, but more upon the form and dimensions of the nasal and other cavities, through which the vibrating air passes.

The cat is distinguished from other mammifers by the almost equal development of the inferior and superior vocal cords. Many of its notes are almost human. The horse and ass are supplied with only two vocal cords. Animals which howl, and are heard at great distances, have generally large laryngeal ventricles.

Birds are furnished with two larynx, a superior and inferior, which serve at the same time for the entrance and exit of air, and for the purposes of vocalization. The upper larynx, which corresponds to the larynx in mammifers, can only be regarded as an accessory of the voice. The lower larynx is the true larynx; it is placed at the lower part of the trachea, where it branches. Those birds in which it is absent are voiceless. The voice of birds is produced, like that of mammifers, by the vibration of the cords of the glottis.

Insects, in general, produce sounds remarkable for their acuteness. Their sounds are produced in a great number of ways, some effecting it by percussion, and some by the friction of exterior horny organs upon each other, as, for example, in the grasshopper. In others, the swiftly recurring beatings of the wings produce sounds, as with the musquito. Many insects produce sound by the action of some of their organs on the bodies around them, as, for example, the various insects which gnaw wood.*

II. THE EAR.—HEARING.

393. **Auditory apparatus of man.**—In the ear, impressions are not at once made upon the sensory nerve, by the body which originates the sensation, but they are propagated to it through the medium of the atmospheric air.

The organ of hearing in man is composed of three parts: the external ear, the middle ear, or tympanum, and the internal ear, or labyrinth.

The external ear consists of (1) the pinna, or pavilion, *a*, fig. 298, which collects the soniferous rays, and directs them into (2) the auditory canal, or meatus auditorius, *b*.

The peculiar form of the pinna, with its numerous elevations and depressions, has not as yet been satisfactorily shown to be related to the principles of acoustics.

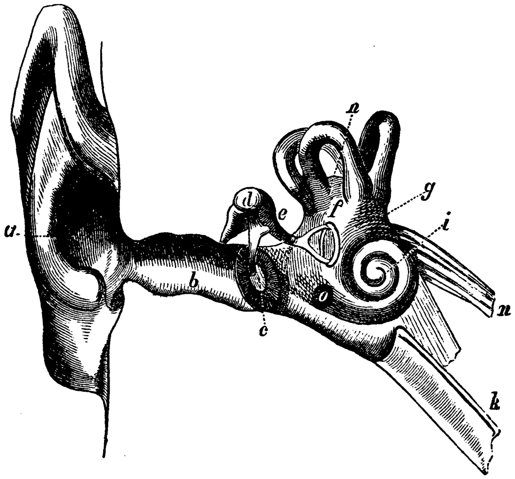
The auditory canal proceeds inwards from the pinna, to the tympanum, *c* it is an elliptical tube, about an inch long. Its interior is protected by hairs, and by a waxy secretion.

---

* See Appendix, p. 668.

The middle ear, tympanum, or tympanic cavity.—The middle ear is a cavity in the temporal bone filled with air, and somewhat hemispherical in form; it measures about half an inch in every direction. It extends from the tympanum, *c*, fig. 298, to *o* and *f*, and encloses the chain of bones, *c d e f*, called the *ossicles*, or little bones, of the ear.

298



The middle ear is separated from the auditory canal by a thin oval membrane, the *membrana tympani*, which is placed obliquely across the end of the canal, at an angle of about 45°, its outward plane looking downwards and forwards.

The *Eustachian tube* is a membranous canal leading from the anterior portion of the middle ear downwards and forwards to the pharynx, with which it communicates by a valvular opening that is generally closed.

The Eustachian tube gives exit to the mucus which forms in the middle ear, and also permits the entrance of air to preserve equality of pressure on the external and internal surfaces of the membrana tympani.

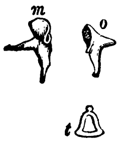
The discomfort produced by unequal pressure on the two surfaces of the membrana tympani may easily be understood by closing the nose and performing the act of swallowing, when the air will be partially exhausted from the middle ear. The external pressure on the membrana tympani then becomes greater than the internal, an unpleasant sensation is produced, and the sense of hearing is obscured. Forcible distension of the pharynx in blowing a horn or trumpet often produces a disagreeable fullness in the ears, by forcing air through the Eustachian tube into the middle ear. A cold often impairs the sense of hearing by obstructing the Eustachian tube.

A chain of three small bones, the ossicles of the tympanum, passes through the middle ear from the membrana tympani to the entrance of the internal ear. These bones are shown separated from each other in fig. 299.

The malleus, or hammer-bone, *m*, the incus, or anvil, *o*, and the stapes, or stirrup, *t*. They are connected with each other in such a manner as to allow of slight movements. This chain of bones is attached at one end, as is shown in fig. 298, by the handle of the malleus, to the tympanic membrane, and at the other by the foot of the stirrup, to the membrane of the fenestra ovalis.

The muscles which act upon these small bones are supposed to have the power of giving more or less tension to the membranes which they connect, and thus rendering them more or less sensitive to sonorous undulations.

299



The internal ear, called, from its complicated structure, the labyrinth, has its channels curved and excavated in the petrous bone, the hardest of any in the body. The labyrinth consists of three parts; the vestibule, the semicircular canals, and the cochlea.

The vestibule, *g*, fig. 298, is a central chamber, formed in the petrous bone; in it are a number of openings, for branches of the auditory nerve, small arteries, &c. In its external wall, the fenestra ovalis is found.

The semicircular canals are three in number, opening into the vestibule at its posterior and upper part, and placed in planes at right angles to each other. Within these canals are placed flexible tubes, of the same form, called membranous canals, filled with fluid.

The cochlea, *i*, is a conical tube, wound spirally, making two and a half turns. It resembles a snail's shell in appearance; whence its name. Its interior is divided by a spiral lamina, called the lamina spiralis, into two passages which communicate by a little hole in the upper part of the helix. Between the membranous and the bony labyrinths, a peculiar liquid (the perilymph) intervenes, which also fills the cavities and cochlea; the membranous labyrinth is distended by another liquid (the endolymph). Within the labyrinth thus filled with liquid, the terminal filaments of the auditory nerve are placed. They are expanded in the vestibule, spread out upon the lamina spiralis, and also in certain enlargements, called ampullæ, at the entrance of the semicircular canals; but they do not traverse the semicircular canals.

300

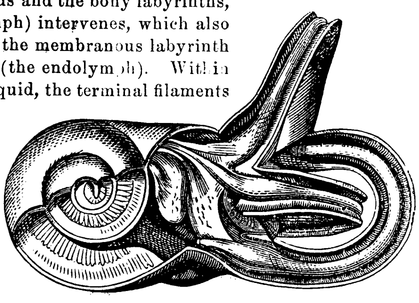


Fig. 300 is a magnified view of the labyrinth, showing the form and relation of the vestibule, semicircular canals and cochlea, partly laid open, so as to display their interior construction.

394. **Functions of the different parts of the ear.**—1. The waves of sound passing into the external ear, are collected and directed into the auditory canal, and strike upon the tympanic membrane, which is thus thrown into vibration.

2. The chain of bones connecting the membrana tympani with the fenestra ovalis, receives the vibrations and transmits them to the vestibule through the membrane which covers the fenestra ovalis.

3. Vibrations thus excited in the fluid which fills the labyrinth, are received by the expanded filaments of the auditory nerve, and the sensation of sound is thus transmitted to the brain.

In considering the uses of the different parts of the middle and internal ear, it is necessary to refer to the following principles, which have been fully demonstrated by experiment.

1. Atmospheric vibrations lose much of their intensity when transmitted directly to either solids or liquids.

2. The intervention of a membrane greatly facilitates the transmission of vibrations from air to liquids.

3. Vibrations are readily transmitted from air to a solid body, if the latter is attached to a vibrating membrane, so placed that the vibrations of the air act upon it.

4. Sonorous vibrations are communicated from air to water without any perceptible loss of intensity, when to the membrane forming the medium of communication there is attached a short solid body, which occupies the greater part of its surface, and is alone in contact with the water.

5. A solid body fixed in an opening by means of a border membrane, so as to be movable, communicates sonorous vibrations, from air on one side to water or the fluid on the other side, much better than solid media not so constructed. But the propagation of sound to the fluid is rendered much more perfect if the solid conductor thus occupying the opening is by its other end fixed to the middle of a tense membrane which has atmospheric air on both sides.

These principles enable us to understand that vibrations are communicated to the internal ear with greater intensity, by means of the membrana tympani and the chain of tympanic ossicles, than if these organs did not intervene between the atmospheric air and the internal ear. We find that in the lower orders of animals, where hearing is less acute than in man, the middle ear and tympanic ossicles are wanting. The air in the cavity of the tympanum serves also to insulate the chain of small bones, and preserve the purity and intensity of the vibrations which are transmitted. The communication between the middle ear and the external air, by means of the Eustachian tube, is thought to prevent reverberation and echoes in that cavity.

The sound of a tuning-fork, or other sonorous solid body, applied to the teeth, or any bone of the head, is heard more distinctly than when the sound is transmitted to the ear by means of the air; it has therefore been concluded that the cochlea, and especially the lamina spiralis, facilitates the appreciation of such sounds. In regard to the use of the semicircular canals, the opinions of physiologists are as yet divided.

For full discussions of the functions of different parts of the ear, the student is referred to Carpenter's Physiology and to Todd & Bowman's Physiology.

395. **Natural diapason.**—Cagniard de Latour, one of the best

modern authorities in acoustics, satisfied himself that he heard the sound *la* (A of the musical scale) sounding within his head when he agitated it from side to side. Mr. Jobard suggests that this natural *la* is caused by the contact of the *malleus* against the *incus* in the ear, a contact easily made by a rapid movement of the head, the neck being disembarrassed of clothing. (Am. Jour. Sci. [2] XXVI. 97.)

396. **Organs of hearing in the lower animals.**—The zoophytes appear to be wanting in the sense of hearing, and no special auditory apparatus has been discovered in insects, although they do not appear to be altogether insensible to sound. In the mollusca, the organ is a sac, filled with liquid, in which the last fibrils of the acoustic nerve are diffused, or a nerve fibril, in connection with a little stony body (an otolith), included in a sac of water. These animals can only distinguish one noise from another, or their quality, and that imperfectly, and have no perception of musical notes. This organ, corresponding, as is assumed, to the semicircular canals, increases in complexity as we rise in the scale of being. In lizards and scaly serpents, the ear commences with the tympanic membrane; and there is added a conical cochlea. As we pass through them, the plan is further developed; the tympanic cavity, Eustachian tube, the chain of bones, &c., appear. In birds there is a continued improvement, and all the aërial tribes of mammals have external ears, while a full development of all the auditory parts is reached only in man.

Problems in Acoustics.

Velocity of Sound.

153. The rumbling of thunder was heard $7\frac{1}{4}$ seconds after the corresponding flash of lightning was seen; what was the distance of the discharge?

154. Calculate the velocity of sound in air at a temperature of 90°; also at 40° below zero of Fahrenheit's scale.

155. What time would be required to transmit sound ten miles in the waters of a quiet lake?

156. In what time would sound travel a distance of $3\frac{1}{2}$ miles in each of the following substances: iron, wood, carbonic acid, hydrogen gas, vapor of alcohol at 140°, vapor of water at 154°?

157. What time was required to transmit the sound of the explosion of the volcano at St. Vincent's to Demerara (see page 260), supposing the sound to have travelled in the air alone?

158. At what distance from the source of sound must a reflecting surface be placed that an echo may be heard three seconds after the original sound?

159. From the top of a precipice a stone was let fall, and after $5\frac{1}{2}$ seconds it was heard to strike the bottom. What was the height of the precipice?

160. What was the distance of the meteor which was heard at Windsor Castle, in 1783, ten minutes after it disappeared, assuming the air at 50° F.?

161. An observer supposes himself in the range of a distant cannon, the report of which he hears 19 seconds after seeing the flash; how soon may he apprehend danger from the ball, supposing it to fly at the rate of a mile in eight seconds?

162. The flash of a gun throwing shells was seen due east from the observer; after three seconds the report was heard; after another interval of three seconds the shell was seen to explode 40° south of east, and the explosion of the shell was heard three seconds later; to what distance was the shell thrown? and what was its velocity of flight?

Physical Theory of Music.

163. A metallic wire, placed upon the sonometer, vibrates 300 times in a second; by how much must its length be diminished that it may make 370 vibrations per second?

164. What number of vibrations per second are required to give the note G 1 of the Italian opera?

165. What is the length of a wave in air when an instrument sounds E 3 of the Berlin opera?

166. What are the relative numbers of vibrations required to form the notes E and D sharp?

167. What is the interval between C sharp and D flat when both notes are correctly sounded?

168. How does the interval of four perfect fifths differ from a major third in the scale two octaves above the key note?

169. What is the fractional expression for the chromatic semitone? What for the grave chromatic semitone?

170. What is the number of beats in a minute formed by two tones whose vibrations are as 24 to 25, when the higher note makes 750 vibrations per second?

171. Calculate the number of vibrations per minute at the Holyoke Falls, for 1, 2, and 4 nodes respectively, the breadth of the dam 1008 feet, the enclosed column of air being entirely open at both ends.

172. Estimating the velocity of sound at 340 metres per second, what number of vibrations per second will be produced in a square organ tube whose length is 1·13 metres and its breadth 0·08 metres?

173. The organ of the church of Saint Denis, in Paris, is tuned to the normal *la* (A 3) of 880 simple vibrations per second, and a square tube 9·566 metres long and 0·48 metres broad, was constructed to play do_{-2} (C_{-2}), but on trial it was found too flat. What alteration of its length would correct its tone?

174. Calculate the respective lengths of a series of square organ tubes to play the scale commencing with C 1, the longer tube having a breadth of 4 inches, and each tube in the ascending scale having a breadth $\frac{1}{8}$ of an inch less than the preceding; the normal *la* being reckoned at 856 simple vibrations.

175. Calculate the dimensions of a series of cylindrical organ tubes for the scale commencing with C 4, the longer tube having a diameter of 1 inch, and the others in the series diminishing regularly in diameter by $\frac{1}{20}$ of an inch; the organ to be tuned as in the last example.

176. What are the names of the next three higher notes in the scale which the tube playing G 4 in the last example would give by sufficiently increasing the strength of the blast?

PART THIRD.

PHYSICS OF IMPONDERABLE AGENTS.

LIGHT, HEAT, AND ELECTRICITY.

CHAPTER I.

LIGHT, OR OPTICS.

§ 1. General Properties of Light.

397. **Optics.—Light.**—Optics (from the Greek verb *οπτομαι,* to see) is that branch of physical science which treats of the nature and properties of light.

Light is a mysterious agent, acting upon the organs of vision, and imparting to us a knowledge of external things. It brings us into relation with surrounding objects, enlarging the sphere of our habitation, in a measure annihilating distance, unfolding to us the beauties of nature, and acting as a perpetual source of enjoyment.

398. **Nature of light.—Theories.**—In regard to the nature of light, a great diversity of opinion has prevailed among philosophers.

(*a*) *Corpuscular theory.*—Sir Isaac Newton maintained that the phenomena of light are produced by luminous corpuscles thrown off from burning bodies, each particle producing, in its flight, vibrations in the surrounding ether similar to the waves produced by a stone falling into the water.

(*b*) *Undulatory theory.*—Huyghens maintained, in opposition to Newton, that light consisted solely of vibrations in an ethereal medium, without the onward progress of any substance whatever. This theory has been investigated and defended by many of the ablest philosophers; by Young, Malus, Fresnel, Brewster and others, and is now generally received.

The undulations producing the phenomena of *sound* take place in the same direction that the sound itself moves; but the vibrations of *light* are supposed to move at right angles to the direction in which light is propagated. It is difficult to explain all the phenomena of light even on this theory.

(*c*) An *oscillatory theory of light* has been proposed by Mr. Rankine, of Glasgow.* In this theory, the particles of luminiferous ether are supposed to rotate on their axes, by the influence of a species of magnetic force, which is wholly destitute of effect in producing resistance to compression, so that it is no longer necessary, as in the undulatory theory, so suppose the luminiferous medium to have the properties of an elastic body. The same mathematical formulæ are employed, with this hypothesis, as for the undulatory theory. Whether this theory can be applied to explain all the phenomena of physical optics, remains to be proved.

399. **Sources of light.—Phosphorescence.**—The sources of light are the sun and stars, heat, chemical combinations, phosphorescence, and electricity.

We know not the real cause of the light emitted by the sun and stars, but we know that bodies become luminous at a high temperature, and shine more vividly in proportion to the intensity of the heat, from which we are accustomed to suppose that heat and light are only modifications of one and the same cause. Artificial lights depend, in general, upon combustion, or the union of the oxygen of the air with burning bodies. This chemical action is attended with the disengagement of considerable heat as the burning body becomes luminous. Other chemical combinations are attended with light, and it is doubtful whether any bodies become luminous without chemical action of some kind.

The term *phosphorescence* is given to a pale light emitted in the dark by certain substances which do not appear to emit any sensible heat. Phosphorescence has been observed in animals, vegetables, and even in minerals. During the heat of summer the glow-worm and fire-fly emit a brilliant light.

In tropical regions phosphorescent insects are very numerous. The waters of the ocean, especially in warm latitudes, are often covered with little animalcules which become luminous at night when the water is agitated, shining in the wake of a vessel like a track of living fire.

---

* See transactions of the British Association for 1853, p. 9.

In certain circumstances also rotten wood and decaying flesh become phosphorescent. By friction, or by long exposure to the rays of the sun, certain minerals, as the diamond, white marble, and fluor spar, acquire the property, it is said, of shining, for a brief period, in the dark. The cause of phosphorescence is not known, but in some cases it appears to depend upon electricity.

Electricity is a source of light so intense that its brightness is equal, in some cases, to one-fifth, or even one-fourth that of the sun.

400. **Relation of different bodies to light.**—All bodies are either luminous, transparent, translucent, or opaque.

(*a*) *Luminous bodies* are those in which light originates, as the sun, and burning bodies.

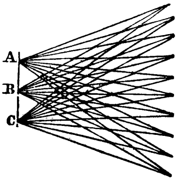
(*b*) *Transparent bodies* allow light to pass freely through them, thus permitting the form of other bodies to be distinctly seen through them. Such are water, air, and polished glass. Such substances are also said to be *diaphanous* (from διὰ, through, and φαίνω, to shine).

(*c*) *Translucent bodies* permit only a portion of light to pass, and in so irregular or imperfect a manner, that the outline of other bodies cannot be clearly seen, as rough glass and oiled paper.

(*d*) *Opaque bodies* are those which do not ordinarily allow any light to pass through them, as wood and the metals. But all bodies, even the metals, may be made so very thin as to become partially transparent or translucent. *Opacity* is not absolute in metals, as is proved in the case of gold-leaf on glass, through which a beautiful violet-green light is seen. This light is found by optical experiments to be truly transmitted light, and not a color caused by the minute fissures of the gold-leaf. It is worthy of remark, that this greenish color is complementary to the red, which is the color of gold when seen by successive reflections.

401. **Rays, pencils, and beams of light.**—A single line of light is called a *ray*. A *pencil* of light is a collection of rays diverging from a common source, or converging to a point. A *beam* of light is a collection of parallel rays. Diverging rays are those which gradually separate from each other. Converging rays are those which tend to meet in a common point; hence we have the terms diverging pencils, and converging pencils of light.

301



402. **Visible bodies emit light from every point** and in every direction, the rays diverging from each point in right lines.

Let A B C, fig. 301, be three points in any visible object; from each of these points, light is emitted in diverging pencils, as partially represented in the figure.

In this figure certain points are seen, where rays from A B C cross each other and between them are vacant spaces. No such vacant spaces exist, but the rays from all points in the object are crossing each other at every point in the space where the object is visible.

403. **Propagation of light in a homogeneous medium.**—A *medium* is something existing in space, capable of producing phenomena. A medium is called *luminiferous*, which is capable of transmitting light; and it is said to be homogeneous when the composition and density of all its parts are the same. All space is supposed to be pervaded by a luminiferous medium, called luminiferous ether, and yet the particles of this ether may act upon each other at great distances. In a homogeneous medium, light always moves in straight lines. If any opaque body is placed in a direct line between the eye and a luminous body, the light is intercepted.

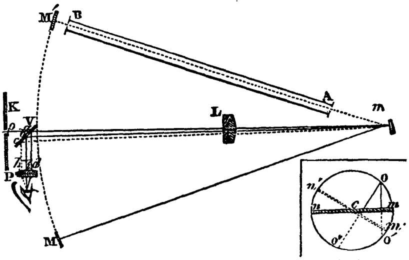
When light enters a dark chamber by a very small opening, the course of the light becomes visible by illuminating the fine particles of dust always floating in the air. Rays of sun-light are thus easily demonstrated to move in straight lines.

404. **Velocity of light.**—Light travels with such amazing velocity, that, for any distances on the surface of the earth, the time occupied in its passage from one point to another is totally inappreciable by our unaided senses.

In 1676, Roemer, a Danish astronomer, observed that the eclipses of the first satellite of Jupiter, which occur at uniform intervals of time when the earth is moving in that part of her orbit nearest to, or most remote from Jupiter, are constantly retarded when the earth is moving from that planet, and as regularly accelerated when the distance between the earth and Jupiter is diminishing. He found that when the earth was in that part of her orbit most distant from Jupiter, the eclipses of the first satellite take place 16 m. 36 s. later than when in the opposite part of her orbit.

By this means the velocity of light was ascertained to be about 192,000 miles in a second.

302



303

Foucault's apparatus for measuring the velocity of light.—Notwithstanding the prodigious velocity of light, M. Foucault has succeeded in measuring it, by employing a revolving mirror, according to the method devised by Wheatstone for measuring the velocity of electricity. In describing this apparatus, we shall suppose the properties of mirrors and lenses to be already understood.

The apparatus of M. Foucault is represented in fig. 302. The shutter of a dark chamber is pierced with a square opening, K, behind which a fine platinum wire, *o*, is stretched vertically. By means of a mirror, a beam of solar light is made to enter the chamber, and being divided by the platinum wire, it falls upon an achromatic lens, L, of long focus, placed at a distance from the platinum wire less than double the distance of its principal focus. The image of the platinum wire would be formed in the axis of the lens, somewhat enlarged. But the beam of light, after passing the lens, falls upon the plane mirror, *m*, which revolves with great velocity, and being reflected by it, an image of the platinum wire is formed in space, which image is displaced with an angular velocity, double the velocity of the mirror.* This image is received by a concave mirror, M, so fixed that its centre of curvature coincides with the axis of rotation of the revolving mirror, *m*. The pencil reflected by the mirror, M, returns backward and is again reflected by the mirror, *m*, and passes back through the lens, L, and forms an image of the platinum wire, coinciding with the wire itself, if the mirror, *m*, revolves slowly. In order to view this image without obscuring the pencil of light which enters the chamber by the opening K, a piece of plate glass, V, with parallel faces, is placed between the lens and the platinum wire, inclined in such a manner that the rays reflected fall upon a powerful eye-glass, P. If the mirror, *m*, remains stationary, or if it revolves slowly, the returning ray, M *m*, falls upon the mirror, *m*, in the same position it occupied at the first reflection, and returning in the direction it came, it meets at *a* the plate glass, V, and is partially reflected, and forms in *d*, at a distance *a d*, equal to *a o*, an image which is seen by the eye by means of the eye-piece, P.

The revolving mirror, *m*, causes this image to be repeated at each revolution, and if the velocity of rotation is uniform, the image does not change its position. When the velocity does not exceed thirty revolutions per second, the successive appearances of the image are distinct, but when the velocity is greater, the impressions upon the eye are continuous, and the image appears constant.

When the mirror, *m*, revolves with great rapidity, its position is sensibly changed during the interval occupied by the light in passing from *m* to M, and back again from M to *m*, and the returning ray, after reflection by the mirror, *m*, takes the direction *m b*, and forms an image in *i*; thus the image has deviated from *d* to *i*. Strictly speaking, there is some deviation even when the mirror turns slowly, but it is appreciable only when it has acquired a certain magnitude, by making the rotation of the mirror sufficiently rapid, or by taking the distance, M *m*, sufficiently great. By means of the deviation in the position of the image and the velocity of rotation, the time required for the light to pass from *m* to M, and back again, becomes known, making $l =$ M *m*, $l' =$ L *m*, $r =$ O L, $n =$ the number of revolutions per second, D $=$ the absolute deviation *d i*, and V $=$ the velocity of light per second. M. Foucault obtained the following formula for the velocity of light,

$$V = \frac{8\pi l^2 nr}{D\,(l + l')}$$

---

* To demonstrate this, let *m n*, fig. 303, be the revolving mirror, O, an object placed before it, and forming its image at O′; when the mirror arrives at the position *m′ n′*, the image will be formed at O″. But the angles, O′ O O″, and *m c m′* are equal, because their sides are perpendicular to each other. But the inscribed angle O′ O O″ is measured by half the arc O′ O″, and the angle *m c m′*, is measured by the entire arc *m m′*; hence the arc O′ O″, is double *m m′*, which thus demonstrates that the angular velocity of the image is double the angular velocity of the mirror.

In the experiments of M. Foucault, *m* M, was only about four yards, but by giving the mirror, *m*, a velocity of 600 or 800 revolutions per second he obtained a deviation of from eight one-hundredths to twelve one-hundredths of an inch. Owing to the vibration of the apparatus revolving with such great rapidity, the results yet obtained by this method for the absolute velocity of light are not considered as entirely correct, although of the highest interest.

Experiments have been made with the same apparatus to determine the velocity of light in liquids as compared with the velocity in air. For this purpose a tube, A B, three yards long, is filled with distilled water, or any other liquid, and placed between the revolving mirror, *m*, and the concave mirror, M′, similar to M. The rays of light reflected by the revolving mirror in the direction *m* M′, pass twice through the column of fluid in the tube, A B, before returning to the mirror, V. The returning ray is reflected at *c*, and forms an image at *h*. The deviations of the rays which traverse the liquid are greater than the deviation of the rays which are propagated in air alone, which shows that the velocity of light in fluids is less than in air.

Fizeau's method.—Another method of direct determination of the velocity of light has been devised by M. Fizeau, of Paris, in 1849. An exposition of this method, by Prof. A. Caswell, will be found in the Smithsonian Report for 1858, p. 130.

Results.—From these and other methods the velocity of light has been determined to be in air 192,000 miles per second; in water 144,000 miles; in glass 128,000 miles; and in diamond 77,000 miles.

405. **No theory of light is entirely satisfactory.**—In the corpuscular theory of light, advocated by Newton, it was supposed that fluids and solids attracted the light, and refraction was explained by supposing that light moves faster in dense bodies than in air, as is known to be the case in regard to sound. According to the undulatory theory, it is known that transverse waves or undulations must move slower in dense bodies than in rarer media.

The discovery of Foucault, as just explained, that light actually moves slower in denser media, tends to confirm the undulatory theory.

The immense power of resisting compression which a medium ought to possess, in order to transmit transverse vibrations with a velocity so much greater than the motions of the swiftest planets or comets, is an objection against the undulatory theory that has not yet been satisfactorily answered.

The discussion of the theories of light belongs to the higher departments of mathematics.

406. **Properties of light.**—(*a*) *Absorption.*—Light falling upon any substance is either absorbed, dispersed, reflected, or refracted. A part of the light disappears and is said to be *absorbed;* as when light falls upon black substances. No substances absorb all the light, for the fact that the blackest substance is still visible, shows that its different parts emit some of the light which they receive.

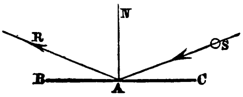
(*b*) *Dispersion.*—Light falling upon opaque bodies, causes them to become luminous, or to emit light in all directions, and thus become visible. Such bodies are said to *disperse* light, because they scatter the light in all directions from which they are visible.

Bodies owe the property of dispersing light to the innumerable little facets of the particles composing their rough surfaces. Only part of the light is thus irregularly reflected or dispersed, while much of it is probably absorbed or destroyed.

(*c*) *Reflection.*—When light falls upon polished surfaces, or on bodies having naturally smooth and uniform surfaces, it is thrown off in a regular manner, as a ball rebounds from a hard floor.

If a ray of light, S A, fig. 304, falls upon a polished surface, B C, it will be reflected in the direction A R. If N A is drawn perpendicular to B C, S A N will be the angle of incidence, and N A R will be the angle of reflection, and the two angles will be equal. The lines S A, N A, and A R, will lie in the same plane; we have therefore the following rules:—

304



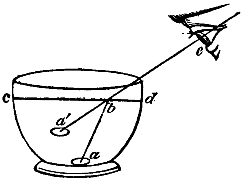
1st. *The incident ray, the perpendicular at the point of incidence, and the reflected ray, are all situated in the same plane.*

2d. *The angle of incidence and the angle of reflection are equal.*

(*d*) *Refraction.*—If a straight rod is placed obliquely, partly immersed in water, it appears broken or bent just where it enters the water. If a coin, *a*, fig. 305, is placed in a cup, in such a position that it is just hidden from view, and water is then gently poured into the cup, the coin will appear to be lifted up and will become visible.

Let *c d* be the surface of the water, the ray, *a b*, is so bent or refracted, at the surface of the water, that the coin appears as if placed at *a'*.

305

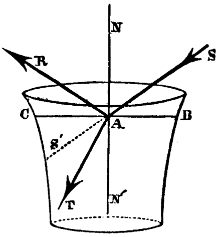


This bending of the rays at the surface of any transparent medium is called *refraction.*

Let C B, fig. 306, be the surface of water in a vessel, S A a ray of light incident at A, and N A N' the perpendicular, A R the reflected ray, and A T the direction of the ray which enters the water and is refracted, then:—

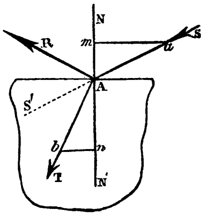
The angle S A N is called the *angle of incidence* of the ray S A. The angle N A R is called the *angle* of *reflection*, which is in all cases equal to the angle *of* incidence. The line N A N', is called the *normal.* The angle T A N' is called the *angle of refraction.*

306



If we take A *a*, fig. 307, equal to A *b*, and draw *a m* and *b n*, each perpendicular to N A N', then *a m* is the *sine* of *the angle of incidence,* and *b n* is the *sine* of the angle of refraction, and *a m* divided by *b n* is invariably the same for any given medium, whether the angle of incidence is increased or diminished. The quotient obtained by dividing *a m* by *b n*, is called the *index of refraction*, and it is represented by *n*. The index of refraction varies for different media; thus for light passing from air into water, it is about $\frac{4}{3}$, for light passing from air into glass, about $\frac{3}{2}$, and about $\frac{5}{2}$ when light passes from air into diamond. These fractions inverted give the index of refraction for light passing out of water, glass, and diamond, into air.

307



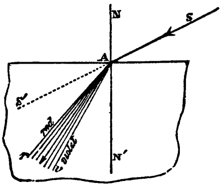
When light passes from a rare to a denser medium, it is refracted towards the perpendicular or normal, and when it passes from a dense to a rarer medium, it is refracted from the perpendicular or normal.

The general law of the refraction of light is thus stated. *The incident ray, the refracted ray, and the perpendicular to the refracting surface at the point of incidence, lie in the same plane; and the sine of the angle of incidence bears a constant ratio, in the same medium, to the sine of the angle of refraction;*

$$\text{or, } \frac{am}{bn} = n.$$

When a ray of ordinary daylight or sunlight is refracted by a dense transparent medium, the refracted light is not confined to a single line, but it is spread out into a fan-like form, as shown in fig. 308, between A *r* and A *v*, and the different parts of the refracted pencil show different colors, the most strongly refracted part being violet, and the least refracted part being red. The index of refraction, for a single color, is uniform for any given medium; but the index of refraction in the same medium varies for differently colored light.

308



407. Amount of light reflected at different angles of incidence.—When light falls upon a transparent medium perpendicular to its surface, nearly all the light enters the medium, and only a small portion is reflected. As the light falls more and more obliquely upon the medium, the amount of light refracted diminishes, and the amount reflected increases.

If we look at the image of the sun in water at midday, and again near sunset, we shall see a remarkable difference. Near sunset the image is so brilliant, the eyes can scarcely bear to look at it, while at midday we observe it without difficulty. The image of objects at a little distance are seen in water more distinctly than the images of near objects, because the light from distant objects falls more obliquely upon the water and a greater amount is reflected.

If we look very obliquely at a sheet of white paper, placed before a candle, an image of the flame may be seen reflected from the surface of the paper, but the image disappears when the rays fall upon the paper nearer to the perpendicular.

When light falls upon any polished metallic surface, the greatest amount of reflection takes place when the incident rays are perpendicular to the surface, and the amount of light reflected diminishes as the angle of incidence increases.

Different substances, polished with equal care, differ in their power of reflecting light. The amount of light reflected depends also upon the nature of the medium in which the reflecting body is placed. Bodies immersed in water reflect less light than in air.

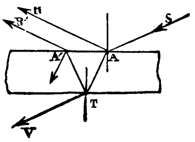
Table showing the number of rays of light reflected out of 100 *rays incident, by different kinds of glass and metals used for optical purposes.**

Angle of incidence.	Crown glass, Sp. gravity, 2·541. $n = 1·524$. Specific heat, 0·38.	Plate glass, Sp. gravity, 2·511. $n = 1·517$. Specific heat, 0·39.	Flint glass, Sp. gravity, 3·225. $n = 1·570$. Specific heat, 0·43.	Glass of Antimony.	Speculum metal, Sp. gravity, 8·9. Specific heat, 0·67.	Polished steel, Sp. gravity, 7·8. Specific heat, 0·88.
0°	3·452	3·380	3·615	8·20	72·30	
10°	3·608	3·546	3·819	8·36	70·85	60·52
20°	3·837	3·790	4·117	8·60	69·43	
30°	4·189	4·164	4·574	8·98	68·11	58·69
40°	4·767	4·778	5·320	9·59	66·91	
50°	5·810	5·882	6·656	10·68	65·87	54·96
60°	7·964	8·155	9·369	12·93	65·03	
70°	13·448	13·891	16·015	18·52	64·41	
80°	32·396	33·155	36·422	36·65	64·04	
85°	56·202	56·204	57·559	57·07		
90°	75·776	74·261	72·074	72·20	63·91	53·60

408. **Internal reflection.**—When light passes through a transparent medium, a portion of the light is reflected at each surface.

In fig. 309, S A is a ray of light incident upon the first surface of a transparent medium. A portion is reflected in A R. A T is the refracted ray, and T V the emergent ray, but a portion of the light is reflected at the second surface in the direction T A′, of which a part emerges in the direction A′ R′, a part suffers a second reflection downward from A′, a part emerges from the second surface, and another portion suffers successive internal reflections before it is either lost by absorption or finally emerges on one or the other side of the medium. In general only the rays A R, T V, and A′ R′, have sufficient intensity to be visible to the naked eye.

309



409. **Total reflection.**—When light passes from a dense to a rarer medium, the angle of refraction is greater than the angle of incidence, and when the angle of refraction is 90°, the angle of incidence is much less. For water it is 48° 35′, for ordinary glass it is 41° 49′, conse-

---

* From Potter's Physical Optics.

quently a ray of light traversing water or glass at greater angles cannot escape into the air, but is *totally reflected*, obeying the ordinary law of reflection. The proportion of light suffering internal reflection from a surface of glass or water, constantly increases from the perpendicular to the point where total reflection takes place.

Since the angle of incidence for a dense medium is always greater than the angle of refraction, when the angle of incidence is 90° the angle of refraction must be considerably less than 90°. If the angle of incidence is 90°, its sine will be unity. The sine of the angle of refraction will be unity divided by the index of refraction, $= \frac{1}{n}$, hence the angle of total internal reflection for any medium is the angle whose sine $= \frac{1}{n}$.

310

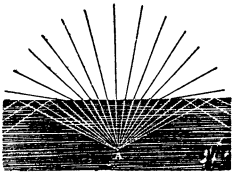


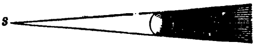
Fig. 310 shows light radiating from a point below the surface of water and escaping into the air, the angle of emergence increasing much faster than the angle of incidence, until the light emerges parallel to the surface of the water, after which total reflection takes place.

To an eye placed below the surface of the water, all objects above the horizon would be seen within an angle of 97° 10′, or double the angle of total reflection for water.

410. **Irregular reflection.—Diffused light.**—The reflection from polished surfaces, which follows the two laws already announced, is called *regular reflection;* but only a part of the light is reflected regularly from any surface, when the reflecting body is more dense than the surrounding medium. A part of the light is scattered in all directions, and is said to be *irregularly reflected* or *diffused.* This is the portion of light which renders objects visible. Light regularly reflected gives an image of the object which *emits* the light, while light irregularly reflected gives only an image of the body which *reflects* it. When a mirror becomes dim by the accumulation of light dust, or anything which tarnishes its surface, the amount of regular reflection diminishes, and the irregular reflection increasing, all parts of the mirror become distinctly visible.

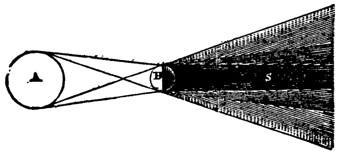
411. **Umbra and penumbra.**—When an opaque object is held in a pencil of light proceeding from a luminous point, as *s*, fig. 311, a dark and well-defined shadow is produced, which increases in size as it becomes more distant. The dark shadow is called an *umbra.* If the light proceeds from a luminous body having a sensible magnitude, as A, fig. 312, besides the dark shadow, or *umbra*, where no part of the luminous body is visible, there will be a much broader partial shadow, called the *penum-*

311

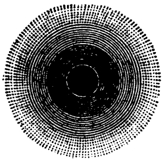


bra, where a part only of the luminous body is visible. The breadth of the penumbra increases with the diameter of the light, and with the distance which the shadow extends behind the opaque object. The darkness of the penumbra gradually increases from the extreme border, which is too faint to be easily seen, to the umbra or full shadow, as is shown in a section of the shadow, at fig. 313.

312

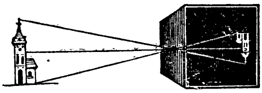


313



412. **Images produced by light transmitted through small apertures.**—If a white screen is placed near a small opening in a dark chamber, the rays of light which pass through the opening will form on the screen inverted images of external objects.

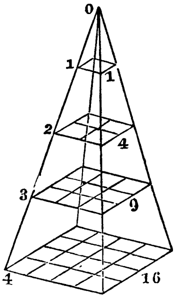
314



It will be seen in fig. 314, that the rays of light from the top and the bottom of the object cross each other in the small opening, and thus invert the image. If the aperture is small, the image will be formed in the same manner, whatever be the form of the aperture. But if the opening is large, the image is indistinct, or entirely disappears.

413. **Intensity of light at different distances.**—The intensity of light at any distance from a luminous body, is in an inverse proportion to the square of the distance.

315



Let 0, fig. 315, be a luminous point; at 1 1, place a board one foot square; it will cast a shadow that will cover a space two feet square at double the distance, three feet square at three times the distance, and four feet square at four times the distance. The areas will therefore be, 1, 4, 9, 16, and the intensity of the light at the distances 1, 2, 3, 4, will therefore be in the proportions of 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$.

If I and I' represent the intensity of a light at the distances D and D', we shall have

$$I : I' = \frac{\text{I}}{D^2} : \frac{\text{I}}{D'^2}, \text{ or } \frac{\text{I}}{\text{I}'} = \frac{D'^2}{D^2}$$

Hence the intensity of a light at different distances will be inversely as the squares of those distances.

414. **Photometers** are instruments employed to measure the comparative intensity of different lights. The principle on which they are constructed is, to so place the lights that they will illuminate a single surface, or two adjacent surfaces, with equal intensity. The relative

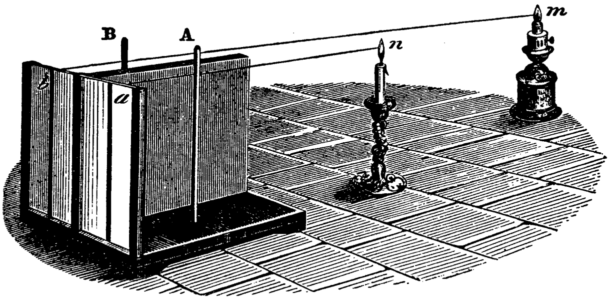
intensities of the two lights are then as the square of their distances from the illuminated surfaces.

Bunsen's Photometer is the simplest and most convenient photometer yet invented. A disk of paper four or five inches in diameter, is rendered translucent by washing it with paraffine or stearine, dissolved in oil of turpentine or naphtha, except a spot about an inch in diameter at the centre. When this disk is held between two lights, at a point where their intensity is unequal, the translucent part of the paper is easily distinguished from the central part, but when moved to a point where the two lights have equal intensity, all parts of the paper have a uniform appearance. No light appears to shine through, because the illumination is equal on both sides. By means of a graduated bar, on which the lights and disk are mounted, the distance of each light from the paper is determined, and their respective intensities are calculated on the principles above mentioned.

This principle may be applied in many ways to determine the intensities of lights; as, for instance, the portion which is transmitted or reflected from different substances.

Rumford's Photometer.—Rumford's photometer is composed of two plates of ground glass, before which are fixed two opaque rods, A and B, separated by a screen, fig. 316. The lights to be compared, as a lamp and a candle, *m n*, are so placed opposite the rods that each plate is illuminated by only one of the lights, and a shadow of the corresponding rod falls upon each plate, as shown in the figure. If the two shadows, *a* and *b*, are of unequal intensity, by moving one of the lights backward or forward a position is obtained where the shadows appear equally dark, and the glass plates are thus known to be equally illuminated. The relative intensities of the lights are determined as in the preceding case.

316



Silliman's Photometer.—Silliman's photometer is the reverse of Rumford's, comparing two discs of light thrown up by two equal triangular glass prisms, upon a disc of roughened glass in the body of a dark chamber moving on a graduated bar. (Am. Jour. Sci. [2] XXII. 315.)

§ 2. Catoptrics, or Reflection by Regular Surfaces.

I. MIRRORS AND SPECULA.

415. **Mirrors** are solid bodies bounded by regular surfaces, highly polished, and capable of reflecting a considerable portion of the light which falls upon them.

The term mirror is generally applied to reflectors made of glass and coated with an amalgam of tin and quicksilver.

416. **Specula are metallic reflectors**, having a highly polished surface. The best speculum metal consists of 32 parts of copper, and 15 parts of the purest tin. Specula are also made both of silver and of steel.

In the use of glass mirrors, a portion of light reflected from the first surface, interferes with the perfection of the image; hence, where the most perfect instruments are required, metallic reflectors are employed. In treating of reflectors, we shall notice only the action of the principal reflecting surface, and use the term mirror to comprehend all regular reflectors.

417. **Forms of mirrors.**—Mirrors are either plane or curved. Curved mirrors may be spherical, elliptical, or paraboloid. The properties of elliptical and paraboloid reflectors have been mentioned in sections 324 and 325. A *concave* spherical mirror is a portion of the surface of a sphere, reflecting from the internal side. A *convex* spherical mirror is a portion of the surface of a sphere, reflecting from the outside. Curved mirrors, whether concave or convex, may be regarded as made up of an infinite number of plane mirrors, each perpendicular to a radius drawn through it from the centre of the mirror.

317

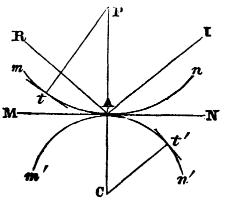


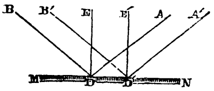
Fig. 317 shows a plane mirror, M A N, a concave mirror, *m* A *n*, and a convex mirror, *m′* A *n′*, having a common point, A, and the line, P A C, perpendicular to each at the point A. If a ray of light, I A, is incident upon either mirror at the point A, the reflected ray, A R, will make the same angle with the perpendicular as is made by the incident ray. At any other points, as *t* or *t′*, the curved mirrors will act like little plane mirrors, perpendicular to the radii *P t* and *C t′*.

II. REFLECTION AT PLANE SURFACES.

418. **Reflection by plane mirrors.**—*Parallel rays* of light, falling upon a plane mirror, will be parallel after reflection.

If parallel rays of light, A D, A′ D′, fig. 318, fall upon the plane mirror, M N, they will each make equal angles with the perpendiculars, E D, E′ D′, and as the angles of incidence and reflection will be equal, the reflected rays, D B, D′ B′, will make equal angles with the perpendiculars, and will consequently be parallel after reflection.

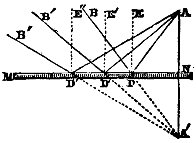
318



If A D represent the upper side of the beam of light before reflection, it will become, after reflection in D B, the lower side of the beam. Hence a beam of parallel light is inverted in one direction by reflection from a plane mirror.

Diverging rays of light, falling upon a plane mirror, will continue to diverge after reflection, and will appear to emanate from a point as much behind the mirror as the luminous point is before it.

319



Let A be a radiant point in front of the plane mirror M N, fig. 319. If the perpendiculars, E D, E′ D′, E″ D″, be drawn, the reflected rays will make the same angles with the perpendiculars as the incident rays, and hence the reflected rays will make the same angles with each other as they did before reflection, but they will appear to diverge from the point A′, behind the mirror.

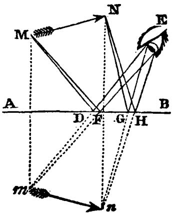
Converging rays continue to converge after reflection from a plane mirror. After reflection they will converge towards a point as much in front of the mirror as the distance of the point behind the mirror, towards which they converged before reflection.

This is easily seen by tracing the rays of light backward in the preceding figure.

Reflection from a plane mirror changes the direction of the rays of light, and removes the point of apparent convergence or divergence to the opposite side of the mirror.

419. **Images formed by plane mirrors.**—Let M N be an object placed in front of the plane mirror, A B, fig. 320, and E the place of the eye. From the great number of rays emitted in every direction from M N, and reflected from the mirror, a few only can enter the eye at E. These will be reflected from those portions, D F, G H, of the mirror, so situated with respect to the eye and the points, M N, that the angles of incidence and reflection will be equal. If the rays, D E, F E, are continued backward, they will meet at *m*, and they will appear to the eye to radiate from that point. In the same manner the rays G E, H E, will appear to radiate from *n*; a virtual image of the object will therefore be formed between *m* and *n*.

320



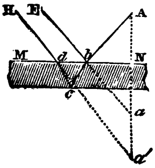
This is called a *virtual* image, because it is not formed of rays of light actually coming from the position of the image, but by rays so changed in their direction, that they appear to the eye as though originating from an object situated at *m n*, behind the mirror.

If the eye is moved about, the image remains stationary, hence it is seen by means of rays reflected from other parts of the mirror. Two or more persons may see the image at the same time and in the same position, but by different rays of light.

The position of the image behind the mirror may be found by drawing lines from prominent points in the object, perpendicular to the mirror, extending them as far behind the mirror as the points from which they are drawn are situated before it, then uniting the extremities of the lines, the outlines of the image will be delineated. The images of all objects seen in a plane mirror have the same form and distance from the mirror as the objects themselves.

420. **Images multiplied by two surfaces of a glass mirror.**—Glass mirrors produce several images. This may be readily demonstrated by looking very obliquely at the image of a candle in a glass mirror. The first image, caused by partial reflection from the first surface of the glass, is comparatively faint. The second image is formed by reflection from the quicksilver, which covers the second surface, and is very clear and distinct.

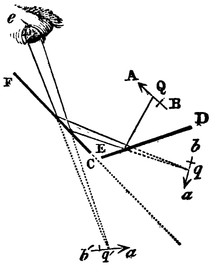
321



When rays of light from any object fall upon the first surface of a plate of glass, M N, fig. 321, a portion of the light being reflected, forms the first image, *a*. The principal part of the light penetrates the glass, and is reflected at *c*, by the silvering which covers the back of the mirror, and coming to the eye in the direction *d* H, produces the image, *a′*, at a distance from the first image equal to about once and a third the thickness of the glass. This image is much brighter than the first, because the metallic coating of the mirror reflects a greater amount of light than the first surface of the glass.

Other images, more and more obscure, are formed by rays which emerge from the glass after successive interior reflections from the two surfaces of the glass. As this multiplicity of images diminishes the distinctness of vision, metallic reflectors are often employed in optical instruments.

322



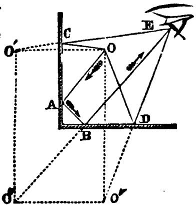
421. **Images formed by light reflected by two plane mirrors.**—Let A B, fig. 322, be an object, and C D, E F, two plane mirrors, making an angle with each other less than 180°. The light falling upon the mirror C D will form an image at *a b*, the position of which may be determined by the method explained at section 419. A portion of this light, after reflection, will fall upon the mirror E F, and be reflected as if coming from an image *a′ b′*, which will be seen by the eye at *e*

To trace the course of the rays which enter the eye from any point, Q, in the object A B; let *q* be the corresponding point in *a b*, and *q′* a similar point in *a′ b′* · the light will enter the eye as if it came from *q′*, therefore draw the lines *q′ e*, and they will show the final course of the pencil by which the point Q is seen. From the points where these lines meet the mirror, E F, draw lines to *q*, and they will represent the course after reflection by C D; from the points where these lines meet the mirror C D, draw lines to the point Q, and they will show the course of the rays which, after reflection by each of the mirrors C D, E F, form the pencil by which the eye at *e* sees the point *q′* in the secondary image *a′ b′*.

The inversion of parts by the two mirrors are now seen to correct each other, and all the parts of the image, *a′ b′*, have the same relation to each other as in the object A B. The peculiar excellence of Wollaston's *Camera lucida* (518) depends upon the fact that by means of two reflections all parts of the image preserve their natural relations.

422. **Multiplicity of images seen by means of inclined mirrors.**—When an object is placed between two mirrors, which make with each other an angle of 90° or less, several images are produced, varying in numbers according to the inclination of the mirrors. If they are placed perpendicular to each other, three images will be seen, situated as in fig. 323.

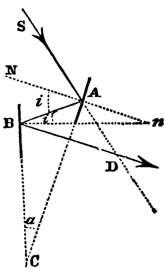
323



The rays O C and O D, from the point O, form, after a single reflection, one the image O′, the other, the image O″; and the ray O A, which undergoes two reflections at A and B, gives a third image, O‴. When the inclination of the mirrors is 60°, five images are formed; and when they are placed at an angle of 45°, seven images are produced. The number of images continues to increase as the inclination of the mirrors diminishes, and when the mirrors become parallel, the number of images is *theoretically* infinite, but as some of the light is lost at every reflection, and the successive images appear more and more distant, only a moderate number of images are visible.

423. **Deviation of light reflected by two mirrors.**—When a ray of light reflected by a mirror is again reflected by a second mirror, in a plane perpendicular to the intersection of the two mirrors, the deviation of the ray from its original direction is equal to twice the angle formed by the two mirrors.

324



Let two mirrors, A and B, fig. 324, be inclined to each other, so that their directions shall meet at some point, C, forming an angle $A\,C\,B = a$. Let the ray of light, S A, be reflected by the first mirror in the line A B, and falling upon the second mirror be again reflected in the direction B D, meeting the original direction S A D in D. Let the angle of deviation $A\,D\,B = d$. Draw N A *n* perpendicular to the mirror A, and B *n* perpendicular to the mirror B. The angle between these perpendiculars will be equal to the angle formed by the inclinations of the mirrors, or $A\,n\,B = A\,C\,B = a$.

Let $SAN = NAB = i$, and $ABn = i'$; then since $NAB = ABn + AnB$, we have.—

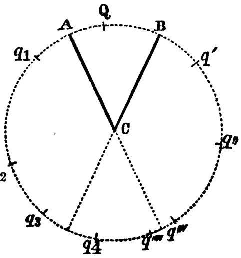
$$i = i' + a \quad \therefore \quad a = i - i';$$
$$\text{also} \quad BAD + ABD + ADB = 180^\circ,$$
$$\text{or } 2(90^\circ - i) + 2i' + d = 180^\circ \quad \therefore \quad d = 2(i - i') = 2a;$$

Or the deviation of any ray after two reflections is equal to twice the angle between the mirrors.

424. **Kaleidoscope.**—This beautiful toy depends upon the multiplication of images by inclined mirrors. Two mirrors, inclined at angles of 30°, 45°, or 60°, are placed in a paper tube, one end of which is closed by plain and the other by ground glass. Various objects, as fragments of colored glass, tinsel, twisted glass, &c., are placed in a narrow cell, at the end of the tube, closed with ground glass, just room enough being left to allow the objects to tumble around as the tube is moved. On looking through this instrument towards the light, multiplied images of every object are seen, beyond all description splendid and beautiful; an endless variety of symmetrical combinations appearing to the view as the instrument is moved, but never recurring with the same form and color.

Let A C and B C, fig. 325, be the two mirrors of the kaleidoscope, and let the dotted circle, described about C as a centre, represent the tube in which they are placed; let Q be the position of an object within the angle formed by the mirrors. If Q is in the circumference of the circle described about C, the two series of images of Q will be formed in the circumference of the same circle, $q_1\, q_2\, q_3\, q_4$ being formed by the mirror A C, and $q'\, q''\, q'''\, q''''$ being formed by the mirror B C. Since q_1 is in a line perpendicular to A C, and at the same distance from A C behind it as Q is before it, that perpendicular is the chord of the arc Q q_1, and q_1 is in the circumference of the circle drawn about C as a centre. For the same reason q' is also in the same circumference; so also q'' being the image of q_1, is as far behind B C as q_1 is before it, and as the line joining q_1 and q'' is perpendicular to B C, it must be the chord of the circle, and hence q'' is in the circumference. In the same manner it may be shown that every image, formed by repeated reflections from A B and B C, is also in the circumference of the circle described about C. When we arrive at any image, q_4 or q'''', falling, as in the figure, between the directions of the mirrors produced, such an image being situated at the back of both the mirrors must be the last of its series, as no light from such an image can fall upon either mirror.

325

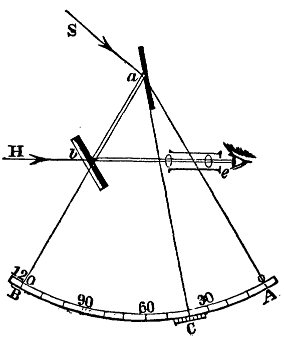


According to § 423, the distance between any two images, formed by an even number of reflections, will be equal to twice the angle between the mirrors. It is evident that images formed by an odd number of reflections will be situated between each two of the former series;

hence the entire number of images seen in the kaleidoscope, including the object itself, will be equal to 360° divided by the angle contained between the mirrors. If the inclination of the mirrors is 60°, the number of images, including the object, will be six; if the inclination is 45°, the number will be eight; and for 30° every object will appear as twelve. If the inclination of the mirrors is small, the images formed by many successive reflections become too faint to be distinctly seen.

425. **Hadley's sextant** is an instrument depending on reflection from two mirrors, and used chiefly by seamen for measuring the altitudes and angular distances of the heavenly bodies.

326

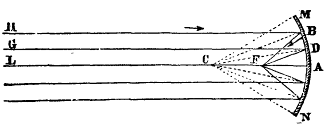


Two mirrors, *a* and *b*, fig. 326, are so mounted that the angle of inclination can be varied at pleasure. The mirror *a* is attached to a movable arm, *a* C, which turns about the centre of the graduated arc A B. This arm carries at C a vernier by which minute divisions of the graduated arc are easily distinguished. The mirror *b* is firmly attached to the frame of the instrument, and the outer portion has the silvering removed, so that an eye placed at *e* sees the distant horizon, or any other object to which it is directed, in its true position. The mirror *a* is turned with the index arm *a* C, until any other object, as the sun, moon, or a star, whose light is twice reflected in the directions S *a b e*, appears to coincide in direction with the horizon or other object, H, seen by direct light, from which its angular distance is to be measured. The telescope at *e* is used to facilitate accurate observation. The divisions of the graduated arc and vernier are also read by the aid of a magnifying lens, not shown in the figure. The deviation of the ray S *a*, after being twice reflected, is, by § 423, twice the angle contained between the mirrors, or twice the degrees contained between A C; half degrees on the scale are therefore marked as whole degrees. The reading by the vernier gives the altitude or angular distance of the observed object.

III. REFLECTION AT CURVED SURFACES.

426. **Concave and convex spherical mirrors.**—If an arc of a circle, M N, fig. 327, is made to revolve around a line, A C L, drawn through its centre of figure A, and its centre of curvature C, it will generate a curved surface, which will be a segment of the surface of a sphere. Internally, such a polished surface is called a concave mirror, and externally a convex mirror. The line, A C, is called the principal axis of the mirror, and any other line drawn through the centre of

327



curvature, C, is called a secondary axis. The angle M C N is called the angular aperture of the mirror. A section made by a plane passing through the principal axis, A C, is called the principal section, or a meridional section.

The theory of reflection from curved mirrors is easily deduced from the laws of reflection by plane mirrors. Every point in the curved mirror may be regarded as a point in a plane mirror so situated that its perpendicular, where the ray of light falls upon it, coincides with the radius of the curved mirror at that point.

A line drawn from any point in a spherical mirror to the centre of curvature, will be perpendicular to the mirror at that point, and also perpendicular to any plane mirror touching the curved mirror at that point.

427. **Foci of concave mirrors for parallel rays.**—The focus of a concave mirror is the point towards which the reflected rays converge.

(*a*) *Parallel rays* falling near the axis of a concave mirror, fig. 327, converge, after reflection, to a point equidistant between the mirror and the centre of the sphere, of which the mirror forms a part. This point is called *the principal focus*.

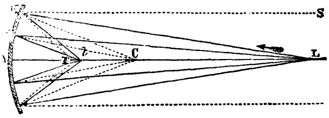
Rays of light emanating from the principal focus of a concave mirror, will be reflected parallel to each other.

Demonstration.—The lines C M, C B, C D, fig. 327, drawn from the centre of curvature of the mirror, M N, are perpendicular to the mirror at those points. The parallel rays, H B, G D, will converge, after reflection, to the point F. It is evident that the angle of reflection, C D F, for any ray, will be equal to the angle of incidence, G D C; but G D C is equal to D C F, which is the alternate angle formed by a line D C, meeting two parallel lines, G D, L A; hence in the triangle, C F D, the angles, F C D and F D C, are equal, and therefore the sides, C F and F D, are equal. If the point, D, gradually approaches the point, A, C F + F D will differ less and less from C A, until their sum will be sensibly equal to C A, and F A will be sensibly equal to one-half of C A; or the focus of parallel rays, after reflection from a concave mirror, will be equal to one-half the radius of curvature. If the point of incidence, D, recedes from A towards M, or N, the point, F, will gradually approach A, or the focal distance will diminish. A concave spherical mirror will therefore reflect parallel rays to a single focal point only when the diameter of the mirror is small. Practically it is found that the diameter of the mirror, or the angular aperture, M C N, should not exceed 8 or 10 degrees.

428. **Foci of diverging rays.**—If rays of light falling upon a concave mirror diverge from a point beyond the centre of curvature, they will converge, after reflection, to a point between the principal focus and the centre of curvature. This point of convergence is called the conjugate focus, because the distance of the radiant point and the focus

to which the rays converge, after reflection, have a mutual relation to each other.

328



Let rays diverging from a point, L, fig. 328, fall upon a concave mirror, the angle of incidence, L K C, will be smaller than S K C, which is the angle of incidence for parallel rays falling upon the mirror at the same point. The angle of reflection, C K l, will also be smaller than C K F; hence the ray, L K, will be so reflected as to cross the principal axis at a point, l, between F, the principal focus, and C, the centre of curvature of the mirror.

The relation between the radiant point and point of convergence is easily determined. In the triangle L K l the radius K C bisects the angle L K l, hence by a well known principle of geometry :—

$$\mathrm{CL} : l\mathrm{C} = \mathrm{LK} : l\mathrm{K} \;\therefore\; \frac{\mathrm{CL}}{\mathrm{LK}} = \frac{l\mathrm{C}}{l\mathrm{K}}.$$

When the incident pencil is very small, L K = L A, and lK = lA, very nearly, hence we have,

$$\frac{\mathrm{CL}}{\mathrm{LA}} = \frac{l\mathrm{C}}{l\mathrm{A}} \text{ nearly.}$$

Let L A = u, lA = v, C A = radius of the mirror = r, and A F = the principal focal length = f. Then $f = \frac{r}{2}$, and by substituting these values, the above equation becomes

$$\frac{u-r}{u} = \frac{r-v}{v}. \text{ Dividing by } r \text{ we have } \frac{1}{r} - \frac{1}{u} = \frac{1}{v} - \frac{1}{r};$$

$$\text{Or, } \frac{1}{v} = \frac{2}{r} - \frac{1}{u} = \frac{1}{f} - \frac{1}{u}.$$

$$v = \frac{uf}{u-f} = \frac{ur}{2u-r}.$$

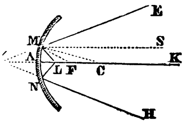
From this formula we may deduce the value of v, or the focus of reflected rays, whatever may be the point of divergence of the incident rays.

If the luminous point is removed to l, the reflected rays will meet at L. If the luminous point is placed at the centre of curvature, C, all the rays will fall perpendicularly upon the mirror, and be reflected back to the point C, from whence they came.

If the luminous point is situated between the centre of curvature and the principal focus, the conjugate focus will be removed beyond the centre of curvature, and become more and more distant as the luminous point approaches the principal focus. When the luminous point arrives at the principal focus, the conjugate focus will be removed to an infinite distance, or, in other words, the reflected rays will become parallel. While the radiant point has removed from C to F, the conjugate focus has removed from C, to an infinite distance.

429. **Converging rays.—Virtual focus.**—If the radiant point passes from the principal focus, F, towards the mirror, as in fig. 329, it is evident that the reflected rays will *diverge*, as though emanating from a point *l*, behind the mirror, called the *virtual focus*.

329

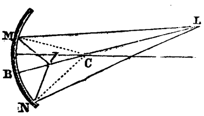


When the radiant point is near the principal focus, between it and the mirror, the virtual focus of the divergent reflected rays will be at a very great distance. As the radiant point continues to approach the mirror, the virtual focus also approaches it. While the radiant point passes from the principal focus to the mirror, the conjugate virtual focus, or point from which the reflected rays appear to diverge, passes from an infinite distance behind the mirror, to the surface of the mirror, or to the radiant point itself.

These propositions may be easily proved by giving to u appropriate values in the formula.

430. **Secondary axes.—Oblique pencils.**—If the luminous point, L, fig. 330, is not situated in the principal axis of the mirror, a line drawn from the radiant point through the centre of curvature, as L C B, will constitute a *secondary axis*, and the focus of the *oblique pencil* of rays diverging from L, will be found in this secondary axis. In the same manner we may draw secondary axes, and determine the foci, whether real or virtual, for any number of points in a luminous object.

330



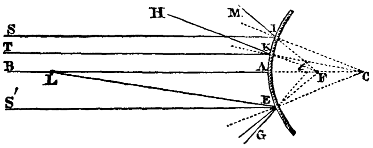
431. **Rule for conjugate foci of concave mirrors.**—*Multiply the distance of the radiant point from the mirror, by the radius of curvature, and divide this product by twice the distance of the radiant point, minus the radius of curvature of the mirror, and the quotient will be the distance of the conjugate focus from the mirror.*

If the quotient given by this rule is negative, or if twice the distance of the radiant point is less than the radius of curvature, the conjugate focus will be a virtual focus behind the mirror, and the reflected rays will diverge.

432. **Convex spherical mirrors.**—The effects attending the reflection of diverging, converging, or parallel rays of light by convex reflectors, are, in general, the opposite of the effects produced by concave reflectors. The foci of parallel and diverging rays of light,

reflected by a convex reflector, are at the same distance as for concave mirrors, but they are situated behind the reflector, and are, hence, only virtual foci. Light converging towards any point behind a convex mirror, more distant than the centre of curvature, will diverge, after reflection, from a virtual focus between the centre of curvature and the principal focus. Rays converging toward the principal, virtual focus, will be reflected parallel; but rays converging towards a point nearer to the mirror than the principal focus, will be reflected to a real focus in front of the convex reflector. These phenomena will be readily understood by an examination of fig. 331. The ray S I is reflected in the direction F I M; L E is reflected in the direction *l* E G, and reciprocally, G E is reflected in the direction E L, and M I in the direction I S.

331



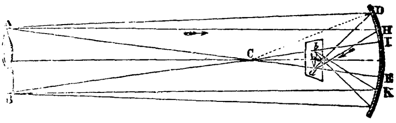
The formula for the convex mirror may be determined in the same manner as for the concave mirror, or we may deduce it at once from the formula for the concave mirror. Since the focus of parallel rays is behind the convex mirror, if we call the value of f for the concave mirror positive, it must be negative for the convex mirror. If therefore we insert $-f$ instead of f in the formula for the concave mirror, it will become for the convex mirror:—

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{u};$$

from which it appears that the value of v must also be negative when u is positive, that is, u and v are on opposite sides of the mirror. Now by putting $-v$ instead of v in the above formula, it will represent the absolute value of the focus of reflected rays reckoned on the back side of the convex mirror, and we have for the convex mirror, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} \therefore v = \frac{fu}{f+u}$.

433. **Images formed by concave mirrors.**—The principles already explained enable us to understand the formation of images by concave mirrors. Let A B, fig. 332, represent an object placed before a concave mirror, beyond its centre of curvature. The lines, A C and B C, drawn through the centre of curvature from the extremities of the object, are the secondary axes in which the extremities of the image, *a b*, will be formed, at a distance from the mirror equal to the conjugate foci for the extreme points of the object.

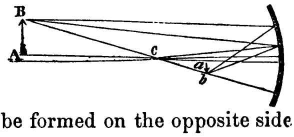
332



This image is real, inverted, smaller than the object, and placed between the centre of curvature and the principal focus.

If *a b* is regarded as the object, placed between the centre of curvature and the principal focus, an enlarged image will be formed at A B. If the object is placed at the principal focus, no image will be formed, because the rays from each point of the object will be reflected parallel to an axis drawn through the centre of curvature from the points where they originate.

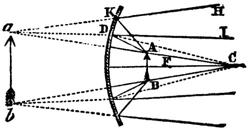
333



If the object, A B, is placed entirely on one side of the principal axis, as in fig. 333, it is evident that its image, *a b*, will be formed on the opposite side of the principal axis.

434. **Virtual images.**—If the object, A B, fig. 334, is placed between the mirror and the principal focus, the incident rays, A D, A K, take, after reflection, the directions, D I, K H, and their prolongations backward, form at *a*, a virtual image of the point A. In the same manner the image of B is formed at *b*, so that the image of A B is seen at *a b*. The image, in this case, is a virtual image, erect, and larger than the object.

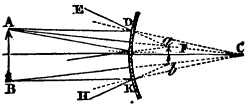
334



From the preceding illustrations, it is evident, that, when an object is placed before a concave mirror, more distant than the centre of curvature, the image is real, but inverted, and smaller than the object; as the object approaches the centre of curvature, the image enlarges and becomes equal to the object and coincides with it; when the object approaches nearer to the mirror than the centre of curvature, the image becomes larger than the object, and more distant from the mirror. When the object arrives at the principal focus, the image becomes infinitely distant, and disappears entirely: when the object approaches nearer to the mirror than the principal focus, an erect virtual image, larger than the object, appears behind the mirror.

435. **Formation of images by convex mirrors.**—Let A B, fig. 335, be an object placed before a convex mirror, at any distance whatever. If we draw the secondary axes, A C, B C, it follows, from what has been said (433) concerning the construction of foci of convex mirrors, that all the rays emitted from the point A, diverge after reflection, and that their prolongations backward converge to a point, *a*, which is a

335



virtual image of the point A. In the same manner, rays emitted from the point B, form a virtual image of that point in *b*.

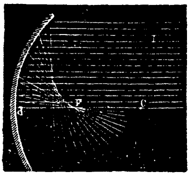
Whatever may be the position of an object before a convex mirror, the image is always formed behind the mirror, erect and smaller than the object.

436. **General rule for constructing images formed by mirrors.**—To construct the image of a point; 1. *Draw a secondary axis from that point;* 2. *Take from the given point any incident ray whatever; join the point of incidence and the centre of curvature of the mirror by a right line; this will be the perpendicular at that point, and will show the angle of incidence;* 3. *Draw from the point of incidence, on the other side of the perpendicular, a right line, which shall make with it an angle equal to the angle of incidence. This last line represents the reflected ray, which, being prolonged until it crosses the secondary axis, determines the place where the image of the given point is formed.* 4. *Determine the position of any other point in the object in the same manner.*

437. **Spherical aberration of mirrors.—Caustics.**—The rays from any point of an object, placed before a spherical mirror, concave or convex, do not converge sensibly to a single point, unless the aperture of the mirror is limited to 8° or 10°. If the aperture of the mirror is larger than this, the rays reflected from the borders of the mirror meet the axis nearer to the mirror than those which are reflected from portions of the mirror very near to the centre. There results, therefore, a want of clearness or distinctness in the image, which is designated *spherical aberration by reflection.*

The reflected rays cross each other successively, two and two, and their points of intersection form in space a brilliant surface, called *a caustic* by reflection, curving towards the axis, as shown in fig. 336, where C is the centre of curvature, F the principal focus, and *d* the centre of figure.

336

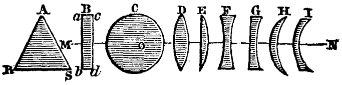


§ 3. Dioptrics, or Refraction at Regular Surfaces.

I. DEFINITIONS.

438. **Prisms and lenses,** are bodies having certain regular forms, sections of which are shown in fig. 337.

337



A *prism* is a solid having three or more plane faces variously inclined to each other as shown at A, fig. 337. The angle formed by the faces, A R, A S, is the refracting angle of the prism. For some purposes prisms are used having more than three plane faces.

A *lens* is a portion of some transparent substance, as glass or crystal, of which the surfaces are *generally* either both spherical, or one plane and the other spherical. The axis of a lens is the line joining the centres of the spherical surfaces when both are curved, and the line perpendicular to the plane surface which passes through the centre of the other surface when one side is plane. When the surfaces of lenses are of different kinds, they are named in reference to the side on which the light first falls.

If the figures C, D, E, F, G, H, I, were revolved around the axis, M N, they would severally describe the solid lenses they are intended to represent.

In explaining the properties of lenses, and showing the progress of light through them, we make use of such sections as are shown in the figure, for every plane passing through the axis has the same form, and what is true of one section is true of all.

A *plane glass*, B, is a plate of glass having two plane surfaces, *a b*, *c d*, parallel to each other.

A *sphere*, shown in section at C, has all parts of its surface equally distant from a certain point within, called the centre.

A *double convex lens*, D, is a solid bounded by two convex surfaces, which are generally spherical.

A *plano-convex lens*, E, has its first surface plane, and the other convex.

A *double concave lens*, F, has two concave surfaces opposite to each other.

A *plano-concave lens*, G, has its first surface plane, and the other concave.

A *meniscus*, shown at H, has one surface convex, and the other concave, their curvatures being such that the two surfaces meet, if continued. As this lens is thicker in the centre than at its edges, it may be regarded as a convex lens.

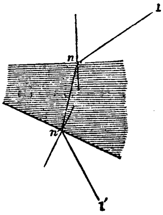
A *concavo-convex lens*, shown at I, has its first surface concave, and the other convex, but the curvatures are such that the surfaces, if continued, would never meet. As therefore the concavity exceeds the convexity, it may be regarded as a concave lens.

II. REFRACTION AT PLANE SURFACES.

439. **Refraction by prisms.**—If a ray of light, *l n*, fig. 338, falls obliquely upon a transparent medium, whose opposite plane faces are not parallel, the ray will be refracted at the first surface, and take

a direction nearer to the perpendicular. Now if the position of the incident ray, and the inclination of the faces of the medium, are as shown in the figure, it is obvious that the emergent ray, *n′ l′*, will be turned still further from its original direction. It is evident that any other position of the second refracting surface would cause a corresponding alteration in the direction of the emergent ray.

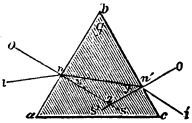
338



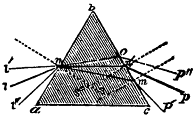
Let *a b c*, fig. 339, be a section of a triangular prism, *l n* a ray of light incident at *n*, O *n s* the perpendicular at that point, *n n′* will be the course of the ray of light through the prism, and *n′ l′* the emergent ray.

If the prism is more dense than the surrounding medium, the light will enter the prism, whatever may be the angle of incidence, but if the angle of incidence, *l n* O, diminishes, then the ray, *n n′*, will fall more obliquely upon the second surface of the prism, until it may arrive at an inclination where it will suffer total internal reflection.

339



340

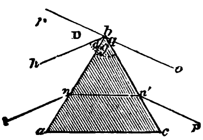


If the incident ray, *l n*, fig. 340, falls upon the prism at such an angle, that, after refraction, it takes the direction, *n n′*, parallel to *a c*, the base of the prism, the angles at which it enters and leaves the prism will be equal, and the deviation of the emergent ray from the course of the incident ray, will be the least possible. The ray, *l′ n*, will emerge in the direction *m p′*, and *l″ n* will emerge in the direction *o p″*, each deviating more from the direction of the incident ray than *n′ p* deviates.

If a candle is viewed through a triangular prism, on slowly turning the prism about its axis, a certain position will be found where the apparent position of the candle differs least from its real position. In whichever direction the prism is now turned, the difference between the real and apparent position of the candle increases.

440. **Method of determining the index of refraction.**—Let *l n n′ p*, fig. 341, be the direction of the ray of light when the deviation caused by the prism is a minimum. Draw *h b* parallel to the incident ray, *l n*, and *r b o* parallel to the emergent ray, *n′ p*. Let D = *h b r*, the entire deviation caused by the prism; *d* = *h b n*, the complement of the angle of incidence; *g* = *a b c*, the refracting angle of the prism; *q* = *n′ b o* = *c n′ p*, the complement of the angle of emergence. In this case the angles of incidence and emergence are equal, hence $d = q = 90° - i$, *i* being the angle of incidence, $D = 180° - d - g - q$; substituting in this equation the values of *d* and *q*, we have $D = 2i - g$, and $i = \frac{1}{2}(D + g)$. Let *x* and *y*, as in fig. 339, represent the angles formed with the perpendiculars by the ray traversing the prism, $x + y = g$, and when the angles of incidence and emergence are equal, $x = y$. If *n* equals the index of refraction, we shall have:—

341



$$n = \frac{\sin. i}{\sin. x}, \quad \text{or,} \quad n = \frac{\sin. \frac{1}{2}(D + g)}{\sin. \frac{1}{2} g}.$$

Now when the angle of minimum deviation and the refracting angle of the prism are measured, this formula enables us at once to determine the index of refraction. In this manner the index of refraction of any substance is easily determined.

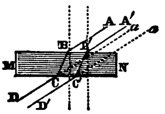
441. Plane glass.—A ray of light passing through a plane glass, or any other medium of uniform density bounded by parallel faces, will have the emergent ray parallel to the incident ray. Parallel rays of light passing through plane glass are parallel after emergence, and the emergent rays are parallel to the incident rays.

If the two surfaces of the transparent medium are parallel, it is evident that the ray of light traversing the medium will make equal angles with the perpendicular at both surfaces. Let I be the angle of incidence, R the angle of refraction at the first surface, and also the internal angle of incidence on the second surface, and E the angle of emergence. Then, if n represents the index of refraction, we shall have:—

$$\text{Sin. } I = n \text{ sin. } R = \text{sin. } E \quad \therefore \quad I = E:$$

Or the angles of incidence and emergence are equal, and the incident ray is parallel to the emergent ray. The same is true for any number of rays; hence also parallel incident rays will, after passing through the glass, emerge parallel.

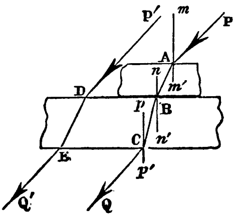
342



Let M N, fig. 342, be a plane glass, or any medium bounded by parallel surfaces, the rays A B, A′ B′, will be refracted towards the perpendicular, on entering the medium, and emerging at C, C,′ they will be refracted from the perpendicular, and take the directions, C D, C′ D′, parallel to each other, and parallel to their directions before entering the medium. The displacement, A a, A′ a', is the lateral aberration produced by transmission through a homogeneous medium bounded by parallel surfaces. The amount of lateral aberration increases with the thickness of the medium, and it also increases with the obliquity of the incident rays.

442. Light passing through parallel strata of different media.—It is found by experiment that if a ray of light passes through a series of plates of dense media, all the refracting surfaces being parallel planes, that the emergent ray is parallel to the incident ray. It therefore follows, that the direction of the ray in passing through any one of the plates is parallel to the course it would have taken if it had entered the plate directly, or if that plate had been the first in the series.

343



Let P A B C Q, fig. 343, be the course of a ray of light passing through two parallel strata of dense media, the second medium being more dense than the first, and P′ D E Q′ the course of a ray passing through the second medium without entering the first; if P A is parallel to P′ D, C Q will be parallel to E Q′, and also B C will be parallel to D E. We may consider the ray of light as passing in the opposite direction, and make n'' the index of refraction for the medium traversed by the ray C B, then sin. Q C $p' = n''$ sin. B C p, hence the angle B C p depends only on the direction of the emergent ray C Q, parallel to the incident ray P A, and upon the index of refraction, n'', of the lower medium. Let m A m', n B n', p C p', be perpendicular to the refracting surfaces at A, B, and C, and let n be the absolute index of refraction for the first medium, n'' that of the second medium, and n' the index of refraction for light passing from the first medium to the second:—

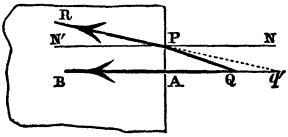
Then, $$n = \frac{\text{sin. PA}m}{\text{sin. BA}m'}; \; n'' = \frac{\text{sin. QC}p'}{\text{sin. BC}p};$$

$$n' = \frac{\text{sin. AB}n}{\text{sin. CB}n'} = \frac{\text{sin. BA}m'}{\text{sin. BC}p} = \frac{\text{sin. BA}m'}{\text{sin. PA}m} \times \frac{\text{sin. QC}p'}{\text{sin. BC}p} = \frac{n''}{n}.$$

Hence: *The index of refraction for light passing from one medium to another, is equal to the index of refraction of the second medium divided by the index of refraction of the first medium.*

443. **Pencils of light refracted at plane surfaces.**—When a pencil of light falls upon a plane surface of any dense medium, it is so changed by refraction that a diverging pencil is made to diverge from a focus without the medium more distant from the dense medium than before it entered it; and a converging pencil is made to converge to a focus within the dense medium more distant from its surface than before.

344



Let Q, fig. 344, be the focus of incident rays, and Q A B the ray which enters the medium perpendicularly to its surface, suffering no deviation. Let Q P be any oblique ray meeting the surface at P; let N P N′ be drawn perpendicular to the refracting surface at P, it will also be parallel to Q A B; let P R be the refracted ray which being extended backward meets the line A Q at q'. The angle of incidence QPN = PQA, and the angle of refraction RPN′ = Pq'A.

Also the index of refraction, $$n = \frac{\text{sin. QPN}}{\text{sin. RPN}'} = \frac{\text{sin. PQA}}{\text{sin. P}q'\text{A}}.$$

$$\text{Sin. PQA} = \frac{\text{PA}}{\text{PQ}}; \; \text{sin. P}q'\text{A} = \frac{\text{PA}}{\text{P}q'} \quad \therefore \quad n = \frac{\text{P}q'}{\text{PQ}}.$$

If the pencil is very small, PQ = AQ, and Pq' = Aq' nearly, hence Aq' = n.AQ. If we let AQ = u, and Aq' = u', then $u' = nu$, which determines the point q', from which the pencil diverges after refraction.

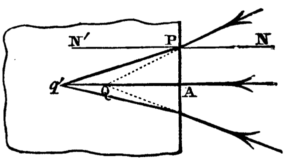
When the pencil is large, and P is so far from A that we cannot consider QP = QA, let i = the angle of incidence, i' = angle of refraction:—

$$\text{We have QP} = \frac{u}{\text{cos. } i}; \; q'\text{P} = \frac{u'}{\text{cos. } i'};$$

$$\text{And since } q'\text{P} = n.\text{QP} \;\; \therefore \;\; u' = n.\frac{\text{cos. } i'}{\text{cos. } i}\, u.$$

Since the cosine of i' is greater than cosine of i, this last value of [illegible] is greater than nu, the first value; this shows that a pencil of light suffers aberration when refracted at a plane surface. The formula also shows that u' is greater than u, or that the focus of the pencil after refraction is more distant than the focus of the incident rays. If the pencil of incident rays converges to a point, Q, within the dense medium, as in fig. 345, the pencil of the refracted rays will converge to point q'. Solving the triangle Q P q', we should find the same result as before; or $Aq' = n.AQ$.

345



Therefore: *When a pencil of light is refracted at a plane surface, the focus of the refracted rays is on the same side of the refracting surface as the focus of incident rays, and at a distance equal to the distance of the focus of incident rays multiplied by the index of refraction.*

If the rays had been proceeding from the dense medium to a rarer medium, as from q', fig. 345, or towards q', fig. 344, then the focus of refracted rays would be at Q, or nearer to the refracting surface than the focus of incident rays.

If the rays proceed from a dense to a rarer medium, and if n still represent the index of refraction for light entering the dense medium, the index of refraction for light passing from the dense to the rare medium will be $n' = \frac{1}{n}$.

The index of refraction for light passing from air into water is $n = \frac{4}{3}$, and hence $n' = \frac{1}{n} = \frac{3}{4}$, for light passing from water into air.

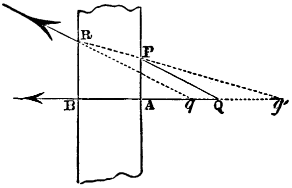
If therefore u represents the actual distance of an object below the surface of water, and u' its apparent distance, $u' = n'.u = \frac{3}{4}u$, that is, the apparent distance below the surface of the water is only three-fourths of the real distance; or water is a third deeper than it appears to be. As every point in an object appears elevated one-fourth as much as its distance below the surface of the water, a pole or cane thrust obliquely into the water appears bent, or broken (406), just at the surface of the water.

It follows also, from the preceding considerations, that an object immersed in water, or any other transparent dense fluid, appears larger than when seen in the air.

As the atmosphere diminishes in density very rapidly above the earth's surface, a man upon the top of a steeple or tower looks much smaller than when seen at an equal distance on level ground; and an object at the foot of the tower, will, for the same reason, appear larger when viewed from the top than if placed at the top of the tower and viewed from below.

444. **Pencils of light transmitted through plane glass.**—When a pencil of light is transmitted through a plane glass, the focus of the emergent rays is removed from the focus of the incident rays, in the direction that the light is moving, a distance equal to the quotient arising from dividing the thickness of the glass by the index of refraction, and multiplying the quotient by the index of refraction diminished by unity.

346



Let a pencil of light fall upon a plane glass, fig. 346, so that Q A shall be perpendicular to the surface of the glass, and Q P an oblique ray, Q A will be transmitted in the line Q A B without deviation, and Q P will be refracted in the direction q' P R, and emerge in the direction q R, q being the focus of the emergent rays. Let Q A $= u$, q' A $= u'$ B $q = v$, and A B $= t$.

By the formula already demonstrated (443), if the pencil is small $u' = nu$; and if the pencil had entered the other side of the plate, converging to q, we should have,

$$\mathrm{B}q' = n.\mathrm{B}q\ ; \quad \text{or,}\ t + u' = nv = t + nu,$$

Hence $v = u + \frac{t}{n}$, by which the position of q is determined. The displacement of the focus $\mathrm{Q}q = \mathrm{BQ} - \mathrm{B}q = u + t - v = t\left(1 - \frac{1}{n}\right) = \frac{n-1}{n}t.$

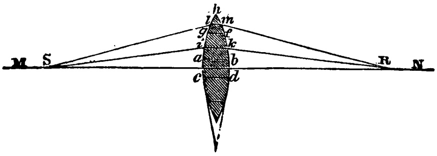
Or the rays diverge, after emerging from the glass, from a point nearer to the glass than the focus of the incident rays.

If we suppose the rays to proceed in the opposite direction, we shall have the case of a converging pencil, and the focus of the rays after emergence will be more distant from the first surface of the glass than before. In both cases the focus of the rays is removed in the same direction that the light is proceeding. If we take the case of plate glass, for which $n = \frac{3}{2}$, the distance to which the focus is removed is equal to $\frac{1}{3}$ the thickness of the glass.

III. REFRACTION AT CURVED SURFACES.

445. **Principles determining the foci of lenses.**—A double convex lens may be regarded as composed of a number of segments of prisms, the faces of each prism more inclined as we proceed from the centre to the borders of the lens, as shown in fig. 347. The central portion, $a\,b\,c\,d$, may be regarded as a plane glass, having its faces, $a\,c$, $b\,d$, parallel, $a\,g\,f\,b$ has its face, $a\,g$, inclined towards $f\,b$, and the triangular prism, $g\,h\,f$, has

347

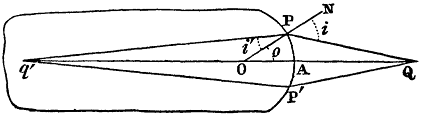


its sides still more inclined. Now since the deviation of any ray passing through a prism increases as the inclination of the two faces of the prism increases, *s l* will deviate more than *s i*, and if the form of each prism is properly adjusted to its distance from the axis, M N, the rays, *s l* and *s i*, or any number of rays, may be made to meet at a common point, R, in the axis M N.

If the segments of prisms, of which we suppose such a lens to be composed, are made sufficiently small, so that each face shall receive but a single ray of light, the sides of the successive prisms will form a regular curve, which, if the lens be of small diameter, will correspond almost exactly with a segment of a sphere.

On account of the great difficulty of grinding lenses with any other than spherical or plane surfaces, other forms are seldom employed, and require no discussion in an elementary work.

446. **Small pencils of light refracted at a spherical surface** have the position of their foci changed.



348

Let P A P′, fig. 348, be a convex spherical surface of a dense medium, O being the centre of curvature of the dense medium, Q the focus of the incident rays, and q' the focus of the refracted rays. Let Q A q' be the ray which enters the medium perpendicular to its surface, and Q P another ray which is refracted in P q', so as to meet Q A continued in q'. Draw O P N perpendicular to the curved surface through the centre of curvature and point of incidence.

We shall then have the angle of incidence i = Q P N, the angle of refraction i' = q' P O. Let P O A = o, then from the triangle, Q O P, we have:—

$$\text{Sin. } i : \text{sin. } o = \text{Q O} : \text{Q P},$$

and from the triangle, q' O P, we have:—

$$\text{Sin. } o : \text{sin. } i' = q' \text{P} : q' \text{O}.$$

By compounding these proportions we have:—

$$\frac{\text{sin. } i}{\text{sin. } i'} = n = \frac{\text{Q O}}{\text{Q P}} \times \frac{q' \text{P}}{q' \text{O}}; \text{ or, } \frac{\text{Q O}}{\text{Q P}} = n . \frac{q' \text{O}}{q' \text{P}}.$$

Since the pencil of rays is very small, we may consider Q P = Q A, and q' P = q' A nearly. Let Q A = u, q' A = u', A O = r, then the last formula becomes $\frac{u + r}{u} = n . \frac{u' - r}{u'}$, which may be reduced to $\frac{n}{u'} = \frac{n - 1}{r} - \frac{1}{u}$.

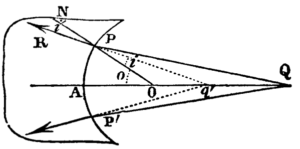
If we suppose Q to be situated at an infinite distance from A, the incident rays will be parallel, and we shall have u = infinity, and:—

$$\frac{n}{u'} = \frac{n - 1}{r}; \quad \therefore \quad u' = \frac{nr}{n - 1} = f'.$$

Making this value of $u' = f'$, the general formula will be $\frac{n}{u'} = \frac{n}{f'} - \frac{1}{u}$.

Refraction at a concave surface.—If the surface of the dense medium is concave, as shown in fig. 349, let Q, as before, be the focus of the incident rays, q' the virtual focus of the refracted rays, and O the centre of curvature. Then the angle of incidence i = Q P O; the angle of refraction i' = R P N = q' P O; let the angle P O A = o. In the triangle Q P O we have:—

349



$$\text{Sin. } i : \text{sin. } o = \text{Q O} : \text{Q P},$$

and from the triangle q' P O we have:—

$$\text{Sin. } o : \text{sin. } i' = q' \text{ P} : q' \text{ O}.$$

Combining these proportions we have:—

$$\frac{\text{sin. } i}{\text{sin. } i'} = n = \frac{\text{Q O}}{\text{Q P}} \times \frac{q' \text{P}}{q' \text{O}}; \quad \text{or,} \quad \frac{\text{Q O}}{\text{Q P}} = n . \frac{q' \text{ O}}{q' \text{ P}}.$$

The pencil being small, we may put Q A = Q P, and q' A = q P nearly, and putting Q A = u, q' A = u', and O A = r, we have:—

$$\frac{\text{Q O}}{\text{Q A}} = n . \frac{q' \text{ O}}{q' \text{ A}}; \quad \text{or,} \quad \frac{u - r}{u} = n . \frac{u' - r}{u'}.$$

From this we obtain $\frac{n}{u'} = \frac{n - 1}{r} + \frac{1}{u} = \frac{n}{f'} + \frac{1}{u}$; in which f' represents the value u' when u = infinity, or the incident rays are parallel.

We may take the general formula for refraction at a convex surface of a dense medium, and, by applying proper values to the letters, deduce formulæ for all other cases, whether the medium be dense or rare, and the refracting surface convex or concave.

In the formula $\frac{n}{u'} = \frac{n - 1}{r} - \frac{1}{u}$,

we have supposed the value of u measured from the convex surface of the dense medium, in the direction A Q in the rarer medium. Calling this direction positive, if the focus of incident rays were taken in the dense medium u should be considered negative; u' has been reckoned positive when measured in the dense medium, therefore if it is measured in the rare medium, as in the example of the concave dense surface, it should be reckoned negative; we also reckon r positive when it lies in the dense medium, and negative when it lies in the rare medium.

Therefore, to apply the general formula for a convex surface of a dense medium to the case where the incident rays converge to a focus in the dense medium, we make u negative, and the formula becomes $\frac{n}{u'} = \frac{-n - 1}{r} + \frac{1}{u}$.

To adapt the formula to the case of diverging rays refracted at the concave spherical surface of a dense medium, we make r negative, and the formula becomes $\frac{n}{u'} = -\frac{n-1}{r} - \frac{1}{u}$, which shows that u' is essentially negative, or that it lies on the same side of the refracting surface as the centre of curvature.

If then we change the sign of u' in the formula, it becomes $\frac{n}{u'} = \frac{n-1}{r} + \frac{1}{u}$, in which u' represents the distance of the focus of refracted rays measured in the direction of the rarer medium. This formula is the same as was deduced from fig. 349, where the same conditions were applied to the analysis of the diagram.

To apply the formula to the case of rays of light proceeding from a dense to a rarer medium, we have but to let u and u' change places in the formula, and change the sign of r. Making these changes in the general formula for a convex surface of a dense medium, the formula for diverging rays refracted at a convex surface of a rare medium will become :—

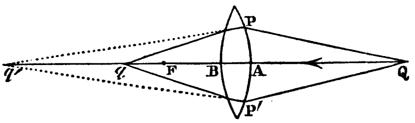
$$\frac{n}{u} = -\frac{n-1}{r} - \frac{1}{u'}, \text{ or } \frac{1}{u'} = -\frac{n}{u} - \frac{n-1}{r}.$$

The formula for diverging rays, refracted at a concave surface of a rare medium (by similar changes), will become $\frac{1}{u'} = \frac{n}{u} - \frac{n-1}{r}$.

The formula for converging rays issuing from a convex surface of a dense medium, or, which is the same thing, entering a concave surface of a rare medium, will become $-\frac{n}{u'} = \frac{n-1}{r} - \frac{1}{v}$; u' being the focus of rays traversing the dense medium, and v the focus of rays issuing from a dense medium or entering a rare medium.

447. Action of a double convex lens upon small pencils of light.—Let P A P′ B, fig. 350, be a double convex lens, of which r is the radius of the first surface, and s the radius of the second surface. Let Q be the focus of the incident rays, q' the focus of the rays after refraction at the first surface of the lens, and q the focus of the rays as they emerge from the second surface of the lens. Also, let $QA = u$, $Aq' = u'$, $Bq = v$, and the thickness of the lens $AB = t$.

350



After refraction at the first surface, we shall have (446) $\frac{n}{u'} = \frac{n-1}{r} - \frac{1}{u}$.

By refraction at the second surface, we have (446) $-\frac{n}{Bq'} = \frac{n-1}{s} - \frac{1}{v}$.

If the thickness of the lens is so small that when compared with Bq' it may be neglected, we make $Bq' = Aq' = u'$ nearly; adding the two preceding equations,

$$0 = \frac{n-1}{r} + \frac{n-1}{s} - \frac{1}{u} - \frac{1}{v}. \quad \text{Or } \frac{1}{v} = (n-1)\left(\frac{1}{r} + \frac{1}{s}\right) - \frac{1}{u}.$$

For parallel rays, $u =$ infinity, $\frac{1}{u} = 0$, and $\frac{1}{v} = (n - 1)\left(\frac{1}{r} + \frac{1}{s}\right)$.

Let $f =$ this value of v when the incident rays are parallel, and the general formula for a double convex lens becomes $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$.

If t is small, but not small enough to be neglected, we shall have :—

$$-\frac{n}{Bq'} = -\frac{n}{u' - t} = -\frac{n}{u'} - \frac{nt}{u'^2} + \left(\frac{nt^2}{u'^2} \div (u' - t)\right);$$

but as t^2 must be very small compared with u'^2, the quantity contained in the brackets may be neglected; hence,

$$-\frac{n}{u'} - \frac{nt}{u'^2} = \frac{n - 1}{s} - \frac{1}{v}; \quad \frac{n}{u'} = \frac{n - 1}{r} - \frac{1}{u}.$$

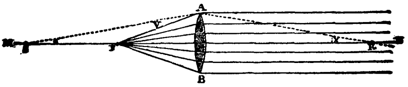
Adding and transposing, $\frac{1}{v} = (n - 1)\left(\frac{1}{r} + \frac{1}{s}\right) - \frac{1}{u} + \frac{nt}{u'^2}$;

$$\frac{n}{u'} = \frac{n - 1}{r} - \frac{1}{u} \therefore \frac{nt}{u'^2} = \frac{t}{n}\left(\frac{n - 1}{r} - \frac{1}{u}\right)^2:$$

$$\text{And } \frac{1}{v} = \frac{1}{f} - \frac{1}{u} + \frac{t}{n}\left(\frac{n - 1}{r} - \frac{1}{u}\right)^2.$$

Conclusions deduced.—Analysis.—1. *Parallel rays* of light falling upon a convex lens, A B, fig. 351, will be refracted to some point, as F, on the other side of the lens. The distance of the focus, F, from the lens, will depend upon the amount of curvature, and also upon the refractive power of the substance, of which the lens is composed. If the two surfaces of the lens have the same curvature, and the index of refraction, as for ordinary glass, is one and a half, the focus of parallel rays, called the *principal focus*, will be at a distance from the lens equal to the *radius of curvature* of either surface of the lens.

351



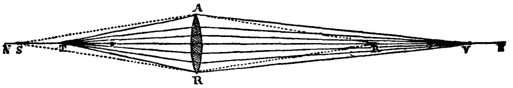
In the formula $\frac{1}{r} = (n - 1)\left(\frac{1}{r} + \frac{1}{s}\right) - \frac{1}{u}$, let $n = \frac{3}{2}$, the index of refraction for ordinary glass, then since the incident rays are supposed to be parallel, $u = \infty$, and $\frac{1}{u} = 0$, and if the two surfaces of the lens have the same curvature, $r = s$, and the formula becomes $\frac{1}{v} = \frac{1}{2}\left(\frac{1}{r} + \frac{1}{r}\right) = \frac{1}{r} \therefore v = r$, or F the focus of parallel rays is at a distance from the lens equal to the radius of curvature.

2. *Diverging rays.*—If the rays falling upon the lens come from a point, R, at a distance from the lens equal to twice the principal focus, they will converge to a point, S, at an equal distance on the other side of the lens.

It will be easily seen from fig. 351, that the angles, X and Z, are equal to each other (being the alternate angles formed by the straight line, R A, meeting two parallel lines), and also that the angles, X and O, are equal. In the triangle, A S F, the sides, F A and F S, are equal, hence the angles, O and Y, are equal, and Y equals Z, therefore if the incident ray is bent inward to a distance represented by the angle, Z, the refracted ray must be bent outward by an equal angle, Y, by which means the radiant point is removed from F, the principal focus of parallel rays, to S, which is at double the distance of F.

The formula shows the same thing. Making $u = 2r$, we find $v = 2r$. If the radiant point is taken more distant than R, as at V, fig. 352, the conjugate focus will be removed from S, to some point, T, between S and the principal focus.

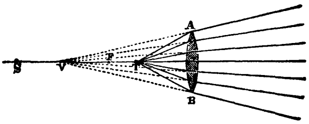
352



The formula will then give $\frac{1}{v} = \frac{1}{r} - \frac{1}{u > 2r}$; $\frac{1}{v} > \frac{1}{2r}$; or $v < 2r$.

3. *Converging rays.*—If rays of light falling upon the lens, A B, fig. 353, converge towards a point, V, before refraction, they will converge, after refraction, towards a point, T, between the principal focus, F, and the lens. Conversely, if rays of light diverge from a point, T, between the lens and its principal focus, they will diverge after passing through the lens, from a virtual focus, V, more distant than the principal focus.

353



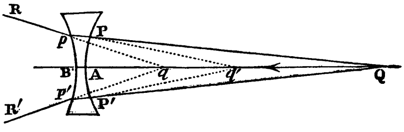
In the first case u becomes essentially negative, and with the same values of n, r, and s the formula becomes $\frac{1}{v} = \frac{1}{r} + \frac{1}{u}$; and as $\frac{1}{v}$ is greater than $\frac{1}{f}$, v must be less than f, hence T lies between the principal focus and the lens.

4. *Plano-convex lenses.*—The action of a plano-convex lens is in general the same as that of the double convex lens, but its foci are at double the distance, the principal focus being at a distance equal to twice the radius of the curved surface.

To adopt the formula to this case, we make $n = \frac{3}{2}$, and $r = \infty$, hence $\frac{1}{v} = \frac{1}{2s} - \frac{1}{u}$. If the rays are parallel, $\frac{1}{u} = 0$, and v, or $f = 2s$.

448. Action of a double concave lens upon small pencils of light.—Let P A P′ p' B p, fig. 354, be a double concave lens of a dense medium, r being the radius of the first surface and s the radius of the second surface. Let Q be the focus of the incident rays, q' the focus of the rays after refraction at the first surface, and q the focus of the emergent rays p R, p' R′. Let Q A $= u$, A $q' = u'$, B $q = v$, and A B $= t$. According to the formula for refraction at a concave dense surface (446):—

354



$$\frac{n}{u'} = \frac{n-1}{r} + \frac{1}{u};$$

and by the formula for rays emerging from a concave dense surface,

$$\frac{n}{\mathrm{B}q'} = -\frac{n-1}{s}. + \frac{1}{v}.$$

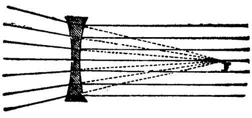
If the thickness of the lens is so small that when compared with B q' it may be neglected, and that we may consider B $q' =$ A $q' = u'$, combining these two equations we have $\frac{1}{v} = (n-1)\left(\frac{1}{r} + \frac{1}{s}\right) + \frac{1}{u}$. Or, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$.

If the thickness of the lens is too great to be neglected, we find by the same method as for a convex lens $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} - \frac{t}{n}\left(\frac{n}{u'^2}\right)^2$.

This formula for the double concave lens may be deduced directly from the formula for the double convex lens, by substituting in that formula for r and s, $-r$ and $-s$, and as the value of v would then be negative, changing that sign also when its positive value is reckoned on the same side as u.

Conclusions deduced from analysis.—A concave lens produces, upon rays of light transmitted through it, an effect the opposite of that produced by a convex lens.

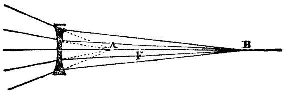
355



1. *Parallel rays* of light, transmitted through a double concave lens, diverge from a virtual focus in front of the lens, as shown in fig. 355; the virtual focus being at the centre of the sphere of which the first surface forms a part. This is its principal focus.

2. *Diverging rays.*—If the radiant point is more distant than the principal focus, as at B, fig. 356, the virtual conjugate focus, A, will be between the principal focus, F, and the surface of the lens, and the rays will diverge after refraction.

356



3. *Converging rays*, transmitted through a concave lens, will be ren-

dered less convergent, parallel, or divergent, depending upon the distance of the point towards which they converge before entering the lens.

The above propositions are easily proved by reference to the formula for a double concave lens, $\frac{1}{v} = (n - 1)\left(\frac{1}{r} + \frac{1}{s}\right) + \frac{1}{u}$.

449. **Rules for determining the foci of lenses.**—When lenses are made of glass whose refractive index is one and a half, their foci may be determined by the following rules:—

Rule for the Principal Focus.

Divide twice the product of the radii by their difference, for the meniscus and concavo-convex lenses, and by their sum, for the double convex and double concave lenses. The quotient will give the focus for parallel rays. The focus of parallel rays, or principal focus, of the plano-convex or plano-concave lens, is double the radius of curvature.

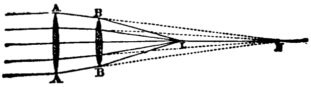
Rule for the Conjugate Focus, when the Focus of the Incident Rays is given.

Multiply the length of the principal focus, with its proper sign, by the focus of the incident rays, and divide the product by the difference between the principal focus and the focus of incident rays, and the quotient will be equal to the conjugate focus.

If the distance of the focus of incident rays is less than the principal focus, the value of the conjugate focus will be positive, and it will lie on the same side of the lens as the focus of incident rays; but if the value of the focus of incident rays is greater than the principal focus, the value of the conjugate focus will be negative, and the focus of refracted rays will lie on the other side of the lens.

450. **Combined lenses.**—If two convex lenses, A A, B B, are placed near together, as in fig. 357, their combined focus will be shorter than that of either lens used alone.

357



Let parallel rays be refracted by the first lens, A A, to a focus at N; represent the distance of this point from the first lens by f', and let the distance between the lenses be represented by a, let f'' represent the corresponding focal length of the second lens for parallel rays, and f the distance of the focus L from the second lens. In the general formula $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, considered with reference to the second lens, $u = -(f' - a)$, and f becomes f'', $v = f$, and we have:—

$$\frac{1}{f} = \frac{1}{f''} + \frac{1}{f' - a} \quad \therefore \quad f = \frac{f'' \times (f' - a)}{f'' + f' - a}$$

If the distance between the lenses is nothing, then for the focus of parallel rays, $f = \frac{f'' \times f'}{f'' + f'}$. For rays not parallel, the formula will be:—

$$\frac{1}{v} = \frac{1}{f''} + \frac{1}{v' - a} = \frac{1}{f''} + \frac{u - f'}{f'u - a\,(u - f')}.$$

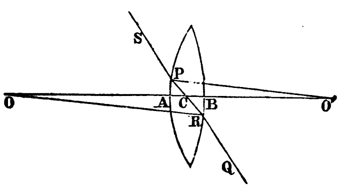
When the lenses are in contact, $\frac{1}{v} = \frac{1}{f'} + \frac{1}{f''} - \frac{1}{u}$. For any number of lenses in contact, $\frac{1}{v} = \frac{1}{f'} + \frac{1}{f''} + \frac{1}{f'''} + \frac{1}{f^{iv}} + \&c., - \frac{1}{u} = \Sigma\left(\frac{1}{f}\right) - \frac{1}{u}$.

451. **Oblique pencils**, when transmitted through lenses, have their foci in secondary axes, and their foci are determined by the same rules as the foci of direct pencils in the principal axis.

It has been shown, in § 439, that a ray of light transmitted through a prism in a direction parallel to its base, suffers the least deviation possible; hence in every other position the deviation is increased. From this principle it follows that the foci of oblique pencils transmitted through lenses will be somewhat shorter than the foci of direct pencils. This fact requires consideration in the formation of the images of large objects. (See § 455.)

452. **The optical centre of a lens** is a point so situated that every ray of light passing through it will undergo equal and opposite refraction on entering and leaving the lens. It will, therefore, be found where a line joining the extremities of two parallel radii of the opposite surfaces cuts the axis of the lens.

358



Let S P R Q, fig. 358, be a ray of light passing through a double convex lens so that the radii O′P, O R, drawn from the points of incidence and emergence are parallel. Let C be the point where this ray intersects the axis of the lens. The triangles O′ C P, O C R, are similar, hence:—

$$O'P : O'C = OR : OC;$$
$$O'P - O'C : OR - OC = O'P : OR;$$
$$AC : BC = O'P : OR;$$
$$AC + BC : O'P + OR = AC : O'P.$$

Putting $O'P = r$, $OR = s$, and $AB = t$.

$$AC : t = r : r + s \quad \therefore \quad AC = \frac{tr}{r + s}, \quad BC = \frac{ts}{r + s}.$$

If the lens were double concave, r and s both become negative, but the values of A C and B C remain unchanged. Since these values are both positive and constant, whatever may be the positions of the points P and R, the optical centre of a double convex or double concave lens will be a fixed point in the lens. For a plano-convex or plano-concave lens, $r = \infty$, $AC = t$, $BC = 0$.

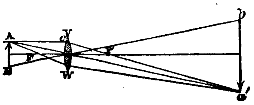
The optical centre will be at the intersection of the axis with the curved surface. For a meniscus, either r or s will be negative, and the formulæ show that in that case the optical centre will be situated without the lens at a point depending upon the relative values of the two radii.

All rays of light passing through the optical centre emerge from the lens parallel to the incident rays. The position, form, and foci of all pencils of light passing through a lens are determined by their relation to some line, or secondary axis, passing through the optical centre of the lens, whether any ray of light from the radiant point actually passes through that centre or not.

453. **Images formed by lenses.**—If an object is placed before a convex lens at a greater distance than the principal focus, an image of the object will be formed on the other side of the lens.

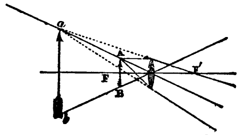
If from the extremities of the object A B, fig. 359, the secondary axes, A *a*, B *b*, are drawn through the optical centre of the lens, the image will be formed between these axes prolonged, at a distance equal to the conjugate focus of the lens, estimated separately for every point of the object. If the object is placed beyond the principal focus, and at less than twice this distance, the image will be more distant and larger than the object. If the object recedes from the lens, the image will approach it. When the object is removed from the lens, more than twice the principal focus, the image will be smaller than the object, and it will gradually approach the lens, and diminish in size as the object recedes. The image can never approach nearer to the lens than the principal focus. The linear magnitude of the image as compared with the object will be proportional to their respective distances from the lens.

359



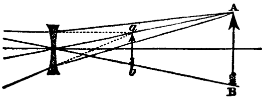
If the object is placed nearer to the lens than the principal focus, as A B, fig. 360, the rays will diverge after passing the lens, and a *virtual image*, *a b*, will be formed on the same side of the lens as the object. The virtual image formed by a convex lens is always larger than the object.

360



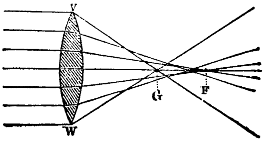
If an object, A B, fig. 361, is placed before a concave lens, the rays from every point of the object will diverge after refraction more than they did before entering the lens; consequently a *virtual image*, smaller than the object, will be formed on the same side of the lens. The size of the virtual image will be in proportion to its distance from the lens.

361



454. **Spherical aberration of lenses.**—It has been assumed that spherical lenses bring rays of light issuing from a point to a sensible focus. For many purposes, however, greater accuracy is required, and it becomes necessary to consider the imperfections of spherical lenses.

362



If the diameter of the lens V W, fig. 362, is large in proportion to its radius of curvature, rays of parallel light will not be brought to an accurate focus, but while the central rays cross the axis at F, the extreme rays will intersect the axis at G, and intermediate rays will intersect the axis at every possible point between F and G. The distance, F G, is called the *longitudinal spherical aberration* of the lens.

For lenses of small aperture, the aberration is nearly in proportion to the square of the angular aperture of the lens; but for lenses of larger aperture, the aberration increases more rapidly than would be required by this proportion. If the length of the principal focus be taken as unity, the longitudinal aberration for lenses of different angular apertures will be as follows:—

For 15° the aberration will be 0·025,
" 22° " " " " 0·062,
" 30° " " " " 0·150,
" 45° " " " " 0·375.

This effect of spherical lenses causes images to be formed at every point between F and G, the rays going from each image, more or less interfering with the distinctness of all the others.

The amount of spherical aberration depends also on the form and position of lenses. If $n =$ index of refraction, $r =$ the radius of the anterior surface, and $R =$ the radius of the posterior surface, then for parallel rays, the form of least aberration will be expressed by the following equation:—

$$\frac{r}{R} = \frac{4 + n - 2n^2}{2n^2 + n},$$

If $n = 1\frac{1}{2}$, the form of least aberration will be a lens whose surfaces have their radii in the proportion of 1 to 6, the side of deeper curvature being towards parallel rays. If the spherical aberration of such a lens, in its best position, is taken as unity, the aberration of other lenses will be as follows:—

Plano-convex with plane surface towards distant objects, 4·2.

Plano-convex with convex surface towards distant objects, 1·081.

Plano-concave the same as plano-convex.

Double convex or double concave with both faces of the same curvature, the aberration will be 1·567.

The spherical aberration of a convex lens is called *positive*, and the aberration of a concave lens is called negative because it is in an opposite direction from that produced by a convex lens.

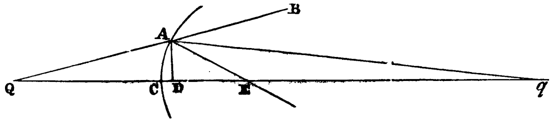
Accurate estimate of spherical aberration.*—To estimate with accuracy the amount of spherical aberration in any given case, it is necessary to calculate the exact course of a ray which falls upon the border of the lens.

---

* Microscopical Journal, Vol. VIII. p. 21.

Let A C, fig. 363, be a section of a curved refracting surface in the plane of refraction, Q C q being its axis. The refractive index $= n$. Here the distance

363



Q C of the radiant point from the refracting surface is given. Also C D and D A co-ordinates of the point A. Hence, also the normal A E and sub-normal D E may be found. Let A q be the refracted ray required, cutting Q C q in q.

$$\text{Now sin. incidence is to sin. } A\,E\,C = Q\,E : Q\,A.$$

$$\text{Sin. E is to sin. refraction} = q\,A : q\,E.$$

$$\therefore \text{Sin. incidence : sin. refraction} = m : 1 = \frac{q\,A}{q\,E} : \frac{Q\,A}{Q\,E}.$$

$$\therefore \frac{q\,A}{q\,E} = \frac{m.Q\,A}{Q\,E} = c, \quad \text{(a known quantity).}$$

$$\therefore q\,A^2 = c^2.q\,E^2.$$

$$i.\,e., \quad q\,E^2 + E\,A^2 + 2\,q\,E.E\,D = c^2.q\,E^2.$$

$$\therefore (c^2 - 1)\,q\,E^2 - 2\,E\,D.q\,E = E\,A^2.$$

$$\therefore q\,E^2 - \frac{2\,E\,D}{c^2 - 1}.q\,E = \frac{E\,A^2}{c^2 - 1}.$$

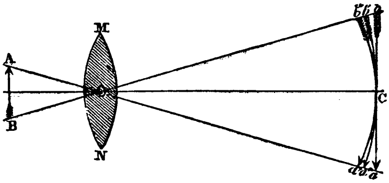
$$\therefore \left\{ q\,E - \frac{E\,D}{c^2 - 1} \right\}^2 = \frac{E\,D^2}{(c^2 - 1)^2} + \frac{(c^2 - 1)\,E\,A^2}{(c^2 - 1)^2}$$

$$\therefore q\,E = \frac{1}{c^2 - 1} \left\{ E\,D \pm \sqrt{E\,D^2 + (c^2 - 1)\,E\,A^2} \right\}.$$

From this formula the accurate value of q E for any surface may be calculated.

455. Aberration of sphericity; distortion of images.—When a straight object is placed before a lens, the extremities of the object not being in the principal axis, if the images of the extreme points are formed in the secondary axes at the same distance from the optical centre of the lens, as the central portions of the image, the image will not be straight, but formed on a curve, the centre of which is at the optical centre of the lens, as a' b', fig. 364. But as an object recedes from the lens, the image will approach it, therefore as A and B are more distant from the lens than the centre of the object, the extremities of the image must be nearer than the centre, and instead of a' C b', we shall have the image a'' C b'' described around a centre, somewhere between the lens and the centre

364

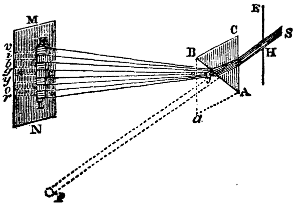


of the image. Oblique pencils are also more strongly refracted than pencils which belong to the principal axis; hence this cause must tend to curve the image still more. This curvature, or distortion of images, is called *aberration of sphericity.* For ordinary purposes this imperfection of lenses may be disregarded. The practical method of overcoming these difficulties will be best explained in connection with the description of achromatic lenses.

§ 4. Chromatics.

456. **Analysis of light.—Spectrum.—Primary colors.**—A beam of sunlight, S H, fig. 365, admitted into a dark chamber, through a small opening in the shutter, E, forms a round white spot, P, upon a screen or any other object upon which it falls. If a triangular prism, B A C, is interposed in its path, as shown in the figure, the light will be refracted both on entering and leaving the prism, but instead of forming only a circular white spot on the screen, M N, it will be spread over a considerable space from S to K, called the *solar spectrum,* in which will be seen all the colors of the rainbow. Beginning with the color most refracted, they are *violet, indigo, blue, green, yellow, orange,* and *red.*

365



If an opening is made in the screen so as to permit only the rays of a single color to pass, and we attempt to analyze this color by passing it through a second prism, we find it cannot be further decomposed by refraction; hence the colors of the solar spectrum produced by the refraction of a triangular prism are generally called *primary colors.*

457. **Recomposition of white light.**—If a second prism, A B *a,* exactly similar to B A C, is placed behind the first, but in a reversed position, as shown in the figure, the differently colored rays will be reunited and form white light at P, as though no prism had been used.

Moreover, if, instead of the second prism, a double convex lens is so placed as to receive the colored rays and converge them to a focus, a round spot of white light will be again formed in the focus of the lens.

If colored powders are mixed in the proportions that the several colors occupy in the solar spectrum, the color of the compound will be a grayish white. That the resulting color is not pure white is probably owing to the fact that we cannot procure artificial colors that will accurately represent the colors of the solar spectrum.

458. **Analysis of colors by absorption.**—Although the colors of the prismatic spectrum cannot be further divided by refraction, Brewster

has shown, that any of the colors may be still further decomposed by transmission through variously colored glass. He thus ascertained that *red*, *yellow*, and *blue* light are found in various proportions in all parts of the spectrum, and that any other color whatever may be formed by suitable combinations of these three. Brewster and other eminent philosophers have hence inferred that there are really only *three primary colors*, red, yellow, and blue.

Dr. Young considered red, green, and violet, primary colors. According to Herschel, any three colors of the spectrum may be taken as primary, and all other colors may be compounded from them, with the addition of a certain amount of white. The distinction of colors into primary and secondary, should therefore be considered to a certain extent as arbitrary, and as adopted principally for convenience of illustration.

459. **Complementary colors.**—Any two colors which by their union would produce white light, are said to be complementary to each other. If we take away from the solar spectrum any color whatever, we may reunite all the remaining colors, by means of a double convex lens, or by a second prism, and the resulting color will obviously be complementary to the first, because it is just what the first wants to make white light. In this manner it is found that,

Red is complementary to			. . . . Green.
Violet red	"	"	. . . . Yellowish green.
Violet	"	"	. . . . Yellow.
Violet blue	"	"	. . . . Orange yellow.
Blue	"	"	. . . . Orange.
Greenish blue	"	"	. . . . Reddish orange.
Black	"	"	. . . . White.

The subject of harmony and contrast of colors, will be treated in connection with the phenomena of vision.

460. **Properties of the solar spectrum.**—In the solar spectrum there are found three distinct properties which exist in various degrees of intensity in the differently colored rays. See fig. 368.

(*a*) *Luminous rays.*—According to Herschel, Fraunhofer, and others, it is found that the maximum illuminating power resides in the yellow rays, and the minimum in the violet.

(*b*) *Calorific, or heating rays.*—The position of greatest intensity for the calorific rays varies with the nature of the material of the prism with which the spectrum has been produced. In the spectrum produced by a prism of *crown glass*, the greatest heating power is found in the *pale red.* If a prism filled with *water* is used, the greatest heating power is found connected with the *yellow rays.* If the prism is filled with *alcohol*, the greatest heat is connected with the *orange* *yellow.* With prisms, formed of highly refracting *gems*, the maximum heating power is found beyond the *red ray.* Flint glass resembles the gems in this respect.

(*c*) *Chemical rays.*—In a great variety of phenomena, solar light acts as a chemical agent. Under the influence of solar light, plants decompose carbonic acid, evolving pure oxygen, and most vegetable colors are destroyed; phosphorus is changed to its red or amorphous state, and loses its power of emitting light; chlorine and hydrogen may be safely mixed in the dark, but combine with an explosion when exposed to the sun's light; the green color of plants disappears in the dark, and the nature of the vegetable juices is changed when withdrawn from the chemical action of light; and the wonderful phenomena of photography depend upon the action of light upon sensitive chemical substances.

The maximum chemical effect, produced by solar light, appears to be connected with the violet rays, or with rays between the violet and the blue. Some chemical effect is produced by rays refracted entirely beyond the extreme border of the visible violet rays. The *lavender light* of Herschel results from the concentration of the so-called invisible rays, beyond the border of the violet. A large convex lens gathers these otherwise invisible rays into a faint beam of lavender colored light.

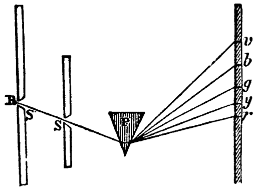
461. **Fraunhofer's dark lines in the solar spectrum.**—In 1802, Dr. Wollaston first discovered the existence of dark lines in the solar spectrum, but the discovery excited no special attention, and was applied to no practical purpose.

Unacquainted with Wollaston's observations, the late celebrated Fraunhofer, of Munich, rediscovered the dark lines of the spectrum, now distinguished as *Fraunhofer's dark lines.* Viewing through a telescope the spectrum formed from a narrow line of solar light, by the finest prisms of flint glass, he noticed that its surface was crossed by dark lines of various breadths. None of these lines coincide with the boundaries of the colored spaces.

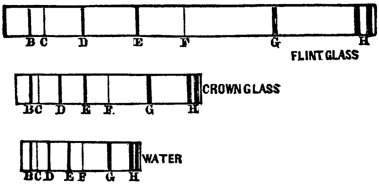
From the distinctness and ease with which they may be found and identified, seven of these lines have been distinguished by Fraunhofer by the letters B, C, D, E, F, G, H. Numerous other lines—varying from 600 to 2000 in number, according to the power of the telescope with which they are viewed—have since been counted in the solar spectrum.

To view these lines with the naked eye, a ray of sunlight is admitted into a dark chamber through narrow openings in two screens, one placed behind the other, as shown in fig. 366, and is then refracted by a prism of the purest flint glass. The lines, or some of them, will then be seen on the screen. The positions of these lines in the colored spaces of the spectrum is perfectly definite, but their distances from each other vary with the substance of which the prism is formed.

366



367



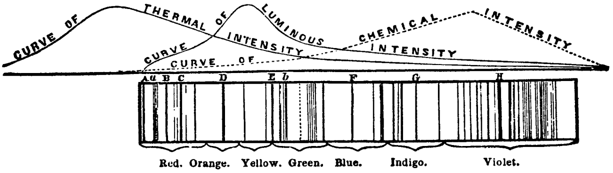
Prof. O. N. Rood states (Am. Jour. Sci. [2] XVII. 429), that fine flint glass prisms are by no means indispensable for viewing the fixed lines, as he found no prism among twelve in a candelabrum which did not show several of them.

Fig. 367 shows the arrangement of the dark lines in the spectrum, formed by prisms of flint and crown glass, and also by a prism filled with water. These dark lines answer the important purpose of landmarks for determining the indices of refraction for various substances. The exact limits of the several colors in the spectrum are not well defined, but the dark lines establish definite points from which the practical optician estimates the refractive power of any medium, and also the comparative refrangibility of the differently colored rays in which the dark lines occupy fixed positions.

Lines in light from different sources.—In the spectrum produced by the light of the sun, whether reflected by the moon or planets, or from the clouds or any terrestrial object, the position of the dark lines is invariable. But the light of the stars differs from that of the sun, and the light of one star differs from other stars in regard to the number and position of the dark lines in the spectrum. Electrical light, and the light of flames produced by any burning body whatever, give bright lines instead of the dark lines in the spectrum formed by solar or stellar light.

The relation of the dark lines to the colors of the spectrum is shown in fig. 368. B lies in the red portion near the end; C is farther advanced in the red; D in the orange is a strong double line easily recognised; E in the green; F in the blue; G in the indigo; and H in the violet. Besides these, there are also others very remarkable; thus b is a triple line in the green, between E and F, consisting of three strong lines, of which two are nearer each other than the third; A is in the extreme border of the red, and *a* is a band of delicate lines between A and B.

368


CURVE OF THERMAL INTENSITY
CURVE OF LUMINOUS INTENSITY
CURVE OF CHEMICAL INTENSITY
A a B C D E b F G H
Red. Orange. Yellow. Green. Blue. Indigo. Violet.

462. **Fixed lines in the spectra from various colored flames.**—It is well known to chemists that characteristic colors are imparted to the flame of alcohol by the salts of various metals. It has been lately observed that the spectra from flames thus colored possess characteristic fixed lines. Thus the spectrum of a soda flame is characterized by two bright lines in the position of the two dark lines at D in the solar spectrum. Lithium gives a brilliant red line between B and C and potash salts give bright lines corresponding to the dark lines A *a* B shown in fig. 368. The spectrum from lime (in the Drummond light) gives at first two bright lines like salts of sodium, which however soon disappear as the heat is continued; but if an alcohol-sodium flame is held in the path of the rays, two dark lines assume the place of the original bright lines in this spectrum, corresponding exactly in position to the two dark lines D of the solar spectrum.

These variously-colored flames held in the path of the rays producing the solar spectrum render the dark lines more distinct although these flames alone would produce bright lines.

From similar observations Kirchoff deduces the inference that the sun's atmosphere contains compounds of sodium and potassium but no lithium.*

463. **Intensity of luminous, calorific, and chemical rays.**—Fig. 368 also shows how the intensity of the luminous, calorific, and chemical rays, varies in different parts of the spectrum. The greatest illuminating power resides in the yellow part of the spectrum. The heating power is almost entirely absent in the violet and the blue, where the chemical agency is at its maximum, and it is greatest beyond the red, and extends a considerable distance, where no illuminating chemical power is ordinarily manifest. The relative positions of the maximum illuminating, chemical, and heating powers of the solar spectrum, vary somewhat with the nature of the substance composing the prism with which the spectrum has been produced.

464. **Refraction and dispersion of the solar spectrum.—Kalychromatics.**—If a glass tube, retort neck, drinking glass, or any similar instrument of glass, be held in the path of the colored rays from a triangular prism in a dark chamber, a beautiful system of colored rings will be formed, varying their form, position, and color, with every change in the position or form of the glass interposed.

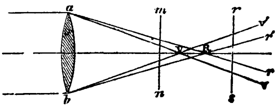
This experiment exhibits, in a surprising and agreeable manner, the wonderful resources of color contained in the solar beam. Language fails to express the exquisite and wonderful beauty of this simple experiment, involving only the refraction and dispersion of the solar spectrum. *Kalychromatics* (from the Greek for beautiful colors) has been suggested as a word to distinguish these phenomena.

---

* Monthly notices of the Berlin Academy, 1859, p. 662.

465. **Chromatic aberration.**—When rays of ordinary white light are refracted by a lens of any form, consisting of a single transparent substance like glass, or a transparent gem, the rays are each acted upon as by a prism, and dispersed into all the colors of the solar spectrum.

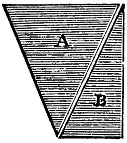
369



This effect is shown by fig. 369, where V is the focus of the violet rays which are most refracted, and R is the focus of red rays which are least refracted. A violet image is formed at V, and a red image at R, and as the other colors are situated between the violet and the red, all the space between V and R is occupied by images of intermediate colors. If an image of a point or line is formed at V, its color will be violet, but it will be surrounded by fringes composed of all the colors of the spectrum, the outer border of the fringe being red. This defect of all single lenses, formed of whatever substance, is called *chromatic aberration.*

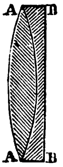
466. **Achromatism.**—We have seen, § 461, fig. 367, that the spectrum formed by flint glass is nearly twice as long as that formed by crown glass. If therefore we take a prism of crown glass, A, fig. 370, and another prism of flint glass, B, having a refractive angle so much smaller than the refractive angle of A, that the solar spectrum formed by it will exactly equal in extent the spectrum formed by the first prism, we may place the two prisms in opposition, as shown in the figure, and the colored rays separated by transmission through one prism, will be exactly reunited by the other. The light transmitted through the two prisms, thus placed, will therefore be of the same color as before transmission. But while the color of the transmitted light is unaltered, its direction will be changed by about one-half the refractive power of the prism A; for while the prism, B, has neutralized all the dispersion of color produced by A, it has neutralized only about half of its refractive power.

370



Applying these principles to lenses, a double convex lens of crown glass, A A, fig. 371, may be united with a plano-concave lens of flint glass, B B, having a focus about double the focus of the convex lens. These two lenses will act like the prisms in the preceding figure. The concave lens of flint glass will correct the chromatic aberration of the double convex lens of crown glass, and leave about one-half of the refractive power of the convex lens as the effective refracting power of the compound lens.

371



An achromatic lens, formed of a double convex lens of crown glass, equally convex on both sides, joined with a plano-concave lens of flint glass, having its concave side ground to fit one side of the double convex lens, will have the focus of a simple plano-convex lens, with its convexity equal to one side of the double convex lens.

467. **Formulæ for achromatism.**—Let it be required to determine the forms of two thin lenses composed of different media, which will together form a compound lens free from chromatic aberration. In this problem we will consider only two colors of the spectrum, as red and violet.

Let m and n be the indices of refraction for the mean ray in the two media; m' and n' the indices for one of the colored rays, and m'' and n'' the refractive indices for the other ray.

Let f and f' be the mean focal lengths of the two lenses, of which r and s and r' and s' are the radii of the surfaces. By the principles already established we shall have:—

$$\frac{1}{v} = (m'-1)\left(\frac{1}{r}+\frac{1}{s}\right) + (n'-1)\left(\frac{1}{r'}+\frac{1}{s'}\right) - \frac{1}{u};$$

$$\frac{1}{v} = (m''-1)\left(\frac{1}{r}+\frac{1}{s}\right) + (n''-1)\left(\frac{1}{r'}+\frac{1}{s'}\right) - \frac{1}{u}.$$

Subtracting the first equation from the second,

$$0 = (m''-m')\left\{\frac{1}{r}+\frac{1}{s}\right\} + (n''-n')\left\{\frac{1}{r'}+\frac{1}{s'}\right\}.$$

This equation may be put under the form,

$$0 = \frac{m''-m'}{m-1}.\frac{1}{f} + \frac{n''-n'}{n-1}.\frac{1}{f'}.$$

The coefficients $\frac{m''-m'}{m-1}$ and $\frac{n''-n'}{n-1}$, are called the dispersive powers of the two media, and may be represented by p and p', hence $0 = \frac{p}{f} + \frac{p'}{f'}$, and as p and p' are either both positive or both negative, under any supposition this equation can only be satisfied by making either f or f' negative, that is, one of the lenses must be concave. We shall then have; $\frac{f}{f'} = \frac{p}{p'}$, Or $f : f' = p : p'$.

We therefore obtain the following conclusions:—

1st. *An achromatic combination must be composed of two or more lenses formed of media having different dispersive powers.*

2d. *One of the lenses must be concave and the other convex.*

3d. *The two lenses forming an achromatic combination must have focal lengths directly proportional to the dispersive powers of the media of which they are respectively composed.*

As it was stated in § 454 that the spherical aberration of a concave lens is the opposite of the aberration of a convex lens, it is easy to see that the combination of such lenses as are required to produce achromatism will also wholly, or in part, correct the spherical aberration.

§ 5. Vision.

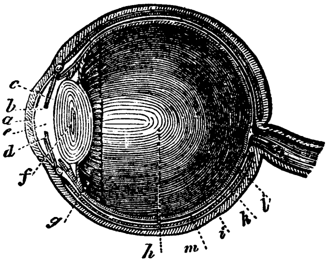
468. **Structure of the human eye.**—The human eye is the most perfect of all optical instruments. By means of this organ, stimulated by the light reflected or refracted from external objects, we recognise their presence, nearness, color, and form. Some knowledge of the structure and action of the eye is essential to a proper understanding of the uses of other optical instruments.

The eye, situated in its bony cavity called the orbit, is maintained in its position by the optic nerve and its sheath, by muscles which serve to move it or hold it steady in any required position, and by the delicate membrane called the conjunctiva, which covers its anterior surface and lines the eyelids. The eyelids serve to protect the organ from external injuries, and also to shut out light which might otherwise be troublesome or injurious by its excess, or too long-continued action.

Fig. 372 shows a horizontal section of the eye, the lower part of the figure representing the side of the eye towards the nose. The globe, or ball of the eye, is nearly spherical, though the anterior portion is more convex than the other portions, as shown in the figure.

The principal portions of the eye which require consideration, are the sclerotic coat, the cornea, the choroid coat, the retina and optic nerve, the iris, the pupil, the crystalline lens, the aqueous humor, the vitreous humor, and the hyaloid membrane.

372



The sclerotic coat, i, is a strong opaque structure, composed of bundles of strong white fibres, interlacing each other in all directions. This membrane covers about four-fifths of the eyeball, and more than any other structure, serves to preserve the globular form of the eye. It has a posterior sieve-like opening, for the transmission of the fibres of the optic nerve, *n*; anteriorly, a transparent membrane called the *cornea, a,* is set into a groove in the sclerotic coat, as a watch crystal is set in the case, but these two membranes are so firmly united that they are separated only with considerable difficulty. The cornea is more convex than the sclerotic coat.

The choroid coat, k, is a strong vascular membrane, lining the sclerotic coat, and covered internally by a dark pigment, the *pigmentum nigrum,* which prevents any reflection of light from the internal parts of the eye.

The third, or inner membrane of the eye, is the *retina, m,* which is merely an expansion of the optic nerve, *n,* uniting it to the brain. It is on this delicate lining membrane (the retina) that the images of external objects are formed.

The iris, d, which forms the colored part of the eye, is a dark annular curtain or diaphragm, adherent at its outer margin, with a central opening which, in man, is circular; in cats and the feline tribe generally it is elongated vertically; in the ox and other ruminating animals, in a horizontal direction. The central opening of the iris, *e*, which allows light to penetrate the eye, is called the pupil. It varies from one-eighth to one-quarter of an inch in diameter. In a strong light the pupil contracts, but where the light is diminished it expands.

Every one knows the sensation produced by entering a house after spending hours in the open air exposed to the light of the sun reflected from snow. In this case the pupil becomes so contracted, and the eye so accustomed to a strong light, that objects within doors are almost invisible until the pupil expands and the eye recovers its sensitiveness in ordinary light. The movements of the iris are involuntary.

The pupil of the owl is so very large that it sees distinctly at night, while in the day-time the pupil cannot contract enough to protect the eye from the blinding effect of the solar rays, and hence the owl is nearly blind by day.

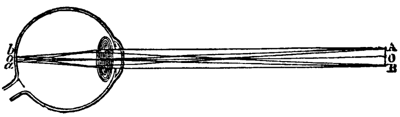
The crystalline lens, f, is a transparent body, placed behind the iris and very near to it; it is enveloped in a transparent membrane or capsule, which adheres by its borders to the ciliary process, *g*. The posterior surface of the crystalline lens, is more convex than the anterior. The crystalline lens is made up of serrated fibres, arranged in layers which increase in density from the circumference to the centre of the lens.

Aqueous humor.—The space between the cornea and the crystalline lens is filled with a transparent liquid called the aqueous humor. The iris divides this space into two chambers, the anterior chamber, *b*, between the cornea and the iris, and the posterior chamber, *c*, between the iris and the crystalline lens. These two chambers communicate freely with each other through the pupil, *e*. The free edge of the iris floats in the aqueous humor.

Vitreous humor.—The posterior compartment of the eye, *h*, behind the crystalline lens, constitutes by far the larger part of the internal cavity of this organ, and is filled with a transparent gelatinous fluid, enclosed in exceedingly delicate cellular tissue, which is condensed externally, and forms a delicate *hyaloid membrane*, everywhere covering the retina and the posterior surface of the crystalline lens. The vitreous humor, enclosed in its cellular tissue, and enveloped by the hyaloid membrane, is called the *vitreous body*.

469. **Action of the eye upon light.**—The eye may be compared to a dark chamber, the pupil being the opening to admit the light, the crystalline lens being a converging lens to collect the light, and the retina a screen upon which is spread out the image of external objects. The effect is the same as when a double convex lens forms, at its conjugate focus, an image of any object placed in the other focus.

373



Let A B, fig. 373, be an object placed before the eye, and consider that rays are emitted from any point, as A, in all directions; only those rays which are directed towards the pupil can penetrate the eye, or contribute to the phenomena of vision. The rays, on entering the aqueous humor, are refracted towards the axis, O *o*, drawn through the optical centre of the crystalline lens; but on entering the lens, which is more dense than the aqueous humor, they are still further refracted, and undergoing yet another refraction on leaving the crystalline lens, they converge towards a point, *a*, where they form an image of the point A The rays of light emitted from B form its image in the point *b*, and in the same manner every part of the object, A B, is delineated in the very small image *a b*, which is a real image, inverted, and formed exactly upon the retina.

470. **Inversion of the image formed in the eye.**—To prove that the image formed in the eye is really inverted, take the eye of an ox, cut away the posterior part of the sclerotic and choroid coats; fix the eye thus prepared in an opening in the shutter of a dark chamber, and look at it with the aid of a magnifying glass, when external objects will be seen beautifully delineated in an inverted position, on the retina at the posterior part of the eye.

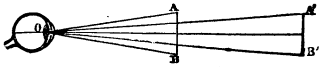
Philosophers and physiologists have proposed various theories to explain how we come to perceive objects erect, when their images in the eye are actually inverted. The most rational of these theories are the two following: 1st. That we judge of the relative position of objects, or of different parts of the same object, by the direction in which the rays come to the eye, the mind tracing them back from the eye towards the object. 2d. That the image formed on the retina, gives correct ideas of the relation of external objects to each other; up and down being, in reference to impressions on the retina or brain, merely the relative directions of the sky and earth; and we see all bodies, including our own persons, occupying the same relations to these fixed directions as our other senses demonstrate that they really occupy.

471. **Optic axis.—Optic angle.**—The principal axis of the eye, called the *optic axis*, is its axis of figure, or the right line passing through the eye in such a position that the eye is symmetrical on all sides of it. In a well formed eye this is a right line, passing through the centre of the cornea, the centre of the pupil, and the centre of the crystalline lens, as O *o*, fig. 373. The lines A *a*, B *b*, which are sensibly right lines, are secondary axes. Objects are seen most distinctly in the principal optic axis.

When both eyes are directed towards the same object, the angle formed by lines drawn from the two eyes to the object, is called the *optic angle*, or the *binocular parallax.*

To appreciate this difference of direction, look at two objects that are situated in a line with one eye, the other being closed; then, without moving the head, look at the same objects with the other eye, and the objects will not both appear in the same line, but will seem suddenly to change their positions. By such experiments it will readily be found that some persons see principally with the right, and others chiefly with the left eye, when both eyes are open. Others will find that a part of the time the direction of objects is determined by one eye, and part of the time by the other.

374



472. **Visual angle.**—The angle formed between two lines drawn from the eye to the two extremities of an object, is called the *visual* angle, as A O B, fig. 374. If the object is

removed to twice the distance, the visual angle A′ O B′ will be only one-half as great as A O B, and the breadth of the image formed on the retina will be proportionally decreased.

The apparent linear magnitude of an object is in inverse proportion to its distance from the eye, or in direct proportion to the visual angle. The *apparent superficial magnitude* is always the square of the apparent linear magnitude, and is in inverse proportion to the square of the distance.

473. **The brightness of the ocular image** of any object will be in direct proportion to the intensity of the light emanating from each point in the object.

The amount of light received by the eye from any point in the object, or from the entire object, will be inversely as the square of the distance, and directly as the intensity of the light from each point (413). But the superficial magnitude of the image will diminish as the square of the distance increases: Hence, *the apparent brightness of the image will remain constant, whatever may be the distance of the object.*

As the object recedes from the eye, the size of the image formed on the retina diminishes, the details of the various parts become crowded together, and only the bolder outlines occupy sufficient space to make a sensible impression, or to be clearly discerned.

475. **Conditions of distinct vision.**—It may be stated in general, that two conditions are essential to distinct vision. 1st. That an object should be situated at such a distance as to form on the retina an image of some appreciable magnitude. 2d. That the object shall be sufficiently illuminated to produce a distinct impression upon the retina.

The distance at which an object can be seen varies with the color of the object, and the amount of illumination. A white object illuminated by the light of the sun can be seen at a distance of 17,250 times its own diameter. A red object illuminated by the direct light of the sun can be seen only about half as far as though it were white, and blue at a distance somewhat less. Objects illuminated by ordinary day-light can be seen only about half as great a distance as when illuminated by the direct rays of the sun. The smallest visual angle under which an object can be seen with the naked eye, is estimated at twelve seconds. All these calculations will vary for different eyes.

Persons having dark-colored eyes can generally see much farther than those who have light-colored eyes. Those whose eyes are trained to view distant objects, as sailors and surveyors, will see objects that are far too distant to be seen by the eyes of inexperienced persons.

476. **Background.**—The distance at which the outline of any object can be distinguished, depends very much upon the color of adjacent objects, or of the background on which the object appears projected. Objects are most distinctly seen when the color of adjacent objects, or the background, presents a strong contrast to the colors of the object we wish to see.

Colored signals.—For signal flags used at sea, the colors *red, yellow, blue,* and *white* are employed, because they are readily distinguished, and are easily seen, with the water or the sky for a background. For railroad signals, the colors *red, white,* and *black* are mostly used.

477. **Sufficiency of illumination.**—It is not enough for distinct vision, that a well-defined image of the object shall be formed on the retina. This image must be sufficiently illuminated to affect the senses, and at the same time not so intensely illuminated as to overpower the organ. An image may be so faint as to produce no sensation, or it may be so intensely brilliant as to dazzle the eye, destroy the distinctness of vision, and produce absolute pain.

When we look at the meridian sun, its light is so brilliant as to overpower the eye and render it impossible even to see distinctly the solar disc, but if a sufficient stratum of vapor or a colored or smoked glass is interposed, we see a well-defined image of the sun.

Many stars are so distant that the rays which enter the pupil, when converged to a point on the retina, produce no appreciable sensation, but when the amount of light from the same stars falling upon a large lens is concentrated upon the retina, it produces sensation, and the stars become visible.

On passing from a dark room to one brilliantly illuminated, or on going out into the open air at night from a well-illuminated room, the sensations experienced are owing partly to the contraction and expansion of the iris, as explained in § 468, and also to the fact that the *sensibility* of the retina is diminished by long exposure to intense light, and increased by remaining a long time in feeble light.

478. **Distance of distinct vision.**—Although the human eye is capable of seeing objects at both great and small distances, most persons, when they wish to see the minute structure of an object clearly, instinctively place it at a distance of from six to ten inches from the eye. This point, called the *limit of distinct vision,* sometimes varies for the two eyes of the same person. Persons who see objects at very short distances are called *near-sighted,* while those who see objects distinctly only at greater distances, are said to be *long-sighted.*

479. **Visual rays nearly parallel.**—When we consider that the diameter of the pupil, when the eye is adjusted for viewing near objects, is only about one-tenth of an inch, if we take the limit of distinct vision at six inches, it will be found that the cone of rays entering the eye, from any single point, is included within an angle of one degree. If we take the limit of distinct vision at ten inches, the angular divergence of the cone of rays entering the eye from a single point will be little more than half a degree. In either case, therefore, the rays differ but slightly from parallel rays. For all objects more remote, the rays may properly be considered as parallel. Distinct vision is therefore obtained only by rays that are *sensibly parallel* or *very slightly divergent.*

480. **Adaptation of the eye to different distances.**—Although there is a definite distance at which minute objects are most distinctly seen, the eye has a wonderful facility of adapting itself to viewing objects at different distances.

Let two similar objects be placed, one three feet from the eye and the other at a distance of six feet: If the eye is fixed steadily upon the nearer object for a few moments, it will be distinctly seen, while the more remote object will appear indistinct, but if the eye is steadily fixed upon the remote object, that object will soon be clearly seen, and the nearer object will appear indistinct. We thus see that either the converging power of the eye is subject to rapid variation, or that the distance of the crystalline lens from the retina is changeable. The means by which the eye thus rapidly adapts itself to viewing objects at different distances, have not been satisfactorily determined.

481. **Appreciation of distance and magnitude:—Aerial perspective.**—The appreciation of the distance and magnitude of objects is entirely a matter of unconscious training, or education, and depends upon a variety of circumstances, as the visual angle, optic angle, comparison with familiar objects, distinctness, or dimness of the image caused by intervening air or vapor.

When the magnitude of an object is known, as the height of a man, a house, or a tree, the visual angle under which it is seen enables us to appreciate its distance. If its magnitude is unknown, we judge of its size by comparing it with other familiar objects situated at the same distance.

In viewing a range of buildings, or a row of trees, the visual angle decreases as the distance increases, and the objects decrease in apparent size in the same proportion, but the habit of viewing the houses or trees, and their known altitude, causes us to correct the impression produced by the visual angle, so that they do not appear to decrease in size as fast as their distance increases.

Thus, when distant mountains are seen under a very small visual angle, occupying but a small space in the field of view, being accustomed to aerial perspective, we unconsciously restore to some extent their real magnitude.

The optic angle, or binocular parallax, is an essential element in appreciating distances. This angle increases or diminishes inversely as the distance; the movement of the eyes required, to cause the optic axes of the two eyes to converge upon any object which we are viewing, gives us an idea of its distance. It is only by habit that we appreciate the relation between the distance of an object and the corresponding movement, required to direct both eyes upon it.

Perfect vision cannot then be obtained without two eyes, as it is by the combined effect of the images produced on the retinæ of both eyes, and the different angles under which objects are observed, that a judgment is formed respecting their solidity and distances.

A man restored to sight by couching cannot tell the form of a body without touching it, until his judgment has been matured by experience, although a perfect image may be formed on the retina of each eye. A man with only one eye cannot readily distinguish the form of a body which he had never previously seen, but quickly and unwittingly moves his head from side to side, so that his one eye may alternately occupy the different positions of a right and a left eye; and, if we approach a candle with one eye shut, and then attempt to snuff it, we shall experience more difficulty than we might have expected, because the usual mode of determining the correct distance is wanting.

Infants plainly have no notions of distances and magnitudes till taught by experience and comparison of optical appearances with the sense of touch.

482. **Single vision with two eyes.**—When both eyes are directed to the same object, images are produced in both eyes, and the inquiry is most natural why all objects thus seen do not appear double? Passing by much learning bestowed on this subject, the simplest and most satisfactory explanation of the phenomenon is deduced from the anatomical structure of the optic nerves, and their relations to each other, and to the brain.

The eyes may be compared to two branches issuing from a single root, of which every minute portion bifurcates, so as to send a twig to each eye. (Müller.) The optic nerve from the right lobe of the brain sends a portion of its fibres to each eye, and also sends some branches across and backward to the left lobe of the brain. A portion of the optic nerve from the right eye, instead of proceeding to the brain, curves around and enters the optic nerve and the retina of the left eye. In the same manner the optic nerve arising from the left lobe of the brain is connected with the right eye, and sends branches also to the left eye.

Branches of the same nerve fibres which go to the external side of the retina of one eye, go to the internal side of the other eye.

It is thus that a perfect sympathy and correspondence is established between similar parts of both eyes. Hence whatever object is observed, if the optic axes of both eyes are directed towards it, the image is formed on corresponding portions of the retina in both eyes, and the mind receives the impression of a single object; but the impression is more vivid than if the same object were seen with only one eye. So perfect is this sympathy between the two eyes, that if one eye only is exposed to a strong light the pupils of both eyes contract. If one eye is diseased and protected from the light, it suffers pain from light entering only the sound eye.

483. **Double vision.**—If both eyes are fixed steadily upon one object, any other object seen at the same time will appear double.

Fix both eyes steadily upon the flame of a lamp or candle, and a finger held between the eyes and the light will appear double.

Drunken persons, or persons about falling asleep, often see objects double, owing to the inability to direct both eyes steadily upon the same object. The same phenomena may occur when, from any cause, the nerves which control the eye become diseased.

484. **Binocular vision.**—A picture of an object is formed on the retina of each eye; but although there may be but one object presented to the two eyes, the picture formed on the two retinæ are not precisely alike, because the object is not observed from the same point of view.

If the right hand be held at right angles to, and at a few inches from the face, the back of the hand will be seen when viewed by the right eye only, and the palm of the hand when viewed by the left eye only; hence the images formed on the retinæ of the two eyes must differ, the one including more of the right side, and the other more of the left side of the same solid or projecting object. Again; if we bend a card so as to represent a triangular roof, place it on the table with the gable end towards the eyes, and look at it, first with one eye then with the other, quickly and alternately opening and closing one of the eyes, the card will appear to move from side to side, because it is seen by each eye under a different angle of vision. If we look at the card with the left eye only, the whole of the left side of the card will be plainly seen, while the right side will be thrown into shadow. If we next look at the same card with the right eye only, the whole of the right side of the card will be distinctly visible, while the left side will be thrown into shadow; and thus two images of the same object, with differences of outline, light and shade, will be formed, the one on the retina of the right eye and the other on the retina of the left. These images falling on corresponding parts of the retinæ convey to the mind the impression of a single object, while experience having taught us, however unconscious the mind may be of the existence of two different images, that the effect observed is always produced by a body which really stands out or projects, the judgment naturally determines the object to be a projecting body.*

485. **Near-sightedness.**—Many persons are unable to see minute objects distinctly unless they are placed within three or four inches of the eye. Such persons are often unable to see ordinary objects distinctly in a large room or across the street; they are therefore said to be near-sighted (478). This defect is owing to a too great convergent power, the eye bringing parallel or slightly divergent rays to a focus before they reach the retina.

To secure distinct vision in such cases, it is necessary to bring the object so near the eye as to render the rays entering the eye considerably divergent, when the image will be formed on the retina. The same object may be accomplished by placing a concave lens before the eye, when the rays from distant objects will be rendered divergent, and the strong convergent power of the eye will form the image on the retina. Concave lenses for near-sighted persons should be such as have a focus a little longer than the distance at which they see objects most distinctly.

486. **Long-sightedness** commonly occurs in old people, when the eye becomes flattened by diminution of its fluids, or some structural change in the crystalline lens occurs, by which its convergent power is diminished. In such cases the rays of light tend to form an image behind the retina, and vision is most distinct when the object, as a book when reading, is held at a considerable distance from the eyes so as to allow the image to be formed on the retina.

This defect of the eyes, when not accompanied by disease, may be entirely remedied by using convex glasses, which make up for the diminished converging power of the eyes, and bring the rays to such a condition that the eye is enabled to bring the light from near objects to a distinct focus upon the retina. In such cases, however, the power of accommodating the eye to different distances is often not as great as in younger persons; hence many people in advanced life find it necessary to use one set of glasses for near, and another for distant objects.

---

* For a discussion of this subject, see Prof. W. B. Rogers on Binocular vision, Am. Jour. Sci. [2] vol. XX.

487. **Duration of the impression upon the retina.**—Every one knows that a lighted stick whirled rapidly around a circle appears like a ring of fire. The rapidity of revolution required to produce this impression is one-third of a second in a dark room, and one-sixth of a second by daylight.

When a meteor darts across the heavens, it appears to leave a luminous track behind it, because the impression produced upon the retina remains after the meteor has passed a considerable distance on its way. The zigzag course of the lightning appears, for the same reason, as a continuous track.

Winking does not interfere with distinct vision, because the continuance of the impression of external objects on the retina preserves the sense of continuous vision.

488. **Optical toys.—Thaumatrope.**—A great number of optical toys and pyrotechnic exhibitions owe their effect to the continuance of the impression upon the retina, when the object has changed its place.

If a horse is painted on one side of a card and a rider on the other side, the rapid revolution of the card causes the rider to appear seated on the horse. In the same manner, if any object which takes a variety of positions in moving is painted in successive positions, at equal distances on a revolving wheel, so arranged that one only of the figures shall be seen at a time, the object is seen performing all the motions of real life. In this manner a horse may be made to appear leaping a gate, or boys playing at leap-frog. These toys are called *thaumatropes* and *anorthoscopes*. Other toys, called *phenakistoscopes* and *phantascopes*, are variations of the same thing, combined with mirrors and other ingenious arrangements on the same principle.

489. **Time required to produce visual impressions.**—If an object moves with sufficient velocity, it is entirely invisible, its image upon the retina not remaining long enough to produce any impression. This is the case with a cannon-ball or rifle-ball, viewed at right angles to the direction of its flight. But if the projectile is going from us, or coming towards us, it preserves the same direction long enough to produce an impression. Motions describing less than one minute of arc in a second of time are not appreciable to us. Hence we do not see the movements of the hour hand of a clock, or of the heavenly bodies.

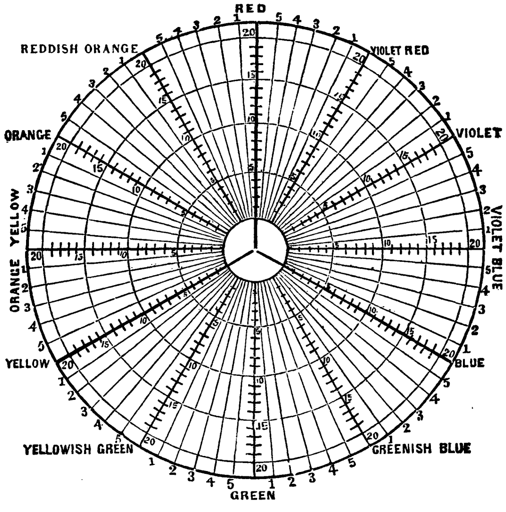
490. **Appreciation of colors.—Color blindness.**—The power of the eye to distinguish colors, varies greatly in different persons. Some eyes fail entirely in this particular, while in every other respect they

are perfect. Such eyes are said to be color-blind Some confound certain colors, as red and green, while they distinguish others, or while they recognise all the colors of the spectrum, they cannot appreciate delicate shades of the same color.

Colors are greatly modified by proper contrast with other colors. Thus the complementary colors mutually enhance, while those not complementary diminish each other's beauty when contrasted. The sensibility of the eye is much diminished by long inspection of any color, and its power of perceiving the complementary color is proportionally increased. This principle is the key to harmony of colors in nature and art, and serves to explain the modification of color by contrast, and proximity of two or more colors.

491. **Chevreul's classification of colors, and chromatic diagram.**—The chromatic diagram of Chevreul, fig. 375, greatly facilitates the study of complementary colors, and the modifications produced by their mutual proximity.

375



Three radii of a circle represent Brewster's three cardinal colors, red, yellow, and blue; between these are placed orange, green, and violet. Between these six colors are placed reddish orange, orange yellow, yellowish green, greenish blue, violet blue, and violet red. We thus obtain twelve principal colors, each of which may be again divided into five scales or hues, which gradually approach the succeeding color.

We thus have the circumference of the circle, which represents the prismatic spectrum, divided into sixty scales of pure colors. Each radius representing a scale of colors is divided into twenty *tones,* to represent the intensity of each color in its own scale. The tone of any color may be lowered by the addition of white, when it will remain in the same radius or scale, but take a position at a lower tone, or nearer the centre of the circle. A color modified by black, is called a broken color, but as the color is deeper, the tone is carried towards the circumference of the circle. To represent the modifications produced by black, Chevreul employs a movable quadrant, not easily introduced in our illustration

When two complementary colors are mixed, their combination produces white, if the colors are pure. The combination of two colors not complementary produces a certain quantity of white, but principally a color which will be found in the diagram intermediate between the two colors, if they are of the same tone, or nearer to the color of deeper tone, when their tones or intensities are different. The complementary color in the diagram is found at the opposite extremity of the diameter of the circle.

This diagram thus explains the effect which two colors produce upon each other by their mutual proximity.

When two colors are placed near each other, each color appears modified as though mixed with a small portion of the complement to the color which is near it.

Examples.—(*a*) Suppose *blue* and *yellow* to be placed side by side; at one extremity of a diameter we read *yellow,* and at the opposite *violet,* hence the proximity of yellow gives to the blue a shade of violet, or makes it approach *violet blue.* In the same manner we find *orange* complementary to *blue;* hence the blue gives a shade of orange to the yellow, or makes it approach *orange yellow.*

(*b*) Let *green* and *yellow* be contiguous, the yellow will receive red, the complement of green, and will become orange yellow, while the green will receive from the yellow its complementary violet. A part of the yellow in the green will thus be neutralized, and the green will appear bluer or less yellow, in fact, greenish blue.

492. **The study of colors** upon the principles here laid down is of great importance to the artist and manufacturer, whether in reproducing the beauties of nature, or in architectural decoration; also in weaving, embroidery, and costume.

The skillful salesman knows how to enhance the brilliancy or beauty of his goods by artfully contrasting the pieces which he hopes to sell by others having complementary colors. Good taste in dress never violates these principles, regarding with care the complexion of the wearer in contrast to the colors selected. Florid skins can bear dark hues in dress, while delicate complexions are made pallid by heavy colors. A green dress or wreath increases the freshness of a rosy complexion. A crimson dress and scarlet shawl worn together appear mutually dull and heavy, while either, with the contrast of an appropriate shade of green, would be attractive and tasteful. These topics will be found fully considered in "Chevreul on Colors."

§ 6. Optical Instruments.

493. **Magnifying glasses.**—Single lenses, used for magnifying small objects, occupy an important place in the arts. They are used by watch-makers, jewelers, engravers, and other artisans, whose labors

are performed upon minute structures. These instruments occupy a middle place between spectacles and the regular microscope composed of a variety of parts.

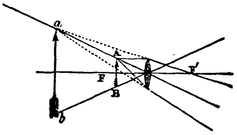
A thorough knowledge of the uses and powers of simple lenses forms the basis of all calculations of the powers and uses of more complex instruments, like the compound microscope and the telescope.

The eye takes no cognisance of real magnitude, which it can only estimate by inference, but notices directly apparent magnitude, which is determined in all cases by the visual angle under which objects are seen (472).

We have seen (479) that it is essential to distinct vision that the rays entering the pupil from any one point of an object should be parallel, or slightly divergent, the distance of most distinct vision being generally from five to ten inches. For near-sighted persons, this distance is as small, sometimes, as two or three inches, and for eyes enfeebled by age, it extends from fifteen even to thirty inches.

494. **The magnifying power of a lens** is found with sufficient accuracy for ordinary purposes by dividing the limit of distinct vision (ten inches) by the distance of the principal focus of the lens.

376



Let A B, fig. 376, be an object placed before a convex lens, so much nearer to the lens than the focus, F, that the rays, after refraction by the lens, shall be in that state of slight divergence best adapted to produce distinct vision, that is, diverging as though emanating from a point at a distance of ten inches or the limit of distinct vision. Let *a b* represent the virtual image, formed where the refracted rays would meet if extended backward, then *a b* will be as much greater than A B, as its distance from the lens is greater than the distance of the object, A B, from the lens. The divergence of rays of light entering the small opening of the pupil, from a point ten inches distant, is so small that we may consider them parallel, and then the object, A B, will be nearly at F, the principal focus of the lens.

To estimate the magnifying power of a lens more accurately; let the distance of most distinct vision be represented by e; with a lens interposed, the eye sees a virtual image of the object, therefore, in the formula for a convex lens, let $v = -e$, and then $-\frac{1}{e} = \frac{1}{f} - \frac{1}{u}$ $\therefore$ $u = \frac{ef}{e+f}$, which is the distance of the object from the lens. If the eye is placed close to the lens, the magnifying power represented by M will be, $M = \frac{e}{u} = \frac{e+f}{f} = 1 + \frac{e}{f}$.

If the eye is placed at a distance from the lens represented by d, we shall have the distance of the virtual image *a b* from the lens represented by $e' = e - d$, and the magnifying power will become, $M = 1 + \frac{e-d}{f}$.

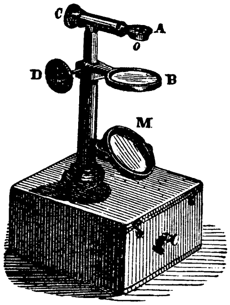
If the eye is placed at a distance from the lens equal to its principal focus, or, $d = f$, then $M = \frac{e}{f}$, and in that case the magnifying power for different eyes varies as the limit of most distinct vision.

If the eye is placed at a distance from the lens equal to the distance at which it sees objects most distinctly, then $\frac{e - d}{f} = 0$, and $M = 1$, or the object is not magnified by the action of the lens.

The superficial magnifying power is equal to the square of the linear magnifying power given by the rule stated above; but the linear magnifying power is alone commonly used in scientific treatises.

495. **The simple microscope** acts in the same manner as the single lens or magnifying glass. Instead of a single lens, a doublet or triplet, acting as a single lens, is often used.

377



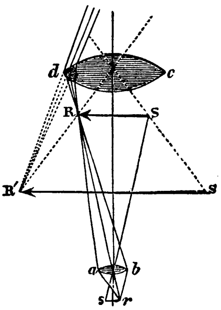
Raspail's dissecting microscope, shown in fig. 377, is the most complete simple microscope. The magnifying lens, *o*, mounted in a dark cup, A, to protect the eye from extraneous light, is fixed in the end of a movable arm which can be rotated on its support, elevated and depressed by the milled head E, or lengthened by turning the milled head C. Below the lens is the stage B, which supports the object to be examined. The concave mirror, M, can be so adjusted as to illuminate the object by a concentrated pencil of transmitted light.

In using this microscope, the eye is placed over the lens *o*, which may be elevated or depressed till the focus is adjusted to give the most distinct view of the object on the stage. Opaque objects are illuminated by a bull's eye lens.

By using lenses of different foci, magnifying powers may be obtained with this instrument, varying from two to one hundred and twenty diameters.

496. **The compound microscope** consists, essentially, of two lenses, so arranged that when an object is placed a little *beyond* the principal focus of the first lens, its image may be formed in the principal focus of the second lens, by which it is viewed as an object is viewed by a common magnifier.

378



The arrangement of the lenses in the compound microscope is shown in fig. 378, and also the position of the object, and the images both real and virtual.

The object, *s r*, being placed near the first lens, *a b*, called the object-glass, an image, inverted and much enlarged, is formed at R S, in the focus of the second lens, *d c*, called the eye-glass. By this lens, the rays are transmitted slightly divergent, and in the exact condition to produce distinct vision when viewed by the eye. The rays transmitted through the eye-glass, if traced backward to the distance of distinct vision, form a virtual image at R′ S′, much larger than the real image R S, formed by the action of the first lens.

Such a compound microscope as the one shown in this figure, is subject to chromatic and spherical aberration, and the image viewed by the eye is not straight as shown in the figure, but curved so as to appear convex towards the eye. These imperfections are almost entirely corrected in the *achromatic* compound microscope described in § 511.

497. **The telescope** is an instrument constructed for viewing distant objects.

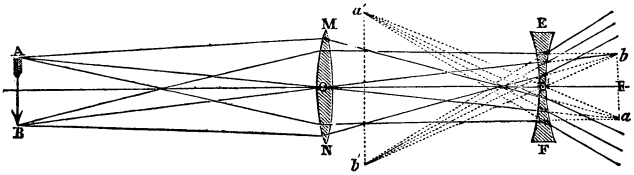
Telescopes are of two kinds. Refracting telescopes are constructed of lenses. Reflecting telescopes contain one or more metallic reflectors.

498. **The telescope used by Galileo** in 1609, is the oldest form of which we have any definite description. The Galilean telescope consists of a convex lens, of long focus, and a concave lens of short focus placed at a distance apart, equal to the difference of their principal foci. The light from distant objects collected by the large surface of the convex field-lens, is brought to such a state of divergence by the concave eye-lens as to produce distinct vision in the eye.

The magnifying power of the Galilean telescope is found by dividing the principal focus of the convex lens by the principal focus of the concave lens.

The convex lens, M N, fig. 379, tends to form an image of a distant object, A B, very near its principal focus, as at *a b*. The concave lens, E F, being placed between the convex lens and the image, *a b*, renders the rays which were converging to *a*, slightly divergent, as though emanating from a point, *a′*, at the distance of distinct vision, about ten inches. The same effect is produced on the rays converging to *b*. The direction of the oblique pencils is changed, and the extremities of the image appear in the secondary axis *a* O′ *a′*, and *b* O′ *b′*, drawn from *a* and *b* through O′, the optical centre of the lens E F. It is especially to be noticed, that while the rays from any one point in the object are rendered parallel, or slightly divergent, by the concave lens, the pencils from the extreme points converge at O′ much more than at O, making the visual angle *a′* O′ *b* , under which the object is seen by the telescope, much greater than the visual angle *a* O *b*, under which the object would appear without the telescope.

379



Since the angle A O B is equal to *a* O *b*, and *a′* O′ *b′* is equal to *a* O′ *b*, the visual angle *a′* O′ *b′* is to the angle A O B as O F is to O′ F, and the image *a′ b′* appears as much greater than the object as the focal length O F of the convex lens exceeds the focal length O′ F, of the concave lens.

The opera-glass consists generally of two Galilean telescopes, placed near together, to allow of distinct vision by both eyes.

Night-glasses, used by seamen, are constructed like large opera-glasses. They serve to concentrate a large amount of light in such a condition as to allow of distinct vision, and thus enable the eye to see objects distinctly in the night. They have a low magnifying power.

With the Galilean telescope in all its forms the object appears erect.

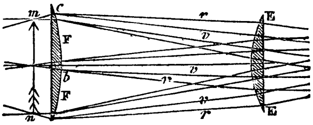
499. **The astronomical telescope** may be constructed with a convex lens placed beyond the image formed by the field-lens. The second lens then magnifies the image formed by the first lens. The object appears inverted, but this occasions very little inconvenience in astronomical observations.

500. **Eye-pieces** are certain combinations of lenses used in both telescopes and microscopes to magnify the image formed by the lens nearest to the object. They have less spherical and chromatic aberration than a single lens, and also enable the eye to take in a larger extent of the object to be examined than could otherwise be seen.

The positive eye-piece, invented by Ramsden, consists of two plano-convex lenses, with their convex surfaces turned towards each other, and placed at such a distance that the object or image to be viewed by it is seen distinctly when brought very nearly in contact with the first lens. To secure this result, the distance between the lenses must be a very little less than one-half the sum of their focal lengths for parallel rays. The spherical aberration produced by this eye-piece is only about one-fourth as much as if a single lens were used. The chromatic aberration also is less than with a single lens.

Let F F, fig. 380, be the field-lens, and E E the eye-lens of the positive eye-piece. Let *m n* be an image formed by the object-glass either of a telescope or a microscope, then each ray from the image on passing the lens F F becomes colored, *c v*, *b v*, representing the violet rays, and *c r*, *b r*, representing the red rays. The red rays, which are least refracted by the first lens, fall near the borders of the second lens, where the refractive power is greater than where the more refrangible violet rays fall; hence the second lens tends to correct the chromatic dispersion of the first, and the violet and red rays enter the eye very nearly as though emanating from a common point. This is an important excellence of the positive eye-piece; but a yet more important advantage of this eye-piece is, that the image is less distorted than when only a single lens is used.

380



The negative eye-piece, which was invented by Huyghens, consists generally of two plano-convex lenses, having the convex surfaces of both turned towards the object-glass. The two lenses are placed at a distance from each other equal to one-half the sum of their focal lengths. The image is formed between the lenses. This arrangement considerably enlarges the field of view, and diminishes the spherical aberration; the chromatic aberration is also less, and it is more equalized in all parts of the field than in other eye-pieces.

In the most perfect form of the negative eye-piece, according to Prof. Airy, the first, or field lens, is a meniscus whose radii are as four to eleven, with the convex side toward the object, and an eye-lens having the form of least spherical aberration (454), with the more convex side towards the object.

The focal lengths of the field and eye lenses should be to each other as 3 to 1, and their distance apart equal to one-half the sum of their focal lengths.

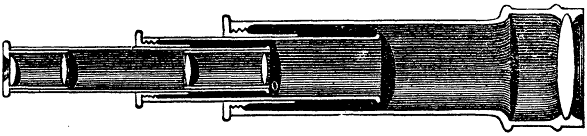
In eye-pieces designed for the microscope, instead of estimating the principal focal length of the field-lens, we must take its conjugate focus when the object is placed in the position of the object-glass of the microscope.

The action of the negative eye-piece will be more fully explained in connection with the compound achromatic microscope (511).

The terrestrial eye-piece consists of four lenses, two of them being added solely to produce an erect image.

Fig. 381 shows a section of the common *spy-glass* or *terrestrial telescope*, with the erecting eye-piece. The several tubes which shut one within another, allow the instrument to be reduced to a convenient length when not in use.

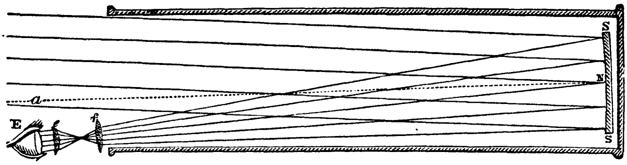
381



501. **Reflecting telescopes** are extensively used for astronomical observations. A variety of forms have been invented by different observers, but in all a metallic speculum is employed to form an image of distant objects, and an eye-piece is used to magnify the image.

502. **Sir William Herschel's telescope**, shown in fig. 382, consists of a speculum, S S, set in a tube somewhat larger than the diameter of the speculum, and an eye-piece, *e f*, placed at one side of the open

382



end of the tube. The axis of the speculum, represented by the dotted line a N, is so inclined that parallel rays, falling on every part of the speculum, will be reflected, converging to the side of the tube where the eye-piece is placed to receive them. The size of the tube, and the inclination of the axis of the speculum, is so adjusted that the eye of the observer may be placed at E without intercepting any part of the light which can fall upon the speculum in such a direction as to be reflected to the eye-piece.

Sir William Herschel's great telescope had a speculum four feet in diameter, three and a half inches thick, weighing two thousand one hundred and eighteen pounds. Its focal length was forty feet, and it was set in a sheet-iron tube thirty-nine and a half feet long, and four feet ten inches in diameter. When directed to the fixed stars it would bear a magnifying power of six thousand four hundred and fifty diameters.

This is called the front view telescope, because the observer sits with his back to the object and looks into the front end of the telescope.

503. **Lord Rosse's telescope.**—By far the largest reflecting telescope ever constructed was made by the Earl of Rosse. It was commenced in 1842, and was so far completed as to be used for the first time in February, 1845.

The great speculum is six feet in diameter, has a focal length of fifty-four feet, and weighs four tons. An additional speculum to be used in the same instrument weighs three and a half tons. The tube is of wood, hooped with iron, seven feet in diameter, and fifty-two feet in length.

This telescope has fittings to mount the eye-pieces either for front view, as in Herschel's telescope, or at the side, as in the Newtonian form : for this purpose a small speculum is placed at an angle of 45°, reflecting the rays at a right angle through an orifice in the side of the tube, where the eye-piece is placed.

The base of the instrument is supported upon a universal joint; and by chains and windlasses this mammoth telescope is moved with ease, between two lofty walls supporting movable galleries, which enable the observer to follow the instrument in any required position.

The amount of light on any surface being as the square of the diameter, if we reckon the pupil of the human eye at one-tenth of an inch in diameter, this telescope will be seven hundred and twenty times as broad as the pupil, or have an area five hundred and eighteen thousand and four hundred times as great as the unaided eye. If one-half the light is lost by reflection from the mirror, we shall still have two hundred and fifty thousand times as much light as commonly enters the eye. We need not wonder therefore at the marvellous power with which this instrument penetrates the remoter regions of celestial space.

504. **Achromatic telescopes.**—The principle of achromatism has been briefly explained in § 466, where it has been shown that a convex lens of crown glass may be combined with a concave lens of longer focus, made of flint glass, which has a higher refractive and dispersive

power, the combination producing refraction without dispersion, and consequently forming an image free from the primary prismatic colors.

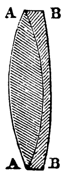
The common form of achromatic compound lens is a plano-concave lens of flint glass, united with a double convex lens of crown glass. Such lenses are found in opera-glasses and spy-glasses, called achromatic, used both on land and at sea. This form of lens is also often employed in the smaller astronomical telescopes. But in such glasses a certain amount of spherical aberration remains uncorrected.

To secure perfect correction of spherical and chromatic aberration at the same time, a double concave lens of flint glass has been placed between two double convex lenses of crown glass, the curved surfaces of the several lenses being carefully estimated in view of the refractive and dispersive powers of the two kinds of glass employed.

The refractive and dispersive powers of glass are so variable, that the optician is obliged to determine them anew for every new specimen of glass, and estimate again, by the formulæ already given, the proportional curvatures of the lenses to be constructed from it.

Sir John Herschel found that an achromatic object-glass of the form shown in fig. 383, will be nearly free from spherical aberration, if the exterior surface of the crown lens is 6·72, and the exterior surface of the flint lens 14·20, the focal length of the combination being 10·00; and the interior surfaces of the two lenses being computed from these data to destroy the chromatic aberration by making the focal lengths of the two glasses in the direct ratio of their dispersive powers (467). The two interior surfaces that come in contact may be cemented together if the lenses are small.

383



Until quite recently, almost insuperable obstacles interfered with the manufacture of flint glass in large pieces of uniform density, free from veins and imperfections.

In 1828, an achromatic lens fourteen inches in diameter was considered a true marvel of optical art. The object-glass in the great achromatic refracting telescope at Cambridge, Mass. (one of the largest in use), is about sixteen inches in diameter, with a clear aperture of fifteen inches, and it cost, unmounted, about $15,000. Mr. Bontemps, a French artist, employed in the glass works of Messrs. Chance, Brothers & Co., Birmingham, Eng., has succeeded in producing a disk of flint glass twenty-nine inches in diameter, two and a half inches thick, weighing two hundred pounds, and pronounced by the most skillful opticians very nearly faultless.

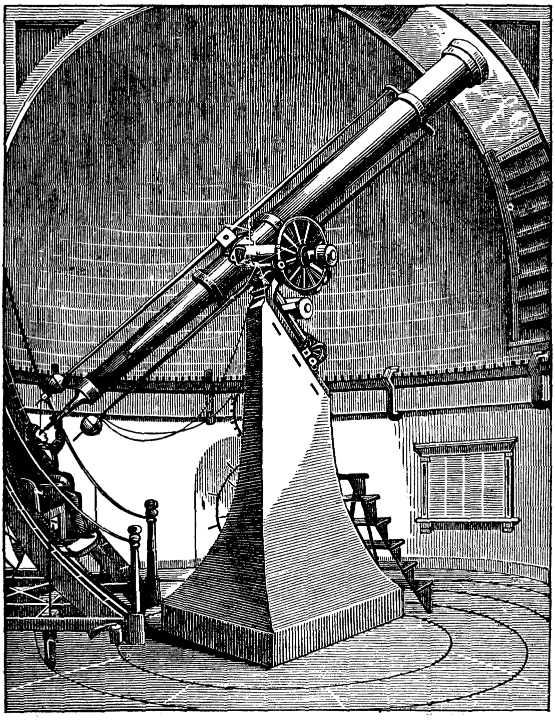
505. **Equatorial mountings for telescopes.**—With telescopes of great power, the diurnal motion of the earth causes a celestial object to pass out of the field of view too rapidly to allow of satisfactory observation. To obviate this difficulty, a system of machinery called an equatorial mounting, has been devised, to give to the telescope such a uniform motion as to keep any celestial object constantly in the field of view.

An axis firmly supported is placed parallel to the axis of the earth, and is caused to revolve by clock-work with a motion exactly equal to the sidereal motion of the heavens. A second axis, across which the telescope is mounted, is fixed upon the first axis, and at right angles with it. The telescope can be elevated or depressed in declination by motion of the second axis, and it can be moved in right ascension by motion on the first axis. When the telescope has been thus directed to any celestial object, it may be clamped on both axes, and the movement of the clock-work will cause it to follow the motion of the object in the heavens.

506. **The Cambridge telescope with equatorial mountings** is shown in fig. 384.

It stands on a granite pier surmounted by a single block of granite ten feet in height, to which the metallic bed-plate of the telescope is secured by bolts and screws. It is covered by a dome moving on a circular railway, which is easily rotated so as to allow the great telescope, twenty-three feet in length, to be directed to any part of the heavens. A narrow window, closed by shutters

384



moved by chains, is opened when the telescope is in use. The hour circle attached to the equatorial axis is eighteen inches in diameter, divided on silver, and reads by two verniers to one second of time. The declination circle is twenty-six inches in diameter, divided on silver, and reads by four verniers to four seconds of arc.

The movable portion of the telescope and machinery is estimated to weigh about three tons, but it is so perfectly counterpoised and adjusted that the observer can direct the instrument to any part of the heavens by a very slight pressure of the hand upon the balance rods. This great achromatic telescope has eighteen different eye-pieces, giving to the instrument magnifying powers varying from 103 to 2000 diameters.

507. **The visual power of telescopes**, or the aid which they afford in viewing distant objects, depends upon the combined effects of increased light and magnifying power.

Sir William Herschel relates that, on a certain occasion, when on account of the darkness a distant steeple was invisible, a telescope showed very distinctly the time by the clock on the tower. Here but little magnifying power was required, and there was a deficiency of illumination, yet the telescope supplied both.

To understand the principles upon which this power of telescopes depends, it is necessary to attend to the following particulars:—

1. *Magnifying power* is measured by the enlargement of the image seen in the telescope, as compared with the apparent dimensions of the object as seen by the naked eye.

2. *The illuminating power* of the telescope is the amount of light which it collects from any object, and transmits to the eye for the purposes of vision, as compared with the amount of light from the same object received by the unassisted eye.

The illuminating power of the telescope should be carefully distinguished from illumination of the object.

3. *Penetrating power* is the ratio of the distances at which the eye and telescope would collect, for the purposes of vision, an equal amount of light. Hence the penetrating power of a telescope is equal to the square root of the illuminating power.

4. *The visual power* of a telescope is found by extracting the square root of the product obtained by multiplying the penetrating power by the magnifying power.

Putting P for the penetrating power of a refracting telescope, x for the proportion of light transmitted by a single lens, n for the number of lenses in the instrument, A the available diameter of the field-lens, and a for the diameter of the pupil of the eye, we shall have the illuminating power $= \frac{A^2x^n}{a^2}$.

The penetrating power, $P = \sqrt{\frac{A^2x^n}{a^2}} = \frac{A}{a}\sqrt{x^n}$.

The value of x in this equation will vary with the thickness of the lenses, the degree of polish, and the amount of curvature; but for ordinary purposes of calculation we may consider its value as varying from $\frac{8}{10}$ to $\frac{9}{10}$.

Let M represent the magnifying power and V the visual power of a telescope, and we shall have generally, $V = \sqrt{MP} = \left\{ M . \frac{A}{a} . x^{\frac{3}{2}} \right\}^{\frac{1}{2}}$.

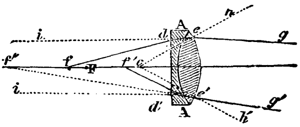
It will be evident that the best effect with the telescope will be obtained when the penetrating and magnifying powers are nearly equal. If the magnifying power is in excess, though the image may be enlarged, it will be too faint to produce a clear impression. If the magnifying power is too small in proportion to the penetrating power, the eyes will be dazzled by the excess of light, while the several parts of the image will not be clearly separated upon the retina.

The magnifying power of the telescope is therefore varied by the use of different eye-pieces (506) to suit the state of the atmosphere and the degree of illumination of the object viewed.

508. **Achromatic object glasses for microscopes**, if constructed of the forms used in telescopes, are very unsatisfactory. In the first place, it is found exceedingly difficult to construct such lenses sufficiently small for the high magnifying powers required in the microscope. Secondly, the largest achromatic lenses for telescopes have but a small diameter in proportion to the length of their foci, and if lenses for the microscope have a diameter equally small in proportion to their foci, they admit too little light to be of much practical utility. But if their diameter is increased, the light admitted through the borders of the lenses produces fringes, with colors in the inverse order of the solar spectrum, showing that while the color is perfectly corrected in the centre, the correction effected by the concave lens is too great at the margin.

509. **Lister's aplanatic foci, and compound objectives.**—The discoveries of Joseph Jackson Lister, Esq., communicated to the Royal Society in 1830, have proved of the utmost value in perfecting the compound achromatic microscope. His preliminary principles are, 1st, that plano-convex achromatic lenses, shown in fig. 371, are most easily constructed. 2d, that if the convex and concave lenses have their inner surfaces of the same curvature, and are cemented together, much less light is lost by reflection than if the lenses are not cemented. Mr. Lister discovered that every such plano-convex achromatic combination as A A, fig. 385, has some point, as f, not far from its principal focus, from which radiant light falling upon the lens will be transmitted free also from spherical aberration. This point is therefore called an *aplanatic focus.* The incident ray, $f\,d$, makes with the perpendicular, $i\,d$, an angle considerably less than the emergent ray, $e\,g$, makes with $e\,h$ the perpendicular at the point of emergence. The angle of emergence is nearly

385



three times as great as the angle of incidence, and the rays emerge from the lens nearly parallel, or converging towards a focus at a moderate distance from the lens.

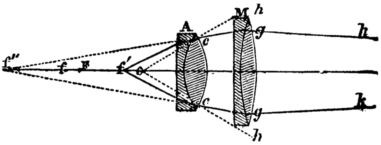
If the radiant point is now made to approach the lens, so that the ray *f d e g* becomes more divergent from the axis, as the angles of incidence and emergence become more nearly equal to each other, the spherical aberration becomes negative or over-corrected. But if the radiant point, *f*, continues to approach the glass, the angle of incidence increases, and the angle of emergence diminishes and becomes less than the angle of incidence, and the negative spherical aberration produced by the outer curves of the compound lens becomes again equal to the opposing positive aberrations produced by the inner curves which are cemented together. When the radiant has reached this point *f′* (at which the angle of incidence does not exceed that of emergence so much as it had at first come short of it), the rays again pass the glass, free from spherical aberration. The point *f′* is called the shorter aplanatic focus.

For all points between the two aplanatic foci *f* and *f′* the spherical aberration is over-corrected, or negative; and for all radiant points more distant than the longer aplanatic focus *f*, or less distant than the shorter aplanatic focus *f′*, the spherical aberration is under-corrected, or positive. These aplanatic foci have another singular property. If a radiant point in an oblique or secondary axis is situated at the distance of the longer aplanatic focus, the image situated in the corresponding conjugate focus will not be sharply defined, but will have a coma extending outwards, distorting the image. If the shorter aplanatic focus is used, the image of a point in the secondary axis will have a coma extending towards the centre of the field. These peculiarities of the coma produced by oblique pencils are found to be inseparable attendants on the two aplanatic foci.

These principles furnish the means of entirely correcting both chromatic and spherical aberration, and of destroying the coma of oblique pencils, and also of transmitting a large angular pencil of light free from every species of error.

Two plano-convex achromatic lenses, A M, fig. 386, are so arranged that the light radiating from the shorter aplanatic focus of the anterior combination is received by the second lens in the direction of *f′′*, its longer aplanatic focus.

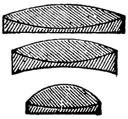
386



If the two compound lenses are fixed in this position, the radiant point may be moved backwards or forwards within moderate limits, and the opposite errors of the two compound lenses will balance each other.

Achromatic lenses of other forms have similar properties. It is found in practice that larger pencils free from errors can be transmitted by employing three compound lenses, the middle and posterior combinations being so united as to act as a single lens, together balancing the aberrations of the more powerful anterior combinations. Fig. 387 shows

387

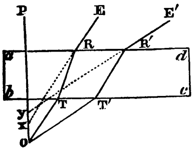


a common form of the triple aplanatic and achromatic objective, used for the compound microscope.

510. **Aberration of glass cover corrected.**—If an object viewed with an achromatic microscope, which has all its aberrations corrected for an uncovered object, is covered with even a thin film of glass or mica, spherical aberration is again produced, thus sensibly impairing the distinctness of vision when a high power is used.

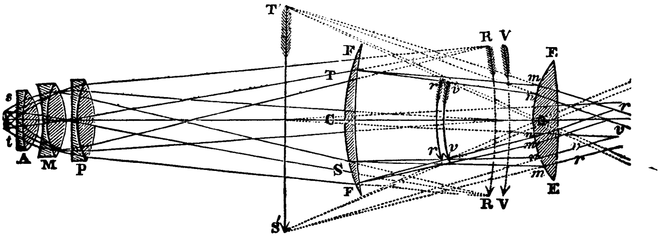
Let *a b c d*, fig. 388, be a film of glass or mica bounded by parallel surfaces. If rays of light, diverging from O, pass through this film, the ray O T′ R′ E′ will suffer greater displacement than the ray O T R E, which makes a smaller angle with the perpendicular O P. If R E and R′ E′ are extended backward, they will cross the axis or perpendicular at the points X and Y. This separation of the points X and Y is exactly similar to the spherical aberration of a concave lens, and is therefore called negative spherical aberration. Chromatic aberration is also produced by the same means. The effect observed by the eye in such cases is, that lines are not so sharply defined, and the outline of an object appears bordered with broader fringes, with colors of the secondary spectrum upon the borders of the object. These errors are easily corrected by diminishing the distance between the anterior and posterior combinations of the compound objective, which is furnished with an adjusting screw for this purpose.

388



511. **The compound achromatic microscope** is composed of the

389



triple achromatic objective, A M P, fig. 389, and the negative eye-piece, formed of the field-lens F F, and the eye-lens E E.

The section drawn in the figure, shows how the light is acted upon in passing through the different parts of the instrument. Pencils of rays from all parts of the object, *s t*, pass through the compound objective A M P, and tend to form a red image at R R, and a violet image at V V, the object-glass being slightly over-corrected, so as to project the violet rays as far beyond the red as may be necessary to make up for the want of absolute achromatism in the eye-piece. The converging pencils S, C, T, being intercepted by the field-lens F, are foreshortened, and at the same time the lateral pencils are bent inward, so that the images *v v*, *r r*, are smaller, nearer together than V V, R R, and curved in an opposite direction. The reversion of the curvature of the images is produced by the form of the field-lens, which meets the central pencil, C, much farther from the images V V, R R, than where it meets the lateral pencils S T; thus the focus of the central pencil is more shortened than the others. The field-lens of the negative eye-piece does not reverse the curvature in every variety of instrument, but it always changes the form of the images so as to improve the definition. The violet rays S *n*, T *n*, fall upon the eye-lens nearer its axis, than the red rays S *m*, T *m*, which are less refrangible, and hence the eye-lens counteracts the divergence of the colored rays which were separated by the field-lens, and causes them to pass to the eye so nearly parallel that they appear to diverge from the same point of the virtual image S T, formed at the distance of distinct vision. The distance between the red and violet images *r r*, *v v*, is just equal to the difference between the red and violet foci of the lens, and these images being curved just enough to bring every part into exact focus for the eye-lens, the eye sees the image at S′ T′ spread out in its true form on a flat field.

By means of this beautiful system of compensations, for the various errors of chromatic and spherical aberration and curvature of the image, which interfere with the performance of a single lens, the compound achromatic microscope has been brought to a degree of perfection unsurpassed by any instrument employed in practical physics.

512. **Solid eye-piece.**—A negative eye-piece, constructed of a single piece of glass, has been patented by Mr. R. B. Tolles, of Canastota, N. Y. In the solid eye-piece there is much less loss of light by reflection, as there are only one-half as many refracting surfaces as in the ordinary eye-piece. The image is of course formed in the substance of the glass. This eye-piece allows the use of a higher magnifying power than the eye-piece formed of two lenses, and it is thought also to give more perfect definition.

513. **Visual power of the achromatic microscope.**—The great distinction between the telescope and the microscope consists in the fact that while the former, practically speaking, is suited to receive parallel rays from a distant object, the latter has to deal with rays which diverge from a closely approximate point. On this account the formula for visual power will require some modification.

Angular aperture.—The angular breadth of the cone of light which a microscope receives from an object, and transmits to the eye, is called its angular aperture.

Illuminating power in the microscope depends upon the square of the angular aperture, due allowance being made for the light lost in its passage through the instrument.

When the formula for visual power is applied to the microscope, A must represent the angular aperture of the instrument measured in degrees; and a will represent the angular breadth of a cone of light which can enter the pupil of the eye from an object at the distance of distinct vision = 1° very nearly. We shall then have:—

The penetrating power of the microscope, $P = A\sqrt{x^n}$; or the penetrating power varies directly as the angular aperture. This is not absolutely correct, for the loss of light by reflection causes x to diminish as A increases.

Let $C = \sqrt{x^n}$, and we shall have:—*The visual power* of the microscope,

$$V = \sqrt{MP} = \sqrt{MAC} = \sqrt{A} \times \sqrt{MC}:$$

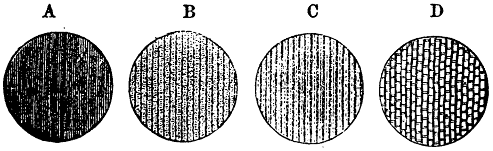
Or (since the magnifying power, or the eye-piece, which may be employed, varies with the angular aperture), generally:—

The visual power of the microscope is proportional to the square root of the angular aperture of the object-glass.

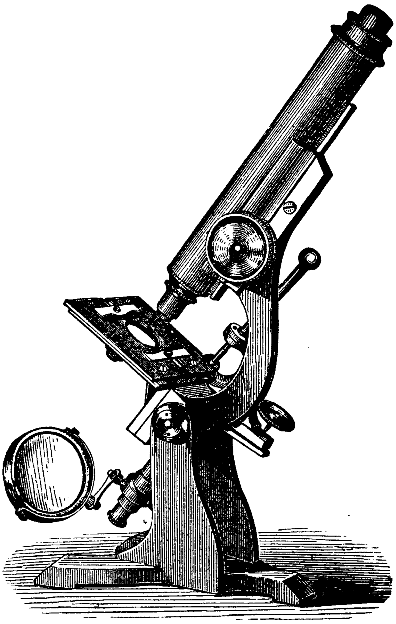
Defining power, or sharpness of minute details in an object seen by the microscope, requires perfect correction of chromatic and spherical aberration.

In fig. 390, A, B, C, D, show the successive appearances of a scale of *Morpho Menelaus,* by regular enlargements of the angular aperture of the microscope with which it was viewed. The available angular aperture of a single lens seldom exceeds fifteen or twenty degrees. In the triple achromatic objective, the aperture for ordinary observations has been extended to 100°. With the highest powers used for viewing infusoria, both English and American opticians have advanced the angular aperture to 150°, and in some glasses to 175°.

390



391



514. **The mechanical arrangement of the microscope** is well exhibited in fig. 391, which has been engraved from a very excellent instrument, manufactured by J. & W. Grunow, New Haven, Conn.

The instrument is mounted on trunnions, which allow it to be inclined at any angle. The body of the microscope is moved in a grooved support, by a rack and pinion motion for adjusting the focus. The stage has a fine, delicate movement, by a screw and milled head, acting upon a lever at the back of the instrument, by which movement the focus can be adjusted with the utmost delicacy.

The stage itself can be moved freely in any direction by a lever at the right.

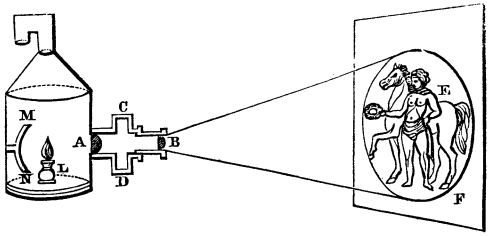
A mirror, concave on one side, and plane on the other, is so mounted below the stage as to illuminate the object with either parallel or converging rays.

Polarizing apparatus, and other accessories, are fitted to the stage, and to the body of the microscope.

515. **The magic lantern** is an instrument for projecting upon a screen, images of transparent pictures painted on glass.

A lamp is placed in a dark box, before a parabolic reflector, M N, fig. 392, which throws the light upon a convex lens, A, by which it is strongly condensed upon the object painted on the glass slide, inserted at C D. The magnifying lens, B, forms an image of the illuminated picture upon a screen E F, placed at its conjugate focus. The picture is placed in an inverted position, to produce an erect image upon the screen.

392

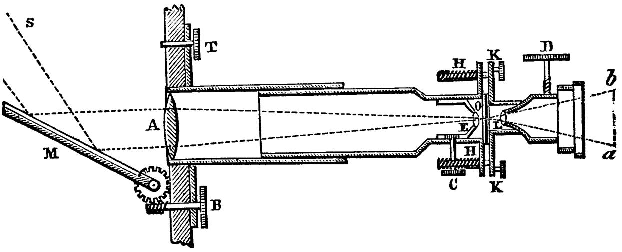


A great variety of objects painted on glass can thus be exhibited either for amusement or instruction. The magnifying power of the magic lantern is equal to the distance of the screen from the lens, B, divided by the distance of the lens from the object.

516. **The solar microscope** is a species of magic lantern illuminated by the sun. It is, however, much more perfect in its structure, and it is commonly employed for viewing on a screen images of natural objects, very highly magnified.

The structure and arrangement of the solar microscope are shown in fig. 393.

393



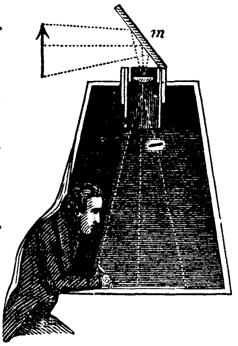
It is mounted over an opening in the shutter of a dark room, on the side towards the sun. A plane mirror, M, is so arranged outside the shutter as to reflect the rays of sunlight, S, through the condensing lens, A, into the microscope. By turning the screw, B, the mirror may be elevated or depressed, and by means of another screw, T, it can be rotated on the axis of the microscope, so as to follow the motions of the sun. A small lens, E, moved by the rack and pinion with the milled head C, serves to condense the light upon the object slide, O. The slide O, which carries the object, is secured between the brass plates K K, by the screws, H H.

The object, strongly illuminated, is adjusted to the focus of the small lens, L (which may be either a small globule of glass, or a compound achromatic objective, of short focus), and an image, *a b*, greatly enlarged, formed in the conjugate focus of the lens, is received upon a white screen placed in a convenient position. By diminishing the distance between the object and the lens, L, the conjugate focus will be more distant, the screen may be placed farther from the lens, and the magnifying power will be proportionally enlarged.

Instead of employing the light of the sun, the solar microscope may be illuminated by the electric, or by the oxyhydrogen light.

517. **The camera obscura** consists of a dark chamber in which images of external objects are formed by the aid of a mirror, and a concave lens. This instrument affords a conenient method of sketching natural scenery.

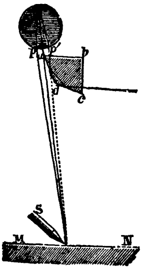
394



A plane mirror, *m*, fig. 394, placed at an angle of 45° with the horizon, reflects the light downward, through a converging lens, placed in the top of the dark chamber. A sheet of paper placed on the table in the focus of the lens, receives the image of a landscape or other object, which can be traced with a pencil by the artist, sitting, as shown in the figure, with his head and shoulders protected from extraneous light by a dark curtain.

The student can easily prepare an instrument of this kind, by inserting a spectacle glass in an orifice in the top of a box about two feet high, and placing a common mirror at the required angle above it. The paper on the table can be placed on a drawing-board, and fixed at such a distance from the lens as gives the most distinct image. A cloak thrown over the side of the box where the observer sits, will darken the chamber so as to permit sketches to be made with great facility.

395



Instead of the mirror and lens shown in fig. 394, a rectangular prism is often used as a reflector, and if one side of the prism is ground in the form of a lens, the two parts of the instrument are combined in one.

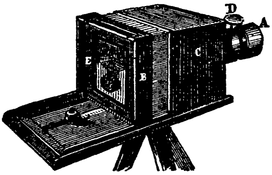
518. **Wollaston's camera lucida** is another instrument used for sketching from nature. It consists of a prism, *a b c d*, fig. 395, of which the angle, *b*, is a right angle, the angle, *d*, is 135°, and the angles at *a* and *c* are each 67½°.

It is mounted on a suitable stand, and the eye, P P', placed as shown in the figure, sees the image of a distant object as though projected upon the paper M N, where the outline may be traced by the pencil S, the eye seeing the image and the pencil at the same time. The light from a distant object entering the prism nearly at right angles with the face, *b c*, twice suffers total reflection, and emerges perpendicular to the face *a b*, when it enters the eye, and appears as if coming from the paper, M N. The image projected upon the paper is as much smaller than the object as its distance from the prism is less than the distance of the object. The image can be made to assume any required dimensions by varying the relative distances of the paper and the object. This instrument is principally employed by artists for sketching landscapes.

A number of other forms of camera lucida are employed to suit different purposes, but in all of them, either the object, or the pencil and paper, are viewed by reflected light, made to coincide in direction with the direct light.

519. **Photography** is the art of producing pictures by the chemical action of light. The daguerreotype, ambrotype, crystallotype, and photo-lithograph, are all produced by modified applications of the camera obscura. Instead of the plain paper and pencil used by the artist for sketching with the camera, a surface of silver or collodion, made sensitive by iodine, bromine, or some other chemical preparation, is placed in the camera and subjected to the action of the light of the image projected there by the lens.

396



A camera employed for photography in any of its forms, requires to be achromatic, and also that the chemical rays shall be brought to a focus at the same point as the visual rays, or at a well-defined distance from them. As objects copied by photography are seldom flat, the objective of the camera requires to be so constructed as not only to give perfect definition of all objects situated in the focal plane, but also it should be adapted to give tolerably good definition of parts of an object that are situated a little anterior or posterior to the focal plane.

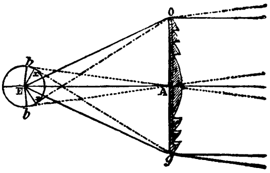
The usual form of the camera employed in photography, is shown in fig. 396. The achromatic compound lens, A, is attached to the box, C, and can be moved backwards or forwards by turning the milled head, D. The second box, B, slides within the first. A plate of ground glass set in the frame, E, is inserted in B, and when the focus is so adjusted as to give a perfect image on the ground glass, this is removed, and the sensitive plate covered by a dark screen is inserted in its place. The dark screen is then removed, and the light produces a chemical change where the image is projected. This image is then made permanent by vapor of mercury or other chemical applications.

520. **Railway illumination.**—For illuminating railroads, it is important to throw upon the track a powerful beam of light, consisting of rays nearly parallel. When the track is thus illuminated, objects upon it are more readily distinguished by contrast with surrounding darkness; it is therefore desirable to limit the light to the immediate vicinity of the track.

The common method of effecting this object is to place an Argand lamp in the focus of a large parabolic reflector (325), situated in front of the locomotive. The light is thus thrown forward in parallel lines, and the lateral illumination produced by light radiated directly from the lamp is comparatively small.

521. **The Fresnel lens**, a section of which is shown at *o* A *g*, fig. 397, is also employed for projecting a powerful beam of parallel light upon objects to be illuminated at a distance. This form of lens, invented and first applied to practical purposes by Fresnel, consists of a central plano-convex lens, surrounded by segmentary rings, with curvatures successively diminishing as much as is necessary to avoid the spherical aberration of a single lens, the central lens, and all the angular segments having their curves so adjusted as to have a common focus.

397

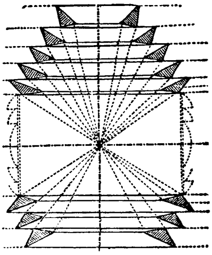


The segmentary rings are sometimes made entire, but generally, when the size is considerable, each ring is composed of several parts. The central lens and lateral segments are all cemented to a plate of glass, as shown in the figure.

For most purposes, where the Fresnel lens is employed, it is necessary to give the illuminating beam of light a slight degree of divergence. It will be easily seen from the figure, that if the centre of the lamp is placed at the principal focus of the lens, F, the divergence of the beam, after passing the lens, will be equal to the angle *b* A *b*, which the flame of the lamp subtends at the surface of the lens. A concave mirror is also placed behind the lamp, to throw forward the light in a condition to be refracted nearly parallel by the lens in front of the lamp. A much more brilliant beam of light is obtained in this manner than by the parabolic reflectors alone. This lens is also used in France for railway illumination.

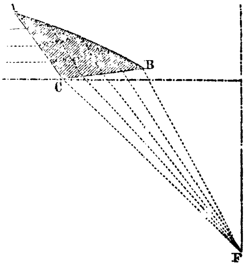
522. **Sea-lights**, designed as beacons to the mariner upon dangerous coasts, or for lighting harbors, are usually placed in towers, called *light-houses*. The great elevation of the light permits it to be seen far out at sea. It is evident that all light thrown out above or below the plane of the horizon, is of no avail to the mariner.

398



By an ingenious application of the principles of the Fresnel lens, a sheet of light is thrown out in every direction in the plane of the horizon. If fig. 398 is revolved about the central perpendicular line, as an axis, it will generate the apparatus known as the *Fresnel fixed light*. The central zone will consist of a series of hoops whose perpendicular section is everywhere the same as a section of the Fresnel lens. This zone will therefore so act upon the light of a lamp placed at the centre, as to project a sheet of light in every direction in the plane of the horizon. Above and below the central zone, are series of triangular hoops. A section of one of these hoops, and its action upon light radiating from the central lamp, is shown in fig. 399; A C and B C are plane faces, while A B is a convex surface. Light from the focus, F, is refracted on entering the face B C, it undergoes total reflection at the surface A B, and a second refraction at A C, from which it emerges in lines parallel to the horizon.

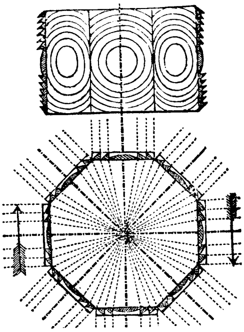
399



The focus of each prismatic hoop is carefully calculated for the place it is to occupy, so that every part of the apparatus throws out the light that falls upon it in a horizontal direction.

523. **Revolving lights.**—To distinguish one lighthouse on the coast from another, the Fresnel light is so modified as to give a steady light, and also revolving flashes of light of very great intensity.

400



In the revolving Fresnel light, the triangular prismatic hoops above and below the central zone are the same as for the fixed light, but the central zone is made of eight Fresnel lenses, fig 400, set as shown in the lower part of figure. The upper part of the same figure shows a front view of the central zone. While the entire apparatus revolves as shown by the direction of the arrows, each of the eight lenses gives a very intense light in certain directions, and between any two there is no light from the central zone of lenses. The light seen from any position appears gradually to increase to very great brilliancy, and then to fade away to much less than half its maximum intensity, after which it again increases to its former brilliancy. These changes are repeated at regular intervals.

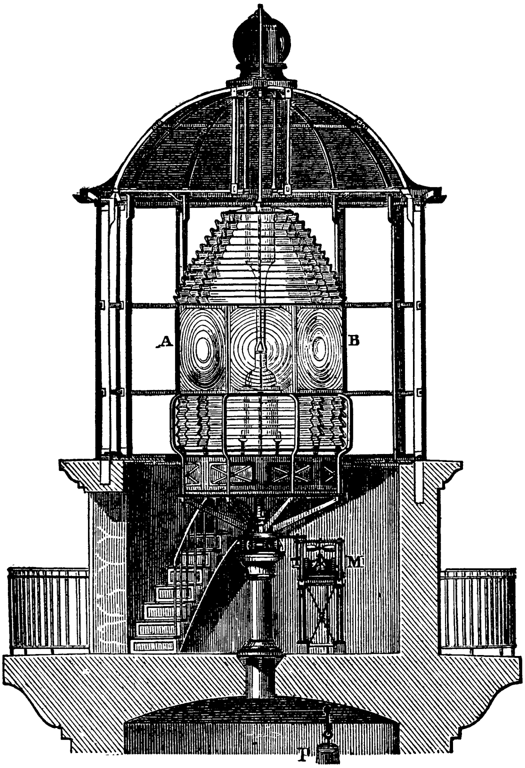
Fig. 401 shows a plan of a revolving Fresnel light fixed in the tower of a lighthouse. At A B are the parts shown in fig. 400 which produce the flashes of light. The whole apparatus is made to revolve by means of the clock-work shown at M, which is moved by the weight P. The balcony surmounting the tower is seen in the lower part of the figure, also the stairs leading to the light. A dome, supported on iron frame-work, protects the illuminating apparatus. The distance at which the light can be seen will depend upon the height of the tower in which it is placed.

The lamp used for the Fresnel light is an Argand burner, with four concentric wicks, with currents of air passing up between them.

The wicks are defended from the excessive heat of their united flames by a superabundant supply of oil, which is thrown up from below by a clock-work movement, and constantly overflows the wicks. A very tall chimney is required to supply a sufficiently strong current of air to support the combustion. The dimensions of the Fresnel light, and the number of lenses and hoops of which it consists are varied to suit the purposes for which it is used; the light pro

duced varying in intensity from twenty-five to three thousand Argand lamps of common size.

401

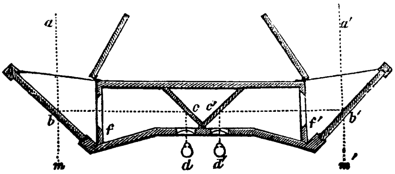


524. **The telestereoscope.**—The image upon the retina of every human eye represents a perspective projection of the objects situated in the field of view. As the positions from which these projections are taken are somewhat different for the two eyes of the same individual, the perspective images themselves are not identical, and we make use of their difference to obtain an idea of the distances from the eye of the different objects in the field of view.

The images of the same object on the two retinæ are more different from each other as the object is brought nearer to the eyes. In the case of very distant objects, the difference between the pictures on the retinæ of the two eyes becomes imperceptible, and we lose the aid just spoken of in estimating their distance and bodily figure.

The telestereoscope is an instrument which causes distant objects to appear in relief. It increases the binocular parallax of distant objects, and by presenting to each eye such a view as would be obtained if the distance between the two eyes were greatly increased, it gives the same appearance of relief, as if the objects were brought near to the observer.

402



Let *b* and *b′*, fig. 402, be two plane mirrors placed at angles of 45° with the line of vision; let *c* and *c′* be two smaller mirrors placed parallel to *b* and *b′*, and let *d* and *d′* represent the position of the two eyes of the observer. It is evident that the light from distant objects falling upon the mirrors in the direction *a b* and *a′ b′*, will be reflected to the small mirrors *c* and *c′*, where it will be again reflected to the eyes at *d* and *d′*. The two views seen by the eyes will evidently be the same as if the eyes were separated to the positions *m* and *m′*. The relief with which objects will be seen by this instrument, will obviously be increased as much as the distance *b b′* exceeds the distance between the eyes *d* and *d′*.

But while this instrument increases the perspective difference of the images seen by the two eyes, the visual angle under which each object is seen remains unchanged, and hence, as the apparent distance of the objects is diminished, their dimensions appear diminished in the same proportion. If the small mirrors are made to rotate on perpendicular axes, while the large mirrors are fixed, the distortion of figure may be easily corrected by turning the small mirrors until objects appear in their true proportions.

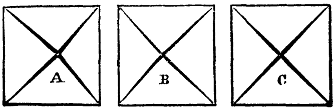
If the lenses of an opera glass are inserted in the instrument, the convex field-glasses being inserted at *f* and *f′*, between the large and small mirrors, and the concave eye-glasses between the eyes and small mirrors, the effect will be to increase the visual angle of every object in the field of view. If the glasses magnify as many diameters as the distance between the large mirrors exceeds the distance between the eyes, every object will appear in its due proportions, and the effect will be surprising. The appearance will be as though the observer had been actually transported to the immediate vicinity of the objects themselves. The distance between the large mirrors of the telestereoscope should not exceed the breadth of an ordinary window, unless it is to be used in the open air, when it may be made of any dimensions that are desired, and the effect produced will be in proportion to its magnitude.

525. **The stereoscope** (from *στερεός*, solid, and *σκοπέω*, to see) is an instrument so constructed that two flat pictures, taken under certain conditions, shall appear to form a single solid or projecting body.

In order to produce this illusion, different images as observed by the two eyes (484) must be depicted on the respective retinæ, and yet appear to have emanated from one and the same object. Two pictures are therefore taken from the really projecting or solid body, the one as observed by the right eye only, and the other as seen by the left. These pictures are then placed in the box of the stereoscope, which is furnished with two eye-pieces, containing lenses so constructed that the rays proceeding from the respective pictures, to the corresponding eye-pieces, shall be refracted or bent outward, at such an angle as each set of rays would have formed had they proceeded from a single picture in the centre of the box to the respective eyes without the intervention of the lenses.

It is an axiom in optics that the mind always refers the situation of an object to the direction from which the rays appear to proceed when they enter the eyes; both pictures will therefore appear to have emanated from one central object. As one picture represents the real or projecting object as seen by the right eye, and the other as observed by the left, though appearing by refraction to have both proceeded from the same object, the sensation conveyed to the mind, and the judgment formed thereon, will be precisely the same as if both images were derived from one solid or projecting body, instead of from two pictures. Consequently the two pictures will appear to be converted into one solid body.

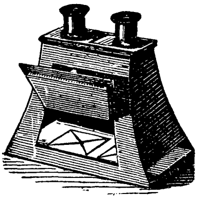
403



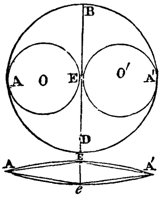
If two pictures of an octahedron, as A and B, fig. 403, such as would be formed on the retinæ of two eyes, are placed in the stereoscope, fig. 404, they give to the observer the idea of a real solid octahedron, instead of the ordinary picture, C. Photographs of natural scenery, taken from two positions, when viewed in this instrument, appear in relief like real objects.

The construction and action of the stereoscope will be readily understood by reference to figs. 405 and 406. From a double concave lens, A B A′ D, two eccentric lenses, represented by the smaller circles, are formed. E A *e*, in the lower part of the figure, represents a transverse section of one of these eccentric lenses, and E A′ *e* the other. Each lens is equivalent to a triangular prism E A *e*, with a plano-convex lens cemented to each refracting face of the prism. Fig. 406 shows a

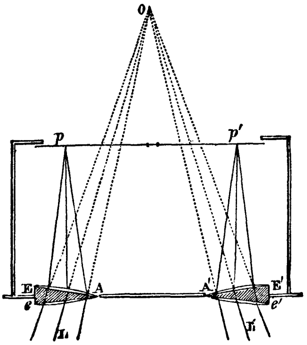
404



405



406



section of the stereoscope, the eccentric eye-lenses A E, A′ E′, being placed at the ordinary distance of the eyes, with their thin edges towards each other Let P and P′ represent two corresponding points in the stereoscopic photographs which are to be examined. The rays of light, diverging from the point P, falling upon the eye-lens, are refracted nearly parallel, and by the prismatic form of the lens are deflected from their course, and emerge from the lens in the same direction as if emanating from the point O. In the same manner the rays from the point P also appear to diverge from the point O. The same is true of all similar parts of the two pictures; thus the pictures appear superimposed upon each other, and together produce the appearance of relief, for which the stereoscope is so much admired.

The eccentric lenses of the stereoscope are sometimes fixed in position, but they are often inserted in tubes, as in fig. 404, which can be extended to adapt the focus to different eyes, or separated to a greater or less distance, to suit the distance between the eyes of different persons.

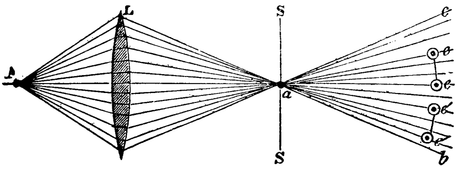
If stereoscopic photographs are taken from positions too widely separated from each other, objects stand out with a boldness of relief that is quite unnatural, and the objects appear like very reduced models. In taking stereoscopic miniatures especially, great care is required to preserve a natural appearance. In general, a difference of a few inches in the two positions of the cameras, gives sufficient relief to the pictures when seen in the stereoscope.

For public buildings and landscapes, two cameras are usually employed, placed on a stand three or four feet from each other. If it is desired to show a great extent of a distant landscape, or to exhibit in miniature the grouping and form of distant mountains, two stations should be selected that are widely separated; but in such cases, care should be taken that no near objects are admitted into the picture.

526. **The stereomonoscope** (described by Mr. Claudet, of London) is an instrument by which a single image is made to present the appearance of relief commonly seen in the stereoscope, and by means of which several individuals can observe these effects at the same time.

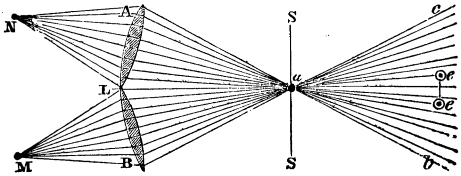
Let A, fig. 407, be an object placed before a large convex lens, L, an image of the object will be formed at *a*, in the conjugate focus of the lens, and from the image *a* the rays of light will diverge as from a real object, which will be seen by the eyes placed at *e e*, *e′ e′*, or any other position, in the cone of rays *b a c*. Thus several persons may at the same time see the image suspended in the air. If a screen of ground glass is placed at S S, the image will appear spread out upon the glass, but it will appear with all the perspective relief of a real object. An image thus formed on ground glass can be seen only in the direction of the incident rays. This is not the case with an image formed on paper, which radiates the light in all directions, and is hence incapable of giving a stereoscopic effect in such circumstances.

407



The stereomonoscope consists of a screen of ground glass, S S, fig. 408, and two convex lenses, A L, B L, so placed as to form images of two stereoscopic pictures, M and N, at the point *a* on the screen S S. Though the two pictures have their images superimposed on the same part of the screen S S, each picture can be seen only by the rays proceeding from the photograph by which it was formed. If the eyes are so placed that the right eye is in the direction of the rays coming from one lens, and the left eye in the direction of rays coming from the other lens, the object will appear in relief as in the stereoscope, and several persons can witness the effect at the same time.

408



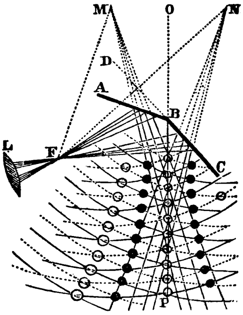
§ 7. Physical Optics.

I. INTERFERENCE, DIFFRACTION, FLUORESCENCE, &C.

527. Interference of light.—The interference of vibrations and waves, has been already alluded to in the *theory of undulations* (328, 333), but the phenomena of luminous interference require some further special consideration.

Let A B, B C, fig. 409, be two plane mirrors, making with each other a very obtuse angle (very near 180°); let a beam of sunlight, entering a dark room by a small opening, be brought to a focus by a lens, L; if this light, diverging from a focus, F, is allowed to fall very obliquely upon the two mirrors, as shown in the figure, it will be reflected as if diverging from two luminous points, M and N, and the light thus reflected will be in a condition to interfere. Draw O P perpendicular to M N, from a point O, midway between them. It is evident that every point in the line B P, will be equally distant from the luminous points M and N; the waves of light which cross each other in the line B P, will therefore be in the same phase of vibration, and consequently produce a line of light of double intensity. Let the smooth circular arcs represent the phases of elevation, and the dotted arcs phases of depression; then where a dotted arc crosses a smooth arc, the two waves should counteract each other and produce darkness. The open dots represent vibrations meeting in the same phase, and the black dots represent vibrations meeting in opposite phases, which produce darkness. The symmetrical curves formed by the intersection of light from the two points M and N, on both sides of the central line, are of the form known in geometry as hyperbolas.

409



The distance on each side of the line B P, where the luminous waves will be again in a like state of accordance represented by the crossing of the smooth arcs in the figure, will depend on the interval between them, which is different for different colors; for red, it is half as much again as for violet light; hence the distance between the curves of double intensity will be least for violet light, greatest for red, and intermediate for the other colors of the spectrum, so that while all the colors are united in the central line B P, they will be separated in the other bars, and form a series of colored fringes. In experiments, this serves to distinguish the central bar, namely, that the other bars are colored symmetrically on each side of it.

Half way between two places of complete accordance there must occur a place of complete discordance, where the difference of distances from M and N is $\frac{1}{2}$ an interval, or $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$, &c.; and according to the undulatory theory, there would be complete darkness. Between these and the places of complete accordance, there would be intermediate stages of accordance and discordance; hence there would be bright bars shading into dark ones, all more or less colored except the central bars, where all the colors are in a state of complete accordance.

By careful measurement of distances between the luminous and dark bars, the lengths of luminous waves of different colors have been very accurately ascertained.

528. **Facts at variance with theory.**—When the atmosphere is free from clouds, and the sunlight is brightest, the *central* bar (which, according to theory, should be *bright*) is found to be a *black one*, whatever be the material of which the mirrors are composed. But when the sun is near setting, the central bar has been seen undoubtedly a *bright* one. It has also been seen as a bright bar when the luminous point was formed at a hole in a thin plate of metal, and the light which had grazed the edge of the hole was used.

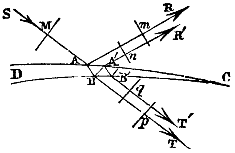
The existence of a *central black bar*, in normal circumstances, where the vibrations must meet in the same phase, is thought to be inconsistent with the undulatory theory of light.

It appears that light is so modified in passing through haze, or at an opaque edge of a small hole, as to acquire an *anatropy* or inversion of properties.*

529. **Interference colors of thin plates** are seen in thin films of varnish, cracks in glass, films of mica, various crystals, and in other transparent substances, as in soap bubbles. The colors of such thin films are due to the interference of light twice reflected by the surfaces of the film.

Two surfaces of glass, pressed together, furnish a thin plate of air between two reflecting surfaces. Let C A D B, fig. 410, be a transparent film, such as a thin blown bulb of glass, or a soap bubble; let S A B T be the transmitted ray, S A R the reflected at the first surface, S A B A′ R′ the portion reflected from the second surface, and emergent at the first surface, S A B A′ B′ T′ the portion emerging from the second surface, after the two internal reflections, then the ray A′ R′ will be retarded behind the ray A R, by the interval *n m*, owing to the

410



---

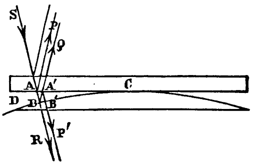
* Potter's Physical Optics.

increased length of path it has to travel in twice traversing the film, and B′ T′ will, in a similar manner, fall behind the ray B T, by the interval *p q*. If these retardations equal the interval of an odd number of half vibrations, they will interfere, as they originated from a common wave, in the ray S A. The reflected rays do not differ greatly in intensity, which is for each about one-thirtieth that of the incident light for glass, and therefore their interference produces blackness where they destroy each other. The transmitted light has the principal beam of little less intensity than the incident beam, having lost only about one-thirtieth part by reflection at each of the points A and B; but the intensity of the twice reflected beam which interferes with it is about one-thirtieth of one-thirtieth, or one nine-hundredth of that of the incident beam; hence the difference of the intensities of the bright and dark bands formed by transmitted light is never as great as in the reflected beams. But the difference between the bright and dark bands is different for different colors of the spectrum, being least for violet light, and greatest for red. This fact is thought to be contrary to what should have been expected, according to the undulatory theory.

530. **Newton's rings.**—If a plane plate of polished glass is pressed against a plano-convex lens whose radius of curvature is known, the interference bands become colored rings, and the exact thickness of the film of air by which each color is produced is easily estimated.

The form of this apparatus is shown in fig. 411. The letters and explanation of the figure are similar to the preceding. When the two glasses are pressed sufficiently near together, the centres appear black by reflected light, and bright by transmitted light. The thickness of the film of air where the first color appears, is equal to one-half the retardation producing that color; hence the length of the wave, or vibration, for any color, is estimated as equal to twice the thickness of the film of air where the color appears. The colors succeed each other in the order of the length of the vibrations required to produce them. A second, third, and fourth series of colored rings will be found, where the thickness of the film is an exact multiple of the thickness required to produce the first series of colors. The distance between the first and second series depends on the rapidity with which the thickness of the film increases. In the case of a lens pressed against a plate of glass, the distance between the glasses, or the thickness of the film, increases as the square of the distance from the centre. The diameters of the bright rings will therefore be as the square roots of the numbers 1, 2, 3, &c., and the diameters of the dark rings will be as the square roots of the numbers 1½, 2½, 3½, &c. The distance between successive rings of violet will be much less than the distance between successive rings of red; one series of colors will therefore overlap some of the colors in the succeeding series of colored images, and by their admixture produce colors, the successive groups of which are designated as Newton's first, second, third, &c., orders of colors.

411



531. **Length of luminous waves or vibrations.**—By such means as we have described, the lengths of the vibrations required to produce different colors have been estimated.

The following table exhibits the numerical results which have been deduced for the length and velocity of luminous vibrations of different colors.

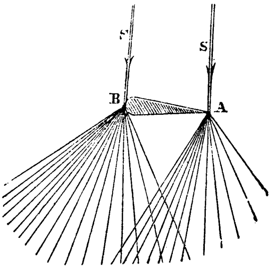
Colors.	Length of undulations in parts of an inch.	Number of undulations in an inch.	Number of undulations per second.
Extreme red, .	0·0000266	37640	458,000000,000000
Red, ...	0·0000256	39180	477,000000,000000
Orange, . .	0·0000240	41610	506,000000,000000
Yellow, . .	0·0000227	44000	535,000000,000000
Green, . . .	0·0000211	47460	577,000000,000000
Blue, . . .	0·0000196	51110	622,000000,000000
Indigo, . . .	0·0000185	54070	658,000000,000000
Violet, . . .	0·0000174	57490	699,000000,000000
Extreme violet,	0·0000167	59750	727,000000,000000

According to Eisenlohr (Am. Jour. Sci. [2] XXII.), the length of the vibrations in the extreme red ray is just double the length of the vibrations of the invisible rays beyond the violet, which, by concentration, produce the lavender light of Herschel. The entire range of visible rays differs in the length of vibrations only by the amount of one octave in music.

When we consider the almost inconceivable velocity with which these wonderfully minute vibrations are propagated, it is evident that absolute demonstration of the real nature of light must be among the profoundest researches of physical science.

532. **Diffraction.**—If a razor is held with its flat surface towards the rays of the sun, the rays that pass in close proximity, both to the edge and to the back, will be deflected as shown in fig. 412. A portion of the rays are deflected outwards, appearing to suffer reflection; the back of the razor deflecting the rays outward, more than the sharp edge; but the edge of the razor deflects more light into the place of the geometrical shadow, than is deflected inwards by the back of the instrument. These differences are represented by the closeness of the lines drawn to represent the rays where the greatest amount of light is deflected. If the body interposed is narrow, like a fine needle or a hair, the rays deflected inwards cross each other, and produce the phenomena of interference in *accordance* with the undulatory theory. The rays deflected outward produce interference with the rays not deflected, but bright lines appear where the undulatory theory would give dark lines. All the bright and dark lines are bordered with colored fringes, as in ordinary cases of interference. These phenomena are best seen in a dark room by

412



looking through an eye-lens at a hair or needle, at a considerable distance from a lamp, or by looking at a beam of sunlight admitted to a dark room between two sharp parallel edges. The rays that have been diffracted or bent into the geometrical shadow, are not as readily deflected again in the same direction, but are more easily deflected in the opposite direction than rays which have undergone no such previous change.

533. **Fluorescence.—Epipòlic dispersion.**—Certain bodies, as fluor-spar, glass colored yellow by oxide of uranium, called *canary glass*, solution of sulphate of quinine, infusion of the bark of the horse-chestnut, and many other vegetable infusions, possess the remarkable property of so dispersing some part of the light passing through them, that the course of the luminous rays becomes visible.

These phenomena are best exhibited by bringing a pencil of light to a focus in the interior of any of these substances, by means of a convex lens, when the course of the rays will become visible, as though the portion through which the light passed had become self-luminous. The rays of light of high refrangibility, especially the violet and the invisible chemical rays, are subject to this kind of dispersion, their refrangibility is at the same time changed, and probably the length of their luminous waves is increased, so that rays previously invisible may be seen by the eye. These phenomena have been called by various names, as internal dispersion, epipolic dispersion, and fluorescence. The latter term, derived from fluor-spar, and adopted by Mr. Stokes, is considered the more appropriate term, as it involves no theory.

This change of the refrangibility and length of luminous waves is analagous to the change of pitch in reflected sounds heard in certain remarkable echoes (§ 355).

534. **Phosphorescence.**—Certain bodies after being exposed to the action of light, acquire the property of shining in the dark (399). The most remarkable phosphorescent bodies are the sulphurets of barium, strontium and calcium, some kinds of diamonds, most varieties of fluoride of calcium, particularly the variety known as chlorophane, compounds of lime, magnesia, soda and potash, salammoniac, succinic and oxalic acids, borax, dried paper, silk, sugar, sugar of milk, teeth, &c.

The time during which these bodies emit light varies from a fraction of a second to several hours, and the intensity of the emitted light varies in a similar manner.

The study of these phenomena requires the use of delicate apparatus adapted to the purpose.

1. The more refrangible rays of the spectrum in general act more powerfully to produce phosphorescence in bodies exposed to their influence than the less refrangible rays. In some cases the invisible rays of the spectrum, *i. e.*, the rays beyond the violet, produce a brilliant phosphorescence.

2. The least refrangible rays, as the red, not only generally produce no phosphorescence, but even counteract the influence of the more refrangible rays when mixed with them.

3. The wave lengths of light emitted in the dark by phosphorescent bodies are in general greater than those of the exciting rays; *i. e.*, the phosphorescent light shows a color belonging to a part of the spectrum nearer to the red than the light which produced it, though in a few cases the color is unaltered.

4. The refrangibility of the light emitted by phosphorescent bodies depends upon their molecular condition, and not merely upon their chemical constitution. Each phosphorescent body appears to be adapted to vibrate in harmony with the wave lengths of some colors more readily than with others.

5. One and the same body may emit rays of very different colors, according to the time which intervenes between the action of light and the moment of observation. This last result shows that vibrations of different velocities are preserved for unequal times in different bodies; sometimes it is the vibrations corresponding to the less refrangible rays which continue longest, as in bisulphate of quinine, double cyanide of potassium and platinum, diamond, &c. Sometimes the most refrangible rays are most durable, as in Iceland spar.

6. Many bodies, such as glasses and certain compounds of uranium, owe their fluorescence entirely to the persistence of the luminous impressions for a very short time, not exceeding a few hundredths of a second; the intensity of the emitted light is then very brilliant.

It is probable that phosphorescence and fluorescence differ from one another only in the time during which a luminous impression is preserved in bodies.

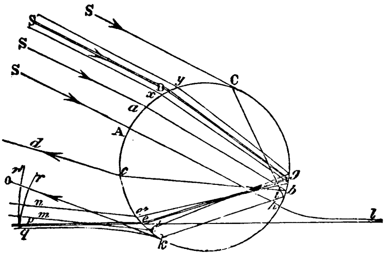
These conclusions, which support the theory of undulation as at present admitted, prove that luminous vibrations, when transmitted to any body, or at least to a great many bodies, compel its molecules to vibrate for a time, and with an amplitude and wave length which depend not only on the chemical constitution of the body but also on its physical condition.*

535. **Colors of grooved plates.**—Fine lines engraved upon polished steel, and lines drawn upon glass with a diamond point, if sufficiently near together, cause a beautiful iridescence by the interference of light reflected from such surfaces. The beautiful play of colors seen upon mother of pearl is caused by the delicate veins with which the surface is covered.

II. OPTICAL PHENOMENA OF THE ATMOSPHERE.

536. **The rainbow** is one of the most wonderful and beautiful phenomena in nature. In it reflection, refraction, dispersion, and interference of light, are all combined. It is seen in that part of the heavens opposite to the sun, when the sun is less than forty-two degrees above the horizon. The shadow of the eye of the observer will always point to the centre of the circle of which the rainbow forms a part; hence, as the sun descends near the horizon, the rainbow

413



---

* Edmond Becquerel, *Bibliotheque Universelle*, vol. XVI. p. 21.

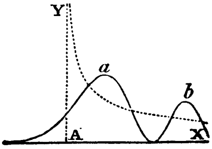
rises higher, and as the sun ascends the morning sky, the height of the rainbow diminishes.

To understand the formation of the rainbow, we must first examine the action of a single drop of water upon parallel rays of light. Let the circle, fig. 413, represent a drop of water, and S A, S B, &c., parallel rays of light falling upon it. The ray S A, which falls perpendicularly upon the drop, will suffer no deviation in its direction, but will be partially reflected backward in the line of incidence, though it will principally pass through the drop. The ray S a, will be refracted to b, where it will be reflected to c, and will emerge in the direction $c\,d$, making a certain angle with the direction of the original ray S a. As the distance of the incident ray from A increases, the emergent ray will make a greater angle with the incident ray, till we arrive at B, where two successive rays will emerge parallel, as shown by the heavy line, S B $g\,e\,p$, which deviates more from the direction A S, than any ray incident at a greater or less distance from A. As we proceed from A, towards C, the deviation of the emergent ray will diminish, and every ray between B and C will emerge parallel to some other ray, which entered the drop between A and B; S y will emerge in $e'\,m$, parallel to $e''\,n$, which entered the drop at the ray S x. The ray S C, which is tangent to the drop, will be refracted to i, and emerge in the direction $k\,o$, making an angle of about twenty-five degrees with $e\,p$, the line of greatest deviation.

The light which enters the drop in parallel rays will therefore emerge, spread over the entire space between $e\,p$ and $c\,d$; but having its greatest intensity near the direction $e\,p$, and rapidly diminishing towards $c\,d$.

If A Y, fig. 414, represent the position of the line $e\,p$ of fig. 413, the dotted curve, by its height above A X, will show how rapidly the intensity of the light fades away, as the distance from $e\,p$ increases toward $c\,d$, where the intensity is zero.

414



Since we have at every angle between $e\,p$ and $c\,d$, parallel rays which have traversed different paths through the drop, we shall have all the phenomena of bright and dark bands, produced by interference.

The intersection of the emergent rays will form a caustic curve $k\,q$, tangent to the circle at k, and approaching constantly to parallelism with the asymptote $e\,p$, which it will never meet. If the emergent rays between c and e were extended backwards, they would form another caustic $h\,l$, having $e\,p$ produced backward for its asymptote. The caustic curve $h\,l$, commences in a direction perpendicular to the surface of the drop, and approaches the asymptote without ever touching it. The curves $q\,r$, formed by unwrapping a thread from the caustic $k\,q$, and $q\,r'$, formed by a thread from the caustic $l\,h$, show, by their gradual separation, the amount of retardation of the wave surface of the two sets of parallel rays which interfere between $e\,p$ and $c\,d$.

According to the undulatory theory, we shall have bright bands where the rays have traversed equal distances, or distances differing by any number of entire vibrations, and dark bands where the rays differ by an odd number of half vibrations.

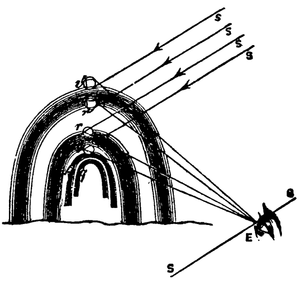
These bright and dark bands are readily seen, with proper precautions, with light reflected from a drop of water suspended at the point of a fine glass tube. When monochromatic light is used, thirty or forty of these bright and dark bands may be counted.

The breadth of the bright and dark bands varies with the size of the drop from which the light is reflected. The interference bands vary also for the different colors, being nearly twice as broad for red as for violet light. When white light is employed, the first red band only is pure, the other bands being more or less confused by the unequal super-position of different colors.

If we consider only the first two bright bands of each color, we can easily explain the common phenomena of the rainbow. A little within the caustic curve, *k q*, fig. 413, on its convex side, we shall have a bright light, represented in intensity by the curve *a*, fig. 414, and a second band of the same color at *b*, of feebler intensity. As the refractive index for red rays is less than for the other colors, the red will diverge more from the incident ray, after refraction, than the violet, and other colors will appear intermediate.

Suppose now that in a shower of rain a ray of light from the sun falls upon a drop of water at *r*, fig. 415, and is reflected from its posterior surface, so as to give to the eye the red ray of maximum intensity, *r* E, a drop below it will give a violet ray of maximum intensity, *v* E, and intermediate colors will be formed in the same manner by intermediate drops. Let the planes of incidence and reflection revolve about a line S E S, drawn from the sun through the eye of the observer; the position of the drop from which light can reach the eye will describe the arch of the rainbow.

415



The radius of the primary rainbow measured from the extreme red was found, by Sir Isaac Newton, to be 42° 4′.

The purity of the several colors in the rainbow is the result of interference, which produces dark bands for each particular color, giving a clear space for the delineation of the other colors of the rainbow before the first color is repeated. When the rain-drops differ greatly in size, as is often the case, the different colors of the first and second interference bands overlap and mingle together, and the bow is but imperfectly developed.

A secondary rainbow, with violet above and red below, is formed by light which has been twice reflected within the drops, as shown in fig. 415, the rays entering the lower border of the drop, and emerging near the upper border. The same principles of interference determine the purity of colors, and angle of maximum intensity, as in the primary bow. The loss of light occasioned by two reflections accounts for the feebler intensity of the secondary bow. In the secondary bow, the order of colors is the reverse of the primary, violet being outermost.

Newton found the distance between the primary and secondary rainbows to be 8° 30′.

A spurious rainbow is often seen within the primary bow, as shown at *p q*, fig. 415. This is formed by the second bright band of each color, the position and intensity of which is represented at *b*, fig. 414. A third and fourth bow is also sometimes seen, still interior to the second, but the colors of the third and fourth orders are so much mingled that only two or three appear in any bow interior to the first spurious bow.

537. **Fog-bows.—Halos.—Coronas.—Parhelia.**—*Fog-bows*, which are sometimes seen, differ from the rainbow by the extreme minuteness of the spherules of water from which the reflection takes place.

Halos are prismatic rings seen around the sun or moon, varying from 2° to 46° in diameter: these are explained by reflection from minute crystals of ice floating in the atmosphere.

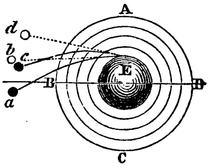
Coronas, encircling the moon, are formed by reflection from the external surface of watery vapor, the light thus reflected interfering with direct light from the same source. They generally indicate change of weather.

Parhelia, and bands of light passing through the sun, are also attributed to reflection from prisms of ice.

Many of these phenomena require for their explanation a refinement of investigation not proper to be introduced in an elementary work.

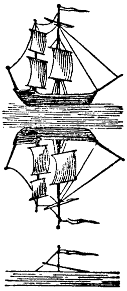
538. **Atmospheric refraction** causes all bodies not directly in the zenith to appear more elevated than they really are.

416



Let A B C D, fig. 416, represent the external surface of the atmosphere, and the inner circles strata of increasing density around the earth, E. Light from any of the heavenly bodies situated at *a* or *c* will suffer refraction by every stratum of air more dense than the preceding; and by a gradually increasing density, it will be made to travel in curved lines, until entering the eye of the observer, the bodies at *a* and *c* will appear situated at *b* and *d*.

417



539. **Looming** is a term applied to the elevation of objects at sea which appear raised above their real position by atmospheric refraction.

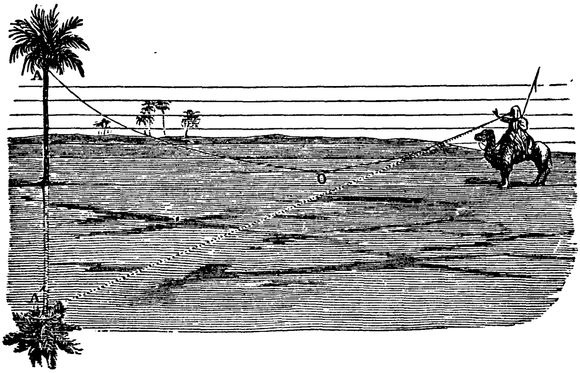
Islands often appear thus raised above the water, and an inverted image is seen below them. Distant vessels sometimes appear above the horizon, when their distance is so great that they would be far below the horizon if they were not elevated in appearance by extraordinary refraction.

In peculiar states of the atmosphere, ships have appeared suspended in the clouds, and occasionally an inverted image has appeared below, when the real ship was mostly below the horizon, as shown in fig. 417.

540. **The mirage**, often seen in Egypt, and sandy deserts, is

caused by rays reflected from strata of air heated by the burning sands. Distant objects are seen reflected by the heated air as in

418



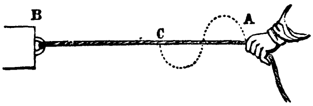
the waters of a beautiful lake, which disappears as the thirsty traveler approaches. The phenomena of the mirage are shown in fig. 418.

III. POLARIZATION OF LIGHT.

541. **Direction of luminous vibrations.**—The phenomena of polarized light are justly regarded as the most wonderful in the whole science of optics. These phenomena are most readily explained and understood by reference to the undulatory theory. It has been stated (398) that the vibrations of light move at right angles with the direction of the rays.

This species of vibration may be illustrated by those of a cord, made fast at one end, and moved rapidly upward and downward by the hand shaking the other extremity, as shown in fig. 419. If we suppose another cord vibrating from right to left, and others in every intermediate direction, we may form a tolerably clear idea of the vibrations of a collection of rays in a beam of ordinary light. A single luminous atom may be supposed to originate vibrations moving in only a single plane, but an infinite number of independent luminous atoms, constituting a luminous body, will produce vibrations moving in every possible plane. which may be illustrated by revolving that plane around the line representing the direction of a ray of common light.

419

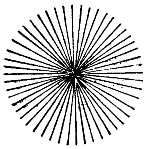


542. **Transmission of luminous vibrations.**—Opaque substances allow no luminous vibrations to pass through them. Some bodies

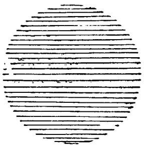
transmit nearly all the luminous vibrations which fall upon them; other bodies are capable of transmitting only those vibrations of light contained in a single plane, or that portion of the vibrating force which can be resolved into vibrations in that plane. Other bodies, capable of vibrating in two directions, reduce all the vibrations which they transmit to vibrations in the two planes in which these bodies themselves are capable of vibrating. Some bodies, by reason of the position in which an incident beam of light falls upon them, alter the direction of the vibrations which they transmit, and thus produce a beam of light, whose vibrations are all limited to a single plane.

543. **Change produced by polarization of light.**—A beam of light is said to be plane polarized when all its vibrations move in a single plane, or in planes parallel to each other. This may be illustrated by a bundle of stretched cords, all vibrating in one direction. If the cords differ in size or tension, the lengths of their vibrations will differ. This may illustrate the vibrations of different colors, which vary in the lengths of their vibrations (531). A round rod may be taken to represent a small beam of common light, and the radii shown in fig. 420 may represent the transverse vibrations by which light is propagated in ordinary media. Fig. 421 will then represent a transverse section of a polarized beam, with vibrations in planes parallel to each other.

420

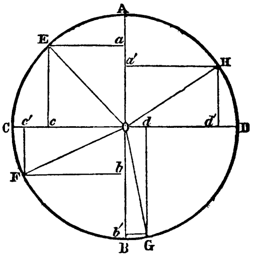


421



544. **Resolution of vibrations.**—The principle of resolution of forces (50) will enable us to understand how vibrations, in an infinite number of planes passing through the general direction of a beam of light, may be resolved into vibrations in two planes, making with each other any required angle. If O E, fig. 422, represents the direction and intensity of a vibration, it will be equivalent to O *a* and O *c*, in axes, at right angles to each other. Vibrations represented by O F, O G, and O H, may, in the same manner, be resolved into vibrations in the axes A B and C D. Then $O a + O a' + O b + O b'$ will represent the intensity of the resulting vibrations in the axis A B, and $O c + O c' + O d + O d'$ will represent the intensity of the resulting vibrations in the axis C D. If we thus resolve vibrations in an infinite number of planes into vibrations in the axes A B and C D, the sum of the resulting intensities in the axis A B will be exactly equal to the sum

422

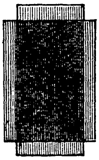


of the intensities in the axis C D. A ray of common light may therefore be considered as consisting of vibrations moving in two planes at right angles to each other. Any medium that will, either by its position or internal constitution, separate light into two parts, vibrating in planes at right angles to each other, will produce that change denominated polarization of light.

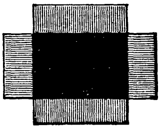
545. **Light polarized by absorption.**—Certain crystals have the remarkable property of polarizing all the light which passes through them in particular directions. They appear to absorb part of the light, and cause the remainder to vibrate in a single direction only.

If a transparent tourmaline is cut into plates one-thirtieth of an inch thick and polished, the plane of section being parallel to the vertical axis of the hexagonal prism in which this mineral crystallizes, the light transmitted through such a plate will be polarized. If a second plate is placed parallel to the first, as shown in fig. 423, the light transmitted through the first plate will also be transmitted through the second plate; but the light will be entirely obstructed if the axis of the second plate is placed at right angles with that of the first, as shown in fig. 424. A plate of tourmaline becomes, therefore, a convenient means of polarizing light, and also an instrument for determining whether a ray of light has been polarized by other means. A tourmaline plate so used is called an analyzer. Crystalline plates of sulphate of iodo-quinine (called Herapathite, from the name of their discoverer, Mr. Herapath), act in all respects like plates of tourmaline.

423



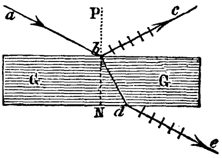
424



546. **Polarization by reflection.**—When light falls upon a transparent medium, at any angle of incidence whatever, some portion of the light is reflected. When the incident light falls upon the medium at a particular angle, which varies with the nature of the substance, all the reflected light is polarized.

Let G, G, fig. 425, be a plate of glass or any other transparent medium, and let a ray of light, *a b*, fall upon it at such an angle that the reflected ray, *b c*, shall make an angle of 90° with the refracted ray, *b d*, then the reflected ray *b c*, which represents but a small portion of the incident light, will be polarized. If the medium is bounded by parallel surfaces, the portion of the light reflected from the second surface will also be polarized.

425

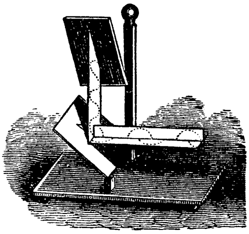


The angle of polarization by reflection may be determined by the following law. *The tangent of the angle of incidence for which the reflected ray is polarized, is equal to the index of refraction for the reflecting medium.* This law supposes the reflecting substance more dense than the surrounding medium. If the light is reflected from the second surface, as when passing from glass or water into air; *the index of refraction equals the cotangent of the angle of polarization.* The polarizing angle for reflection from glass, is 56° 25′, reckoned from the perpendicular

The polarizing angle for water is 53° 11′. As the index of refraction varies for different colors, the polarizing angle varies in the same manner.

If a polarized ray falls upon a reflecting surface at the angle of polarization, and the reflecting surface is rotated around the polarized ray as an axis, when it is so placed that the plane of incidence corresponds with the plane in which the ray was polarized, the polarized light will be reflected just as if it were not polarized; but when the plane of incidence makes an angle of 90° with the plane of polarization, the light is entirely intercepted, as shown in fig. 426. In this respect a reflecting surface, at the proper angle of incidence, serves the purpose of an analyzer, just like a plate of tourmaline.

426



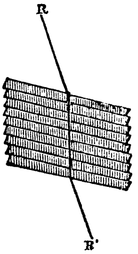
Polarization by metallic reflection.—To obtain a beam of plane polarized light by reflection from metallic plates, the light must be reflected many times at the angle most favorable to polarization. A ray of light once reflected from a metallic plate at the most favorable angle appears to consist of light vibrating in two planes, in one of which the phase of vibration is retarded from 0 to ¼ of a vibration behind the light vibrating in the other plane. This is called *elliptical polarization.*

When the two planes of vibration are at right angles to each other, and the phases of vibration differ by ¼ of a vibration, the light is said to be *circularly polarized.*

The diamond, sulphur, and all bodies possessing an adamantine lustre, produce elliptical polarization, and if employed as analyzing reflectors, they change plane polarized to elliptically polarized light. The investigation of these varieties of polarized light would exceed the limits of an elementary work.

547. **Polarization by refraction.**—When light is polarized by reflection from either the first or the second surface of a transparent medium, a portion of the transmitted light is polarized by refraction. The amount of light polarized by refraction is just equal to the amount polarized by reflection, but as the amount of light transmitted by transparent substances very much exceeds the amount reflected from their surfaces, only a small portion of the transmitted rays are polarized, or, more properly, the light transmitted through a single plate is but *partially polarized.*

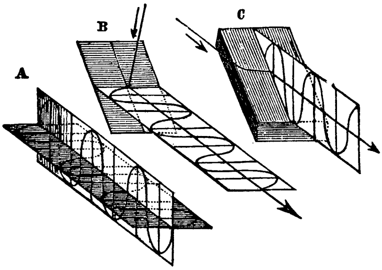
427



548. **Polarization by successive refractions.**—If a ray of light, R R′, is transmitted obliquely through a number of parallel transparent plates, as shown in fig. 427, a portion of the light is polarized at every refraction, and after a sufficient number of refractions the whole of the transmitted light is polarized.

Light polarized by refraction, is polarized in a plane at right angles with the plane of polarization by reflection. Light polarized by reflection vibrates at right angles with its plane of polarization, or its plane of reflection, as shown at B, fig. 428.

428

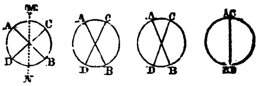


Light polarized by refraction, vibrates also at right angles with its plane of polarization, but parallel to its plane of refraction, as shown at C, fig. 428. Light not polarized, though vibrating in an infinite number of planes, is equivalent to a system of vibrations in two planes at right angles to each, as shown at A, fig. 428.

549. **Partial polarization.**—Light reflected or refracted at any oblique angle is, in general, partially polarized, and by repeated reflections or refractions the degree of polarization is increased, until, after a sufficient number of reflections or refractions, it is apparently completely polarized.

Let M N, fig. 429, represent the plane of refraction, and A B, C D, the axes of vibration for common light, then by repeated refractions these axes will be gradually made to approach each other, until they sensibly coincide, as shown in the figure, when the light is said to be completely polarized. The portion of light reflected, undergoes a similar series of changes, until the axes of vibration sensibly coincide, in a plane at right angles to their position in light polarized by refraction.

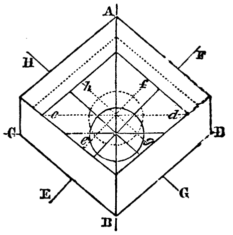
429



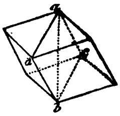
550. **Double refraction** is a property in certain crystals that causes the light passing through them in particular directions to be separated into two portions, which pursue different paths, and which causes objects seen through the crystals to appear double.

The most remarkable substance of this kind with which we are familiar, is Iceland spar, or carbonate of lime, which crystallizes in the rhombic system, as shown in fig. 430. The line *a b*, about which all its faces are symmetrically arranged, is called the major axis of the crystal, and the plane *a c b d*, passing through the axis, and through the obtuse lateral edges, is called the *plane of principal section.*

431



430



If a crystal of Iceland spar, from half an inch, upwards, in thickness, is laid upon a sheet of paper, on which are drawn various lines, they will appear double, as shown in fig. 431. A B, C D, E F, G H, are the real lines, seen in their true positions. The dotted lines show the position of the additional lines, caused by extraordinary refraction. The line A B, in the plane of principal section, is not doubled. Any line parallel to A B, will also appear single.

The index of refraction for the ordinary ray remains constant, in whatever direction light passes through the crystal. The index of refraction for the extraordinary ray, when parallel to the axis, is the same as that of the ordinary ray, and differs most from the ordinary ray when it passes through the crystal at right angles with the axis.

The index of refraction for the ordinary ray in Iceland spar is constantly $1.6543 = m$. The index of refraction for the extraordinary ray, when it makes an angle of 90° with the major axis, is $1.4833 = n$. Let $x =$ the angle which the extraordinary ray makes with the major axis in any other position, and let $N =$ the corresponding index of refraction for the extraordinary ray, its value may be determined by the following formula:—

$$N = \sqrt{m^2 + (n^2 - m^2)\sin^2 x} = \sqrt{2.7367 - 0.5365 \sin^2 x}.$$

551. **Positive and negative crystals.**—*Positive crystals* are those in which the index of refraction for the extraordinary ray is greater than for the ordinary ray, and the extraordinary ray is refracted nearer to the axis than the ordinary ray. Quartz and ice are examples of this class.

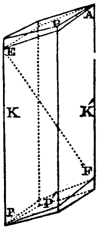
Negative crystals are such as have the index of refraction for the extraordinary ray less than for the ordinary ray, the extraordinary ray being refracted farther from the axis than the ordinary ray. Iceland spar, tourmaline, corundum, sapphire, and mica, are examples of negative crystals.

Some crystals have two axes of double refraction, as nitrate of potash, sulphate of barytes, and some varieties of mica.

552. **Polarization by double refraction.**—When the light transmitted through a doubly refracting substance is examined with an analyzer, it is found that both the ordinary and extraordinary rays are completely polarized, whatever be the color of the light employed. The tourmaline plate, or other analyzer, will, in one position, transmit the ordinary image and wholly intercept the other, but when the tourmaline has been rotated 90°, the ordinary ray is intercepted, and the extraordinary ray is transmitted.

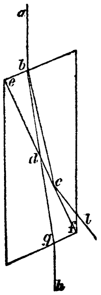
553. **Nicol's single image prism** is an instrument formed of Iceland spar, by which the ordinary image, produced by double refraction, is thrown out of the field, and only a single image (the extraordinary) is transmitted.

432



An elongated prism of Iceland spar is cut through by a plane, E F, at right angles with the principal section, from the obtuse solid angle E, fig. 432, making an angle of 22° with the obtuse lateral edge K. The terminal face, P, is ground away, so as to make an angle of 68° with the obtuse lateral edge, K, and the opposite face, P is ground in the same manner. All the new faces are carefully polished, and the two parts are cemented together again with Canada balsam, in the same position they previously occupied. The lateral faces of this compound prism are all painted black, leaving only the terminal faces for the transmission of light.

433

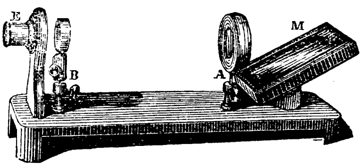


When a ray of light, *a b*, fig. 433, falls upon this prism, it is refracted into the ordinary ray *b c*, and the extraordinary ray *b d*. The index of refraction of Iceland spar, for the ordinary ray, being 1.654, and that of balsam only 1.536, the ordinary ray cannot pass through the balsam, unless the incident ray diverges widely from the axis of the prism, but it suffers total reflection, and is absorbed by the blackened side of the prism. The extraordinary ray has a refractive index in the Iceland spar generally less than in the balsam, varying, for Nicol's prism, between 1.5 and 1.56; therefore it passes through the balsam into the lower part of the prism, and emerges in the direction *g h*, parallel to the incident ray.

These prisms are capable of transmitting a colorless pencil of light, perfectly polarized, from 20° to 27° in breadth.

554. **Polarizing instruments** are made in a variety of forms, to suit particular purposes. A simple instrument, and yet one of the most convenient in use for exhibiting the phenomena of polarized light, is shown in fig. 434.

434



A mirror, M, made of plate glass, covered with black varnish or cloth on the back, or, better, a bundle of ten to twenty thin plates of polished glass, is mounted on a mahogany support at the polarizing angle. A Nicol's prism, or a tourmaline plate, at E, serves as an analyzer. The objects to be examined are mounted in discs of wood or cork, and supported at A or B, where they are most distinctly seen by the eye, looking through the analyzer. The student who has not a tourmaline, or a Nicol's prism, can use as an analyzer a small piece of plate glass, mounted so as to rotate on an axis parallel to the base of the instrument.

Polarized light may be applied to the microscope, by mounting a Nicol's prism beneath the stage as a polarizer, and another for an analyzer, in the body of the microscope, above the object glass.

555. **Colored polarization.**—When a thin plate of selenite, mica, or any other doubly refracting substance, is placed between the polarizer and the analyzer in the polariscope, the light is separated into two beams, which follow different paths, and as the vibrations of one ray are more retarded than those of the other, when they are reunited they interfere, and produce the most beautiful colors, varying with the thickness of the plates, and the position of their axes with reference to the axes of the polarizer and the analyzer.

If the film is rotated, while the polarizer and analyzer remain fixed, the color will appear at every quadrant of revolution, and disappear in intermediate positions. If the film and the polarizer remain fixed, and the analyzer is rotated, the color will change to the complementary at every quadrant of revolution; that is, the same color will be seen in positions of the analyzer differing 180°, and the complementary color will be seen at 90° and 270°, from the first position.

Films of selenite, varying between 0·00124 and 0·01818 of an inch in thickness, will give all the colors between the white of Newton's first order, and white resulting from the mixture of all the colors. If two films of selenite are placed over each other, with their axes parallel, the color produced will be that which belongs to the sum of their thicknesses. But when the two films are placed with their axes at right angles, the resulting tint is that which belongs to the difference of their thicknesses.

556. **Rotary polarization** is a property which some substances possess of changing the plane of vibration in a ray of polarized light, even when it falls perpendicularly upon it. The entire amount of rotation depends upon the thickness of the medium. Quartz, cut transversely to its major axis, solution of sugar, camphor in the solid state, and most of the essential oils, possess the power of rotating the plane of polarization of a ray passing through them.

Different substances, and sometimes different specimens of the same substance, rotate the plane of polarization in contrary directions. When the rotation takes place in the direction of the motion of the hands of a watch, the medium is said to have right-handed polarization. Thus we have right-handed quartz, and left-handed quartz.

In a beam of white light, the vibrations which produce red have their plane of polarization rotated much more than the colors of greater refrangibility. This property varies inversely as the squares of the lengths of the luminous waves which produce the several colors. The power of rotating the plane of polarization becomes a valuable test for speedily determining the nature of various chemical substances, or the strength of a solution of any substance having this power. Soliel's saccharimeter, for measuring the relative amount of cane and grape sugar in solutions or syrups, is constructed on this principle. Such an instrument affords also a ready method of detecting the presence of sugar in diabetic urine.

557. **Arago's chromatic polariscope** is a very simple instrument for testing polarized light, and for determining its plane of polarization. In one end of a brass tube is inserted a prism of Iceland spar; in the other end of the tube a circular opening is covered by two plates of quartz cut parallel to the axis and united by their edges, one of these plates having right-handed, and the other left-handed, rotary polarization.

When polarized light is viewed through this instrument (the Iceland spar being turned towards the eye), the circular opening appears double, and in each image is seen the line dividing the two plates of rotary quartz, with complementary colors on opposite sides of the line.

If the instrument is now rotated, two positions will be found at right angles to each other where both parts of the opening in the same image appear of a uniform tint, though the two images still have complementary colors.

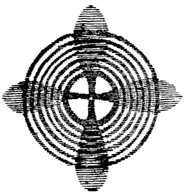
When both segments of the extraordinary image have a uniform red tint, the principal section of the prism is parallel to the plane in which the light has been polarized, and when both segments of the extraordinary image are uniformly green, the plane of polarization is perpendicular to the principal section of the prism.

A plate of selenite or mica, or a single plate of quartz, may be substituted for the two segments of right and left-handed quartz, and two images with complementary colors will be seen as before. All the phenomena of atmospheric polarization may be demonstrated with this form of the instrument, and the plane of polarization may be determined, but with less accuracy than in the first form of the instrument. As before, when the extraordinary image is red, the plane of polarization is parallel to the principal section of the prism. When the extraordinary image is green, the plane of polarization is perpendicular to the principal section of the prism.

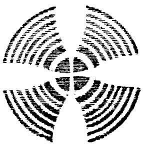
558. **Colored rings in crystals.**—Colored rings of great beauty, with a black cross, are seen in thin plates of doubly refracting crystals, when viewed in certain directions, with polarized light.

Figs. 435 and 436, show the appearance of the rings and cross in thick plates of quartz, in positions at 90° from each other. Other uniaxial crystals show a similar system of rings beautifully colored. Figs. 437 and 438 show the form of the colored rings in biaxial crystals; *e. g.* some micas. Every doubly refracting crystal presents some peculiarity in the form and arrangement of the colored rings seen in its thin sections. This subject is of great interest to the mineralogist.

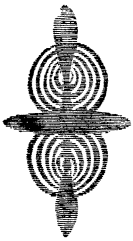
435



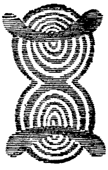
436



437



438



559. **Polarization by heat, and by compression.**—Glass irregularly heated, or heated and irregularly cooled, possesses the power of double refraction, and when viewed by polarized light, it exhibits dark crosses, bands, or rings, varying with the form of the glass, and difference of density in different parts. Similar phenomena may be produced by compression, or by bending rods or plates of glass.

560. **Magnetic rotary polarization.**—If a thick plate f glass is applied to the poles of a powerful electro-magnet, the glass is neither attracted nor repelled; but if a ray of polarized light is transmitted through the plate in a certain direction, the plane of polarization is rotated as by a plate of quartz, or other rotary polarizer, showing that light and magnetism have some intimate relation to each other.

This rotary effect may depend upon change in the tension of the molecules of the glass by the magnetic force, and not upon any direct relation between light and magnetism.

561. **Atmospheric polarization of light.**—The light of the sun reflected by the atmosphere is more or less polarized, depending upon the angular distance from the sun.

If the earth had no atmosphere, the sky would everywhere appear perfectly black. The color of the sky is produced by light reflected by the atmosphere. If we look at the sky through a Nicol's prism, we shall find, on rotating the prism, that light from some parts of the sky is polarized to a very appreciable extent. There are several points in the sky where no polarization is perceptible. The point in the heavens directly opposite to the sun is called the *anti-solar point.* At a distance above the anti-solar point, varying from 11° to 18°, there is a point of no polarization, and another neutral point at an equal distance below the anti-solar point. Another neutral point, or point of no polarization, is found from 12° to 18° above the sun, and a similar one below it; but the latter is observed with great difficulty. When the sun is in the zenith, these two points coincide in the sun. At all other points in the sky, the light is more or less polarized, the degree of polarization amounting sometimes to more than one-half as much as by reflection from glass at the angle of complete polarization.

562. **The eye a polariscope.**—The structure of the crystalline lens is such, that the unaided eye is capable of analyzing a beam of light polarized by reflection or by double refraction. A person accustomed to use his eyes in viewing the phenomena of polarization, can thus detect with ease facts of this nature, which are wholly inscrutable to one not familiar with such observations; another of the numerous proofs we have that the eye is capable of very exact training; but nevertheless it is a proof also of an imperfection in the eye itself.

M. Haidinger has observed a remarkable phenomenon of polarized light, by which it may be recognised by the naked eye, and its plane of polarization ascertained. This phenomenon consists in the appearance of two brushes of a very pale yellow color, the axis of which coincides always with the trace of the plane of polarization; these are accompanied, on either side, by two patches of light of a complementary or violet tint. In order to see them, the plane of the polarization of the light must be turned quickly from one position to another; this may be done by revolving before the eye a Nicol's prism directed towards a white cloud.

The most probable explanation is that given by M. Jamin, in which the production of the phenomenon is ascribed to the refracting coats of the eye. they being compared to a pile of parallel plates of glass.

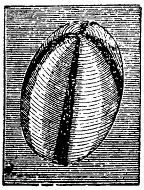
563. **The practical applications of polarized light are numerous.** The *water telescope* consists of an ordinary marine telescope, with a Nicol's prism inserted in the eye-piece.

The light reflected from the surface of the water is the principal obstruction to viewing objects beneath its surface. Nicol's prism, in a certain position, entirely cuts off the polarized portion of the reflected light, and allows objects far below the surface to be seen in the telescope. A Nicol's prism, in the same manner, will enable the fisherman to direct his spear with greater certainty.

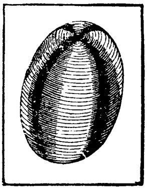
Amateurs, in visiting galleries of paintings, find Nicol's prisms mounted as spectacles of great service. Let the observer place himself in an oblique position, and look at an oil painting; when the sheen of reflected light renders the objects in the painting invisible, he has but to look through a Nicol's prism, set in a proper position, and the entire details of the painting at once become visible in all their proper colors. An opera-glass, provided with Nicol's prisms, would be a valuable instrument in examining a picture gallery. Polarized light is also of great value in microscopic investigations.

Fig. 439 shows the appearance of a grain of starch, brilliantly illuminated on a dark ground, when seen in the microscope with polarized light. By rotating the analyzer, the field becomes light, and the dark cross changes its position, as shown in fig. 440. The appearance of the starch distinguishes it from every other substance. Different kinds of starch are also thus readily distinguished from each other.

439



440



By means of polarized light, the chemist can detect one thirteen-millionth of a gramme of soda, and distinguish it from potassa or any other alkali. In physiological chemistry, especially in the examination of crystals found in various cavities and fluids of both animals and plants, the use of polarized light is especially important.

Instead of a few isolated facts, of interest only to the curious inquirer, the polarization of light presents itself as a great fact in nature, meeting us with wonderful revelations in almost every department of natural science. By this marvelous property of light, the astronomer determines that the planets shine by reflected light, and that the stars are self luminous bodies.

Problems in Optics.

Velocity and Intensity of Light.

177. What time is required for light to come to the earth from the sun, the distance being 95,000,000 miles?

178. How long does it take the light of the north star to reach the earth, the distance being estimated 2,961,000 times greater than the distance of the earth from the sun?

179. Two lights give equal illumination upon Bunsen's photometer, when it is placed between them at a distance of 3 feet from one light, and 4 feet 7 inches from the other; what is their relative illuminating power?

180. A screen is equally illuminated by two flames, when one of them is 40 inches from it, but when a plate of glass is interposed, the same light is required to be brought to a distance of 38 inches that the illumination may be again equal; what proportion of the light is transmitted by the glass?

181. At one extremity of a bar, 100 inches long, is placed a standard candle burning 120 grains per hour (the usual standard in photometric measurements of gas), at the other end is a gas burner consuming 5 cubic feet per hour. A Bunsen's screen, moving on this bar, is distant $17\frac{3}{10}$ inches from the candle, when both sides are equally illuminated; what is the illuminating power of the gas in terms of the candle as a unit?

182. By a similar trial, the photometer is $19\frac{1}{10}$ inches distant from the standard candle when the disc is equally illuminated, but it is found that the candle is burning 129 grains per hour, while the gas burner consumes only $4\frac{1}{2}$ cubic feet of gas per hour; what is the illuminating power of 5 cubic feet of this sample of gas in terms of the standard candle at 120 grains as a unit?

Reflection of Light.

183. At what angle must two mirrors be inclined, so that a ray of light incident parallel to one mirror may, after reflection at each mirror, be parallel to the other?

184. How many images will be seen in a kaleidoscope when the two mirrors of which it is composed are placed at an angle of 20°?

185. A concave mirror collects solar light to a focus 6 inches from its surface; where will it form an image of an object placed 12 feet in front of it?

186. A luminous point is placed at a distance of 3 feet in front of a concave mirror of 1 foot radius; find the distance of the focus of reflected rays.

187. What must be the position of a luminous point before a concave mirror, that the distance between the foci of incident and reflected rays may be equal to the radius of the mirror?

Refraction of Light.

188. Find the thickness of a plane glass mirror silvered at the back, so that an object one foot in front of its first surface may have the image formed by reflection at the second surface twice as distant from the object as if the reflection took place from the first surface; the index of refraction being $n = 1\frac{1}{2}$.

189. A fish is seen in a position known to be 4 feet below the surface of the water, and the direction in which it is seen makes an angle of 45° with the perpendicular; at what angle must a lance be thrown to strike the fish?

190. If a pool of water appears to be 5 feet deep, what should we consider its real depth?

191. A small pencil of solar rays, incident upon the surface of a refracting sphere, is brought to a focus upon the opposite surface; what is the refractive index of the material of which the sphere is made?

192. A small pencil of light falls upon a concave spherical surface of glass ($n = 1\frac{1}{2}$), the radius of which is 2 feet. Supposing the radiant point distant 3 feet from the refracting surface, where will the focus of the refracted rays be found?

193. When divergent rays are incident from a certain point upon a spherical surface of glass, the refracted rays are found to converge to a focus at exactly the same distance on the opposite side of the surface; is the surface convex or concave? and if the position of the point of incidence be 3 feet from the refracting surface, and $n = 1{\cdot}5$, what is the radius of the refracting surface?

194. A double convex lens of glass ($n = 1{\cdot}534$), whose radii are respectively 2 inches and $4\frac{1}{2}$ inches, is placed 15 inches before a luminous point; what is the position of the focus of refracted rays?

195. What is the form of a double convex lens, having the least spherical aberration, when the glass of which it is made has an index of refraction $m = 1{\cdot}635$?

196. What is the distance of the principal focus from a lens of flint glass ($n = 1{\cdot}635$) whose radii are, $r = 2\frac{1}{4}$, and $s = -5$?

197. What single lens is equivalent to a combination of a double convex lens of focal length 2 inches with a double concave lens of focal length 4 inches?

198. Determine the form of two lenses of flint and crown glass, that may be cemented together so as to constitute a plano-convex achromatic combination of 7 inches focal length, using flint-glass in which $m = 1{\cdot}635$, and the dispersive power $p = 1000$, and crown-glass $n = 1{\cdot}534$, and $p' = 625$.

199. Two convex lenses, whose focal lengths are $3f$ and f, are separated by an interval of $2f$; how must a pencil of rays be incident upon the first lens, so as to emerge parallel after refraction through the second lens?

Optical Instruments.

200. Considering the distance of distinct vision 8 inches, what will be the magnifying power of a lens whose solar focus is 3 inches, when it is placed at a distance of 5 inches from the eye?

201. Calculate the radii of the two surfaces of a meniscus of crown-glass ($n = 1{\cdot}5$) to be used as the field-lens of Prof. Airy's eye-piece (500), when the eye-lens has a solar focus of half an inch, and the field-lens is 2 feet from the object-glass.

202. Calculate the illuminating power of Herschel's great telescope, allowing $\frac{7}{10}$ of the incident light to be reflected by the speculum.

203. On the same conditions calculate the illuminating and penetrating powers of Lord Rosse's great telescope (503).

204. Estimate the illuminating and penetrating powers of the Cambridge refracting telescope (506). $A = 15$ inches, $a = 0{\cdot}1$ inch, $x = 0{\cdot}9$, $n = 2$.

205. Compare the illuminating and penetrating powers of two achromatic objectives for the microscope, one of which has an angular aperture of 100°, and the other 150°, calling $n = 6$ in both cases, and $x = 0{\cdot}8$.

Polarization of Light.

206. Calculate the angles of most perfect polarization by reflection from the three kinds of glass whose index of refraction is given in § 407.

207. What proportion of the incident light is reflected in each of the cases considered in the last problem, supposing the increase of reflection uniform between any two angles whose amount of reflection is given in the table, page 299?

208. What is the index of refraction for the extraordinary ray in Iceland spar, when it makes an angle of 54° with the principal axis of the prism?

CHAPTER II.

HEAT.

§ 1. Nature of Heat.

564. **Heat.—Its nature.**—The sensations, which we call heat or cold, are produced by an agent or cause whose real nature is unknown. Whatever this agent may be, it influences matter of all kinds without changing its nature. Scientific opinion is divided, chiefly, between two views of the nature of heat. These are, the *corpuscular* theory, or *theory of emission*, and the *undulatory theory*.

According to the *corpuscular theory*, heat is attributed to a peculiar imponderable fluid, existing in all bodies in combination with their atoms. The particles of this supposed fluid are self-repellent, and thus the atoms of bodies are prevented from coming into absolute contact with each other. This fluid is thrown off from all hot bodies with inconceivable velocity, and upon its absorption by other bodies the effects of heat are manifested. Thus hot bodies lose what colder bodies gain.

By the *undulatory theory*, heat is attributed to the vibratory movements of the molecules of a hot body, communicated to those of other bodies, by means of a highly elastic fluid called *ether*. This ether pervades all space, and in it the undulations of heat are propagated with inconceivable rapidity, in a manner analogous to the slower progress of sonorous waves in air, as already explained. This same medium, by another kind of motion, is supposed to produce light and electricity.

565. **Temperature.—Heat and cold.**—All bodies receive or part with heat, as their conditions change from time to time; the relations which they sustain to heat *at a given moment* are distinguished by the word *temperature*, which term implies nothing as to the *quantity* of heat present in a body, but only its relation at a specific time to an arbitrary standard.

Heat and cold are relative terms; cold implying not a quality antagonistic to heat, but merely the absence of heat in a greater or less degree. There are no bodies so cold that they will not be warm to bodies colder than themselves. Our sensations give us but little evidence respecting actual changes of temperature.

If we place one hand in hot and the other in cold water, and then suddenly transfer both to water having an intermediate temperature, our sensations are at once reversed; one hand will feel cold, and the other warm, although both are exposed to the same temperature.

566. **Action of heat on matter.**—Assuming that cohesion and heat are counteracting forces, it follows that the three states of matter are effects of the relative intensities of these two agencies; and heat being a repellent force, its increase must be accompanied with an enlargement of volume in either of the three states of matter, while a loss of volume must accompany an increase of molecular force, or a loss of heat.

Many familiar facts of daily experience confirm this statement. All bodies (with few exceptions) expand with an increase, and contract with a loss of heat. The expansion may be measured by an increase of length; in which case it is called *linear expansion*, or by an increase of volume, and then it is called *cubic expansion*. The first measure is commonly used for solids, the second for liquids and gases, but the first is easily converted into the second by cubing. Substances vary very much in their degree of expansion for the same increase of temperature: solids expand less than fluids, and liquids less than gases.

But the laws of expansion will be better studied after some acquaintance is obtained with the means of measuring differences of temperature.

§ 2. Measurement of Temperature.

I. THERMOMETERS.

567. **Thermometers.**—The measurement of temperature is accomplished by observing the amount of expansion or contraction in any substance arbitrarily assumed as a standard for the purpose. Such an instrument, whether the substance selected is a solid, a liquid, or a gas, is called a *thermometer*, or measure of temperature. For special purposes, thermometers are constructed with either solids, liquids, or gases. In much the greater number of cases, however, *mercury* is the only substance employed, not only because of its great range of temperature between freezing and boiling, but also because its changes of volume for equal changes of temperature are more nearly proportional to the temperature than those of any other liquid.

568. **Construction of mercurial thermometers.—Thermometer tubes.**—A capillary glass tube of which a thermometer is to be made, should be one whose bore, throughout, is of the same calibre, so that equal lengths within it will contain equal quantities of mercury. The equality of the bore is ascertained by causing a short cylinder of mercury (say one inch) to pass from end to end of the tube, and if it measures an equal length throughout, then the calibre is equal; otherwise the tube is rejected. Only about one in six of thermometer tubes

are found to possess a canal of equal bore. A proper tube having been selected, a bulb (cylindrical or spherical) is blown upon it by means of a gum elastic bag fitted to the open end. The breath would fill the tube with moisture. The form of the bulb is conventional. A cylindrical bulb will be more readily affected by the temperature of the surrounding medium than a spherical one, because it exposes a larger surface.

Filling the tube with mercury is facilitated by tying a paper funnel on the open end of the tube, or a glass reservoir, C, fig. 441, is employed to hold a portion of pure mercury. As so dense a fluid could not enter a capillary opening, the air in the bulb, D, is expanded by the flame of a spirit lamp, holding the tube as seen in the figure. As the air expands, a portion of it escapes, passing the mercury in C. Allowing the bulb D to cool, the pressure of the air soon forces a portion of the mercury from C through the capillary tube into the lower reservoir, exhibiting in its progress the successive stages of the capillary surfaces explained in § 235. The lamp is again applied to boil the mercury in D, and after several minutes, all the air and moisture are expelled from the tube by the mercurial vapor. The bulb is then cautiously cooled once more, when it will be found, as well as the stem, completely filled with mercury. The extremity of the tube, C, is then drawn out to a narrow neck, and broken off preparatory to sealing. A greater or less portion of the mercury remaining in the stem, must now be removed, according to the range designed to be indicated by the thermometer. This is accomplished by gently heating the bulb. When about two-thirds of the mercury contained in the stem has been driven out, and while the stem is yet full, the flame of a blow-pipe is directed upon the end of the stem, the glass melts, and the tube becomes hermetically sealed.

441

569. **Standard points in the thermometer.—Graduation.**— As variations in the height of the mercurial column in the thermometer depend upon the changes of temperature to which it is subjected, it is necessary to graduate the instrument, or construct a scale, whereby these variations may be indicated, and the temperatures indicated by

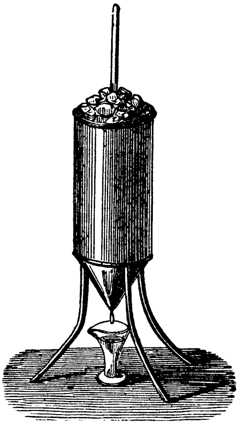
one thermometer compared with those shown by another. If there existed a natural zero, or absolute limit to temperature, the thermometric scale might be numbered upwards from it. But there is no natural zero, and therefore the thermometric scale must be arbitrary, although based upon certain well-determined physical phenomena. Experiment has determined that the melting point of ice, and the boiling point of pure water, under certain given conditions, are always the same, and these points (called, respectively, the *freezing* and *boiling points*) have been adopted in all countries as the two temperatures, with reference to which thermometric scales are constructed.

Freezing point.—To fix the freezing point in a thermometer, a tin vessel, like fig. 442, is filled with pounded ice or snow; a hole in the bottom of the vessel allows the water from the melted ice to escape. Into this vessel the bulb of the thermometer is thrust, with part of the stem.

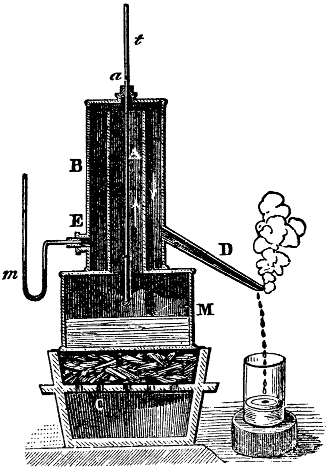
When the mercurial column becomes stationary, that is, when the mercury contained in the bulb has attained the temperature of the melting ice, its level is marked with a diamond point, upon the glass, or with a pencil upon a paper previously attached to the tube. This indicates the melting point of ice, and consequently the freezing of water. This point may be indicated by the expression n_0 for the centigrade scale, or n_{32} for the Fahrenheit scale.

Boiling point.—This point is accurately fixed by immersing the whole thermometer in vapor of boiling water. For this purpose Regnault has contrived the apparatus, a section of which is seen in fig. 443. The course of the steam rising through A, and descending in the outer vessel, is shown by the arrows, and the drawing is so easily understood as to need no description. An equally efficacious apparatus may be made by using a chemical flask, in the elongated

442



443



neck of which the thermometer is suspended by a cork, through which a small glass tube allows also the escape of the steam. The stem of the thermometer (t) is adjusted so that the point a, to which the mercury rises when heated to the temperature of boiling water, is just visible above the cork. This point is definitely marked when the level of the mercury becomes stationary, thus indicating the boiling of water. Let this point be indicated by the expression n_T, which corresponds to the temperature T, at which the observation was made.

This temperature T is equal to 100° C. or 212° F., when the barometric pressure is 30 inches (or 760 millimetres). But as the temperature of ebullition varies with the barometric pressure, it is plain that the value of T will vary with the height, H, of the barometer. It is essential, therefore, to accuracy in graduating the thermometer, that the boiling point should be fixed when the barometer is at 30 inches.

After dividing the space between the freezing and boiling points into as many equal parts as there are designed to be degrees in the scale, the divisions are continued both above and below the fixed points. This mode of graduation involves, however, serious errors, some notice of which is taken in § 576.

570. **Different thermometric scales.**—Having fixed these two standard points in a thermometer, the space between them is next to be subdivided into a certain number of equal parts, called degrees. Unfortunately, in different countries, this interval has been differently subdivided. In scientific researches, the Centigrade scale is almost exclusively in use, while in common life, in the United States, England, and Holland, Fahrenheit's scale alone is used.

Fahrenheit's scale.—In the Fahrenheit scale, the interval between the boiling and freezing points is divided into 180 equal parts, which are called degrees. The zero, or 0°, of this scale, is 32 of these degrees below the freezing point.

Fahrenheit adopted, as the zero of his thermometer, the temperature which had been observed at Dantzic, Holland, in 1709, and which he found he could always reproduce, by using a mixture of ice and salt. At that temperature (which he believed to be an absolute zero of cold) he computed that his instrument contained 11,124 equal parts of mercury, which, when plunged into melting snow, were increased to 11,156 parts. Hence the space included between these two points (viz., $11{,}156 - 11{,}124 = 32$) was divided into 32 equal parts, and 32° indicates, therefore, the freezing point of water. When his thermometer was plunged into boiling water, Fahrenheit estimated that the mercury was expanded to 11,336 parts, or 212 parts above his zero, and therefore 212° ($11{,}336 - 11{,}124 = 212$) was marked as the boiling point of that fluid. In practice, Fahrenheit determined the boiling point of water, and the melting point of ice, and then graduated the tube by equal divisions to his zero. To Fahrenheit belongs the merit of having introduced the use of mercury in thermometers, which had previously been made only with alcohol, water, or air, and his graduation of the thermometric scale, although unscientific, is not irrational as it is often represented.

Centigrade scale.—In the year 1742, the Swedish philosopher Celsius, professor at Upsal, introduced the Centigrade scale (from *centum*, one hundred, and *gradus*, degree). It is adopted universally in France, and in the north and middle of Europe. The interval between the freezing and boiling points in this scale, is divided into 100 equal parts or degrees; the degrees being counted upwards and downwards, from the freezing point of water, which is zero. The temperatures below zero in this, as in all thermometers, are indicated by the negative algebraic sign —; those above, by the positive algebraic sign +; thus — 20° signifies 20 degrees below zero, but + 20° signifies 20 degrees above zero.

Reaumur's scale.—Reaumur, a French philosopher, introduced his scale in 1731. He proposed to use spirits of wine, of such a strength, that between the two standard points, 1000 parts should become 1080. He divided the interval between these points into 80 equal parts; the zero being placed at the freezing point of water. Reaumur's thermometer was the only one used in France before the Great Revolution (A. D. 1789); it is still best known in Spain, and in some of the older European states.

All thermometric scales are purely arbitrary. We know only a small part probably of the vast possible range of temperature, and we select the two great natural phenomena adopted for the fixed points of our scales because they can be readily verified, and because the range between them includes the temperatures which we have most occasion to measure in the common experience of life.

571. **Comparison and conversion of thermometric scales.**—The scale employed in a thermometer is indicated by the name, or by one of the initial letters, F., C., R. The degrees of one thermometric scale are readily converted into those of another. Since 180° F. = 100° C. = 80° R., therefore 1° F. = $\frac{5}{9}$° C. = $\frac{4}{9}$° R.

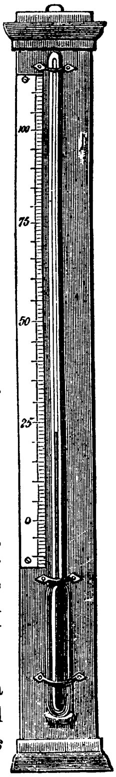
As the value of a degree in Fahrenheit's thermometer is greater by 32 than the number of divisions from the freezing point, 32 must always be subtracted before the + degrees of Fahrenheit are converted into those of the other scales, and added upon the conversion of other degrees into Fahrenheit.

Easy rules for mental calculation are:—1st, to convert Centigrade to Fahrenheit degrees, *double the number of Centigrade degrees, subtract one-tenth, and add thirty-two;* or, *multiply the Centigrade degrees by* 1·8 *and add* 32°. And concisely, to reduce Fahrenheit degrees to Centigrade, *subtract* 32° *from the Fahrenheit degrees, and divide the remainder by* 1·8.

572. **House thermometers.**—For common use, the thermometer is mounted on a plate of brass, ivory, porcelain, or wood, on which the degrees are marked, as in fig. 444. The words *summer heat, blood heat,* and *fever heat,* are often placed opposite the points 68°, 98°, 108° F.

House thermometers are usually graduated by comparison with a standard, and not by determining the two fixed points. For this purpose the standard is immersed in a water bath with the tube to be graduated, and a number of points are thus fixed, at different temperatures, as the bath slowly cools from boiling. The distances between the marked points are divided into equal parts. This mode of graduation is capable of giving results accurate enough for common use, between boiling and freezing, say within a degree on Fahrenheit's scale. But above and below these points little reliance can be placed on it, and at low and high temperatures common thermometers will be observed to vary often several degrees, even when made by the best makers.

444



Some thermometers have their graduated wooden support divided near the lower end into two parts, connected together by a hinge, so that the lower part may be turned back, and the bulb thrust into any liquid whose temperature it is desired to ascertain.

Other thermometers have their stem very small, and completely surrounded by a larger tube; the scale being marked upon a porcelain strip, or a roll of paper, inserted between the two tubes.

In very accurate thermometers, the degrees are marked on the glass tube with fluohydric acid in a manner described in § 577.

Tests of a good thermometer.—In order to ascertain whether a thermometer is correct or not, it is first plunged into melting ice, and then into boiling water; the level of the mercury should indicate upon the scale exactly 32°, and 212° F. When inverted, the mercury should fall with a sudden click, and fill the tube, thus showing the perfect exclusion of air.

To determine whether the value of the degrees is uniform, a slight jerk is given to the thermometer, by which a little cylinder of mercury is detached from the column. On moving the little column through the tube, it should occupy equal spaces in all parts, if the bore is perfectly accurate, and the scale is properly graduated.

Sensibility of thermometers.—The sensibility of a thermometer is of two kinds; it may indicate very small *differences*, or it may be very sensitive to sudden *changes* of temperature.

If the capacity of the reservoir is large, compared with the bore of the tube, a slight change of temperature will affect, considerably, the height of the mercurial column. If the capacity of the reservoir is small, and the glass bulb thin, the mercury contained in it will be *more rapidly* affected than if a larger amount were to be acted upon. A cylindrical reservoir is, therefore, better than a spherical one, because it exposes a larger surface.

The two kinds of sensibility indicated, are obtained in a thermometer which has a small cylindrical reservoir and a very capillary tube.

573. **Displacement of the zero point.**—One source of error in the mercurial thermometer is found in the displacement of the zero point by changes subsequent to graduation. If a thermometer which has

been made some time is thrust into melting ice, the column of mercury will not sink to the original freezing point, but will remain at a distance above it, sometimes as much as two or three degrees, and even more.

Mr. Legrand has found that the cause of this change is, that the capacity of the reservoir is enlarged at a high temperature (as during the construction of the thermometer), and that it does not return to its original dimensions until after a long time, sometimes not until after two or three years.

The effect is also supposed to arise from the pressure of the atmosphere upon the bulb, which, when not truly spherical, seems to yield slightly and in a gradual manner.

This defect may be avoided by giving the bulb a certain thickness. Mr. Crichton's thermometers, of which the freezing point has not altered in forty years, were all made of unusually thick glass.

Before making important observations, therefore, thermometers should be examined as to the position of their freezing point.

574. **Limits of the mercurial thermometer.**—Mercury is by far the most available thermometric fluid. It may easily be obtained pure; it does not adhere to the sides of the tube, and above all, it has a greater range of temperature, between its freezing and boiling points, than any other liquid; freezing at —39°·2, and boiling at 662° F. Between these two points, its expansion for equal increments of heat is very regular, excepting near its freezing point. Owing to this last irregularity, mercurial thermometers cannot be accurately used for temperatures lower than — 31°, or — 35° C. Above the boiling point of mercury, heat is measured by instruments called pyrometers. For very low temperatures, spirit of wine thermometers are usually employed.

575. **Spirit thermometers; other liquid thermometers.**—Alcohol has never been frozen, and is, therefore, generally employed for the estimation of low temperatures.

Thermometer tubes are filled with alcohol (which is generally colored red) by heating the bulb, with the open end of the tube thrust into alcohol, and completing the process in the manner already indicated (568). The tube is graduated by comparison with an accurate mercurial thermometer, exposing both to the same temperature, and marking, successively, upon the alcoholic thermometer, the temperatures indicated by the mercurial thermometer as they are gradually heated. The alcoholic thermometer should not be divided into equal parts between the freezing point of water and its boiling point, because it expands unequally for equal increments of heat. Alcoholic thermometers often differ much from each other, because there is great difficulty in obtaining alcohol perfectly pure, or of exactly the same degree of concentration.

Capt. Parry, in his arctic voyages (whose experience was confirmed by Dr. Kane, Arct. Exp. II., 405), found a difference of 18° F. between alcoholic thermometers constructed by the most celebrated makers; and a difference of 14° F. has been observed even at a temperature of only 15° or 20° F. In consequence of this, other liquids have been proposed for thermometers, intended to indicate low temperatures. From the experiments of M. Pierre, of all liquids, ordinary sulphuric ether, chlorid of ethyle, and bromid of methyle, were found to be best adapted for such instruments.

It is plain from these statements that a more accurate mode of measuring low temperatures is one of the desiderata of science.

576. **Defects inherent in mercurial thermometers.**—Besides those sources of error in this instrument already noticed, there are others inherent in the nature of the materials employed.

As glass and mercury expand unequally by heat, it is plain that we read in the mercurial column not the absolute expansion of the mercury, but the difference between its expansion and that of the glass. If they expanded equally, no movement of the mercurial column would be perceived, and if the glass expanded more than the mercury, the latter would appear to fall when the temperature rose. But as in fact the mercury expands about seven times as much as glass, the apparent expansion of the metal in glass is about one-seventh less than its absolute expansion.

Again, it is proved that the expansion of mercury for equal increments of heat is not absolutely equal, but increases slightly with the temperature. At temperatures between freezing and boiling, this increase is very slight, and may be disregarded, since the division of this distance into parts of equal length gives the degrees a mean length, slightly in excess for the degrees near freezing, and as much too short for those near the boiling point, but exact for the intermediate degrees. But above the boiling point the error is more serious, since while the degrees have the same length, the space occupied by a unit of mercury is constantly increasing, consequently the degrees become too short, and the thermometer reads too high a temperature. For the same reason, below the freezing point, the thermometer constantly indicates a temperature higher than the true temperature.

The error here pointed out could be easily allowed for in the graduation; the coefficient of expansion of mercury for various temperatures being known. But this correction is rendered almost impossible, from the fact that the rate of expansion of the glass is found not only to increase about as rapidly as that of the mercury (and sometimes even more so), but to render the case yet more difficult, it is found that glass of different kinds varies in its rate of expansion, and the same glass under different conditions may also vary.

Regnault, to whom we are indebted for these data, has illustrated the facts by a series of observations, the results of which are shown in the following table.

He has shown that the air thermometer may be relied on as giving results almost invariable and exact. His form of this instrument is described at length in his memoir before quoted (p. 220).

COMPARISON OF DIFFERENT THERMOMETERS (CENTIGRADE DEGREES).

Air Thermometer. True Temperature.	Thermometer without Glass.	Thermometer, Flint-glass.	Thermometer, Crown-glass.	Coefficient of Expansion of Mercury.
°	°	°	°	
0	0	0	0	0·000 1790
50·00	49·65		50·20	0·000 1815
100·00	100·00	100·00	100·00	0·000 1830
120·00	120·33	120·12	119·95	0·000 1850
140·00	140·78	140·29	139·85	0·000 1860
160·00	161·33	160·52	159·74	0·000 1870
180·00	182·00	180·80	179·63	0·000 1880
200·00	202·78	201·25	199·70	0·000 1890
220·00	223·67	221·82	219·80	0·000 1901
240·00	244·67	242·55	239·90	0·000 1911
246·30			246·30	
260·00	265·78	263·44	260·20	0·000 1921
280·00	287·00	284·48	280·52	0·000 1931
300·00	308·34	305·72	301·08	0·000 1941
320·00	329·79	327·25	321·80	0·000 1951
340·00	351·34	349·30	343·00	0·000 1962

The temperatures indicated by an air thermometer, recorded in the first column, were taken as a standard, and are very near the truth. The second column gives the temperatures which would be shown by a mercurial column graduated in the ordinary way, assuming the glass to be without expansion, thus showing the errors attributable only to the varying rate of expansion in that metal. In the third and fourth columns are given the comparative temperatures shown by thermometers of flint and crown glass respectively, showing the discrepancies due to differences of material. The rapidity with which the rate of expansion in the mercury increases with the temperature is shown by column fifth.

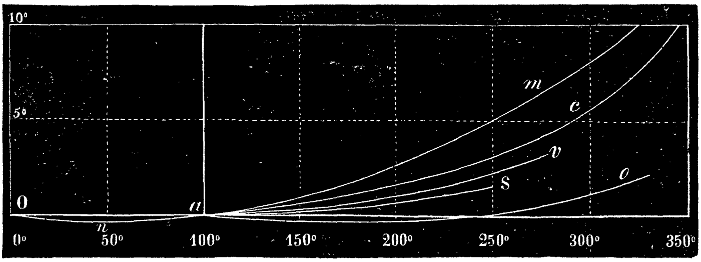
At and between the fixed points of 0° and 100° a perfect accord was observed, the small differences there existing being distributed among all the degrees. Above 100°, however, it will be seen the differences between the true temperatures and the several thermometers are more and more sensible; and most conspicuous in the thermometer without glass. The effect of the expansion of the flint-glass is seen to be approximately to correct the expansion of the mercury. The crown-glass thermometer, owing to the peculiar rate of expansion in crown-glass, is seen to march very closely with the true temperature up to 246°·30, where the coincidence is perfect. Above that point the differences increase, until at 340°, the error is three degrees. A thermometer of crown-glass is plainly to be preferred for accuracy over one of flint-glass.

The facts embodied in this table are made conspicuous by a geometrical construction,* fig. 445, in which the figures on the horizontal line (or axis of ordinates) stand for the temperatures of an air thermometer assumed as invariable, and those on the vertical line (or axis of abscissas), for the differences found between the air thermometer and various mercurial thermometers. The variation of the theoretical thermometer, without glass, from the true temperature, is seen in the curve O*nam*; while the curves O*nac*, O*nav*, O*nas*, O*nao*, show respectively the variation of thermometers made with flint-glass, green glass,

---

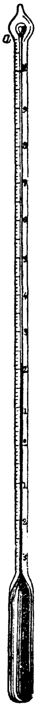
* Cook's Chemical Physics.

445



Swedish glass, and "verre ordinaire" of Paris. The last curve (corresponding to column fourth in the table) is a beautiful illustration of the anomalies indicated by observation.

446



577. **Standard thermometer.**—The unavoidable defects in the mercurial thermometer, just pointed out, are in common instruments greatly exaggerated by errors of construction. Standard thermometers for scientific purposes are constructed with a scale engraved in the glass, as in fig. 446. The divisions of this scale are marked by a dividing engine on the surface of a covering of varnish, with which the tube is coated preparatory to etching. As no calibre is absolutely uniform in any glass tube for a considerable length, the value of the calibre for every part of the tube is exactly measured by a cylinder of mercury drawn in at one end of the open tube, and taken so short that the error in the length chosen may be disregarded. This cylinder of mercury is then gradually moved from end to end of the tube by the force of air from a gum elastic bottle, dividing the tube into lengths equal to the successive lengths of the mercury column. The position of these successive points is marked, and each of these divisions is then subdivided by the engine into an equal number of degrees; varying in length, of course, in proportion to the lengths of the several cardinal divisions. These graduations are then etched into the glass by exposing it to the vapor of fluohydric acid. This graduated tube is then soldered to a cylindrical reservoir prepared of a size proportioned to the diameter of the tube, and the destined range of the thermometer.

The mode of determining the required size of this reservoir is as follows:—

We wish to know what size to give the reservoir for a given graduated tube that N divisions of the thermometer may correspond to 100° C. Weigh the tube empty, and also when containing a column of mercury of an observed length. We thus learn the weight of mercury, w, occupying n degrees of the tube. From this we obtain $N\frac{w}{n}$, the *weight* of mercury which will fill N divisions of the tube, and by (99), we know the corresponding *volume* $= N\frac{w}{n\,(Sp.\ Gr.)}$. But this volume represents the expansion which the mercury in the reservoir of the proposed thermometer must undergo when heated from 0° to 100° C. Now the apparent expansion of mercury under these conditions is known to be $\frac{1}{65}$ of its volume at 0°. Representing, then, by V the unknown volume of the reservoir, we shall have: $\frac{V}{65} = N\frac{w}{n\,(Sp.\ Gr.)}$, and $V = 65\,N\frac{w}{n\,(Sp.\ Gr.)}$. If the reservoir is spherical, $V = \frac{1}{6}\pi D^3$, from which we can calculate the required diameter. If it is cylindrical, $V = \frac{1}{4}\pi D^2 h$, from which the approximate length of h is calculated when the diameter is given.

The thermometer thus graduated is filled, and the fixed points marked as already described (568). Its scale, of course, is arbitrary, and may be reduced by calculation to the Centigrade or any other scale. Observation determines the number of divisions between freezing and boiling, which we call N, and also the point on the arbitrary scale, corresponding to the freezing point (0° C). Call the number of divisions below this point d°, the degrees centigrade C°, and those of the arbitrary scale A°. We then have, $N = 100$, and $C = \frac{100}{N}(A^\circ - d^\circ)$. Suppose there are 379 divisions on the arbitrary scale between the fixed points, and the freezing point is the 147th division from the bottom, and it is required to know to what temperature the 303d division corresponds in Centigrade degrees, we shall have $C = \frac{100}{379}(303 - 147) = 41{\cdot}16$. Every such thermometer has, of course, its own equation; a table is readily calculated for its convenient use.

Every good standard has both the boiling and freezing points included in its range. The cavity *a*, fig. 446, is designed to allow for any sudden expansion of the mercury above the limit of the scale, to avoid the fracture of the instrument. Similar expansions are often introduced at particular points on the stem, to avoid undue length in the stem when a long range is required. The whole scale in this case is distributed between several thermometers, and the swellings so placed as to cover in each case the range of the preceding instrument. It is thus possible to divide each Centigrade degree into twenty or more parts.

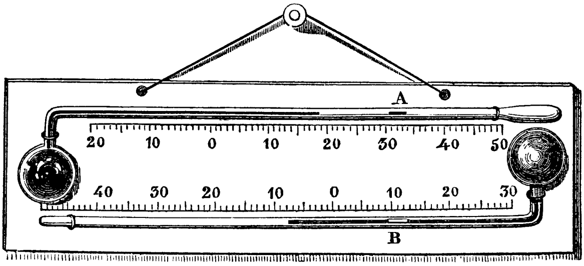
In accurate observation, the whole instrument should be immersed in the medium whose temperature we seek, but when this is not possible, a correction may be calculated by a formula which our space requires us to omit. (See Cooke's Chemical Physics, p. 444.)

II. SELF-REGISTERING THERMOMETERS.

578. **Maximum and minimum thermometers.**—It is often desirable to ascertain, in the absence of an observer, the highest and lowest temperature of the night, or of any other interval of time. This may be done by the employment of what are called maximum and minimum, or self-registering thermometers. One of the most simple instruments of this kind was invented by Rutherford, and is represented in fig. 447.

(*a.*) **Rutherford's maximum and minimum thermometer** consists of two thermometers attached to a plate of glass, or to wood; their tubes are

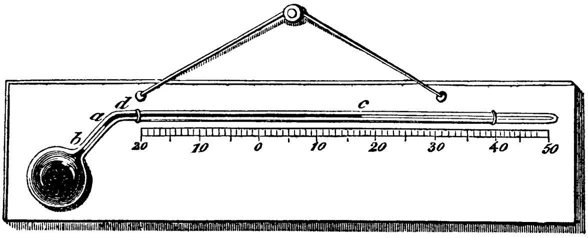
447



bent at right angles, near the bulbs. The maximum thermometer, A, contains mercury; the minimum thermometer, B, contains alcohol. In the tube of the former is a small piece of steel (seen at A); when the mercury expands, it pushes the steel before it, but when the fluid recedes toward the bulb, the wire does not follow it. The steel is thus left at the extreme point to which the mercury may have moved it, and indicates the highest, or maximum temperature, to which it has been exposed. The alcoholic thermometer contains a small piece of enamel (seen at B), sunk below the surface of the liquid. The position of the enamel is not affected by expansion, because the alcohol readily passes it; but by contraction it is drawn back with the column of alcohol, by the cohesive attraction of the particles of liquid at the surface of the column. Thus the enamel is left at the lowest point to which the column has retreated, and represents, therefore, the minimum temperature which has occurred.

(*b.*) **Negretti and Zambra's maximum thermometer.**—A slight agitation given to Rutherford's maximum thermometer will often cause the steel index to become immersed in the mercury, which, upon expansion, will pass by the steel, and thus the instrument will fail to fulfill the purpose for which it was designed. This source of error is avoided in the use of Negretti and Zambra's instrument, fig. 448.

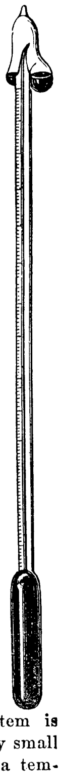
448



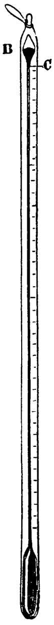
A small rod of glass, *a b*, is introduced into the thermometer tube, which is then bent just above the point where the rod is placed; the rod nearly fills the bore of the tube. When in use, the instrument is suspended horizontally. The mercury, by expanding, will force its way past the obstruction, to the point *c* for example; when the temperature falls, and the mercury contracts, the cohesion of the particles of mercury to each other will prevent the column from passing the rod. The extremity of the column, *c*, will therefore indicate the highest temperature to which the instrument has been exposed. It has been observed that in this form of instrument the mercury does not move steadily, but by jerks.

(*c.*) **Walferdin's maximum thermometer.**—The upper part of the tube of this instrument, fig. 449, terminating with a small orifice, is surrounded by a reservoir which contains mercury. When the instrument is to be used, it is first heated, whereby the mercury rises in the tube and flows over into the reservoir; it is then inverted. The elongated point of the tube thus dips into the mercury of the reservoir. It is now exposed, while inverted, to a lower temperature than the one to be determined. During this cooling, the tube will remain full because its point dips into the reservoir of mercury. The instrument is now placed in its proper position, and it is evident that as the temperature rises, a portion of mercury will pass out of the full tube into the reservoir; and this portion will be greater as the temperature is higher.

449



450



To determine afterwards the highest temperature to which it has been exposed, it is compared with a standard thermometer. Both being placed in a water bath, gradually heated, the temperature indicated by the standard thermometer is observed when the mercurial column has risen to the top of the tube of the maximum thermometer.

579. **Metastatic thermometer.**—Walferdin has applied the same principle to the construction of a thermometer designed to indicate very small differences of temperature. In this instrument, fig. 450, the reservoir, and calibre of the tube are very small, so that the instrument is extremely sensitive to small changes of temperature.

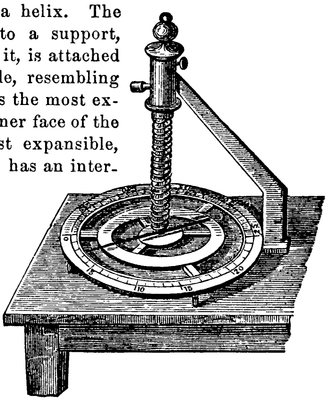
The bulb, B, corresponds to the reservoir in fig. 449. Just below this bulb, the capillary tube suddenly contracts at C. The stem is graduated into parts of equal capacity, each of which represents a very small fraction of a degree. In using this thermometer, it is first heated to a temperature somewhat higher than the one it is desired to estimate. The mercury rises in the tube and partially fills the bulb. A slight jar given to the instrument while cooling, causes the mercurial column to break at the point of contraction, and while a portion of mercury remains in the bulb, the mercurial column sinks down to a point somewhere above the reservoir. The thermometer must now be exposed to a known temperature, very near to that we wish to estimate: the position of the level of the mercury in the tube is then noted. The thermometer is next subjected to the medium, whose temperature is to be estimated. Suppose there is a difference in the level of the mercurial column in the two cases, of 18 divisions of the arbitrary scale, and if 300 of these divisions are equal to one degree, then the difference in temperature must be $\frac{18}{300}$ of a degree. Walferdin has employed thermometers of this kind which indicated one one-hundredth ($\frac{1}{100}$) and even one one-thousandth ($\frac{1}{1000}$) of a degree, Centigrade. ($\frac{1}{55}$ and $\frac{1}{555}$ of a degree F.). By causing more or less mercury to flow into the upper bulb, differences in temperature may be estimated for different points on the scale. A single thermometer of this description may take the place of a series of thermometers with a fractional scale.

III. METALLIC THERMOMETERS AND PYROMETERS.

580. **Breguet's metallic thermometer.**—This instrument, remarkable for the extreme sensitiveness of its indications, depends upon the unequal expansion of different metals; it is represented in fig. 451.

Strips of platinum, gold, and silver, after being soldered together throughout their whole length, are rolled into a thin ribbon, which is then formed into a spiral, or a helix. The upper extremity of this helix is fixed to a support, and to the lower end, at right angles to it, is attached a needle, moving over a graduated circle, resembling the dial of a watch. The silver, which is the most expansible of the three metals, forms the inner face of the helix; the platinum, which is the least expansible, forms the outer face, and the gold, which has an intermediate expansibility, is included between them, and moderates their effects. When the temperature rises, the silver, expanding more than the other metals, unrolls the helix; the contrary effect takes place when the temperature is lowered. This thermometer is graduated by comparison with a standard mercurial thermometer. The instrument is particularly useful when very rapid variations of temperature are to be determined. Another form is sometimes given to it. The compound ribbon being bent into the form of a letter U, one of the extremities is fixed, and the other is left free to move. By means of a lever and toothed wheels, the movements which changes of temperature cause in the free end are communicated to a pointer, moving over a dial.

451



581. **Saxton's deep sea metallic thermometer.**—Mr. Joseph Saxton, of the United States Coast Survey, has adapted the principle of Breguet's metallic thermometer, to the construction of an instrument by which numerous very accurate observations have been made upon the temperature of the sea in deep soundings. Silver and platinum form the compound spiral, and its torsion is registered by an index, moved by multiplying wheels, and carrying forward a tell-tale, or stop-

hand, to the lowest temperature attained. This instrument has been some years in use for deep sea soundings, with the best results. A small correction in the readings is made (not exceeding one degree for 600 fathoms) proportionate to the depth of the sounding.

The term pyrometer is sometimes applied to instruments intended to measure changes of dimensions in bodies at low temperatures by the expansion of solid rods ; such is :—

582. **Saxton's reflecting pyrometer.**—In the measurement of the base lines of the larger triangles, in the survey of the coast of the U. S., the greatest accuracy is required. These lines are sometimes forty or fifty miles in length. An instrument constructed by Saxton, under the direction of Prof. Bache, accomplishes this object perfectly.

The measuring rods are compound bars of iron and brass, so proportioned in their cross section as to equalize their differences of specific heat and conductibility, while their unequal expansions compensate for each other, and preserve an invariable length. To verify these bars, the ends are brought into contact with two blunt knife edges; one immovable, the other forming the shorter end of a compound lever; having at the other end a rotating mirror. Any variation of length in the bar, by changing the angular position of the mirror, gives evidence of the change to an observer, whose eye is placed at a telescope directed towards the mirror, in which the one twenty-five thousandth of an inch on a scale is magnified into a unit of graduation, about one-fourth of an inch long, from which the one hundred-thousandth of an inch, or about one four-millionth of a metre is easily read. If desirable, this degree of minuteness might be greatly increased. This simple and beautiful contrivance has superseded all methods previously known for verifying rods of any length. It is sensibly affected in bars six metres long, by changes of temperature otherwise quite inappreciable, and it then becomes the most sensitive of thermometers. This method is good only for end measurement.

583. **Wedgewood's pyrometer.**—The range of the mercurial thermometer is limited by the boiling point of mercury ; higher temperatures are measured by the effects of heat upon solids with instruments called pyrometers.

The celebrated English potter, Wedgewood, invented the first pyrometer used, founded upon the *contraction* which clay undergoes when exposed to high temperatures. He assumed this contraction to be as much greater as the temperature was higher. The results obtained with this instrument are, however, inaccurate, as it is now known that the contraction which clay undergoes depends rather on the duration than on the intensity of the heat, and is much modified by the particular sort of clay employed.

584. **Daniell's pyrometer** is an instrument capable of exact measurements of high temperatures by the expansion of a bar of platinum encased in a sheath of black lead. The bar and its case are adjusted both before and after the experiment to a measure which indicates on a graduated arc the degree of expansion. The degrees of temperature are then calculated from the known rate of expansion of platinum.

585. **Draper's pyrometer.**—This instrument registers its results by the expansion of a little strip of platinum, heated (in free air) by a measured current of voltaic electricity.

The platinum strip is connected with the short end of a lever, whose longer limb marks upon a graduated arc the degree of expansion. With this delicate instrument, Prof. Draper conducted a series of experiments upon the temperatures at which bodies become visibly red, in the dark and in diffused light, the temperatures being determined from the coefficient of expansion in the several metals. (Am. Jour. Sci. [2] IV. 388.)

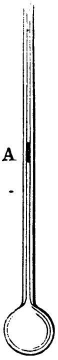
586. **Estimation of very high temperatures.**—According to Bunsen's calculations (Gasometry, p. 236–243), the temperature of a hydrogen flame burning in free air is 3259° C. (= 5898° F.), and of olefiant gas 5413° C. (= 9775° F.). Since it is probable that at high temperatures the radiating power of a body for heat is proportional to its radiating power for light, we are in possession of the means of comparing the intensity of glow of a coil of platinum heated in a furnace, or in a stream of lava, with the glow from a like coil heated in a flame of known temperature, and thus approximately of estimating the temperature of a furnace or of a volcano.

IV. THERMOSCOPES.

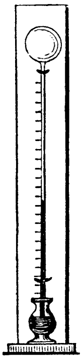
587. **Thermoscopes.**—This name (from θέρμη, temperature, and σκοπέω, to see) is applied to a class of instruments designed to *indicate* small differences of temperature, and not to measure them in degrees.

Air thermometers.—As air contracts and expands uniformly and quickly, it is often used where slight and sudden variations of temperature are to be observed. The contractions and expansions which it undergoes, are rendered visible by the movements that it causes in liquids. Such instruments are often called air thermometers, but are not to be confounded with the form of air thermometer described by Regnault, which is the most accurate measure of temperature yet made known. The results in the first column in the table on p. 404 were obtained by the air thermometer here alluded to, some notice of which will be found in the section on expansion.

452



453

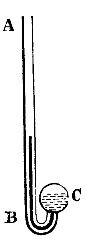


The simplest air thermometer is that represented by fig. 452, and is often called the thermometer of Sanctorius, an Italian philosopher of the 17th century. It is a bulbed tube, filled with air, having for an index a drop of colored liquid in the stem at A. The movements of the index show the variations of temperature. Another form of the same instrument is represented by fig. 453. The extremity of the tube rests in the colored liquid contained in the open vessel. If the bulb is heated, the liquid falls in the tube, and rises if the bulb is cooled.

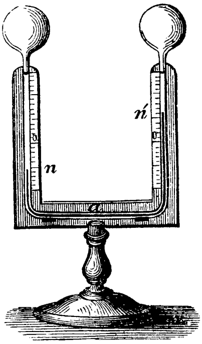
Amontons' thermometer, fig. 454, is essentially the same as the last; the bulb, C, is partially filled with colored liquid. Expansion of the air contained in the upper part of the bulb, C, causes the liquid to rise in the tube A B.

These instruments are necessarily imperfect, owing to the varying pressure of the atmosphere, and they serve only as means for the illustration of principles in the class-room.

454



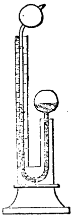
455



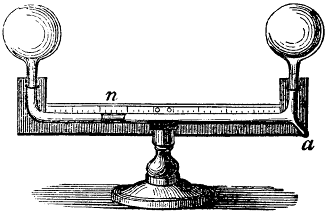
(*a.*) **Leslie's differential thermometer.**—This instrument, fig. 455, avoids the objection to the open air thermometer. It was used by Leslie in his experiments on radiant heat, and consists of a two-bulbed tube filled with air, bent twice at right angles. It contains a column of sulphuric acid in the stem, which stands at the same height, if both bulbs are equally heated, but if one is heated more than the other, the difference is seen in the unequal height of the two columns as shown in the figure.

(*b.*) **Howard's differential thermometer**, fig. 456, contains ether, and the vapor of ether, in place of common air. It is by far the most sensitive instrument of its class. It was invented by Professor Howard, of Baltimore, in 1819.

456



457



(*c.*) **Rumford's thermoscope**, fig. 457, is an instrument resembling Leslie's, and like it, contains air. The horizontal tube is longer, and the bulbs larger, than in Leslie's, and a short column of sulphuric acid, *n*, separates the two masses of air, and by its motion over a scale of equal parts, serves to indicate differences of temperature.

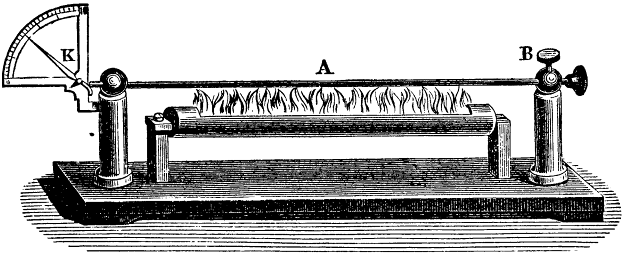
588. **Thermo-multiplier.**—By far the most delicate of all means of measuring small variations in temperature, is the thermo-multiplier, or thermo-electric pile of Nobili and Melloni. Its indications depend on the production of electric currents by small changes of temperature. It was with this instrument that Melloni conducted the remarkable series of researches on the transmission and radiation of heat which are noticed in their appropriate place, farther on.

§ 3. Expansion.

I. EXPANSION OF SOLIDS.

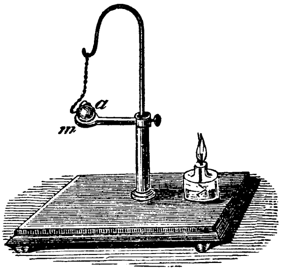
589. **Linear expansion.—Pyrometers.**—The general fact of the expansion of bodies by heat, has already been stated in § 566. Linear expansion, or expansion in a single direction, is illustrated by the apparatus seen in fig. 458. A metallic rod, A, securely held by a screw at the end, B, is heated by the flame of an alcohol lamp. The expansion is shown by the movement of the index, K, over the graduated arc, occasioned by the pressure of the fore end of the rod against the short end of the index. At the beginning of the experiment, the rod A is adjusted by the screw B, so that the index stands at zero; as the rod cools the index returns. Rods of various metals and alloys may be used for comparison. Such an instrument is called a *pyrometer;* but it has no scientific value, being replaced by instruments of far greater delicacy. Those named in §§ 582, 584, and 585 are examples of the accurate application of linear expansion of solids for the exact admeasurement of changes of temperature.

458



590. **Cubical expansion** in solids may be shown by the apparatus, fig. 459. The ring of metal, *m*, allows the ball of copper, *a*, merely to pass through it at the ordinary temperature. If the ball is heated, it expands in all directions, and will then no longer pass through the ring, but rests upon it, as is shown in the figure. As the ball cools, it gradually returns to its original dimensions, and again passes through the ring as before.

459



Different solids expand unequally, but, for the most part, uniformly, in all directions, and return to their original dimensions on cooling.

There are some exceptions to this general statement. *Wood* expands and contracts more in the breadth of its fibres than in their length; and, when it is considerably heated, it contracts permanently. *Clay* also contracts permanently by heating, and becomes vitrified; new chemical compounds being formed. The particles of *lead* slide over each other during expansion, and do not return again on cooling to their original position. Lead pipes, which convey hot water or steam, become permanently elongated; and the leaden linings of bath-tubs and cisterns, which receive hot water, become gathered into ridges from this cause.

Relation between cubical and linear expansion.—The linear and cubical expansion in any homogeneous solid, is so related, that, by the same elevation of temperature, its length, breadth, and depth will be increased in the same proportion. Thus:—

If a solid, heated to a certain temperature, increases in *length* one one-thousandth of its original length, its surface increases two one-thousandths of its original area, and its volume, three one-thousandths of its original bulk.

This theoretical view is found to be nearly, but not quite, true, in fact.

Expansion of crystals.—Crystals of the monometric system (154, *a*), like common salt, fluor spar, &c., expand equally in all directions. In this system, all the crystallogenic axes are equal, and at right angles to each other. In crystals of all other systems, the expansion is the same in only two directions (dimetric system, 154, *b*), or it is different in all three, depending upon the position of the crystallogenic axes to each other. The amount of expansion in some crystalline compound bodies, *e. g.*, fluor spar, aragonite, sulphate of barytes, quartz, &c., is found to be greater than in metals, contrary to the generally-received opinion.

591. **Coefficient of expansion.**—The small gain in length in a rod 1 foot or 1 metre long, when heated from 32° to 33° F., or from 0° to 1° C., is called its coefficient of linear expansion.

1. *Coefficient of linear expansion.* If the length of the bar is l, at the temperature of 32°, its length at 33° is $l + lK$, composed of its original length, l, and a small fraction, lK, variable with the substance experimented on.

If the rod is carried successively through the scale of temperatures, it gains, at each degree, a new elongation, which experiments show to be nearly constant, and equal to lK, so that if the rod is elevated from 32° F., to t degrees above 32° F., its total gain in length is expressed by lKt, and its new length, l_t, is

$$l + lKt, \quad \text{or,} \qquad l_t = l\,(1 + Kt).$$

At any other temperature, t',—this expression becomes, $l_t'\,b = l(1 + Kt')$, and if the value of l_t' (any temperature above 32°), is sought in terms of l_t, we write approximately, $l_t' = l_t\,[1 + K\,(t' - t)]$.

The coefficients of expansion for some of the most frequently occurring solids is given in Table III., in terms of the decimal system.

2. *The coefficient of superficial expansion*, is obtained from the expressions for linear expansion, by substituting S and S_t for l and l_t; thus:—$S_t = S\,(1 + 2Kt)$, where $2K$ replaces K in the formula for linear expansion.

3. *The coefficient of cubic expansion,* is the small fraction of its volume, by which a solid, liquid, or gas is increased when heated from 32° to 33° F. Assuming the expansion to be proportional to temperature, we must admit the volume V at t degrees and at 32 degrees to be proportional to the cubes of their homologous dimensions. By the same reasoning as before, we have, therefore, the formula; $V' = V(1 + 3Kt)$, by which the increased volume (V') of any mass of matter may be calculated when the value of V, t, and K are known.

The coefficient of cubic expansion may also be determined accurately from the specific gravity of the solid taken at different temperatures, thus:—

Let (Sp. Gr.) and (Sp. Gr.)′ represent the specific gravities of any solid at the two temperatures, t and t'; let W be the weight of the solid under trial, V its volume at 32°, and K the co-efficient to be found; then, since by the last expression, we know the value of the solid at $t°$ and $t'°$, $V(1 + 3Kt)$, and $V(1 + 3Kt',)$ we have from § 99 $(Sp.\ Gr.) = \dfrac{w}{V(1 + 3Kt)}$, and $(Sp.\ Gr.') = \dfrac{w}{V(1 + 3Kt')}$. The value of the co-efficient of cubic expansion, is obtained from the reduction and combination of these two equations

$$\text{Thus: } K = \frac{(Sp.\ Gr.) - (Sp.\ Gr.)'}{3(Sp.\ Gr.)'t' - 3(Sp.\ Gr.)t}.^{*}$$

This coefficient may also be obtained from the apparent expansion of mercury.

It is plain that all questions relating to the expansion of solids, may be solved by these expressions, when the value of K is known; and that this quantity must be the subject of exact experimental determination in each solid. Our limits do not permit the description of the various means by which the linear expansion of solids has been measured. In the researches of Lavoisier and Laplace, a bar of the substance under examination was heated in a water-bath. One end was fixed, the other free, and touched the end of a lever, acting by any expansion of the bar, and causing a movement observed in a telescope attached to the lever, as already described in § 582. The expansions, from 32° to 212°, were thus read off upon a scale placed at a proper distance.

The capacity of hollow vessels is increased by the expansion of their walls, to the same amount which a solid mass of the same material and volume would expand by a like change of temperature. Hence it is easy to calculate from the known co-efficient of glass, or any other substance, the changes of capacity of hollow vessels.

The amount of expansion in solids, between freezing and boiling, is, after all, but a very small fraction, being, for zinc, which is the most expansible of all metals, only one three-hundred and fortieth of its length; while glass expands only about one-third of this quantity, for a like change in temperature (1 in 1248). The order of the expansibility of metals and glass is as follows, commencing with the most and ending with the least expansible:—zinc, lead, tin, silver, brass, gold, copper, bismuth, iron, steel, antimony, platinum, glass.

---

* In all these formulæ, t is taken to represent the number of degrees above the freezing point.

This, it is worth while to remark, is also very nearly the order of compressibility of the same substances.

Ice is more dilatable than zinc, in the ratio of $\frac{1}{267}$ to $\frac{1}{323}$. The contraction of ice by cold has been observed for 30° or 40° below the freezing point.

The most expansible solids are, in general, the most fusible, *e. g.*, ice, zinc, &c.; while the least expansible metal, *platinum*, is also the least fusible; but in other cases this comparison fails.

The hardness, ductility, and other physical properties of the metals, appear to sustain no relation to their expansibility.

592. **The ratio of expansion increases with the temperature.**— Between 32° and 212° F., the increase in the coefficient of expansion in solids, is hardly appreciable; but for high temperatures, the increase becomes a considerable quantity. Regnault has determined the mean coefficients for glass, when blown in hollow vessels between zero C. and the following temperatures—the coefficients being in each case ten-millionths of the whole:—

Coefficients. K. =	276	284	291	298	306	313
Temperatures, C.	100°	150°	200°	250°	300°	350°

This increase in the coefficients of expansion of bodies by rise in temperature, is probably due to the distance between the particles augmenting with the heat. Their mutual cohesion is thus more readily overcome.

In the case of glass, which has been more carefully studied than any other solid, it appears from the results of Regnault, not only that glasses of different composition differ in their coefficient of expansion, but the same glass, in solid rods, expands more than in the form of tubes; and that great and sudden changes of temperature, as in making a thermometer (573), may vary the coefficient of expansion, owing probably to slow molecular changes in the glass.

593. **Amount of force exerted by expansion.**—The enormous force exerted by an expanding or contracting solid, may be conceived by estimating (from the coefficient of elasticity, § 161) the power requisite to produce an equal change of length by compression or by traction.

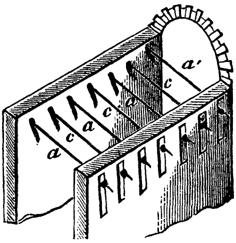
Assuming, in round numbers, the coefficient of elasticity of iron at 212° = 21,000 kilogrammes, a bar of iron, one metre long, expands 0·0012 m., if heated from 32° to 212° F. Therefore a bar of iron, one square inch in section, raised from the temperature of freezing to boiling water, expands with a force of 35,847 pounds; or it exerts a force of 199·15 pounds for every degree Fahrenheit that its temperature is elevated.

When a bar of iron one inch square has its temperature changed 12° F., its expansion or contraction exerts a strain equal to one ton weight; and if varied from 10° to 90°, a common change from winter to summer, it expands with a force of about seven tons.

The force of contraction in a cooling solid, is equal to the force of expansion when it is heated. This force is constantly used in the arts.

The walls of an arched gallery in the Museum of Arts and Trades in Paris, having bulged outwards by the weight of the arch, Molard placed a series of iron bars, fig. 460, through the wall, secured by nuts on the outside. The alternate bars were first heated by charcoal furnaces, and, when they were expanded, the nuts were screwed firmly up to the walls. As the bars cooled, they drew up the walls to an extent equal to their contraction. The other half of the bars were in like manner heated and cooled; and, by a series of such operations, the walls were gradually brought to an erect position. A similar proceeding was adopted in the Cathedral at Armagh, and in a storehouse in Providence, R. I.

460



Wheelwrights and coopers make iron tires and hoops a little smaller than the wheel or barrel for which they are designed; these are applied in a heated state, and quenched; as they contract, they bind the parts firmly together. The heavy wrought-iron rims of the driving-wheels of locomotive engines, are shrunk on in the same way. Rail car wheels are often cast with split hubs, to allow play for the unequal contraction of the heavy rims and lighter arms, or the latter would be broken at the hub, or rim, on cooling. The same precaution is requisite in all castings where heavy and light parts are united. Boiler-plates are riveted together with red hot rivets, which, on cooling, draw the plates together more firmly than any other means could do. When the stopper of a bottle sticks, it may usually be withdrawn by heating the neck of the bottle with a spirit-lamp, or with a cloth dipped in warm water. The neck is thus expanded, and the stopper is released.

594. **Common phenomena produced by the expansion of solids.**—In every-day life may be seen numerous phenomena, caused by the expansion and contraction of substances by variations in temperature.

A stove snaps and crackles when the fire is lighted, and again when it is extinguished, because of the unequal expansion and contraction of the different parts. The pitch of a piano-forte, or harp, is lowered in a warm room, owing to the expansion of the strings being greater than that of the wooden frame which supports them; and for the reverse reason, the pitch is raised, if the room is cooled.

Nails driven into wood often become loose; the expansion and contraction of the nails, through variations of temperature, gradually enlarging the holes. A gate in an iron railing may be easily shut, or opened, in a cold day, but only with difficulty in a warm day, because the gate itself, and the surrounding railings, have become expanded by the heat.

Astronomical instruments, placed on elevated buildings, are sometimes sensibly deranged by the expansion of the walls exposed to the sun. Iron and platinum wires may be successfully soldered into glass, because their mutual expansibility differs very little, while silver, gold, and copper, similarly treated, crack out as the joint cools, because their expansibility is much greater than that of the glass.

Glass and earthen vessels, with thick walls, are liable to break when hot liquids are suddenly poured into them. The surfaces in contact with the hot liquid, expanding before the other parts are affected, have a tendency to warp, or bend the sides unequally, and the brittle material breaks. We use this peculiarity of glass to convert broken vessels in the laboratory to useful purposes. Since, by a red-hot iron, or the point of a burning coal, we can lead a crack in any direction, and thus safely divide the thickest glass.

Bunker Hill Monument, an obelisk of granite, two hundred and twenty-one feet high, moves (as observed by Horsford), at top, with the sun's rays, so as to describe an irregular ellipse with the sun's motion. This movement commences about 7 A. M., of a sunny day, and has its maximum in the afternoon. In a cloudy day, no motion exists, and a shower restores the shaft to its position; showing that the heat which produces the deflection penetrates but a short distance.

Railroad bars must be laid with open joints, or their expansion and contraction between the extremes of natural temperature would destroy the road. Between 4° F., and 100° F., the expansion of one mile of rails (5280 feet) is 5 feet 7 inches.

The two tubes of the Britannia Bridge (172), are secured at the centre to the main pier, called the Britannia Tower; but the other points of support rest on friction rollers, admitting of free motion with changes of temperature. An increase of 26° F., from 32° to 58°, gives a total increase of 3½ inches in the whole length of each tube, or one-half that amount at each end. The daily change of dimensions varies from half an inch to three inches; the maximum and minimum effects being about 3 P. M., and 3 A. M., respectively. The same changes noticed by Horsford in Bunker Hill Monument, are produced in this bridge by the sun's rays. The heated portions of the tube expand, warping the free ends to the cooler side about two and a half inches, both vertically and laterally.

The Victoria Bridge, at Montreal, shows the same phenomena, but not so remarkably, as the several tubes are much shorter (page 137).

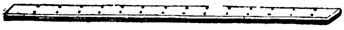
Fire regulators.—The expansion of solid bodies is often used to regulate the temperature of stoves.

A metallic bar, usually of copper, is placed within, or beside the stove or furnace, and as it becomes heated it expands, and moving a lever, turns a damper, or valve, thus regulating or arresting the draught, with perfect fidelity and accuracy.

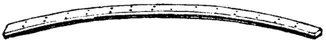
595. **Unequal expansion of solids.**—Breguet's thermometer, already described (580), is a beautiful example of the application of unequal expansion to measurement of temperatures.

If a compound bar of iron and copper, secured together by rivets, fig. 461, is heated, the copper expanding more than the iron, the bar is thereby curved, as seen in fig. 461*b*, to accommodate the irregularity of length resulting. If this compound bar is cooled below the temperature at which the two metals were united, it curves in the opposite direction.

461



461 *b*

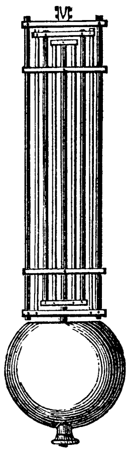


596. **Compensating pendulums.**—The length of a pendulum alone determines its times of oscillation (82). A difference of one one-hundredth of an inch in a seconds pendulum would cause a clock to vary eleven seconds in twenty-four hours, and a difference of 60° F. produces this effect.

In ordinary clocks, this defect in the length of the pendulum is remedied, by raising or depressing the ball at the end of the rod by means of a screw. Pendulums in which this defect is remedied by a self-adjusting arrangement, are called *compensating pendulums.* The compensation is effected by the unequal expansion either of mercury and glass, or of different metals.

Harrison's gridiron compensating pendulum, fig. 462, is one of those most commonly employed. The large weight at the bottom of this pendulum is supported by a series of rods of brass and steel arranged in alternate pairs. The middle rod is of steel, and, like all the other steel rods, is shaded in our figure. The cross-pieces connect the two systems of rods, alternately at top and bottom, in such a way that while the expansion of the steel rods lengthens the pendulum the expansion of the brass rods shortens it. The length of the pendulum is plainly the sum of the length of the steel rods less the sum of the brass rods (the supporting crotchet being added to the length of the steel rods), each pair of rods being reckoned as only one rod. In order that the length of the pendulum should remain invariable with changes of temperature, it is obvious that the expansion of the two systems of rods must exactly balance each other.

462



To determine the length of rods required to effect this, let L and l be the sum of the lengths of the steel and brass rods respectively, and K and K' their respective coefficients of expansion. Then, if the amount of expansion in both systems is equal, LK will equal lK'. But since, at London, the length of the seconds pendulum is 39·14056 inches (82), it follows that $L - l = 39{\cdot}14056$ inches.

If, therefore, we take from Table III., the values of K and K', and combine these two equations, we shall find the respective lengths of L and l.

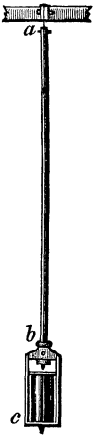
$$L = \frac{K'}{K' - K} \times 39{\cdot}14056 \text{ inches}, \quad l = \frac{K}{K' - K} \times 39{\cdot}14056 \text{ inches}$$

But the position of the centre of oscillation, which determines the virtual length of the pendulum (83), may vary, although the sensible length remains unchanged. Hence the necessity of adjusting the position and mass of the suspended weight after the length of the rods is approximately accurate.

In Graham's compensating pendulum, fig. 463, the rod, a, b, is of glass, and the ordinary weight is replaced by a glass vessel containing mercury sustained in a metallic stirrup. When the temperature rises, the pendulum lengthens, and the mercury also expanding, rises in the glass.

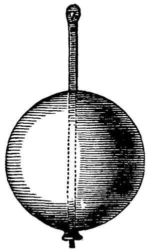
The compensation in this instrument is not quite perfect, since the position of the centre of gravity (which remains unchanged by the construction) does not entirely coincide with the centre of oscillation, on which the virtual length of the pendulum depends.

463

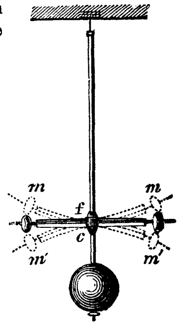


Mr. Henri Roberts' compensating pendulum is remarkable for its extreme simplicity. The rod of the pendulum, fig. 464, is of platinum, and supports at its lower end a disk of zinc. The centre of gravity of this disk will always be preserved at the same distance from the point of suspension, if the expansion of the platinum rod is equal to that of the zinc disk; this condition is obtained when the radius of the disk is equal to one-third of the length of the rod.

464



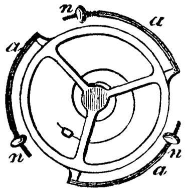
465



Martin's compensating pendulum is a compound bar of iron and copper soldered together throughout their length, and fixed transversely upon the pendulum rod, fig. 465. The copper, being the most expansible, is placed below the iron. When the temperature rises, and the centre of oscillation is, by the expansion of the pendulum, removed to a greater distance from the point of suspension, the copper, expanding more than the iron, bends the rod into the curve, *m f m*, whereby the metallic balls, *m m*, at the extremities of the rods, are raised, and being brought closer to the point of suspension, compensate for the increased distance of the weight of the pendulum from that point. If the temperature is lowered, the rod bends into the curve, *m' c m'*, and the balls are lowered. These balls are of such a size, and placed at such a position upon the compound bar, that the centre of oscillation is not displaced by variations in temperature, and thus perfect compensation is produced.

Compensating balance wheels of watches and chronometers are constructed precisely on the plan of Martin's pendulum. The balance wheel of a watch varies with changes of temperature,—the duration of an oscillation depending on the radius of the wheel, the strength of the spring, and the mass of its rim. The expansion of the wheel, by enlarging the radius, retards the timepiece, and, conversely, cold accelerates it. The three metallic arcs, *a a a*, fig. 466, are designed to counteract and correct the effect of expansion on the wheel. Each arc is composed of two strips of metal, the most expansible being placed outside. Heat, therefore, carries the masses, *n n n*, inward and nearer to the axle of the wheel, while cold throws them outward, thus preserving the virtual length of the radius under all changes of temperature. Any errors of compensation are adjusted by turning the masses, *n n n*, on the screws at the ends of the arcs.

466



II. EXPANSION OF LIQUIDS.

597. **General statement.**—All liquids expand by heat more than solids; thus mercury, the least expansible of all liquids, expands more than zinc, the most expansible of all solids.

The rate of expansion in liquids is not so uniform as it is in solids, and especially near their points of solidification and vaporization they are subject to great irregularities.

598. **Apparent and absolute expansion.**—We have already (576) noticed the fact that it is only the apparent, and not the absolute, expansion of mercury which is read in the thermometer. It is plain that in any case the absolute expansion of a liquid must be the sum of its apparent expansion, and of the increased capacity of the containing vessel (591) at the given temperature. Either two of these quantities being known, the third can be calculated.

The absolute expansion of mercury, being one of the most important constants in physics, and one on which many others depend, has been determined with the greatest accuracy. This determination was originally made by Dulong and Petit, and has been confirmed and corrected more recently by Regnault.

The method giving most exact results depends on the familiar principle of hydrostatics (202), that the heights of liquid columns in communicating vessels are in the inverse ratio of the specific gravities of the liquids. What is here true of different liquids is of course true of the same liquid at different temperatures. To determine this point accurately, a glass tube, bent into a syphon, is filled with mercury, and so arranged that, while the two legs are respectively exposed to the required temperatures, the corresponding heights may be exactly measured by a cathetometer. The coefficient of expansion for each temperature may then be calculated from (591·3) by means of the specific gravities thus determined.

Let C and C' represent the two columns; H and $(Sp.\ Gr.)$ the height and specific gravity of C at 32°, and H' and $(Sp.\ Gr.)'$ the height and specific gravity of the column C' at $t°$. Then, by 202, $H(Sp.\ Gr.) = H'\ (Sp.\ Gr.)'$. Let K represent the coefficient of absolute expansion in mercury, and by 591 and 99, we have $(Sp.\ Gr.) = (Sp.\ Gr.)'\ (1 + Kt.)$ Hence the value of K, obtained by combining these equations, is,

$$K = \frac{H' - H}{Ht}.$$

The mean absolute expansion of mercury was by this method found by Dulong and Petit to be between 32° and 212° F. for 1° F., $K = \frac{1}{9990} = 0{\cdot}0001001$. This number has been corrected by the later researches of Regnault to $K = 0{\cdot}00010085$ for each degree of Fahrenheit's scale; or, $K = 0{\cdot}00018153$ for each degree Centigrade.

The increase of the coefficient of expansion for mercury, with increase of temperature already alluded to (576), is shown in the following table copied from Cooke's Chemical Physics, p. 510. The degrees are Centigrade.

COEFFICIENT OF EXPANSION FOR MERCURY.

True Temperature by Air Thermometer.	Mean Coefficient of Expansion of Mercury from 0° to t°.	Actual Coefficient of Expansion from t° to $(t + 1)$°.	Volumes of Equal Weights.
0°	0	0·00017905	1·0000000
30	0·00017976	0·00018051	1·0053928
50	0·00018027	0·00018152	1·0090135
70	0·00018078	0·00018253	1.0126546
100	0·00018153	0·00018305	1·0181530
150	0·00018279	0·00018657	1·0274185
200	0·00018405	0·00018909	1·0368100
250	0·00018531	0·00019161	1·0463275
300	0·00018658	0·00019413	1·0559740
350	0·00018784	0·00019666	1·0657440

The last column of this table shows the volume to which one cubic centimetre of mercury will expand when heated to the temperatures given in the first column. Whenever the mean coefficient between 0° and t° (as given in the second column) is known, the corresponding volume may be calculated by the formula $V' = V(1 + Kt)$; and, by interpolation, the volume can be calculated for temperatures for which the coefficient has not been determined.

599. **Correction of the observed height of the barometer for temperature.**—As the volume and density of mercury vary with the temperature, the height of the mercurial column in a barometer varies not only with changes of the atmospheric pressure, but also with changes in temperature. Before comparing barometric observations, therefore, made at different times, it is necessary to reduce the observed heights of the mercurial column to the height they would have at some standard temperature.

The principles enunciated in the last section enable us to obtain the following value for the height of the barometer reduced to 32° F.

$H = H' - H' \frac{t}{9916 + t}$, t being the degrees Fahrenheit above freezing; or,

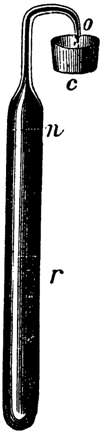
$H = H' - H' \frac{t}{5508 + t}$, when t is given in degrees of the Centigrade scale.

The true height of the barometer is therefore to be obtained by subtracting the correction from the observed height when the temperature is above the freezing point. There is also a small correction to be made for the expansion of the scale, which for present purposes may be neglected.

600. **Apparent expansion of mercury.**—The apparent expansion of mercury in glass is readily determined by means of the simple apparatus, seen in fig. 467, consisting only of a glass tube, *r*, drawn out into a narrow neck, which is recurved so as to dip conveniently into the cup *c*. The weight of the empty tube is first taken, and it is then filled with mercury in the manner described in § 568; taking care to expel, by continued boiling, the last traces of air and moisture. The tube and its contents are then cooled to 32° F. by immersion in melting

ice, the point o being kept constantly beneath the mercury in c. It is then weighed again, and thus by deducting the weight of the empty tube we learn the weight (W) of the mercury it contains at 32°. Lastly, it is exposed to a constant temperature, t° (say of 212° F., see fig. 443), and the weight of the escaping mercury (w) ascertained. The weight of the mercury which fills the tube at t° is therefore $W - w$. From these data the coefficient of apparent expansion is calculated.

467



The volume, V, of a weight of mercury, represented by $W - w$ at 32°, is $V = \frac{W - w}{(Sp.\ Gr.)}$. Now this weight of mercury at t° filled the same volume (*i. e.*, the whole apparatus), not regarding the expansion of the glass, which was filled by the weight, W, at 32°. The volume of the weight $W - w$ at t° is therefore $V' = \frac{W}{(Sp.\ Gr.)}$. Let the coefficient of apparent expansion be K, and then $V' = V(1 + Kt)$; then substituting the values of V and V', and reducing, we have $K = \frac{w}{(W - w)\,t} =$ for common French glass $\frac{1}{6480}$.

A similar mode of experiment gives of course the coefficient of apparent expansion for all other liquids. It is also applicable for the determination of the coefficient of expansion of all solids not acted on by mercury, since it is true that the coefficient of apparent expansion for mercury is equal to the coefficient of absolute expansion less the coefficient of expansion of the material for the containing vessel. As the coefficient of absolute expansion for mercury is known with the greatest accuracy, it follows that, by an application of the reasoning in this section, we have the means of determining the coefficient of expansion in glass and other solids.

601. **Amount of expansion of liquids.**—Liquids expand very unequally for equal increments of heat; the law of their expansion has not been fully determined. Generally the most expansible liquids are those whose boiling points are the lowest. Those whose boiling points are high have usually a small but very regular expansibility, especially at temperatures much below their boiling points.

The rate of expansion in all liquids increases with the temperature, but it varies with each substance according to laws not well understood.

Between 32° and 212°, mercury expands 1 in 55, water 1 in 21·3, sulphuric acid 1 in 17, alcohol 1 in 9 +, &c. See Table IV.

The statement, in the first edition of this work, that in many liquids of analogous chemical constitution, the rate of expansion is nearly uniform at equal distances from their respective boiling points, appears, from the observations of Pierre, not to be sustained.

The expansibility of liquids is not in proportion to their density, but is more nearly the inverse of this than in any other known ratio.

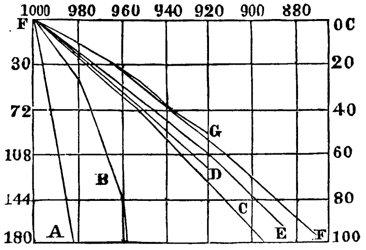
602. **Expansion of liquids above their boiling points.**— The late researches of C. Drion* show that the coefficient of expansion in liquids, above their boiling points, increases at an accelerated ratio, and even surpasses the coefficient of expansion in gases. As long since as 1835, Thilorier, in his memoir on liquid carbonic acid, states that this liquid expands between 0° and 30° C. (32° to 86° F.), four times as much as air expands for the same range of temperature, being as $\frac{116}{267}$ for the liquid gas to $\frac{30}{267}$ for air. Twenty volumes of liquid carbonic acid, between 32° and 86° F., become therefore twenty-nine volumes.

Liquid sulphurous acid, and cyanogen present the same apparent anomaly, as well as certain fluids found in the minute cavities of topaz and quartz crystals.

The experiments of Drion were made on chlorid of ethyl, sulphurous acid, and hyponitric acid.

Curves of expansion of liquids.—The variation in the expansion of liquids may be graphically represented as in fig. 468. The diagram shows the lines representing the expansion of six liquids. In each case 1000 parts of liquid are taken at 212° F. The horizontal lines (reading from above downwards) show the bulk of the liquid at temperatures below the boiling point. These temperatures are represented in degrees of Fahrenheit's scale on the left hand column, and in Centigrade degrees on the right hand. The line A indicates the contraction of mercury; B, that of water; C, for alcohol; D, for wood spirits; E, for formic ether; F, for terchlorid of silicon; G, for ordinary ether.

468



Thus at 108° below the boiling point (at 104° F.), 1000 parts water have contracted into 966 parts; alcohol into 931 parts; and formic either into 918 parts.

603. **The amount of force exerted in the expansion of liquids** is enormous; being equal to the mechanical force required to compress the expanded liquid into its primitive volume.

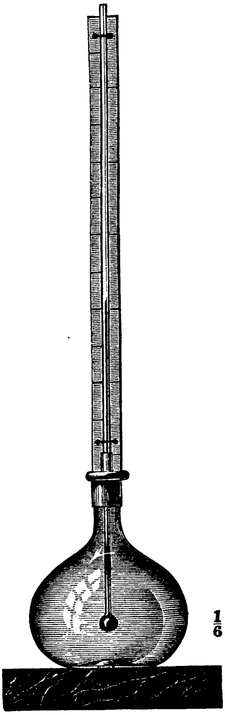
Thus the expansion of mercury for 10° F., is ·0010085. Its compressibility for a single atmosphere is ·0000053. Therefore the amount of force required to restore the mercury to its original bulk, after heating it 10° F. is equal to 190 atmospheres (10085 ÷ 53 = 190), or 2850 pounds pressure to a square inch. Owing to this enormous force exerted during expansion, closed vessels filled with liquid, however strong they may be made, burst when heat is applied.

---

* Ann. de Ch. et de Phys. 1859.

604. **Expansion of water.**—Water, which presents so many remarkable exceptions in its physical history, does so in no respect more than in the singular irregularities observed in its expansion for equal increments of temperature between 32° and 212°. Its *total* expansion for this range is by no means large, while its coefficient of expansion is found, by an examination of Table IV., to be smaller than that of any liquid except that of mercury. The expansion of water, which is irregular through the whole range, from freezing to boiling, is especially so between 32° and 40° F. While all other liquids are most dense at their freezing points, the *maximum density* of water occurs some degrees above that point (39°·2 F.), and above or below this temperature it expands.

469



Maximum density of water.—To illustrate, by experiment, this signal exception in water to the ordinary laws of expansion, a water thermometer, like fig. 469, may be used. The flask, holding about a quart, is filled with water, and the tube passing into it is secured water-tight by a brass cap or well-fitting cork, so that at ordinary temperatures the column of water stands at some convenient point on the scale of equal parts. It is then set in a cold room (below freezing), and the loss of temperature indicated by the fall of the column of water, is more accurately noted by a mercurial thermometer seen in the figure placed within the flask. When that temperature reaches about 42° F. (6° C.) the fall of the water column ceases—it comes to rest for a short time, and at 39° or thereabouts (4° C.), it is seen to mount more and more rapidly as the temperature falls, until it reaches 32° (or even lower, if the apparatus is kept quite still).

If the apparatus is filled with water near the temperature of maximum density, and placed in a warmer room, we have evidence of the converse, and not less remarkable fact, that expansion equally occurs, whether we heat or cool the water. These results are somewhat obscured by the expansion of the glass; but for a few degrees above and below 38°, the density of water is nearly uniform.

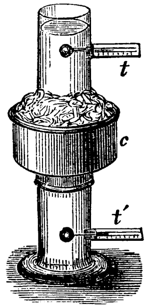
At the moment of freezing, water expands about ten per cent. of its volume, and the fact is often evidenced in the apparatus here figured, by a *jet d'eau* from the tube at the moment of freezing.

Owing to the difficulty of compensating the errors involved in the expansion of the containing vessels, the point of maximum density cannot be settled with absolute accuracy. Hassler assumed it at 39·83 F. in his determination of the value of the United States standards of measure. Gineau determined the French unit of weight at 40° F. (4·5 C.), and Despretz, in 1839, fixed it at 39°·2 or 4° C. The later researches of Plücker and Geissler reduced it to

38°·84; but it is agreed by physicists to assume 4° C., or 39°·2 F., as representing the true point of maximum density for water.

The apparatus shown in fig. 470, serves well to illustrate the effect of this law in the freezing of lakes and rivers. A glass jar, around the central part of which is fitted a metallic vessel, *c*, is provided above and below with two delicate thermometers, *t t'*, entering the sides of the jar horizontally by openings drilled for that purpose. After filling the jar with water, a freezing mixture of ice and salt is placed in *c*, which rapidly cools the water. The two thermometers continue to indicate nearly the same temperature until the water is cooled to 39°·2 F., when it will be observed that the lower thermometer remains at that point, while the upper one indicates a lower temperature, until it finally reaches 32° F., or even lower.

470



The explanation of these phenomena is, that the water, cooled by the freezing mixture, becomes more dense and sinks, while other and lighter portions rise, to be cooled and sink in turn. Thus a system of currents is established, by which the whole of the water gradually reaches the temperature of 39°·2. On cooling below this point, the water expands, and, thus becoming lighter, the colder portion remains at the surface, and is further cooled by the freezing mixture, while the water in the lower part of the vessel, not coming in contact with the freezing mixture, and being no longer disturbed by currents, remains at the temperature of 39°·2.

Effects of the unequal expansion of water.—Under the influence of this law of unequal expansion in water, the cold of our most severe winters produces only a comparatively thin covering of ice upon the lakes and rivers. Water freezes at 32°, but, before that temperature can be reached, expansion sets in at the surface, and the water, although colder, is specifically lighter than the warmer water below, and, consequently, floats buoyantly upon it. Ice is formed only on the surface, but, being a very bad conductor, it cuts off the escape of heat from the water below, and this renders the freezing process a very slow one. In fact, a film of ice may be likened to a blanket, which, although of itself cold, becomes a means of preserving heat by cutting off radiation.

Lake Superior has, uniformly, throughout the year, the temperature of about 40°, at a short distance below the surface; and the deep sea soundings show, that the sea, at the bottom of the ocean, even under the Gulf Stream, is below the temperature of maximum density, which, in saline solutions, is lower than in pure water. The temperature of the deep Alpine lakes is 39°·2 F., at all seasons of the year.

Maximum density of different aqueous solutions.—The solution of various salts in water has the effect of lowering its point of maximum density. Thus, the point of maximum density of sea-water is 25°.70. The point of maximum density of solutions falls more rapidly than their point of congelation, and is proportional to the quantity of salt dissolved.

The volume of water, at different temperatures, has been determined by several experimenters, and the results, according to Kopp, are given in Table XXIV., with the corresponding specific gravities, both when taken at 32°, for the unit of volume and density, and also at 4° C. (39°·2 F.).

III. EXPANSION OF GASES.

605. **General statement.**—Gases and vapors, being under the influence of repulsion, and having little cohesion, expand, for equal increments of heat, much more than either solids or ordinary liquids. (Compare § 602.)

The expansion of air, and of all gases, may be shown by plunging the open end of a bulbed tube into water; a slight elevation of temperature, even the heat of the hand, will expand the air in the bulb, and cause a part of it to escape in bubbles through the water. And when the source of heat is withdrawn, the rise of the water in the tube indicates the amount of expansion (604).

606. **Gay Lussac's laws for the expansion of gases by heat.**—Gay Lussac was the first to discover the general laws of the expansion of gases by heat. The gases on which he experimented were not freed from moisture; but the laws which he deduced are remarkable for their great simplicity and general accuracy, considering the state of experimental science at that time (A. D. 1805). They are as follows:—

1st. *All gases have the same coefficient of expansion as common air.*

2d. *The coefficient of expansion remains the same, whatever may be the pressure to which the gas is subjected.*

These laws, like the laws of Mariotte (274), though sufficiently accurate for ordinary purposes, are found, by the more complete experiments of modern science, to be not strictly correct.

607. **Results of Regnault's experiments upon the expansion of gases.**—Very valuable experiments were made by Dulong and Petit, but the most recent and complete investigation of the expansion of gases by heat, was conducted by Regnault. In all his experiments, the different gases experimented upon were completely deprived of moisture, and the results of his experiments are contained in the following tables:—

EXPANSION OF GASES BETWEEN 32° AND 212° F. (JAMIN).

Gases.	Under Constant Volume.	Under Constant Pressure.
Air	0·3665	0·3670
Nitrogen	0·3668	0·3670
Hydrogen	0·3667	0·3661
Oxyd of Carbon . . .	0·3667	0·3669
Carbonic Acid	0·3688	0·3710
Protoxyd of Nitrogen .	0·3676	0·3719
Sulphurous Acid . . .	0·3845	0·3903
Cyanogen	0·3829	0·3877

From this table it appears that the coefficients of expansion of those gases which have never been condensed to liquids, are very nearly the same as air; while the coefficients of the condensible gases, carbonic acid, sulphurous acid, and cyanogen, are considerably greater, and the greater in proportion as they are more readily condensed into liquids. Each gas has two coefficients of expansion,—the coefficient of expansion for a constant volume being less than for a constant pressure,* except in the case of hydrogen, in which the reverse takes place. This agrees in a remarkable manner with the fact (276) that hydrogen alone is less compressible than the law of Mariotte would indicate.

It is further shown, by the experiments of Regnault, that:—

1st. *The coefficients of expansion are very nearly, but not absolutely, the same for different gases.*

2d. *The coefficients of expansion, for different gases, vary more from each other in proportion as the pressure to which they are subjected is increased.*

3d. *The coefficients of expansion for all gases, except hydrogen, increase with the pressure to which they are subjected, and this increase is most rapid in those gases which deviate most from Mariotte's law* (276).

4th. *For ordinary calculations, under the pressure of the atmosphere, the coefficient of expansion for all gases may be considered as* 0·3666 *between the freezing and boiling points of water, or* $\frac{1}{491}$ *of the volume at* 32°, *for each degree of Fahrenheit's scale.*

For accurate scientific purposes, the coefficient of expansion of every gas considered must be taken from the tables given for that purpose.

Table V., Appendix, gives the coefficients of expansion of common gases under varying pressures.

608. **Formulæ for computing changes of volume in gases.**—In physical researches it is often desirable to ascertain the increase or decrease in volume which a given gas undergoes by measured differences in temperature. This is easily done by the following formulæ:—

Let V represent the volume of the gas at 32° F., V' its volume at the higher temperature, and t the number of degrees between 32° and the higher temperature. The increase in the volume will therefore be expressed by $V' - V$. And since the increase in volume for 1° F. is generally $\frac{V}{491}$, the increase for the higher temperature is $\frac{V}{491} \times t$.

$$\text{Therefore, } V' - V = \frac{V}{491} \times t, \text{ and } V' = V\left(1 + \frac{t}{491}\right).$$

If the gas is subjected to a lower temperature, it suffers a diminution in volume, expressed by $V' - V$, and if t expresses the number of degrees below 32°

---

* This may be due to the action of cohesion.

to which it is reduced, and $\frac{V}{491}$ its diminution for 1° F., then the diminution for the lower temperature will be $\frac{V}{491} \times t$, and $V - V' = \frac{V}{491} \times t$.

$$\text{Therefore, } V' = V\left(1 - \frac{t}{491}\right).$$

If the volume of a gas at 32° F. is known, its volume at any other temperature above or below 32° may be calculated by the following:—

RULE.

Multiply the difference between the number of degrees of temperature and 32°, *by the coefficient of expansion of the gas* (*for ordinary purposes this coefficient equals* 1 *divided by* 491). *Add the quotient to* 1, *if the temperature be above* 32°, *and subtract it from* 1, *if it be below* 32°. *Multiply the number thus found by the volume of the gas at* 32°, *and the product will be the volume of the gas at the observed temperature.*

609. **Formulæ expressing general relation between volume, temperature, and pressure.**—The volume which a gas occupies depends not only on the temperature, but also upon the pressure to which it is subjected (274); the pressure of a gas being inversely as the volume into which it is compressed.

As the volume of a gas at the same temperature is inversely as the pressure, if V and V' be two volumes under the same temperature, and under the pressures P and P'; then,

$$V : V' = P' : P, \text{ and } V' = V \times \frac{P}{P'}.$$

If t and t' express the number of degrees above or below 32°, at which the temperature stands ($+$ being used when above, and $-$ when below), if a gas be simultaneously subjected to changes of temperature and pressure, the relation between its volume, pressure, and temperature, will be expressed by the general formula

$$\frac{V}{V'} = \frac{1 \pm Kt}{1 \pm Kt'} \times \frac{P'}{P} = \frac{491 \pm t}{491 \pm t'} \times \frac{P'}{P}.$$

610. **Relation between expansibility and compressibility.**—It has been found, generally, that the most expansible liquids are the most compressible.

Solids expand less than liquids, and are likewise less compressible, while liquids have a less expansibility and compressibility than gases. Among solids, the most expansible are generally the most easily compressed.

The expansibility of a substance increases with the temperature, as does also its compressibility.

611. **Density of gases.**—The density of gases and vapors is compared with atmospheric air as the standard, air being called 1, or 1000

The method for the determination of the density of gases is, in principle, the same as for the density of liquids. The determinations are made in a glass globe, fig. 205 (§ 258), to which an accurately fitted stop-cock is attached. The globe is first weighed, when filled with dry and pure air, and again after being exhausted of air by means of the air-pump; the difference in the two weights gives the weight of air contained in the flask. The globe is then filled with the perfectly dry gas under examination, and again weighed; the weight found, less the weight of the globe, gives the weight of the gas. The weight of the gas, divided by the weight of the same bulk of air, gives the specific gravity, or density of the gas, as compared with air.

Example: A glass globe held 28·73 grains of atmospheric air, and 43·93 grains of carbonic acid. The specific gravity of the latter is therefore 43·93 ÷ 28·73 = 1·529, or, 28·73 : 43·93 = 1000 : 1·529.

A number of corrections must be made, in order to obtain the true density of the gas under examination. Thus, the barometric height, and the temperature of the air at the time of weighing, must be reduced to the standard barometric height, 30 inches, and the standard temperature, 62° F. Corrections must also be made for the film of hygroscopic moisture, always adhering to the globe, and for the buoyancy of the globe in the air.

Regnault has reduced the number of corrections ordinarily necessary, by counterpoising the globe in which the gas is weighed by a second globe of equal size made of the same glass. Thus, the corrections for the film of hygroscopic moisture, and the buoyancy of the globe in the air, may be dispensed with, as they are equal in both cases.

The most important applications of a knowledge of the density of gases have been made in chemistry. As in demonstrating and elucidating the discovery of Gay Lussac, that the volume of a compound gas is either equal to, or bears a very simple relation to the volumes of its constituent gases. Also, in calculating the atomic weight of numerous elementary substances.

Table XI. c., Appendix, gives the density of the most important gases, as obtained by distinguished authorities.

§ 4. Communication of Heat.

I. CONDUCTION.

612. **Modes in which heat is communicated.**—Heat is communicated in three ways: 1st. By conduction (chiefly in solids). 2d. By convection, or circulation, in liquids or gases. 3d. By radiation.

613. **Conduction of heat.**—Heat travels in solids slowly, from particle to particle. It implies contact with, or close approach to, a hotter body. The end of a bar of iron thrust into the fire, becomes red-hot, while the other end can yet be handled. Things vary very much in their power to conduct heat, every substance having its own rate of conductibility.

A metallic vessel, filled with hot water, is at once as hot as its contents, while an earthen vessel becomes heated slowly. The metal is a good, and the earthenware is a bad, conductor. A pipe-stem, or glass tube, held in a spirit lamp, may be heated red-hot within a short distance of the fingers, where a wire of silver or copper would become at once too hot to hold.

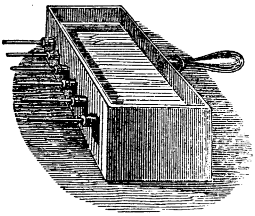
The progress of conducted heat in a solid is easily shown by a metallic rod, to which are stuck by wax several marbles, at equal distances; one end is held in a lamp, when the marbles drop off, one by one, as the heat melts the wax; the one nearest the lamp falling first, and so on. If the rod is of copper, they all fall off very soon; but if a rod of lead, or platinum, is used, the heat is conducted much more slowly.

Solids conduct heat better than liquids, and liquids better than gases, which are the poorest conductors of all. The metals, as a class, are good conductors, and their oxyds, as a class, are bad ones. The more matter, then, is present in a given body (*i. e.* the higher its density), the greater, as a general rule, is its conducting power, and *vice versa.*

614. **Determination of the conductibility of solids.**—The apparatus of Ingenhausz, fig. 471, may be employed to determine the *unequal* conductibility of solids.

This is a small copper box, one side of which is pierced with holes, in which are fitted, by means of corks, small cylinders of different substances, of the same size, covered with wax. When the vessel is filled with boiling water or hot sand, the wax will be melted from the rods in the order of their conductibility, viz., copper, iron, lead, porcelain, glass, wood. Or small bits of phosphorus may be placed at equal distances upon the rods, and these will be fired in corresponding succession.

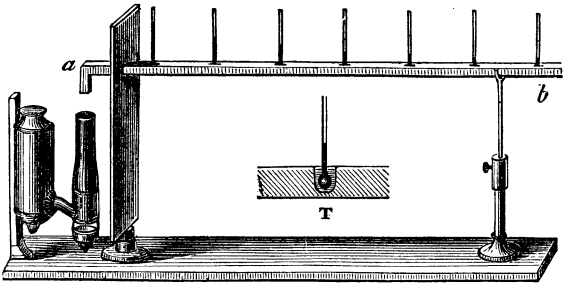
471



To determine the *relative* conductibility of solids, the apparatus of Despretz may be employed, fig. 472.

It is a series of prismatic bars, *a b*, heated at one end, *a*, by an argand lamp. Each bar has a series of small cavities, T, formed in it, at equal distances (1 *c. m.* = ·39 in.) throughout its length, and filled with mercury. In each of these cavities is placed a thermometer, which indicates the progressive propagation of the heat along the bar. Bars of various metals are used. By heating these bars successively over a steady lamp flame, their relative conductibility will be indicated by the times required for them each to attain the same temperature.

472



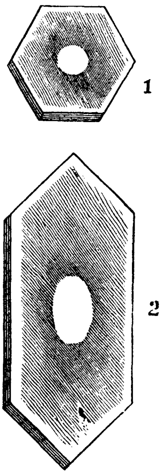
615. **Conductibility of metals, &c.**—Gold is a better conductor of heat than any other metal, or other solid. Its conductibility is represented by 1000. The order of the conductibility of other metals is (according to Despretz) platinum, copper, silver, iron, zinc, tin, lead. The conductibility of the last-named metal is only 179·6.

The precise rate of the conductibility of these metals, according to different authorities, may be seen in the Appendix, Table VII. A., and in Table VII. B , showing the conducting power of different materials used in the construction of houses, as observed by Mr. Hutchinson. The substances are arranged in the order in which they most resist the passage of heat, those substances which are most valuable in construction in this respect (viz., the warmest) being placed first. The substances marked H. P. are the building-stones employed in the construction of the new houses of parliament.

616. **Conductibility of crystals.**—The conductibility of homogeneous solids, and of crystals belonging to the monometric system, is the same in every direction. But in crystals of other systems, the conductibility varies in different directions, according to the relation of the direction to that of the optic axis of the crystal.

Senarmont, in his experiments, took thin plates of crystals, some cut parallel to the optic axis, and others at right angles to it. In the centre of each plate a small hole was drilled for the reception of a silver wire, which was heated by a lamp; the surfaces of the crystals were covered by a thin coating of colored wax. The conduction of the heat was observed by the melting of the wax, the melted portion assuming, with crystals of the monometric system, the form of a circle, 1, fig. 473, and in the other systems, ellipses of different forms, 2, fig. 473.

473



617. **Conductibility of wood.**—The dependence of the conduction of heat upon molecular arrangement is shown as well in organic structures as in crystalline media. This subject was investigated most carefully by Dr. Tyndall, who examined the conducting power of various organic substances, especially wood.

He found that at all points not situated in the centre of the tree, wood possesses three unequal axes of calorific conduction. The first and principal axis, is parallel to the fibres of the wood; the second, and intermediate axis, is perpendicular to the fibres and to the ligneous layers, and the third, and least axis, is perpendicular to the fibres, and parallel to the layers. It may be stated, as a general law, that, *the axes of calorific conduction in wood coincide with the axes of elasticity, cohesion, and permeability to liquids, the greatest with the greatest, and the least with the least.* The heat-conducting power of wood bears no definite relation to its density. American birch, one of the lightest of woods, conducts heat better than any other. Oak wood, which is very dense, conducts nearly as well, but iron wood, which has the great density of 1·426, is very low in the scale of conduction. Green woods conduct heat better than dry.

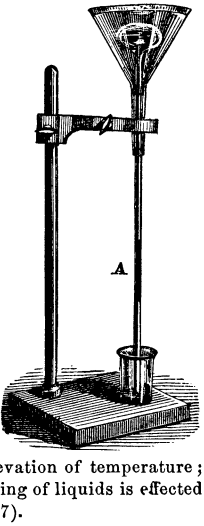
618. **Vibrations produced by conduction of heat.**—When a hot bar of metal, having a narrow base, is supported on knife edges of metal or crystal, or upon metallic points, a vibratory motion of the bar is produced, and continued until the temperature of the bar and the supporting body become nearly the same. This vibration produces a musical tone varying with the nature of the metal and the form of the bar.

It was formerly supposed that these vibrations indicated that heat was produced by molecular vibrations. But it has been shown by Tyndall (Phil. Trans. 1854) that these vibrations are caused by the want of synchronism in the sudden expansion of the points of support, as heat is communicated from the metallic bar.

Dr. Page produced similar vibrations by employing a rocker having a cylindrical surface supported on two narrow bars, using voltaic electricity as a source of heat. *Am. Jour. Sci.* [2] XX., p. 165.

619. **Conductibility of liquids.**—Count Rumford concluded, from his experiments, that liquids were absolutely non-conductors of heat, but later experimenters have determined, that liquids do conduct heat, but only to a very limited degree. That the conductibility of liquids for heat is very slight, is shown by Rumford's apparatus, fig. 474.

474



The glass funnel is nearly filled with water. A thermometer tube, with large bulb, is so arranged, that the bulb is just below the surface of the water. The stem passes through a tight cork, and contains a few drops of colored liquid at A, which will move with any change in bulk of the air contained in the bulb. A little ether poured upon the surface of the water and ignited, does not cause any movement in the column of fluid (as may be found by pasting a line of paper on the stem at one of the drops of liquid), which would be the case if any sensible warmth was communicated. The warmth of a finger, touching the bulb, will at once cause the fluid to move by expanding the air within. As the walls of the glass vessel gradually become hot by conduction, the water will slowly rise in temperature. By heating a vessel on the top, therefore, we should never succeed in creating anything more than a superficial elevation of temperature; at a small depth the water would remain cold. The heating of liquids is effected by means of currents, as will be presently explained (627).

620. **Conductibility of gases.**—Gases are more imperfect conductors of heat than liquids. It is difficult to make accurate experiments upon this subject, from the readiness with which currents are formed, and which thus diffuse the heat, but we know that gases, when

confined, are almost non-conductors of heat. Thus, substances which imprison large volumes of air within their pores, as down, wool, feathers, &c., are very poor conductors of heat.

Air, loaded with moisture, is rendered thereby a much better conductor of heat than dry air, in the proportion of 230 to 80; hence, damp air feels colder to the body than dry air of the same temperature, because it conducts away the heat from the body more rapidly.

The sense of oppression experienced before a thunder storm is due to the combined effect of the heat and moisture of the atmosphere.

621. **Relative conductibility of solids, liquids, and gases.**—If we touch a rod of metal heated to 120° F., we shall be burned; water at 150° will not scald, if the hand is kept still, and the heat is gradually raised; while dry air at 300° has been endured without injury.

The oven-girls of Germany, clad in garments of woolen and thick socks to protect their feet, enter ovens without inconvenience, where all kinds of culinary operations are going on, at a temperature above 300°, although the touch of any metallic article while there would severely burn them.

622. **Examples and illustrations of the different conductibility of solids**, are very evident to common observation.

The crust of the globe is composed of poor conducting materials, and notwithstanding the intensity of the central fires within, the amount of heat which escapes is so inconsiderable, that it has now no sensible influence on the temperature of the surface. It has been calculated, that the quantity of central heat which reaches the surface in a year would not suffice to melt an envelope of ice surrounding the earth one-quarter of an inch in thickness.

Water-pipes laid at a distance of a few feet under ground, are not frozen by the winter's cold, because the soil is a comparatively poor conductor.

Fire-proof safes are boxes of iron, constructed with double or treble walls, the intervening spaces of which are filled with gypsum (plaster of paris), burnt alum, or some other non-conducting material. These linings prevent the exterior heat, in case of fire, from passing to the books and papers within. Furnaces are lined with fire-bricks, because, being of poor conducting and infusible material, they prevent the waste of heat. Ivory and wooden handles are attached to cooking vessels, and to tea and coffee pots, because, being poor conductors, they prevent the heat from passing to the hand so rapidly as to burn it. Hot dishes are placed upon mats that the table may not be injured. Water is sooner heated in a metallic vessel than in one of glass or porcelain, because the first conducts the heat more rapidly from the fire than the others.

Buildings constructed of wood and brick are cooler in summer and warmer in winter than those of iron, because they are poorer conductors of heat.

The hearth-stone feels colder than the wooden floor, and this than the carpet, owing to the difference in their conducting powers, although all are at the same temperature.

623. **Examples drawn from the animal and vegetable kingdoms.**—The covering of animals not only varies with the climate which the several species inhabit, but also with the season. This

covering is not in itself a source of warmth, but prevents the escape of the vital heat from within.

Animals in warm climates are generally naked, or are covered with coarse and thin furs, which in cold countries are fine, close, and thick, and are almost perfect non-conductors of heat. The plumage of birds is likewise formed of substances which are poor conductors of heat, containing also a large quantity of air in their interstices. Besides this protection, the birds of cold regions are provided with a more delicate structure beneath the feathers, called down, which intercepts the heat still more perfectly. The fossil elephant of the White River, in Siberia, was covered with three sorts of hair, of different lengths, the shortest being a fine, close wool, next the body, a protection against the arctic cold. The arctic navigator and the Esquimaux endure the cold of —40°, or —60°, F. with the aid of fur bags and clothes. Animals with warm blood, which live in the water, as the whale and seal, are surrounded with a thick covering of oil and fat, which acts in a manner similar to the furs and feathers of land animals.

The bark of trees is much more porous than the wood, and, being arranged in plates and fibres around the body of the tree, prevents such a loss of heat as would be injurious to its life.

624. **The conducting power of substances in a pulverized or fibrous state**, is less than that of the same thing in a compact mass, partly because the continuity of the substances is diminished, and also because of the air imprisoned among the particles.

Saw-dust in a loose state, is a very poor conductor of heat, much poorer than the wood of which it was formed. Ice-houses are built with double walls, between which dry straw, shavings, or saw-dust are placed, keeping the interior cool by excluding the heat. Ice wrapped in flannel is preserved by excluding the warm air. Refrigerators are generally double-walled boxes, the space between the walls being filled with powdered charcoal, or some other porous non-conducting substance. (See *Ventilation.*) Similarly constructed vessels form the ordinary water-coolers.

Snow is made up of crystalline particles, enclosing a large quantity of air among their interstices, which, being a very good non-conductor, prevents the escape of the heat from the earth and limits the penetration of frost, which always reaches a much greater depth in winters without snow, than when snow abounds. On the flanks of Mount Ætna, the winter snows often reach near to the border of the fertile regions, and it is the practice of the mountaineers to cover those parts of the snow which they wish to preserve for summer use, with two or three feet thickness of volcanic sand and powdered pumice, everywhere abounding. The snow, thus protected, remains all summer under an almost tropical sun, and is distributed from these natural ice-houses over the whole Island of Sicily. There exists even to this day a heavy bed of ice near the summit of Ætna, covered first by an eruption of ashes and sand several yards thick, and subsequently by a flow of molten lava, many centuries since. This store of ice has been opened and used when the supply below on the mountain fell short. Straw-matting, and other fibrous materials, being poor conductors, are used to envelop tender plants and trees to protect them from severe cold.

625. **Clothing.**—The object of clothing, in cold climates, like the furs and feathers of animals, is to prevent the escape of heat from the

body. Fibrous materials, as wool and furs, are best adapted for clothing, because they are themselves very poor conductors of heat, and likewise contain air in their interstices.

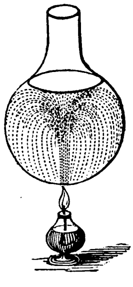
The order of the conductibility of the different substances used for clothing, is as follows:—linen, cotton, silk, wool, furs. Hence a woolen garment is warmer than one of cotton, or silk, or linen. The linen sheets of a bed feel colder than the woolen blankets, because they are better conductors of heat. Fine cloths are warmer than coarse ones, because they are poorer conductors of heat. In summer, coarse linen goods are used, because they allow the escape of heat from the body more readily than other materials, while a dress of fine, close woolen goods, is a better protection from the cold of winter than anything else, excepting furs. A thick dress of non-conducting material is sometimes used to exclude heat, as when workmen enter a hot furnace in certain manufacturing processes.

II. CONVECTION.

626. **Convection.**—Although liquids and gases are very poor conductors of heat, yet they admit of being rapidly heated by a process of circulation called convection, and which depends upon the free mobility of their particles. The particles of liquids and gases in immediate contact with the source of heat, becoming warm, and also specifically lighter, rise, and, moving away, make room for others; this is continued until all the particles attain the same temperature. Currents are thus produced both in water and air.

627. **Convection in liquids.**—The circulation just mentioned may be rendered visible by heating in a flask, water containing a little bran or amber (or other substance of about the same density as water), over a spirit lamp, as shown in fig. 475.

475



The particles of liquid at the bottom of the vessel, where the heat is applied, becoming heated, rise, and other particles of colder liquid come in below, and supply their place. Thus two systems of currents are formed. In the centre of the jar, currents of the hot particles ascend, and descending currents of colder particles, flow down the sides; this circulation continues until the whole mass has attained the same temperature.

Anything that checks this free circulation, and occasions viscidity, impedes the heating of the liquid, and likewise prevents its rapid cooling.

Starch and gum, during boiling, require to be constantly stirred, for the purpose of presenting fresh surfaces to the action of the heat, and preventing portions from adhering to the hot bottom, and thus being charred.

628. **Currents in the ocean.**—In consequence of the unequal heat to which the waters of the ocean in different parts are subjected, currents of great constancy and regularity are formed. Under the tropics,

the waters become highly heated and flow off on either side towards the poles, while other colder currents flow from the poles towards the equator. These currents are modified in their direction by the form and distribution of land and water on the surface of the earth, and the rotation of the earth upon its axis.

One of these currents (called for that reason the Gulf Stream) is directed into the Gulf of Mexico, around the western end of Cuba, and sweeping through it, passes by the narrow channel between Florida and the Bahama Islands. It has a temperature 8° or 10° F. higher than that of the surrounding ocean. This current passes northward, parallel to the coast of the United States, gradually widening and becoming less marked, and finally is directed toward the frozen ocean and British Islands. It carries away the excess of heat from the Antilles, and warm regions near the equator, beyond the western Atlantic, ameliorating the climate of the British Islands and all north-western Europe.

The researches of the U. S. Coast Survey have greatly extended our knowledge of this remarkable river of the ocean (or rather union of many rivers of warm water), first brought to the notice of the scientific world by the illustrious Franklin in 1770.

III. RADIATION.

629. **Radiation of heat.**—Hot bodies radiate heat equally in all directions. Radiant heat proceeds in straight lines, diverging in every direction from the points where it emanates. These diverging lines are called *thermal rays*, or *heat rays*. Heat rays continue to issue from a hot body, through the whole process of its cooling, until it sinks to the actual temperature of the air, or surrounding medium. It is generally by radiation, that bodies become heated at a distance from the source of heat.

Standing before a fire, or in the sun's light, we feel the genial influence of the heat radiated from these sources. A candle, or gas light, gives off its heat as it does its light, in all directions. A thermometer, placed at equal distances around the flame, indicates the same temperature.

630. **Radiant heat is but partially absorbed by the media through which it passes**, and is not sensibly affected by any motion of the media, as of winds in air.

The sun's rays lose about one-fourth (0·277) of their heat in passing through the atmosphere, the remainder being absorbed or reflected at the surface of the earth. The air receives, however, the greater part of its warmth by reflection, conduction, and convection, from the surface of the earth thus heated by the sun.

We receive warmth from the fire upon our persons, although the air remains cold, and may be continually renewed.

The conduction of heat is probably internal radiation from particle to particle; for the material atoms of which any substance consists, are not supposed to be in absolute contact, although held near each other by a strong attraction.

631. **Intensity of radiant heat.**—The intensity of radiant heat is according to the following laws:—

1st. *It is proportional to the temperature of the source.*

2d. *It is inversely as the square of the distance from the source.*

3d. *It is greater in proportion as the rays are emitted in a direction more nearly perpendicular to the radiating surface.*

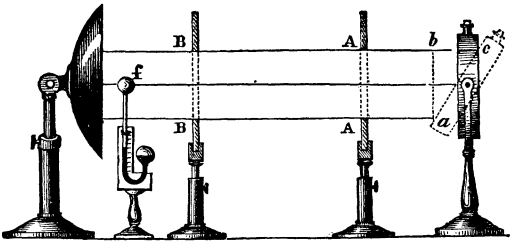
1st. If a thermometer be exposed at the same distance from different sources of heat, having, for example, the temperatures of 100°, 150°, and 200°, the amount of radiant heat will be directly as these numbers.

2d. Thus, the heating effect of a body at a distance of two feet is only one-fourth, at three feet, one-ninth, and at four feet, one-sixteenth of what it is at one foot.

This law may be exemplified by supposing two globes, one of one foot diameter, the other of two feet diameter, having a body equally heated in both. The larger globe exposes four times as much surface as the smaller one; consequently, each square inch of the larger one will receive only one-fourth as much heat as each square inch of the smaller one, while the distance to this surface is only twice as great.

3d. This law may be demonstrated by the apparatus, fig. 476. In the focus of the mirror, a thermoscope, *f*, is placed. A A, B B, are screens, pierced with equal openings. The vessel, *a c*, is filled with hot water. The position of the index of the thermoscope will be the same, whether *a c* is perpendicular, or more or less inclined. And, as in the latter case, there is a greater surface exposed, and consequently a greater number of heat-rays pass through the screen; yet, as the same effect is produced, the oblique rays must be less intense than the perpendicular rays, the intensity diminishing with their obliquity.

476



632. **Law of cooling by radiation.**—Newton supposed that the rapidity of cooling of a body was proportional to the difference between its temperature and that of the surrounding medium. This law is correct only for those bodies differing in temperature not more than 15° or 20° C. (59° to 68° F.)

Dulong and Petit made elaborate investigations upon this subject, and determined that where the heated body was placed in vacuo at temperatures ascending according to the terms of an arithmetic progression, the rapidity of cooling increased according to the terms of a geometric progression, diminished, however, by a constant quantity, this constant being the heat radiated back upon the cooling body from the walls of the confining vessel. If the temperature of the vessel, and that of the heated body, were *both* raised according to the terms of an arithmetic progression, so that the difference between the two was always constant, the rate of cooling increased according to the terms of a geometric progression.

Radiation is found to take place more freely in vacuo than in air.

633. **Universal radiation of heat.**—Heat is radiated from all bodies, at all times, whether their temperature be the same as, or different from, that of surrounding bodies; for it is the tendency of heat to place itself in equilibrium.

In an apartment where all the articles are of the same temperature, each receives as much heat as it radiates, and, consequently, their temperature remains stationary. Where some bodies are warmer than others, the warmer radiate more than they receive, until finally all attain the same temperature. Hence all bodies, however cold, will warm bodies colder than themselves; thus, frozen mercury, placed in a cavity of ice, will be melted by the heat received from the ice.

634. **Apparent radiation of cold** takes place when two parabolic mirrors are placed opposite to each other, having a delicate thermometer in the focus of one, and a mass of ice suspended in that of the other. The temperature of the thermometer will be seen to fall, apparently by the radiation of cold from the ice. The true explanation is, that the thermometer is warmer than the ice, and radiating more heat than it receives, thus loses heat, and the temperature falls. If the thermometer had been at a lower temperature than the ice, the phenomenon would have been reversed.

The following remarkable instance of the apparent focalization of cold, is explained in a similar manner. The experiment is due to the Florentine Academician Porta in the sixteenth century. If a parabolic mirror is placed with its axis pointing towards the sun, the heat-rays will be reflected to the focus of the mirror. But if the mirror be turned so as to face the clear blue sky, its focus becomes a focus of cold, and a delicate thermometer placed at that point will sink, in clear weather, a few degrees in the day time, and as much as 17° F. at night. This phenomenon is thus accounted for:—the thermometer is constantly radiating heat in all directions; the mirror, being a paraboloid, reflects to its focus only those rays that come in a direction parallel to its axis. In that direction no rays come, for there is no source to reflect them, consequently the temperature of the thermometer falls. If a cloud passes over the axis of the mirror, the thermometer instantly rises to its usual height.

§ 5. Action of different Bodies upon Heat

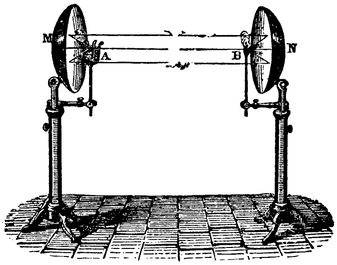
I. SURFACE ACTION.

635. **Reflection of heat.—Conjugate mirrors.**—Radiant heat, like light, is reflected at the same angle at which it falls upon any reflecting surface. This law in respect to light has been fully illustrated in the chapter on that subject.

If a piece of bright tin-plate is held in such a position as to reflect the light of a clear fire into the face, the sensation of heat will be felt the moment the light is seen.

Conjugate mirrors.—The reflection of heat may be shown in a still more striking manner by the apparatus called the conjugate mirrors, fig. 477, consisting of two similar parabolic mirrors, arranged exactly opposite to each other, at a distance of ten or twelve feet. In the focus of one mirror is placed a heated body, as a mass of red-hot iron, and in the other a portion of an inflammable substance, as gunpowder or phosphorus. Certain of the heat-rays pass directly from A to B; the greater part, however, reach B, by being twice reflected. The rays emitted from A are reflected by the mirror, M, in a direction parallel to its own axis; these rays are received by the second mirror, N, and, by reflection, are conveyed to the focus B, igniting the gunpowder or phosphorus placed at that point.

477

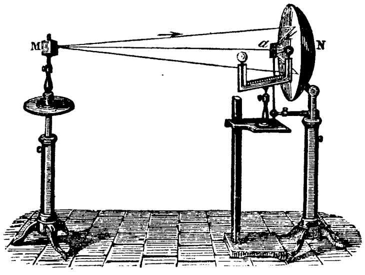


The reflection of heat in vacuo, takes place according to the same laws as in air.

636. **Determination of reflective power.**—Different bodies possess very different powers of reflection. This is well illustrated by the apparatus, fig. 478, designed by Leslie.

The source of heat is a cubical tin canister, M, filled with boiling water. A plate, *a*, of the substance whose reflective power is to be determined, is placed between the mirror and its focus. The rays of heat emitted from M, which are directed upon the mirror N, are reflected upon the plate *a*, and from this, upon the bulb of the thermoscope, placed at the point where the rays are brought to a focus. The temperature indicated by the thermoscope is found to vary with the nature of the plates.

478



The causes which modify the reflective power of bodies will be given hereafter.

637. **Absorptive power.**—Different bodies possess very different powers of absorbing the heat thrown upon them. The absorptive power of a body is always in the inverse ratio of its reflective power; that is, the best reflectors are the worst absorbents, and *vice versa.*

The absorptive power of bodies may be determined by a modification of the apparatus, fig. 478. At the focus of the mirror, N, is placed the bulb of a thermoscope, which is successively covered with different substances, as with lampblack, Indian ink, gum lac, metallic leaf, &c. Leslie has been the principal experimenter in this department of heat. A smoke-blackened surface, and a surface covered with carbonate of lead, absorb nearly all the radiant heat thrown upon them; glass, $\frac{90}{100}$; polished cast iron, $\frac{25}{100}$; tin, $\frac{14}{100}$; silver, $\frac{3}{100}$. Table no. VIII., Appendix, gives the results obtained by Messrs. de la Provostaye and Desains.

All black and dull surfaces absorb heat very rapidly when exposed to its action, and part with it again slowly by secondary radiation. The different powers of absorption, possessed by the different colors, may be illustrated by repeating Franklin's experiment. Pieces of the same kind of cloth, of different colors, were placed upon the snow; the black cloth absorbed the most heat, so that after a time it sunk into the melted snow beneath it, while the white cloth produced but little effect; the other colored cloths produced intermediate effects. Ranged according to their absorbent powers, we have, 1. Black (warmest of all); 2. Violet; 3. Indigo; 4. Blue; 5. Green; 6. Red; 7. Yellow; and 8. White (coldest of all).

638. **Emissive or radiating power.**—The earliest, and some of the most valuable, observations upon this subject, were published by Sir John Leslie, in his Essay on Heat, in 1804. Leslie proved that the rate of cooling of a hot body is more influenced by the state of its *surface,* than the nature of its *substance.* It also varies greatly with different substances, as may be seen in the table below.

Leslie employed in his experiments the apparatus, fig. 478. A bulb of a thermoscope was placed in the focus of the mirror, the other bulb being protected from the radiant heat by a screen. The cubical vessel containing boiling water, has its lateral faces covered with different substances, which are successively turned toward the mirror.

The table below gives the results as obtained by Leslie. Lampblack, possessing the greatest emissive power, is called 100.

Lampblack . . .	100	Indian ink . . .	88	Polished lead . . .	19
Water (by calc'n) .	100	Ice	85	Mercury	20
Writing paper . .	98	Minium	80	Polished iron . . .	15
Sealing wax . .	95	Plumbago	75	“ silver, tin,	
Crown glass . . .	90	Tarnished lead . .	45	“ copper, gold,	12

Messrs. De la Provostaye and Desains, and also Melloni, have obtained results differing somewhat from those of Leslie. See Table VI., Appendix. Melloni found that the radiant and absorbent powers of surfaces were not always proportional, as the following table shows:—

	Lampblack.	Carbonate of Lead.	China Ink.	Isinglass.	Lac.	Metallic Surface.
Absorptive power .	100	53	96	52	52	14
Radiant power .	100	100	85	91	72	12

Melloni has also found that the absorbent power of surfaces varied considerably, according to the source of the radiation, and the temperature of the radiant body. (See Table IX.)

From Melloni's experiments may be drawn the following conclusions —

1. That bodies agree very nearly, but not exactly, in their emitting and absorbent powers.
2. That their absorbent power varies very remarkably with the origin and intensity of the calorific rays.
3. That they approach each other more and more in their power of emitting and absorbing rays of heat, when the temperature approaches that of boiling water; and that, when at exactly that temperature, the emitting and absorbent powers coincide.

639. **Causes which modify the emissive, absorbent, and reflective powers of bodies.**—Not only do different bodies possess the powers of reflection, absorption, and emission in different degrees, but the physical condition of the material affects them in an important manner. So also the obliquity of the incident rays, the source of heat, and the thickness of the superficial layer, exercise great influence.

The absorbent and emissive powers of metallic plates are diminished if they are hammered or polished. The opposite effect is produced if the plates are scratched or roughened. This is doubtless owing to the change in density which the superficial layers of the plates undergo by these operations. For the same reason, the reflective power of a substance is generally increased by polishing or hammering, and diminished by roughening or scratching it; which latter also causes a portion of the heat to be irregularly reflected. That this is the true explanation is probable from the fact, that if such materials as ivory or coal are taken, whose density will not be changed by roughening or polishing, the reflective and absorbent powers remain the same.

The thickness of the superficial layer has an influence on the reflective power of bodies. Leslie covered a mirror with successive coatings of varnish; the reflection diminished as the number of layers increased, until their thickness amounted to twenty-five thousandths of a millimetre, when it remained constant. While a vessel covered with layers of varnish or jelly had its emissive power increased with the number of layers, until they reached sixteen (with a thickness of 0·034 m. m.), when it remained constant, even upon the addition of other layers. The absorbent power of substances varies with the nature of the source of heat. Thus a substance covered with white lead, absorbs nearly all the thermal rays from copper, heated to 212° F.; 56 of those from incandescent platinum; and 53 of those from an oil lamp. Lampblack is the only substance which absorbs all the thermal rays, whatever be the source of heat. This subject has been ably treated by Prof. A. D. Bache.*

The absorptive power varies with the inclination of the incident rays; the smaller the angle of incidence the greater is the absorption. This is one of the reasons why the sun heats the earth more in summer and less in winter.

The reflective power of glass increases with the angle of incidence, but with metallic surfaces the proportion of heat reflected diminishes with the angle of incidence, and is the same as the proportion of light reflected, § 407.

640. **Applications of the powers of reflection, absorption,**

---

* Am. Jour. Sci. [1] XXX. 16, 1836.

and radiation are often made in the economical use of heat. We shall refer only to the more familiar examples.

Meat-roasters and Dutch-ovens are constructed of bright tin, to direct the heat from the fire upon the article cooking.

Hoar frost remains longer in the presence of the morning sun upon light-colored objects than upon the dark soil, because the latter absorbs much of the heat, while the former, reflecting it, remain too cold to thaw the frost. Water is slowly heated in bright metallic vessels, as in a silver cup or a clean bright kettle, because they are poor absorbents, but if the sides and bottom of the vessels become covered with soot, the water is heated quickly.

To keep a liquid warm it should be contained in a vessel composed of a poor radiating material. Hence if tea and coffee pots, &c., are made of polished metal they retain the heat much longer than those which have a dull surface or are composed of earthenware.

Stoves of polished sheet-iron radiate less heat, but keep hot longer than those made of cast-iron with a rough and dull surface.

Pipes conveying steam should be kept bright or thoroughly covered with felt or cloth until they reach the apartments to be warmed, and there their surfaces should be blackened in order to favor the process of radiation.

II. DIATHERMANCY.

641. **Transmission of radiant heat.**—Light passes through all transparent bodies from whatever source it may come. The rays of heat from the sun also, like the rays of light from the same luminary, pass through transparent substances with little change or loss. Radiant heat, however, from terrestrial sources, whether luminous or not, is in a great measure arrested by many transparent substances as well as by those which are opaque.

The glass of our windows remains cold, while the heat of the sun, passing through it, warms the room. A plate of glass held before the fire stops a large part of the heat, although the light is not sensibly diminished.

Melloni terms those bodies which transmit heat *diathermanous*, or *diathermic* (from the Greek, διά, though, and θερμαίνω, to heat); those bodies which do not allow this transmission of heat are termed *athermanous*, or *adiathermanic* (from *alpha*, privative, and θερμαίνω).

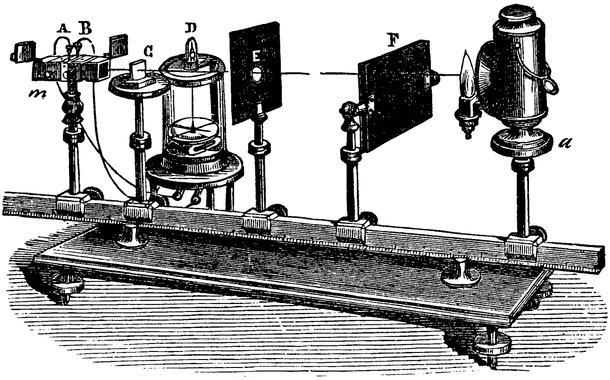
It appears that many substances are eminently diathermanous, which are almost opaque to light; smoky quartz for example.

Prevost of Geneva, and De la Roche, in France, in 1811 and 1812, discovered many of the phenomena of diathermanous bodies, but it is from the beautiful researches of Melloni, in 1832—1848, that our knowledge upon this subject has been chiefly derived. Melloni, called by De la Rive "the Newton of heat," died of cholera at Naples, in August, 1854.

642. **Melloni's apparatus.**—The apparatus used by Melloni in his researches upon the transmission of heat, is represented in all its essential details in fig. 479.

At one end of the graduated metallic bar, L L, is placed the thermo-multiplier, *m*, and in connection with it, by fine wires, A B, the anastatic galvanometer, § 905, D. Upon the stand, *a*, is placed the source of heat; in this case a Locatelli lamp: F is a double screen to prevent the radiation of the heat from the source, and is lowered at the moment of observation: E is a perforated screen which allows only a certain quantity of rays to pass through it and to fall upon C, which represents the substance whose diathermancy is to be determined.

479



In experimenting, the source of heat is placed at such a distance, that when F is removed, the heat directed upon *m*, will cause the needle of the galvanometer to move through 30°. The screen, F, is then raised, and the plate, C, to be experimented upon, is placed upon the stand. When the needle of the galvanometer, D, has returned to 0° (its normal position), the screen, F, is removed. The proportion of heat transmitted through the plate, C, is then indicated by the arc of vibration of the needle, over the dial plate of D. The construction of the galvanometer and thermo-multiplier is more particularly described in the chapter on thermo-electricity.

643. **Influence of the substance of screens.**—In experimenting with liquids, they were placed in glass cells. The stratum of liquid was 9·21 m. m. (·362 in.) in thickness. The source of heat used was an argand oil lamp.

The independence of transparency and diathermancy was clearly shown in these researches, for it was found that the bisulphid of carbon transmitted three times as many heat-rays as ether, four times as many as alcohol, and more than five times as many as water, although these liquids are equally transparent and colorless. Table X gives the diathermancy of different liquids.

It is found that those solids which are transparent to light do not necessarily allow the passage of heat, and *vice versa.* Thus sulphate of copper transmits the blue rays of light, but entirely arrests the rays of heat. Again, black mica, smoked rock-salt, and opaque black glass, transmit a considerable portion of the heat-rays, but prevent the passage of light.

Rock-salt is the only substance that permits an equal amount of heat from all sources to pass through it. Melloni experimented with plates of this substance of a thickness varying from one-twelfth of an inch to two or three inches, and in all cases 92·3 of 100 rays incident upon them were transmitted. The loss of 7·7 per cent. being due to a uniform quantity which is reflected at the two surfaces of the plate. Rock-salt is, therefore, to heat, what clear glass is to light, and well deserves the name which Melloni gave it, of *the glass of heat.*

The diathermanic power of different solids for different sources of heat may be found in detail in Table XIII.

644. **Influence of the material and nature of the source.**—The quantity of heat transmitted through different solids of the same thickness is very variable. The nature of the source of heat exercises a great influence on the diathermanic power of bodies. Melloni, in his experiments, used four sources of heat, viz.: 1. The naked flame of a lamp; 2. Incandescent platinum; 3. Copper heated to 700° F.; and, 4. Copper heated to 212° F.

645. **Other causes which modify the diathermanic power of bodies** are the degree of polish, the thickness and number of the screens, and also the nature of the screens through which the heat has been previously transmitted.

The quantity of heat which a diathermanic body transmits, increases with the degree of polish of its surface. The diathermanic power of a body diminishes with its thickness, although according to a less rapid rate. Thus with four plates whose thickness was as the numbers 1, 2, 3, 4; of 1000 rays, the quantity absorbed by each was, respectively, 619, 577, 558, 549: so that beyond a certain thickness of the body, the quantity of heat it can transmit remains nearly constant. Rock-salt is the only exception to this law; it always allows the same quantity of heat to pass through it, at least for thicknesses between 2 and 40 m. m. (·0787 and 1·575 in.)

The increase of the number of screens produces an effect similar to an increase of thickness. If many plates of the same kind are placed together, they absorb more heat than one plate having the combined thickness of several, owing to the numerous surfaces.

The thermal rays which have passed through one or more diathermanic bodies, are so modified, that they pass with more facility through other diathermanic bodies than direct rays do. Thus the heat from an argand lamp, where the flame is surrounded with a glass chimney, differs much in its transmissibility from the heat of a Locatelli lamp, where the flame is free and open. Thus in making use of an argand lamp surrounded with a glass chimney, and a Locatelli lamp which is not thus protected, Melloni obtained the following results.

TABLE OF HEAT TRANSMITTED FROM DIFFERENT SOURCES.

Of 100 rays.	Argand lamp.	Locatelli lamp.
Rock-salt transmitted	92	92
Iceland spar "	62	39
Quartz (limpid) blackened, transmitted	57	34
Sulphate of lime "	20	19
Alum "	12	7

646. **Thermochrosy**, or *heat-coloration* (*θερμος*, heat, and *χρωμα*, color).—As Newton has shown that a pencil of white light is composed of different colored rays, which are unequally absorbed and transmitted by different media, and which may be combined together or isolated, so Melloni argues from his results, that there are different species of calorific rays emitted simultaneously in variable proportions by the different sources of heat, and possessing the property of being transmitted more or less easily through screens of various substances.

If a pencil of solar light falls successively upon two plates of colored glass, one red and the other bluish-green, it will be wholly absorbed, the second plate absorbing all the rays transmitted by the first. This is precisely analogous to what may happen with a thermal pencil, its entire absorption being caused by passing it through two media successively, each of which absorbs the rays transmitted by the other. Viewed in this manner, it may be said that rock-salt is colorless as respects heat, while alum, ice, and sugar-candy, are almost black. It is a fact of common observation, that snow melts more quickly under trees and bushes than in those spots which receive the direct rays of the sun. This is proved by Melloni to be owing to the fact, that the rays emitted by the heated branches are of a different nature from the direct rays of the sun, and more easily absorbed by snow than the latter.

647. **Applications of the diathermancy of bodies.**—The air is undoubtedly very diathermanic, or else the upper layers would be heated by the solar rays passing through them, while we know that they are only slightly heated by this means.

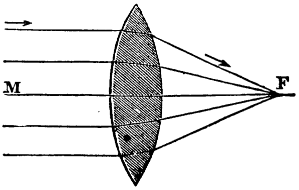
In certain processes of the arts, workmen protect their faces by a glass mask, which allows the passage of the light but arrests the heat

In certain physical experiments, where heat is to be avoided, the light is first passed through a solution or plate of alum, whereby the *heat is arrested.* On the contrary, if the heat is directed upon rock-salt covered with lampblack, the *light is arrested* but the heat passes through but slightly diminished.

648. **Refraction of heat.**—Heat, like light, is refracted, or bent out of its course, in passing obliquely through diathermanic bodies, as is shown by the burning-glass. A double convex lens, fig. 480, con-

centrates the rays of heat from the sun, or other heated body, in the same manner as it concentrates the rays of light. It is only with a lens of rock-salt, that the rays of all our sources of heat can be condensed, for a lens of glass concentrates only the solar rays, and becomes itself heated by artificial heat.

480



A lens of *ice* was made in England in 1763, having a diameter of 3 metres (118·112 in.), at whose focus gunpowder, paper, and other combustibles were inflamed. Burning-glasses have generally more power than mirrors of equal diameter. Both produce their more intense effects on high mountains after a fall of snow, for then the air is free from moisture, and the solar rays lose less of their intensity in passing through it.

649. **Polarization of heat.**—Heat is polarized in the same manner as light. It undergoes double refraction by Iceland spar, and the two beams are polarized in planes at right angles to each other. A pencil of heat, polarized by a plate of tourmaline, or by a Nicol's prism, is transmitted or intercepted by another tourmaline plate or Nicol's prism, in the same circumstances that a pencil of polarized light would be transmitted or intercepted.

Heat also suffers a rotation of its plane of polarization, by plates of right or left-handed quartz, in the same direction, and to the same extent as light of the same refrangibility. Polarization of heat is also effected by reflection from plates of glass, or by repeated refraction, also by reflection from the atmosphere, in which points of no polarization and of maximum polarization exist corresponding with similar points in regard to polarized light. The phenomena of magnetic rotary polarization of heat have also been observed.

Prof. Forbes of Edinburgh first demonstrated the polarization of heat.

Knoblauch has obtained distinct evidence of the diffraction and interference of the rays of heat.

§ 6. Calorimetry.

650. **Calorimetry.**—The amount of heat required to produce a given temperature varies greatly for the different bodies to which it is applied. Calorimetry (from *calor*, heat, and *μετρον*, measure) is the measurement of the quantity of heat which different bodies absorb or emit during a known change of temperature, or when they change their state. Water absorbs or emits a much greater quantity of heat during a change of temperature than the same weight of any other substance. It is therefore selected as the standard of comparison.

Unit of Heat.—The quantity of heat which is required to raise a pound of pure water from 32° to 33° F., is reckoned as the *unit of heat*, or *thermal unit*, both in this country and in England.

In France, and in Europe generally, the thermal unit is the quantity of heat necessary to raise one kilogramme (2·20486 lbs.) of water from 0° to 1° C.

651. **Specific heat.**—If equal weights of water and mercury at the same temperature be placed over the same source of heat, it will be found, that the mercury becomes heated much more quickly than the water. That when the water is heated 10° the mercury will have become heated 330°; the capacity of water for heat is, therefore, 33 times as great as that of mercury. Each substance in this regard has its own capacity for heat. This relation is called *caloric capacity*, or more commonly, *specific heat*. Table XI. contains the specific heats of certain solids and liquids as determined by Regnault.

Three methods have been devised for determining the specific heat of bodies: these are, 1st, the method of mixture; 2d, by the melting of ice; 3d, by cooling.

Method of Mixture.—This method is exceedingly simple in theory, and, with suitable care, exact in its results.

In determining the specific heat of solids by this method, a weighed mass of each substance is heated to the proper degree, and is then plunged into a measure of water of known temperature and weight. The elevation of temperature produced in each case is carefully noted.

If a pint of water at 150° be mixed quickly with a pint at 50° F., the two measures of water will have a temperature of 100°, or the arithmetical mean of the two temperatures before mixture. If, however, a measure of mercury at 50° be mingled with an equal measure of water at 150°, the temperature of the mixture will be 118°. The mercury has gained 68° while the water has lost 32°. Hence it is inferred, that the same quantity of heat will raise the temperature of mercury through twice as many degrees as that of an equal volume of water, and that the specific heat of water is to that of mercury as 1 : 0·47 when compared by measure.

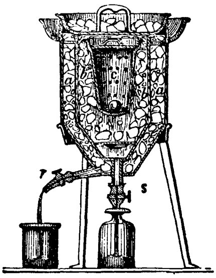
If, however, equal weights of these bodies be taken, the resulting temperature is then still more in contrast. A pound of mercury at 40°, mixed with a pound of water at 156°, produces a mixture whose temperature is 152°·3. The water loses 3°·7, while the mercury gains 112°·3, and therefore, taking the specific heat of water as 1, that of the mercury will be 0·033, since,

$$112°\cdot3 : 3°\cdot7 = 1 : x = (0\cdot033.)$$

Method by Fusion of Ice.—This method is founded on the quantity of ice melted by different bodies in cooling through the same number of degrees.

Lavoisier and Laplace contrived the apparatus, fig. 481, used for this purpose, and called a calorimeter. It consists of three vessels made of sheet tin or copper. In the interior vessel, *c*, pierced with holes and closed by a double cover, is placed the substance whose specific heat is to be determined. This is entirely surrounded by ice contained in the second vessel, *b*, and also on the cover In order to cut off the heat of the surrounding air, the exterior vessel, *a*, is also filled with ice The water from the ice melted in this outer vessel, passes off by the stop-cock, *r*. The body in the interior vessel, cooling, melts the ice surrounding it, and the water from it flows off through the stop-cock, *s*, and is weighed.

481



The specific heat of different substances is determined in this apparatus by the comparative weights of the water produced during the experiments; in which a certain weight of each body cools from an agreed temperature, *e. g.* (212° F.), to 32°, the constant temperature of the vessel C.

The specific heat of a liquid is determined by placing it in a vessel, as of glass, whose specific heat is known. The amount of ice melted by the liquid, is the whole quantity of water produced, minus that which would be melted by the glass alone.

This method, though excellent in principle, is subject to many inaccuracies, and is now seldom employed.

The method of cooling is founded on the different rates of cooling of equal masses of different substances; those having the greatest specific heat cooling most slowly.

The application of this method is also attended with so many sources of error that it is seldom employed, and need not be described.

Specific heat affected by change of state.—A body in the liquid state has a greater specific heat than when it is in the solid form, as might be concluded from the fact that the addition of heat is necessary to convert the solid into a liquid.

Thus ice has a specific heat of 0·505, water being 1·000; sulphur solid, 0·2026, fluid, 0·2340; phosphorus, between 45° and —6°, 0·1887, at 212°, 0·2045, &c.

The high specific heat of water moderates very greatly the rapidity of natural transitions from heat to cold and from cold to heat, owing to the large quantity of heat emitted or absorbed by the ocean, and other bodies of water, in accommodating themselves to variations in external temperature.

652. **Specific heat of gases.**—If a unit of weight of any gas, allowed to expand freely without change of pressure, is heated from the freezing point one degree, the amount of heat thus absorbed, measured in fractions of the unit, is called the *specific heat under constant pressure.* If the same gas is heated one degree, when so confined that its volume cannot be increased, the amount of heat required to produce the change of temperature is called the *specific heat under a constant volume.*

When the heat required to raise the temperature of equal volumes of different gases one degree, is determined, the results obtained are called *specific heat by volume.* In these determinations the unit of volume is the volume of a unit of weight of air when the barometric pressure is 30 inches.

The determination of the specific heat of gases is a problem involved in the greatest practical difficulties, and authorities vary somewhat in the results obtained.

The most valuable researches in regard to the specific heat of gases have been made by Regnault. He has established the following very important preliminary principles:—

First. *The specific heat of gases is sensibly the same at all temperatures.*

Second. *The amount of heat required to raise the temperature of a given weight of any gas one degree does not vary with the pressure to which it is subjected, and hence the specific heat of gases is the same for all densities.*

Regnault experimented on air and other gases under pressures varied from one to ten atmospheres, and found no sensible difference in the quantity of heat which the same weight of a gas lost under these different pressures in cooling the same number of degrees. Nevertheless he thinks it possible that slight differences may exist.

Table XI. c. gives the specific heats of different gases and vapors as determined by Regnault. The specific heat by weight being determined under a constant pressure, the gas being allowed to expand freely.

The specific heats by volume given in the table were obtained by multiplying the specific heat by weight, by the specific gravity of the several gases and vapors, as compared with air taken as unity.

653. **Specific heat of gases under a constant volume.**—It is well known that the temperature of a confined mass of air can be raised sufficiently high to ignite tinder by mechanical condensation, § 739, and it seems reasonable to suppose that the same amount of heat is expended in producing an equal degree of expansion when a gas is heated.

It has been stated (608) that gases expand $\frac{1}{491}$ part of their volume for an elevation of temperature of 1° F. Let t represent the small increase of temperature which a mass of gas undergoes when compressed $\frac{1}{491}$ of its volume, and if S represent the specific heat of the gas under a constant pressure, and S' the specific heat under a constant volume, we shall have for the specific heat under a constant volume:—

$$S' = \frac{S}{1 + t}.$$

It is obvious that if the value of t could be determined by condensing a gas, and observing the increase of temperature the value of S', the specific heat under a constant volume could be readily calculated. The unavoidable loss of heat absorbed by the walls of the containing vessel, when a gas is compressed, has rendered it hitherto impossible to obtain accurate values of t by this method, and similar difficulties have attended the determination of the specific heat under a constant volume by other direct methods.

The principles of acoustics have happily furnished an indirect method of determining the specific heat of gases under a constant volume with great accuracy.

Specific heat determined by the laws of acoustics.— By considering the conditions of an elastic fluid during the transmission of a sonorous wave, Newton obtained the following formula for the velocity of sound in any gas: $V = \sqrt{g . \frac{H}{d}}$.

In this formula V is the velocity of sound, g the force of gravity, H the height of the barometer, and d the density of the gas referred to mercury as unity. This formula gives for the velocity of sound in dry air at 32° F., when the barometer stands at 30 inches, $V = 883$ feet, which is less than the true velocity of sound (1086 feet, § 344) by more than one-sixth of the whole.

Laplace discovered that this error resulted from the effect of heat developed and absorbed by alternate compression and rarefaction of the air in the transmission of sonorous waves, and he showed that the formula for the velocity of sound, taking into account this effect of heat, should be, $V = \sqrt{g . \frac{H}{d} . \frac{S}{S'}}$, in which S represents the specific heat of the gas under a constant pressure, and S' the specific heat under a constant volume.

From this formula we obtain, by transposition, $S' = \frac{gHS}{V^2 d}$, from which we readily obtain the value of the specific heat of a gas under a constant volume, when the velocity of sound in the medium, and the other constant quantities, are known.

By this method Dulong has obtained for the specific heat of gases, under a constant volume, the values given in the following table; but the results obtained are regarded only as approximations:—

SPECIFIC HEAT OF EQUAL VOLUMES.*

Name of Gas.	Under Constant pressure.	Under Constant volume.	Difference $S - S'$	$1 + t$.
Air, ...	0·2377	0·1678†	0·0699†	1·417†
Oxygen, . . .	0·2412	0·1705	0·0707	1·415
Hydrogen, . .	0·2356	0·1675	0·0681	1·407
Oxyd of carbon,	0·2399	0 1681	0·0718	1·428
Carbonic acid, .	0·3308	0·2472	0·0836	1·338
Olefiant gas, . .	0 3572	0·2880	0·0692	1·240

Comparing these results in the case of air, we see that when air is heated in a situation where it is free to expand, only about $\frac{5}{7}$ of the heat applied is expended in producing elevation of temperature—as in heating a room—while about $\frac{2}{7}$ of the heat is expended in producing expansion of the air, to be given out again as the room cools.

---

* Cooke's Chemical Physics.

† Corrected according to the most recent experiments.

Dulong has deduced from his experiments the following conclusions:—

1. *Equal volumes of all gases, measured at the same temperature and pressure, set free or absorb the same quantity of heat when they are compressed or expanded the same fractional part of their volume.*

If all gases had the same specific heat, the same change of volume would be attended by the same change of temperature. But this is the case only with oxygen, hydrogen, and nitrogen. The specific heats of compound gases differ considerably from each other, and change of volume causes less change of temperature in proportion as the specific heat of the gas is greater.

2. *The variations of temperature which result, are in the inverse ratio of the specific heats under a constant volume.*

Whether these laws are the exact expressions of the truth, or only approximately correct, remains to be determined by further investigation.

654. **Relation between the specific heat and atomic weight of elements and compounds.**—Dulong and Petit, from their researches upon the elements, were led to conclude, that the ultimate atoms of all elements possessed the same capacity for heat, and they accordingly announced the law, that:—

The specific heat of elementary substances is in inverse ratio to their atomic weights.

This law appears to be true for most of the elements, as will be seen by examining Table XI. of Atomic Weights and Specific Heats. It will be noticed, that the one increases in almost the exact proportion in which the other diminishes, and that by multiplying them together, a very nearly constant product is obtained. Some elements, as those given in the lower part of the table, give a product ($C \times p$) double of the others. So that equivalent weights of these would contain twice as much heat as equivalent weights of those first given.

The relation between the specific heat and atomic weight of compounds is expressed by Regnault in the following law:—

In all compound bodies containing the same number of atoms, and of similar chemical constitution, the specific heats are in inverse ratio to their atomic weights.

§ 7. Liquefaction and Solidification.

655. **Latent heat.**—During the conversion of a solid into a liquid, or of a liquid into a gas or vapor, a certain quantity of heat is absorbed or disappears. As the thermometer and the senses give no evidence of the existence of this heat, it is called latent heat.

Let a pound of ice and a pound of water, each at the temperature of 32°, be exposed to the same source of heat in precisely similar vessels; it will be found, at the moment when all the ice is melted, that the water into which it is converted has still the temperature of 32°; while the temperature of the other pound of water has risen from 32° to 174°. As both have received the same amount of heat, it follows, that the 142° which have disappeared, have been used in converting the ice into water, and have become latent or insensible.

If a pound of water at 212° be mixed with a pound of powdered ice at 32°, when the ice is melted the two pounds will have the temperature of only 52°; the ice gains only 20°, while the water loses 160°. Here again 142° have disappeared or have become latent.

656. **Liquefaction and congelation are always gradual,** owing to the absorption or evolution of heat during these processes.

If this was not so, water at 32° would immediately become ice, upon losing the smallest additional portion of its heat, and on the other hand, ice would suddenly pass from the solid to the liquid state by the smallest addition of heat.

This fact, coupled with the law of irregular expansion of water, will explain why ice never acquires any very great thickness. The high specific heat of water acts to moderate the natural changes of temperatures.

657. **Freezing mixtures.**—Solids cannot pass into the liquid state without absorbing and rendering latent, a certain amount of heat. If the heat necessary for the liquefaction is not supplied from some external source, the body liquefying will absorb its own sensible heat. A knowledge of this fact enables us at pleasure, in the hottest seasons and climates, to produce extreme degrees of cold.

The so-called *freezing mixtures* are compounds of two or more substances, one of which is a solid. These, when mixed together, enter into combination and liquefy. The operation should be so conducted, that no heat can be absorbed from external sources, and hence, as the substances liquefy, a depression of temperature results proportional to the heat rendered latent. (See Table XII.)

The most convenient freezing mixture is salt 1 part, and ice or snow 2 parts, universally used in the freezing of ices and creams. With this freezing mixture, a temperature of 4° or 5° below zero can be maintained for many hours. A solution of equal parts of nitre and sal-ammoniac will reduce the temperature from 50° to 10° F. Very well constructed ice-cream freezers are now commonly sold in the shops, in which an adroit use has been made of the laws of radiant heat and conduction, to facilitate the rapidity of this operation.

Thilorier, with a mixture of solid carbonic acid and sulphuric acid, or sulphuric ether, obtained a temperature 120° below zero. More lately, Mitchell obtained by the same means a temperature of —130° and —146° F. At the former temperature, alcohol (Sp. Gr. 0·798) had the consistency of oil, and at the latter temperature resembled melting wax.

In the liquefaction of metallic alloys, a similar depression is observed. When an alloy composed of 207 parts lead, 118 tin and 284 bismuth, is dissolved in 1617 parts mercury, the temperature will sink from 63° to 14° F.

In producing extreme degrees of cold, the substance to be operated upon is first cooled to a certain degree by a less powerful freezing mixture, before the more energetic one is used; the full effect of the latter is thus obtained.

658. **Laws of fusion and latent heat of fusion.**—Expansion (the first effect of heat) has a limit, at which solids become liquids. The powers of cohesion are then subordinate to those of repulsion, and fusion results.

Fusion takes place in accordance with the following laws:—

1st. *All solids enter into fusion at a certain temperature, invariable for the same substance.*

2d. *Whatever may be the intensity of the source of heat when the fusion commences, the temperature remains constant until the whole mass is fused.*

3d. *The latent heat of fusion is obtained by multiplying the difference between the specific heat of the substance in its liquid and solid form, by the quantity obtained by adding the number* 256 (*an experimental constant furnished by researches upon the latent heat of water*) *to the melting point of the substance in question.*

The fusion points and latent heat of fusion of a number of the more important substances are given in Table XV. of the Appendix, drawn from the labors of Regnault and others.

659. **Peculiarities in the fusion of certain solids.**—Certain solids soften before they become liquefied; such are tallow, wax, and butter, while others never become entirely fluid. This is because the former are composed of several substances, which melt at different temperatures. Metals, like iron and platinum, that are capable of welding, soften before they fuse. Glass, and certain metals, never attain perfect fluidity. The fusion of sulphur presents striking peculiarities. (See *Chemistry.*)

660. **Refractory bodies.**—Substances difficult of fusion are called refractory bodies.

Among the most refractory bodies are silica, the metallic oxyds, lime, baryta, alumina, &c. Their fusion may be effected by the oxy-hydrogen blow-pipe, or by the use of the voltaic battery. By these means, also, the fusion of platinum is effected, which resists the heat of a powerful blast-furnace, although a thin wire of this metal can be melted by the mouth blow-pipe.

Carbon is the most refractory of all bodies. Its fusion has not yet been perfectly effected; although, by means of the voltaic battery, Professor Silliman obtained (in 1822) unequivocal evidences of the volatility and partial fusion of this substance: and more lately these results have been verified by Despretz, with a carbon battery of 600 cups; boron and silicon also yielding to the same power.

661. **Solution.—Saturation.**—When a solid immersed in a liquid gradually disappears, the process is termed solution. Thus, sugar and salt dissolve in water, camphor in alcohol, &c. Solution is the result of an attraction existing between the particles of a liquid and those of

a solid. A liquid is said to be saturated when, at a given temperature, it has dissolved as much as possible of a solid.

The causes which diminish cohesion among the particles of a solid, generally facilitate solution. Thus, a pulverized body dissolves quicker than the same quantity in large masses. Heat also facilitates solution by diminishing the cohesive force and producing currents. The solubility of some bodies is diminished by heat, and the precipitation of bodies from solution is sometimes hastened by heat,—sulphate of soda and hydrate of lime are examples of the former.

662. **Laws of solidification.**—The passage of a body from the liquid to the solid state, always occurs in accordance with the following laws:—

1st. *The solidification of a body takes place at a certain fixed temperature, which is also that of its fusion.*

2d. *The temperature of a body remains constant from the commencement to the end of its solidification.*

663. **Elevation of temperature during solidification.**—When liquids return to the solid state, the heat which has been absorbed during their liquefaction, and rendered latent, is given out.

If the solidification takes place suddenly, the heat evolved is often very apparent. Thus water may be cooled to 22° or 23°, and yet remain liquid, but if in that state it is shaken, it becomes at once a confused mass of ice crystals, and rises to 32°, the freezing of a part giving out heat enough to raise the temperature of the whole 8° or 10°. Thus we arrive at the seeming paradox, that freezing is a warming process; and, owing to the absorption of heat during liquefaction, it is equally true, that melting is a cooling process. Hence, in part, the cooling influence of an iceberg, or of a large body of snow on a distant mountain.

664. **Change of volume during solidification, and its effects.**—Mercury, and most metals, contract while solidifying; hence, the freezing of a mercurial thermometer does not burst its reservoir. Water expands during freezing to the amount of one-eleventh of its bulk; hence, ice floats on the surface of water, and close vessels, even of iron, are burst, if frozen when full of water.

This fact is familiar to housekeepers, who prevent the bursting of their water-casks during winter, by a stick of wood placed in the cask, about which the bulge from expansion takes place. Aqueduct service-pipes are often saved from the same accident in cold weather, by allowing the water to flow uninterruptedly, thus preventing the formation of ice crystals, both by motion and the supply of warmer water.

A brass globe filled with water burst at 32°, in the experiments of the Florentine Academicians, who estimated the force exerted as equal to 28,000 pounds on the square inch. A bomb-shell, filled with water, and tightly closed by an iron plug, when exposed to severe cold in Montreal, discharged the plug to a distance of 400 feet, and a cylinder of ice eight inches in length protruded from the hole. All metals which, like water, assume the rhombohedral form on solidification, produce sharp casts. Such are cast-iron, antimony, tin, zinc, and bismuth. All alloys capable of producing sharp casts, must contain such a metal. Type-metal (3 lead and 1 antimony), brass (2 copper and 1 zinc), and bell-metal (7 copper and 3 tin), are familiar examples. Copper, lead, gold, silver, and indeed most metals, except those above enumerated, crystallize in the monometric system, and occupy less space as solids than as fluids, producing imperfect casts. Hence, coins are stamped, and gold, silver, and copper utensils, and ornamental wares are wrought by the hammer, or stamped, to secure sharpness and beauty.

665. **Freezing of water.**—Water ordinarily freezes at 32°; but it has already been stated (663) that, under certain circumstances, it may be cooled near to 22°, and remain liquid. If, however, water is turbid, or contains carbonic acid, it always freezes at 32°.

Certain experiments made in France indicate that the temperature to which water may be exposed without freezing, falls in proportion as it is exposed in tubes of smaller diameter. This remarkable circumstance seems to throw light upon the fact, that plants, whose capillaries are full of juices, resist frost in a manner so noticeable as many of them do. Nevertheless, in very severe weather, the trunks of large trees are sometimes burst open by frost.

Water, containing salts in solution, freezes at a lower temperature than pure water. Thus, sea water freezes at 27°. The ice formed from salt water, and from impure or turbid water, is comparatively fresh and pure, since it is the water which freezes, and not the foreign bodies it contains. Frozen ink, and other colored fluids, precipitate the coloring matter, and are spoiled as colors, until, by boiling, the precipitate is again diffused. Likewise, the watery portion of cider, and other weak alcoholic liquors, exposed to moderate cold, congeals; and the alcoholic part may thus be obtained in a more condensed state.

Some absorbent rocks are pulverized, and gradually covered by a thick bed of soil, by the effects of freezing water in breaking down their solid mass. The value of building-stones, in our climate, depends much upon the resistance they offer to the action of frost. In hot climates the effect is not seen, and the crags and summits of mountains are there generally more sharp. Experiments to determine the resistance of rocks to frost, are made by saturating cubes of the material with water, and repeatedly freezing them. But the same result is more conveniently obtained by using a solution of sulphate of soda. This salt, crystallizing on exposure to the air, effects the same results.

666. **Absolute zero.**—Since the permanent gases contract $\frac{1}{491}$ of their volume at 32°, for each degree of Fahrenheit below that point (or expand that quantity for each like increment of heat above 32°), it has been inferred by Clement and Désormes that, at the temperature of —459° F. they would cease to exist as gases, since the amount of contraction would then be equal to their initial volume. Likewise, since the volume of a gas is doubled by heating from 32° to 523°, they further inferred that the quantity of heat added must be equal to that held by the initial volume, and that at —459° F. there must be an absolute zero.

§ 8. Vaporization and Condensation.

667. **Vaporization.**—Liquids become vapors upon receiving a certain quantity of heat. Thus, water at 212° is rapidly converted into steam, which, at or above that temperature, remains as an invisible

vapor. This change of state presents some of the most interesting and important phenomena of physics.

Evaporation occurs only at the surface of liquids, quietly, as in the insensible changes of water to vapor in an open vessel. *Boiling* or *ebullition*, is the rapid formation of vapor throughout the whole mass of liquid, producing more or less agitation. *Sublimation* is the change of solids to vapors without the intermediate liquid condition. Arsenic, iodine, and camphor are examples of solids which may be so changed.

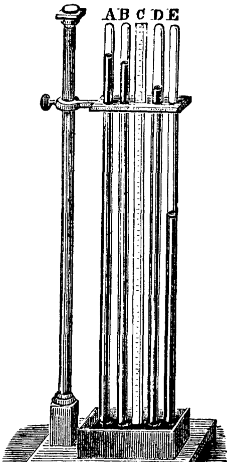
The remarkable disappearance of nearly one thousand degrees of heat when water is turned into steam (and correspondingly for other liquids), will be considered under Latent Heat of Steam, § 683.

668. **Formation of vapors in a vacuum.**—Evaporation takes place slowly in the open air, owing chiefly to the atmospheric pressure. In a vacuum, however, it occurs instantaneously, because the vapor then meets with no resistance. This phenomenon occurs in obedience to the following laws:—

1st. *All volatile liquids, in a vacuum, volatilize instantly.*

2d. *At the same temperature the vapors of different liquids possess unequal elastic force.*

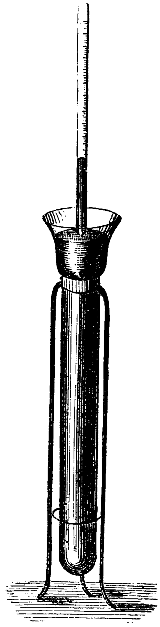
482



These laws are illustrated in the apparatus, fig. 482, where four barometer tubes, originally filled with pure dry mercury, are supported by the stand in a mercurial cistern, and will all indicate upon the scale, C, the same height of column. A drop of ether passed up to E, instantly flashes into vapor, and depresses the column perhaps half its height or more. This illustrates the first law. A drop of bisulphid of carbon introduced into D; of alcohol into B; and of water into A, will also be respectively changed to vapor, wholly, or in part, and will depress the mercury unequally, in the order of their volatility as enumerated. This illustrates the second law. If all the ether introduced into E has disappeared, then successive small portions may be added, and with each addition an increased depression of the mercury will be observed, until, finally, a point is reached where the ether remains liquid. This is the point of *saturation*, or *maximum tension* of ether vapor for that temperature. A change of temperature will, of course, vary these conditions. If either of the tubes is surrounded by one of larger diameter dipping under the mercury, and so affording a cell into which hot water may be poured, the liquid ether in E, for example, will be vaporized, still further depressing the mercury, according to the temperature. If a freezing mixture were similarly used, the reverse would be seen—a portion of vapor would be liquefied, and the mercury will rise in proportion.

669. **Saturated space or maximum tension of vapors.**—The meaning of these terms may be still further illustrated by the use of the apparatus in fig. 483, which is provided with a well, filled with mercury, and deep enough to allow the tube to be depressed nearly its whole length.

483



Suppose the tube to have the condition of E in the last paragraph; that is, the vapor of ether has nearly filled the whole tube, and is at its point of saturation or maximum tension. If the tube is now depressed, the contained vapor is subject to increased tension, in proportion to the amount of depression; and the result is, that a portion of it becomes liquid, and the mercury takes the place of the vapor. If the tube is raised, then the pressure is again diminished, and a fresh portion of ether is vaporized. There is, therefore, a maximum tension or elasticity for the vapor of different liquids at every temperature; so that, in a saturated space, at a given temperature, the maximum tension is the same, whatever may be the pressure to which the vapor is subjected.

670. **Dalton's law of the tension of vapors** is as follows:—

The tension or elasticity of different vapors is equal, if compared at temperatures the same number of degrees above or below the boiling point of their respective liquids.

This law does not perfectly accord with the results of experiment, but it is nearly correct (except for mercury), at short distances above and below the boiling point. See Table XIV.

671. **The tension of vapors in communicating vessels unequally heated** is the same, and is equal to that of the lower temperature.

Thus, if a vessel containing water at 32°, communicates by a tube with a vessel in which the water is boiling, the pressure in both of the vessels will be the same, as may be ascertained by a manometer. This is explained by the condensation which the vapor constantly suffers in the colder vessel. Application is made of this principle in the condenser of the steam-engine.

672. **Temperature and limits of vaporization.**—The evaporation of liquids takes place at temperatures much below their boiling points, as common experience testifies. Even at the ordinary temperature of the air, water, many liquids, and even some solids vaporize.

Even mercury, whose boiling point is 662°, evaporates at all temperatures above 60° F., as was proved by Faraday. He suspended from the cork of a flask containing mercury, a slip of gold leaf. After six months, the gold leaf was found to be whitened by the mercury which had risen in vapor. A dew of metallic globules is sometimes seen in the Torricellian vacuum. Iodine, camphor, and other solids, rapidly evaporate at the ordinary temperature. Snow and ice disappear from the surface of the earth during cold weather when there has been no thawing. Boyle found that two ounces of snow, in a very cold atmosphere, lost ten grains in six hours.

The experiments of Faraday, however, appear to show that vaporization does not occur at all temperatures.

Thus, mercury gives off no appreciable vapor below 60°. Sulphuric acid undergoes no appreciable evaporation at ordinary temperatures. Faraday proved that several substances which are volatilized by heat at temperatures between 300° and 400°, did not suffer the slightest evaporation when kept in a confined space at the ordinary temperatures during four years.

The limit of evaporation is reached when the cohesive force of the particles of the solid or liquid overcomes the feeble tendency to evaporation.

673. **Circumstances influencing evaporation.**—Evaporation, as has been said, is the slow production of vapor from the surface of a liquid. The elastic force of a vapor which saturates a space containing a gas (like air) is the same as in a vacuum. The principal causes which influence the amount and rapidity of evaporation, are as follows:—

1st. *Extent of surface.* As the evaporation takes place from the surface, an increase of surface evidently facilitates evaporation.

2d. *Temperature,* by increasing the elastic force of vapor, increases the rapidity of evaporation; therefore, the temperature of ebullition marks the maximum point of evaporation.

3d. *The quantity of the same liquid already in the atmosphere,* exercises an important influence on evaporation. When the air is saturated, evaporation ceases; it is, therefore, greatest when the air is free from vapor.

4th. *Renewal of the air* facilitates evaporation, since new portions of air, capable of absorbing moisture, are presented to it; hence evaporation is more rapid in a breeze than in still air.

5th. *Pressure on the surface of the liquid* influences evaporation, because of the resistance thus offered to the escape of the vapor.

Prof. Daniell, from a series of researches on the rate of evaporation, deduced the following law, viz.:—

The rapidity of evaporation is inversely as the pressure upon the surface of the evaporating liquid.

674. **Dew-point.**—If air saturated with moisture is cooled, a portion of the moisture will be precipitated as dew. The temperature at which this deposition of moisture commences, is called the *dew-point.* The dew-point is nearer the temperature of the atmosphere, the more fully the air is saturated with moisture. The methods of determining the amount of moisture contained in the atmosphere, will be described in the chapter on Meteorology.

675. **Ebullition.**—The elasticity of the vapor from a boiling liquid is equal to the pressure of the superincumbent atmosphere.

When water is boiled in a glass vessel, the phenomena of ebullition may be distinctly seen. On first heating a liquid, the dissolved air is expelled in small bubbles. As the heat is continued, bubbles of transparent and invisible steam are formed in the lower part of the vessel where the heat is applied. These grow smaller and smaller as they rise, and finally condense in the colder liquid with a series of little noises, producing what we call simmering. After a time, when the mass of liquid attains a nearly uniform temperature, these bubbles increase in size as they rise to the surface, owing to the evaporation from their interior surfaces, as well as from the less pressure to which they are there subjected. As they reach the external air, at the surface of the liquid, they condense in a cloudy vapor, which is commonly called steam, but which, in reality, is water in exceedingly minute globules, steam itself being invisible.

When a liquid has reached the boiling point, a comparatively small quantity of heat maintains it at that temperature. Water, or any other liquid, boiling moderately, has the same temperature as when it is in violent ebullition; the excess of heat only causing a more rapid evaporation of the water. The boiling points of certain liquids are shown in Table XVII.

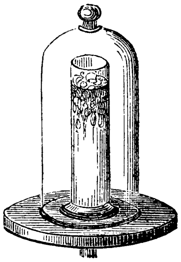
676. **Circumstances influencing the boiling point.**—The principal of these are:—

1. *Adhesion.* It is probably owing to the different degrees of adhesion between the liquid and the surfaces of the vessels, that the boiling point of water varies in vessels of different materials.

2. *Solids in solution* in liquids raise their boiling points in proportion to the quantity dissolved. Thus, a saturated solution of common salt boils at 227° F.; of nitre at 240°; of carbonate of potash at 275°; and of carbonate of soda at 220°. This is probably owing to the adhesion existing between solids and liquids, which opposes itself to the repulsive force of heat. The vapor rising from boiling solutions, Rudburg says, has only the temperature of steam from pure water boiling in free air. According to Regnault, the temperature of the vapor of a boiling saline solution, appears very nearly equal to that of the liquid. It is extremely difficult to obtain accurate results, because the bulb of the thermometer becomes covered with a film of condensed water.

3. *Pressure.* As ebullition consists in the rapid formation of vapor of the same elasticity as the superincumbent atmosphere, it is evident, that if the pressure is diminished, the boiling point will be lowered; and if it be increased, that the boiling point will be raised.

484



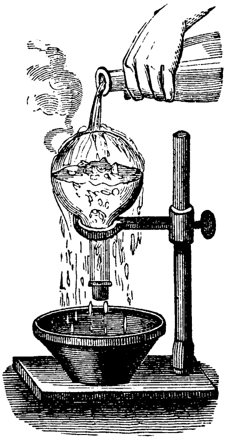
The influence of pressure on the temperature of ebullition, is strikingly shown by placing a vessel of water, which has cooled considerably below the boiling point, beneath the receiver of an air-pump and exhausting the air, fig. 484. As the air is removed, the water enters into violent ebullition, even at a temperature of 125°. Liquids generally boil in vacuo at a temperature of from 70° to 140° below their point of ebullition under the ordinary atmospheric pressure; Black says 140° for all liquids.

Table XVI., from Regnault, shows the temperature at which water boils under different pressures, represented by the corresponding heights of the barometric column. These results have been confirmed by direct observation.

In Table XVIII. is given the boiling point of water at different places, with their corresponding elevation above the level of the sea.

677. **The culinary paradox** is an excellent illustration of the phenomena of boiling under diminished pressures.

485

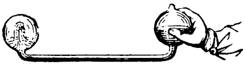


A small quantity of water is boiled in a glass flask until the steam has driven out the air. When the water is in active ebullition, a good cork is firmly inserted in the mouth, and the heat is removed. The flask is then supported in an inverted position, as is shown in fig. 485. The water still continues to boil more violently than when over the flame. If cold water be poured upon the flask, the ebullition becomes still more violent, but will be speedily arrested by the application of hot water.

The cause of this seeming paradox is plain. When the flask was corked, there was only the vapor of water above the liquid, the air being driven out by the previous boiling. By the application of cold water, a portion of this vapor becomes condensed, and the water within being under diminished pressure, boils at a correspondingly low temperature. But hot water thrown upon the flask increases the elasticity of the vapor, and the water being thus subjected to a greater pressure, ceases to boil.

Franklin's pulse glass, a double bulbed glass, fig. 486, partly filled with ether and closed while boiling; boils from the heat of the hand, a sensible coolness being felt as the last portions of fluid rush out of the empty bulb, the hand furnishing the heat needed to vaporize the ether.

486

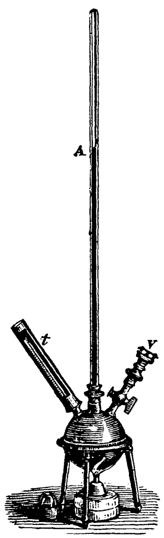


678. **Useful applications in the arts** are constantly made of the facts just explained, to concentrate vegetable extracts, cane-juice, &c., under diminished pressure, and consequently at a temperature below the point where there is any danger of injury from heat. Sugar is usually concentrated thus in large close copper vessels, called vacuum-pans, at a temperature of 150° F., aided by a powerful air-pump and condenser to remove the vapor rapidly. There is no economy of fuel by boiling under diminished pressure, as will be understood from what is said hereafter.

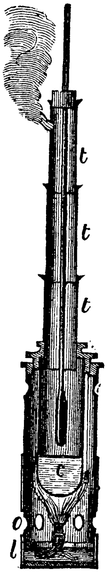
679. **Measurement of heights by the boiling point.—Hypsometer.**—On ascending mountains, the boiling point of liquids falls, because the atmospheric pressure is less, and conversely in descending into mines, it rises. Accurate observations show, that a difference of about 543 feet in elevation produces a variation of 1° F. in the boiling point of water. The metastatic thermometer (579) is used in these observations. Fahrenheit first proposed determining the heights of mountains by the depressed temperature of boiling water.

Regnault has designed an apparatus called a hypsometer, fig. 487, for determining elevations by the boiling point of water. It consists of a copper vessel, C, containing water. This is surmounted by a brass cylinder which supports and encloses a thermometer. The upper part of this cylinder is formed in pieces, *t*, which slide into each other like the tubes of a telescope, and serve to confine the steam about the thermometer tube, as in fig. 443. Air is supplied to the lamp, *l*, by the holes, *o*, *o*. The steam escapes by a lateral orifice in the upper part of the instrument.

488



487



680. **High pressure steam.**—The boiling point rises as the pressure increases. This fact is readily demonstrated in a general way by Marcet's apparatus, fig. 488.

A spherical boiler is supported over a lamp upon a tripod of brass. A thermometer, *t*, enters the upper hemisphere, and its bulb is exposed directly to the steam. A stop-cock and safety valve, V, opens a communication to the outer air. A manometer tube, A, with confined air (280) descends into some mercury placed in the boiler (whose lower hemisphere is for that reason made of iron). The boiler is filled with water to the equator. When the water boils and the air has been expelled, the open stop-cock is closed and the steam commences to accumulate. The thermometer, which stood previously at 212°, begins to rise higher and higher as the column of mercury rises in the gauge. When the mercury has risen in the gauge a little less than half the height of the tube, the thermometer will indicate 249°·5 F., when two-thirds of the way 273°·3, and so on. Table XIX. gives the boiling point of water at different atmospheric pressures as ascertained by Regnault.

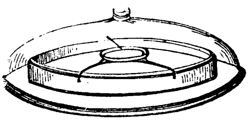
Advantage is taken of the temperature of high steam in the arts to extract gelatine from bones, and to perform other difficult solutions and distillations which, at 212°, would be impossible. Papin, a French physicist, who died in 1710, first studied these effects of high steam with an apparatus known as Papin's digester. It is only a boiler, of great strength, provided with a safety valve (then first used).

681. **Production of cold by evaporation.**—A liquid grows sensibly colder, if while evaporating it does not receive as much heat as it loses, and the more sensibly so, as the evaporation is more rapid.

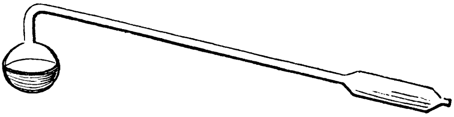
Eau de cologne, bay-rum, or ether, evaporating from the surface of the skin, produces very sensible coldness, due to the rapid absorption of the bodily heat in the evaporation. Portions of body may be thus benumbed and rendered insensible to pain during surgical operations.

A summer shower cools the air by absorbing heat from the earth and the air during evaporation. Curtains wet with water, called *tatties*, much used in India; leafy branches of trees, mossy banks, and fountains draped by climbing plants, are cool for the same reason. Fanning the surface produces coolness both from conduction and evaporation. Wet clothes are pernicious, chiefly from the rapid loss they cause of animal heat during evaporation, thus impeding the circulation. In hot climates, where ice is rare, water is cooled to an agreeable temperature by the use of jars of porous earthenware placed in a draught of air. The surface moisture is rapidly evaporated by the dry air, and the water in the vessels falls 20 or 30 degrees below the exterior air, even at 80 or 90 degrees. Water is readily frozen in a thin narrow test-tube by the constant evaporation of ether from a muslin cover drawn over the outside of the tube. In the East Indies, water is frozen by its own evaporation, aided by radiation, in cool serene nights, when the external air is not below 40°. For this purpose shallow earthen pans are used, placed in a slight pit or depression of the earth upon straw to cut off terrestrial radiation.

489



490



Water is endowed with a remarkable emissive power, and will, as shown by Melloni, lose 7° below the atmosphere by simple radiation in serene nights. Compared to this remarkable Indian result, Leslie's experiment of freezing water in the vacuum of an air-pump (over sulphuric acid to absorb the vapor, fig. 489) seems simple; and easier still is the same effect produced in the cryophorus (or *frost-bearer*) of Dr. Wollaston, fig. 490, where a portion of water in one bulb of a vacuous glass tube is frozen by its own rapid evaporation due to cooling the empty bulb in a freezing mixture.

Twining's ice machine.—An apparatus has been successfully contrived by Prof. Alex. Twining for producing ice upon a commercial scale in those hot climates where it cannot be carried from colder countries, by the rapid evaporation of a portion of ether confined in metallic chambers contiguous to the water vessels—the process, by aid of an air-pump and condenser, being continuous and without sensible loss of ether. This plan is equally applicable to cooling the air of apartments, either for the preservation of provisions or for the comfort of the occupants.

682. **Latent heat of steam.**—A large amount of heat disappears or is rendered latent during evaporation. According to Regnault, the latent heat of steam is 967°·5. Its determination is made in a number of ways.

If a vessel containing water at the temperature of 32° is placed over a steady source of heat, it receives equal additions of heat in equal times. Let the time be noted that is required to raise the temperature to 212°. If now the heat is continued until all the water is converted into steam, it will be found that the time occupied in the evaporation was 5½ times that required to heat the water through the first 180°, *i. e.*, from 32° to 212°. Consequently 5½ times as much heat is absorbed during the evaporation of water as is required to bring it to boiling point. The latent heat of steam is therefore about (180° × 5½) 990°.

Again, the latent heat of steam is determined by distilling a certain amount of water and condensing the steam in a large volume of the same liquid. If the temperature be noted before and after the experiment, it will be found that the heat from the steam formed from a pound of water, was sufficient to raise the temperature of ten pounds of water 99°. The latent heat of steam is therefore again found to be (99° × 10) 990°.

Experiments conducted in the simple manner just mentioned cannot be entirely accurate, owing to a certain loss of heat by vaporization, conduction, and radiation. Numerous precautions are therefore to be adopted to insure the accuracy science demands in such an investigation, the details of which are inconsistent with our limited space.

The latent heat of steam obtained by different experimenters, varies somewhat as follows:—Watt, 950°; Lavoisier, 1000°; Despretz, 955°·8; Brix, 972°; Regnault, 967°·5; Fabre and Silbermann, 964°·8.

683. **Latent and sensible heat of steam at different temperatures.**—The whole amount of heat in steam is the *latent heat,* plus the *sensible heat.* Thus the heat of steam at the temperature of ebullition is 967°·5 + 212° = 1179°·5. It has heretofore been generally stated, that the heat absorbed in vaporization is less as the temperature of the vaporizing liquids is higher. So that if the sensible heat of steam at any temperature is subtracted from the constant 1179°·5, the remainder is the latent heat of steam at that temperature. For example: the latent heat of steam at 279°·5, is 900°, at 100°, 1079°·5, &c. This statement however is found to be somewhat inaccurate, although in practice it may be assumed to be nearly correct.

From the experiments of Regnault, it appears that the sum of the latent and sensible heat increases with the temperature by a constant difference of 0°·305 for each degree F., as is shown in Table XXII.

684. **Mechanical force developed during evaporation.**—During the conversion of a liquid into vapor, a certain mechanical force is exerted. The amount of this force depends on the pressure of the vapor and the increase in volume which the liquid undergoes.

Equal volumes of different liquids produce unequal amounts of vapor at their respective boiling points.

1 cubic inch of water expands into 1696 cubic in. vapor at boiling point.
1 " " alcohol " " 528 " " " " "
1 " " ether " " 298 " " " " "
1 " " turpentine " 193 " " " " "

Now although the latent heat of equal weights of other vapors is less than that of steam, yet no advantage would arise in generating vapor from them in place of water in the steam-engine. For equal volumes of alcoholic and aqueous vapor contain nearly the same amount of latent heat at their respective boiling points, and such is the case to a great extent with other liquids. The cost of the fuel in generating vapor would be in proportion to the amount of latent heat in equal volumes of the vapor.

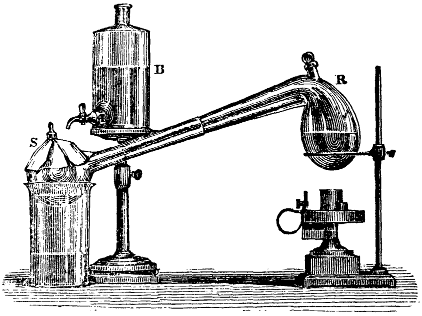
685. **Liquefaction of vapors**, or the conversion of vapors into liquids, is accomplished in three ways. 1st, by cooling; 2d, by compression; and 3d, by chemical affinity. Only the first two of these methods will be spoken of. When vapors or gases are condensed into liquids, the same amount of heat is given out as sensible heat which was absorbed and rendered latent when they assumed the aeriform condition.

686. **Distillation** is the successive evaporation and condensation of liquids. The process depends on the rapid formation of vapor during ebullition, and the condensation of the vapor by cooling.

Distillation is used, first, for the separation of fluids from solids, as the distillation of ordinary water, to separate the impurities contained in it; 2d, for the separation of liquids unequally volatile, as in the distillation of fermented liquors, to separate the volatile spirits from the watery matter.

687. **Distilling apparatus** of various kinds is employed according to the special purpose to which it is applied. The most ancient is the *alembic;* its invention is attributed to the Arabs. It consists of a boiler of copper or iron, furnished with a dome-shaped head; to the upper part of this is attached a metal tube which passes through a vessel of cold water, whereby the vapor (as it passes over when heat is applied to the boiler) is condensed, and flows into a proper receptacle.

491



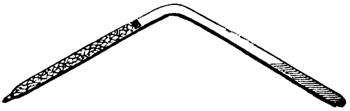
Where small quantities of liquid are to be distilled, glass retorts, fig. 491, or flasks are used. These are heated by alcohol lamps, or by small charcoal furnaces. The receiver may consist of a small flask connected with the neck of the retort, as represented by S. By means of water flowing continually on it from B, a proper cooling is effected.

688. **Physical identity of gases and vapors.**—The difference between gases and vapors is merely one of degree, and their identity in many physical properties has already been shown. Thus the ratio of their expansion by heat is the same as that of the permanent gases. A permanent gas may be considered as a *super-heated vapor;* the vapor of a liquid which volatilizes at very low temperatures.

Theory of the liquefaction and solidification of gases.—By the last section, if the excess of heat is removed from a gas, it is in the same condition as an ordinary vapor, containing only sufficient heat to maintain it in the aeriform condition. By the compression of a gas, heat is evolved, by rendering sensible the heat before latent. If the compressed gas is then surrounded by a freezing mixture, the further abstraction of heat causes the condensation of a corresponding portion of gas into a liquid. It is thus by condensing and cooling gases, that their liquefaction and solidification have been effected.

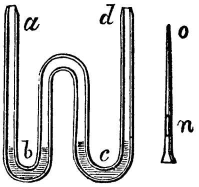
689. **Methods of reducing gases to liquids.**—In 1823, Faraday liquefied chlorine, cyanogen, ammonia, carbonic acid, and some other gases, by the following simple means.

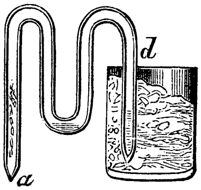
492



The materials from which the gas was to be evolved, provided they were solids, were placed in a strong glass tube, bent at an obtuse angle near the middle, fig. 492, and the open ends hermetically sealed. Heat was then applied to the end containing the materials (*e. g.* cyanid of mercury), while the empty end was cooled in a freezing mixture. The pressure of the gas evolved in so small a space, united with the cold, liquefied a portion of it. Otherwise, if fluids were to be employed, the tube had the shape seen in fig. 493. The fluids were introduced by the small funnel *o n,* into the curves *c* and *b*, and the ends, *a, d,* were then sealed by the blow-pipe. By a simple turn of the tube, all the fluid contents are transferred to the end, *a*, fig. 494, and the empty end, *d*, is placed in a freezing mixture where the liquid gas collects. Any fluid which distils over from *a*, collects in the bottom of the middle curve. A minute manometer was introduced by Faraday into these tubes, in order to determine the pressure at which liquefaction occurred. The manometer was a small glass tube sealed at one end, and holding a drop of mercury; the mode of reading the pressure has been before explained (280).

493





494

Later researches of Faraday.—In 1845, Faraday published the results of his experiments on the liquefaction of gases by means of solid carbonic acid. A mixture of this solid with ether, in the vacuum of an air-pump, gave him a temperature as low as —166° F.

In such a bath, at the ordinary pressure of the atmosphere, chlorine, oxyd of chlorine, cyanogen, ammonia, sulphuretted hydrogen, arseniuretted hydrogen, hydriodic acid, hydrobromic acid and carbonic acid, were obtained in the liquid form under moderate pressures. These liquids were colorless, with the exception of those from chlorine and oxyd of chlorine, which are colored gases in the ordinary state. A number of the liquefied gases were solidified. The results obtained by Faraday on the liquefaction and solidification of gases may be found in Table XX.

690. **Thilorier's and Bianchi's apparatus for condensation of**

gases.—To avoid the danger of explosion in the use of glass tubes and at the same time to obtain large supplies of liquid gases in a manageable form, a powerful apparatus of iron has been contrived by Thilorier; and, more lately, another by Bianchi with mechanical compression, for a description of which reference may be had to the Author's *Chemistry.*

691. **Properties of liquid and solid gases.**—Liquid carbonic acid is colorless, like water, and has a density of 0·83. Its coefficient of expansion is more than four times that of air. Twenty volumes of the liquid at 32°, becoming 29 volumes at 86°.

The solidified acid obtained by the evaporation of a portion of the liquid, appears in the form of snow; when congealed by intense cold alone, it is clear and transparent like ice. It melts at a temperature of —70° F., and is heavier than the liquid bathing it. The solid acid may be preserved for many hours if it be surrounded with cotton or some other poor conductor of heat. It gradually vaporizes without assuming the liquid form. The temperature of this solid, as determined by Faraday's experiments, is about 106° below 0° F. Although so intensely cold, it may be handled with impunity, and when thrown into water, the latter is not frozen. By moistening it with ether, to which it has a strong adhesion, its low temperature is at once manifested. If mercury is placed in a wooden basin and covered with ether, and then solid carbonic acid be added, the mercury will soon be frozen. The temperature required to freeze the mercury is about —40° F. This frozen mercury may be drawn into bars, or moulded into bullets, or beaten into thin plates, if the operations be performed with wooden instruments.

NATTERER, with a mixture of liquid protoxyd of nitrogen and bisulphid of carbon, records a temperature of —220° F. Even at this low temperature, liquid chlorine and bisulphid of carbon preserve their fluidity.

In protoxyd of nitrogen gas, combustibles burn with nearly as great intensity as in pure oxygen; combustion also takes place in liquid protoxyd of nitrogen, notwithstanding the intense cold. A fragment of burning charcoal, thrown into this liquid, burns with brilliant scintillations, and thus almost at the same point there is a temperature of about 3600° above and 180° below Fahrenheit's zero.

692. **Latour's law.**—From his experiments on the conversion of liquids into vapors, Caignard de Latour announced the following law:—

There is for every vaporizable liquid a certain temperature and pressure at which it may be converted into the aeriform state, in the same space occupied by the liquid.

In these experiments, strong glass tubes, furnished with interior manometer gauges, were partially filled with water, alcohol, ether, and other liquids, and hermetically sealed. The temperature of the tubes was then gradually raised. Ether becomes a vapor at 328°, in a space equal to double its original bulk, exerting a pressure of 37·5 atmospheres; alcohol at a temperature of 404°·5, with a pressure of 119 atmospheres, and water disappeared in vapor, in a space four times its own bulk, at the temperature of about 773^{c}. If Mariotte's law held good in these cases, the pressures exerted would have been very much greater than were actually observed. Even before a liquid wholly disappears, the elasticity of the vapor is found to increase in a proportion far greater than is the case with air at equally elevated temperatures. It is not therefore surprising that mere pressure fails to liquefy many bodies which exist ordinarily as gases. Compare the statements respecting Mariotte's law in §§ 274–277.

693. **Density of vapors.**—The accurate determination of the density of vapors, is of much importance in Chemical Physics. It is accomplished by filling a globe, or other vessel of glass, with the vapor at a given temperature, and weighing it; this weight, divided by the weight of an equal volume of air, under the same circumstances of temperature and pressure, gives the density of the vapor. The details of the methods in use for this purpose, belong more appropriately to chemistry.

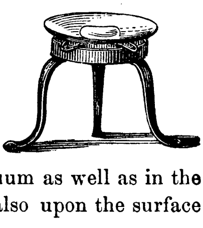
§ 9. Spheroidal condition of Liquids.

694. **Spheroidal state.**—Drops of water scattered on a polished surface of heated metal do not immediately disappear, but assume the form of flattened spheres, rolling quietly about, until they gradually evaporate. If the metal has not a certain temperature, it is wetted by the water with a hissing sound. This observation was made in 1746, and ten years after, Liedenfrost called particular attention to the phenomenon. Döbereiner, Laurent and others, also experimented upon this subject. They found that saline solutions, as well as simple liquids, would act in the same manner as water. It is, however, to Boutigny that we are particularly indebted for the investigation of the phenomena of the spheroidal state of liquids.

Illustration of the spheroidal state.—The above experiment may be variously performed, according to the ingenuity of the experimenter.

A small smooth brass or iron capsule is heated over a lamp, fig. 495, and a few drops of water allowed to fall upon it from a pipette; the drops do not wet the metallic surface, but roll about in spheroidal globules, uniting together after a time into a single mass, which, it will be seen, has the form of an oblate spheroid, and evaporates but slowly. This is the condition distinguished by Boutigny as the spheroidal state. If the metal is allowed to cool gradually, when the temperature falls to a certain point, the liquid will burst into violent ebullition and quickly evaporate.

495



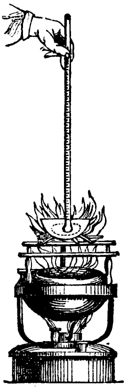
The spheroidal state may be produced in a vacuum as well as in the air, upon the smooth surface of most solids, and also upon the surface of liquids.

Noticeable phenomena connected with the spheroidal state.—There are several important points to be noticed as regards this curious subject. The chief of these are, that,—

1. *The temperature of the plate* must be greater than the boiling point of the liquids, in order to produce the spheroidal state, and it varies with the boiling point of the liquid employed.

Thus, with water, the spheroidal state is produced when the plate is at a temperature of 340°, and may attain it even at 288°; with alcohol and ether, the plate must have at least the temperature of 273° and 142° respectively.

496



2. *The temperature of the spheroids* is always lower than the boiling points of the liquids. This was determined by Boutigny, by immersing a delicate thermometer in the spheroid, as shown in fig. 496.

Thus, 205°·7 is the temperature of the spheroid of water; 168°·5 that of alcohol; 93°·6 that of ether; 13°·1 that of sulphurous acid.

The temperature of a spheroid is not quite as definite as the temperature of ebullition of the liquid, but rises somewhat as the plate upon which it rests is more intensely heated.

3. *The temperature of the vapor from a spheroid* is nearly the same as that of the plate upon which it rests, which proves that the vapor is not disengaged from the mass of the liquid.

4. *The rapidity of evaporation from a spheroid, increases with the temperature of the plate upon which it rests,* as is proved by the following experiments of Boutigny. The same quantity of water (0·10 gramme, or 1·534 grs.) was evaporated in each case. With the plate at the temperature of 392°, the water evaporated in 207 seconds. With the plate at the temperature of 752°, the water evaporated in 91 seconds. With the plate at dull red heat, the water evaporated in 73 seconds. With the plate at bright red heat, the water evaporated in 50 seconds.

Water, in the spheroidal state, evaporates much more slowly than at the temperature of ordinary ebullition. Thus, when the plate was at the temperature of 212°, 0·10 grms. of water evaporated in 4 seconds; and when at the temperature of 392°, in 207 seconds, or about one-fiftieth part as rapidly.

695. **Spheroidal state produced upon the surface of liquids.**—A highly heated liquid may cause the spheroidal state in another liquid of lower boiling point than itself.

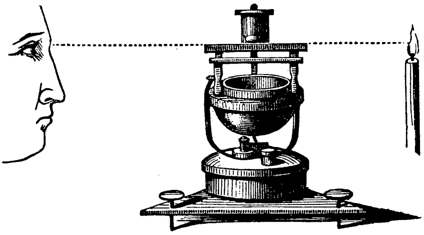
Thus, Pelouze found that water assumed the spheroidal state on very hot oil of turpentine, although the water is the denser liquid. Boutigny has thus sustained water, alcohol, and ether on sulphuric acid, nearly at its boiling point. With sufficient precautions, a number of liquids may be thus piled one upon the other.

696. **A liquid in a spheroidal state is not in contact with the heated surface beneath.**—This must appear, on reflection upon the facts already stated, and may be demonstrated as follows:—

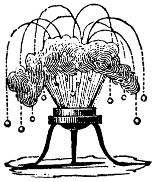
A horizontal silver plate is surmounted by a tube of the same metal, fig. 497, whose lower edges have two longitudinal slits opposite to each other. The plate is placed upon the eolipile (704) containing alcohol, which is nicely adjusted to a perfect level by the screws in the triangular base. Silver is employed to avoid the formation of scales of oxyd of copper, which would interfere with the observation by interposing themselves to the light.

When the plate heated over the lamp reaches the proper temperature, a portion of water is placed upon its centre, and immediately assumes the spheroidal condition. Placing the eye on a level with the surface of the plate, and looking through the apertures in the sides of the tube, the flame of a candle opposite

497



498



may be distinctly seen. This could not happen if the liquid was in contact with the plate. If a thick and heavy silver capsule is heated to full whiteness over the eolipile, it may, by an adroit movement, be filled entirely with water, and set upon a stand, some seconds before the heat declines to the point when contact can occur between the liquid and the metal. When this happens, the water, before quiet, bursts into steam, with almost explosive violence, and is projected in all directions, as shown in fig. 498.

697. **A repulsive action is exerted between the spheroid and the heated surface.**—This proposition follows, indeed, as a consequence of the last. It has already been demonstrated, that a liquid does not wet a surface, when the cohesion which exists between its particles is double of their adhesion for the solid (234). This adhesion is not only diminished by heat, but a repulsive action is exerted between the hot body and the liquid, which becomes more intense as the temperature is higher. This repulsive action is strikingly demonstrated by the following experiment of Boutigny:—

A few drops of water were let fall into a basket, formed of a net-work of platinum wires, heated red-hot. The water did not pass through the meshes, even when the basket was rapidly rotated. But when the metal was sufficiently cooled, the water immediately ran through in a shower of small drops, or was quickly dissipated in vapor. It would also seem, that vapors, like liquids, are repelled from the heated surface, for Boutigny found that a hot silver dish was not attacked by nitric acid, or one of copper by sulphuric acid or ammonia. The latter substance had no action upon either iron or zinc at a high temperature. The suspension of chemical affinity under certain conditions of high temperature, is a fact of great interest in the physics of the globe.

698. **The causes which produce the spheroidal form** in liquids are at least four:—

1st. *The repulsive force of heat* exerted between the hot surface and the liquid, and which is more intense as the temperature rises.

2d. *The temperature of the plate* is so high, that the water in momentary contact with it, is converted into vapor, upon which the spheroid rests as upon an elastic cushion.

3d. *The vapor is a poor conductor of heat,* and thus prevents the conduction of heat from the metal to the globule. Another cause which prevents the liquid from becoming highly heated is, that the rays of heat from the metal are reflected from the surface of the liquid. This is shown by the fact, that, if the water be colored by lampblack, heat is absorbed, and the evaporation is much more rapid.

4th. *Evaporation from the surface of the metal* carries off the heat as it is absorbed, and thus prevents the liquid from entering into ebullition. The form of the oblate spheroid, which the liquid assumes, is the combined result of the cohesion of the particles to each other, and the action of gravity upon the mass.

699. **Freezing water and mercury in red-hot crucibles.**—The remarkable phenomena of freezing water, and even mercury in red-hot crucibles, are striking examples of the production of the spheroidal state of liquids.

Boutigny placed a portion of liquid sulphurous acid in a red-hot vessel. It assumed the spheroidal state immediately, at a temperature below that of its ebullition, that is, below 14° F. A little water placed in the spheroid becomes, therefore, cooled below 32° F., its freezing point, and is converted into ice.

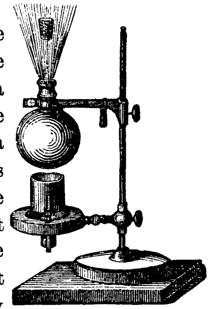
Faraday placed in a heated crucible a mixture of solid carbonic acid and ether, which immediately assumed the spheroidal state. Into it was plunged a metal spoon containing mercury; almost immediately the mercury was frozen into a solid mass. The temperature in this case was probably as low at —148° F.

700. **Remarkable phenomena connected with the spheroidal state.**—On the principle explained, the hand may be bathed in a vase of molten iron, or passed through a stream of melted copper unharmed, or one may stir fused glass under water without danger. In all similar cases, if the temperature be sufficiently high, the moisture of the hand assumes the spheroidal state, and does not allow of contact with the heated mass. If, however, the hand is drawn rapidly through the melted metal, contact is mechanically produced, and injury follows this rashness. The finger, moistened with ether, may be, for the same reason, plunged into boiling water without injury.

701. **Explosions produced by the spheroidal state.**—The experiment illustrated by fig. 499, may be modified to illustrate explosions, and some other interesting facts consequent on the spheroidal state.

A copper bottle, fig. 499, is heated as hot as possible over a double current lamp, and in this state a few grammes of pure water are introduced by a pipette. The water at once assumes the spheroidal condition, and has a temperature (as may be ascertained by a thermometer) below that of its ebullition. If the neck of the bottle is now tightly closed by a good cork, the evaporation is so slight, that the pressure of the vapor within is not immediately sufficient to drive out the cork. If, however, the lamp is withdrawn, the metal will soon cool sufficiently to allow contact of the water with it. There will then be so sudden an evolution of a large volume of vapor as to drive the cork from the bottle with a loud explosion.

499



Steam boiler explosions may sometimes be explained by a knowledge of the principles here elucidated. Thus, whenever from any cause a deficiency of water occurs in a boiler, as when the pumps fail of a supply, or when, by careening, a part of the flues are laid bare while the fire is undiminished, a portion of the boiler may become heated even to redness. Water coming in contact with such over-heated surfaces, would first assume the spheroidal state, and, almost at the next instant, burst into a volume of vapor so suddenly as to rend the boiler with frightful violence. Numerous accidents are on record where the explosion has been so sudden as not to expel the mercury from the open gauges. The fact that explosions on our American rivers have occurred most frequently just at or after starting from a landing, is explicable on the view here presented; the vessel, while landing and receiving freight, being careened, so as to render the exposure of some part of the flues possible.

702. **Familiar illustrations of the spheroidal state**, and effects of the spheroidal state of liquids, are not unfrequent in common life and in manufactures.

The most common example of the spheroidal state, is that of a drop of water on a heated stove, which moves around in a spheroidal mass, slowly evaporating. The laundress determines whether her flat-irons are heated sufficiently for her purpose by touching the surface with a drop of saliva on the finger. If it bounds off, the iron is judged to be heated to a proper temperature. In the manufacture of window-glass, constant application is made of the principles here explained. The masses of glass are first formed into a rude hollow cylinder by blowing them in wooden moulds. In order to prevent the charring of the mould, its interior is moistened with water, which, assuming the spheroidal state, protects the wood, while it does not injuriously cool the glass.

Saline solutions are more efficacious for tempering steel than pure water. Now, as the point of ebullition of saline solutions is higher than that of pure water, contact between the liquid and the metal is produced sooner, and thus the steel is cooled more quickly, and the temper is better.

Melted metals, like iron or copper, allowed to fall into water, do not throw the water into violent ebullition, as might be supposed, but pass in a brilliant stream to the bottom of the vessel, the water in contact with the metal assuming the spheroidal state.

§ 10. The Steam-Engine.

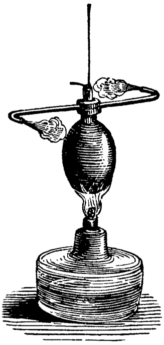
703. **Historical.**—The principles involved in the construction and theory of the steam-engine, have already been sufficiently discussed. A few words must suffice respecting their practical applications in the discovery and perfecting of this remarkable machine.

For the first rudiments of our knowledge of steam as a motor, we must go back, as upon many other so-called modern inventions, to Egypt, where, 130 years B. C., Hero, or Heiro, describes in his "Spiritalia seu Pneumatica," among many other curious contrivances, what he calls the *eolipile.*

704. **The eolipile** is a metallic vessel, globular, or boiler-shaped, containing water, and provided at top with two horizontal jet pipes, bent into the form of an S.

This apparatus, fig. 500, is suspended over a flame, and being free to move, when the water boils, the steam rushing out, strikes against the atmosphere, and the recoil drives the apparatus around with great rapidity. This is in fact a direct-action rotary steam engine, and undoubtedly the earliest mechanical result achieved by steam power. It has often been re-invented, in numberless forms, in modern times. In another form the eolopile is made to blow by its jet the flame of a lamp, and in this case the boiler is fixed and filled with alcohol in place of water, the jet descending through the flame of the lamp as in the apparatus seen in fig. 497. Hero describes also other devices where steam was the moving power.

500



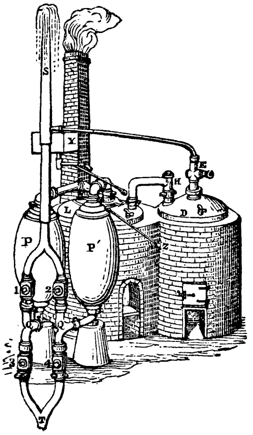
705. **First steamboat.**—Blasco de Garay, a sea-captain of Barcelona, in Spain, in 1543, moved a vessel of 200 tons burthen three miles an hour by paddles propelled probably by steam, as the moving force came, it was said, from a boiler containing water, and liable to burst.

This experiment was made on the 17th day of June, 1543, in presence of Commissioners appointed by the king, Charles V., whose report secured the favor of the crown to the projector. But what is unaccountable, nothing more ever came from this singular success. De Garay probably employed Hero's eolipile on a large scale, as Hero's work above named was about that time translated into several languages and generally diffused. Passing the early efforts of Baptista Porta and De Caus (A. D. 1615), of Brancha (in 1629), Otto V. Guerick (1650), and the Marquis of Worcester (1663), we come to the first efficient steam apparatus (that of Savary).

706. **Savary's engine.**—In 1698, Capt. Thos. Savary obtained a patent "for raising water and occasioning motions to all kinds of mill work by the impellent force of fire." His apparatus can hardly be called an engine, or machine, since it has no moving parts.

Fig. 501 is Savary's engine. Two boilers, L and D, are connected together by the pipe, H. Two "condensers," P and P′, are connected with the larger boiler, L, by pipes entering at top of both, and capable of being alternately shut off from the boiler by a valve, moved by the lever Z. By two branch pipes beneath the condensers, communication is established at pleasure, by the aid of the cocks 1, 2, 3, 4, alternately with the well by T, and the open air by the outlet pipe S. The boiler, L, being in action, the condenser, P, for example, was filled with steam, the cocks 1 and 3 being closed. By moving Z, the condenser, P′, was next filled with steam also, cocks 2 and 4 being closed, and at the same instant cock 3 being opened, the water rushed up through T, to fill the vacuum occasioned by the condensation of the steam in P. The lever, Z, was then moved to close P′ and open P again to the boiler. Cock 4 now admitted cold water to P′, and cock 1 being opened, the direct pressure of the steam from the boiler forced the water out of P, in a stream through the discharge pipe, S. The water in P′ was also discharged in the same manner, and so on, alternately, each condenser was filled with cold water, and again discharged, maintaining a continuous stream of water from S. To supply the waste of water in the boiler, L, the contents of the smaller boiler, D, were from time to time forced by superior steam pressure into L, through the pipe H (provided with a valve for that purpose), reaching near the bottom of D, whose capacity was such as to fill L to a suitable height. The boiler, D, was then refilled through the pipe, E, from the supply box, X, attached to the discharge pipe.

501



All the details of Savary's contrivance show a nice adjustment of means to the end to be accomplished.

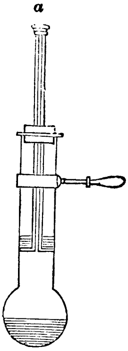
707. **Papin's steam cylinder, Newcomen's engine.**—Denys Papin (Prof. of Mathematics at Marburg), whose name is connected with the high-steam digester, suggested in 1690 the use of steam to produce a vacuum in lieu of the air-pump before used.

For this purpose he constructed the cylinder of sheet iron, and built a fire beneath its bottom, to boil a portion of water there placed. When the cylinder was filled with steam, the piston, before held up by a latch, descended as the steam was condensed. No practical result followed this clumsy contrivance, on which Papin's countrymen rest his claims to be considered as the inventor of the steam-engine.

Thos. Newcomen, in 1710, first put in practice the use of a cylinder and piston in the steam-engine, in which the steam was alternately admitted and again condensed by a stream of cold water. This engine operated against the pressure of the atmosphere, and was effectual in only one direction, *i. e.*, it was a single acting engine.

708. **The atmospheric engine** is well illustrated by the apparatus shown in fig. 502, which was contrived by Dr. Wollaston, to show the nature of Papin's cylinder.

502



A glass, or metallic tube, with a bulb to hold water, is fitted with a piston. This piston-rod is hollow, and closed by a screw at *a*. This screw is loosened to admit the escape of the air, and the water is boiled over a lamp: as soon as the steam issues freely from the open end of the rod, the screw is tightened, and the pressure of the steam then raises the piston to the top of the tube, the experimenter withdraws it from the lamp, the steam is condensed, and the air pressing on the top of the piston forces it down again; when the operation may be repeated by again bringing it over the lamp.

In all the early steam-engines, the steam was condensed within the cylinder, either by water applied externally, or by a jet of water thrown directly into the cylinder. It is very obvious, that a great loss of fuel and time was thus involved in bringing the cylinder up again to 212°, before a second stroke could be made.

NEWCOMEN and SMEATON constructed very large engines, however, on this principle, and applied their power directly to the pumping of mines. Although Smeaton introduced an improved kind of mechanical work and many improvements in minor details, and better boilers, he succeeded only in raising the average duty of steam-engines from about five and a half millions of pounds, raised one foot by a bushel of coal (80 lbs.) burned, to about nine and a half millions, in his best engines. A good pumping engine now raises from ninety to one hundred and thirty millions of pounds for every bushel of coal burned.

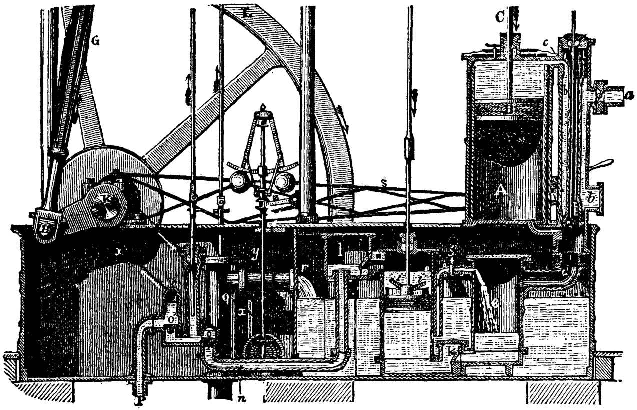
709. **Watt's improvements in the steam-engine.**—The steam-engine as it was left by Smeaton was, as we have seen, only a steam pump, confined to the single function of raising water, and incapable of general use, as well from its imperfections as from the enormous cost of fuel it required.—Watt, in 1763, was a maker of philosophical instruments at Glasgow, and had occasion to repair a model of the Newcomen engine. The study of this machine and its defects, led Watt to construct a new model, in which the steam was condensed in a separate vessel, in connection with which he subsequently found it advantageous to use an air-pump—to aid in keeping the vacuum good, as it was otherwise vitiated by atmospheric air leaking in, and coming from the water of the boiler. These ideas were matured and realized in 1765, and in 1769 he took out his patent, in which all the essential features of our modern steam-engines are included. In connection first with Mr. Roebuck, of Carron Iron Works, and subsequently with Mr. Boulton, of Soho, he put his ideas in practice, and by reserving to the patentees one-third part of the saving of fuel effected by his improvements, his genius was rewarded by the accumulation of a princely fortune

Watt's invention of low pressure condensing engines stands without a parallel in the history of science for the perfect realization of all the conditions of the problems to be solved — the perfect mastery of the laws of nature — the use of matter, by which they were accomplished, and the thorough exhaustion of the subject even in its minutest details, so that to this day we have no improvements in this machine involving a single principle unknown to Watt. In the beauty and perfection of mechanical work, in size of parts, and the strength of boilers, we have machines greatly superior to any Watt ever saw, but it was his genius that rendered those perfections possible, and supplied the very power by which they have been worked out.

710. **The low pressure or condensing engine.**—The low pressure engine is employed in all situations where economy of fuel and the best mechanical effect from it are the ruling considerations, and where lightness and simplicity of construction is unimportant. This machine now remains almost exactly as Watt left it. Owing to the nearly perfect vacuum obtained in it by the condenser and air-pump, much less pressure of steam is required to produce a given mechanical result; *e. g.*, if the vacuum is equal to fourteen lbs. atmospheric pressure, then a steam pressure of six lbs. would give an efficient moving force of twenty lbs. to the machine. Hence the propriety of the term "low pressure" engine; but in practice it is found advantageous to use higher pressures in the condensing engines than Watt ever contemplated.

Fig. 503 is a section of the cylinder, A, condenser, *e*, air-pump, hot and cold well, and a view of the most important attached parts of a modern condensing engine. The cylinder, A, is seen receiving steam at top through the throttle-valve, *a*, driving down the piston, B, with its rod, C. A stream of cold water injected into the condenser, *e*, has completely condensed all the residual steam

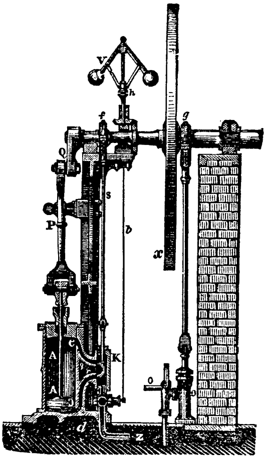
503



of the former stroke which has found its way from A by the eduction pipe, *d*, so that the piston, B, is descending into a nearly perfect vacuum (671). The hot water of this condensation is constantly drawn off through the valve, *k*, by the air-pump whose valves, *i i*, rise to allow its flow into the hot well, *l*, whence it finds its way, solicited by the plunger pump, S, to the boilers by the pipe, P, and its valves, *o o*. The cold water pump, *q*, supplies a steady stream of cold water by the spout, *r*, to the cold well. By the time the piston, B, has reached its lowest point of descent, the valve rod, V, and eccentric bar, S, have moved so as to open the lower steam ports and reverse the direction of the piston, when the steam above, B, is in its turn taken into the condenser, *e*, by the appropriate channels, and removed as already explained for the downward stroke. The piston rod, C, and valve and pump rods, are connected above with the great working-beam, whose further extremity conveys the power of the engine by the pitman, G, through the crank pin, H, to the main shaft, K, on which is the fly-wheel, L, to give steadiness of motion to the whole apparatus. The arrows show the motion of these parts as the piston descends. The governor, *z*, controls the throttle-valve, *a*, by connections not shown in this drawing.

711. **The high pressure engine.**—In this machine, the escape steam is driven out against the pressure of the atmosphere, and no attempt is made to utilize its capacity to form a vacuum, consequently this form of apparatus could be used as well with condensed air, or any other elastic fluid, as with steam, if there was any other that could compete in economy with it. The lightness, simplicity, and low cost of the high pressure engine, make it available in spite of its uneconomical use of steam, in many situations where a condensing engine would be unavailable.

504



The steam arrives by the pipe, Z, fig. 504, to the steam chest, R, and is admitted alternately by the ports *e d*, to the top and bottom of the cylinder, A A, as the valve rod, S, actuated by the eccentric, *f*, on the main shaft, opens and shuts the ports by the slide valve in K. The escape steam makes its exit through *q* to the atmosphere. The pitman, P, conveys the motion of the piston, C, to the crank, Q, and the main shaft, on which is the large fly-wheel, *x*, to accumulate momentum. The flow of steam is regulated by the governor, V, whose balls fly out with the centrifugal force of a more rapid motion, and by the rod, *h b*, close more or less the throttle-valve, *z*, which regulates the supply of steam; the pump, *o o*, supplies water to the boiler, and is moved by the rod and eccentric, *g*, on the main shaft.

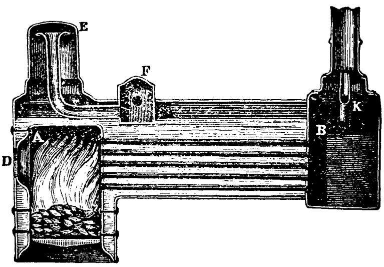
712. **The cut off.**—The supply of steam both to the high and low pressure engine is further regulated by a contrivance called the " cut

off," which may be set to cut off the flow of steam entirely, or at any portion of the stroke, as one-half, or one-third. The expansion of the steam then completes the work, and great economy of fuel is found to follow its use.

713. **Steam boilers.**—The form of steam boilers varies very much with the purpose to which they are to be applied. On land, large boilers may be safely used, which would be wholly valueless at sea, or on a locomotive engine.

Plate-iron strongly riveted and braced, is the material combining the greatest economy and strength. Copper can be used only when the fuel contains no sulphur, and is the best material to resist corrosive agents. Simple cylindrical boilers, laid horizontally, with a fire-flue under the whole lower surface, are commonly used for high pressures. When these are made large enough to receive the furnaces within and distribute the heat in interior flues, they are called Cornish boilers. When their construction is still further modified, with reference to the greatest possible increase of fire surface, they are called locomotive boilers, as in the annexed figure, 505; which is the common locomotive boiler seen in section. D, is the feed-door, for fuel to the furnace or fire-box, A, which communicates by numerous small horizontal tubes, entirely surrounded by water, with the base of the chimney, B, into which the blast of exhaust steam from the engine is driven at K. E, is the steam chamber, where a trumpet tube in the dome conveys the dry steam on its way to the cylinder through F. Steam boilers are supplied with hot water by a force pump, and gauge cocks indicate the water level.

505



714. **Mechanical power of steam.—Horse-power.**—As steam-engines were originally employed to take the place of horses in raising water, it was natural to estimate their power by the number of animals they replaced. The value of any force is correctly stated as the number of pounds raised one foot high in a given time (foot-pounds). As the use of steam became general, the term *horse-power* was retained, but its use was restricted by Watt to mean 33,000 lbs. raised one foot per minute, or nearly 2,000,000 lbs. raised one foot per hour.

As one cubic foot of water converted into steam yields, in round numbers, 1700 cubic inches of vapor, its mechanical effect at atmospheric pressures, is equivalent to raising 15 lbs. 1700 inches (or 142 feet) in a tube of one inch area. But 15 lbs. raised 142 feet, is the same thing as 142 times 15 lbs. raised one foot, or 2130 lbs., or nearly a gross ton. The total mechanical force developed by changing one cubic inch of water into 1700 cubic inches of steam is, therefore, nearly one ton. Only 60 or 70 parts of this power are, however, regarded as actually available in use, deducting friction and loss from other causes. Therefore, the evaporation of a cubic foot of water in an hour, subject to this deduction, will give the full force of about 1000 cubic inches of water converted into steam, as the expression of one-horse power (viz., 33,000 lbs. × 60 = 1,980,000 lbs.), or nearly 2,000,000 lbs. raised one foot. This is a somewhat rough approximation, but it gives constants easily remembered and sufficiently near the truth.

A boiler of one-hundred horse power means, then, a boiler capable of evaporating 100 cubic feet of water per hour.

In practice, it is common to allow, in large land engines, for every horse power, one square foot of fire bars in the grate, three cubic feet of furnace room, ten cubic feet of water in the boiler, and ten cubic feet of steam chamber. In locomotives and steamships, these proportions vary very much.

715. **Evaporating power and value of fuel.**—In England, engineers estimate ten pounds of bituminous coal for every cubic foot of water (*i. e.* every horse power) to be evaporated. In carefully constructed boilers, however, this effect is produced by seven or eight pounds of coal. In the Cornish boilers, where a very large evaporating surface is allowed, five pounds of coal only, and sometimes less, are used per horse power. In the United States, anthracite coal averages ten pounds of water evaporated, for every pound of coal burned. This would give 6·25 lbs. of anthracite for each cubic foot of water evaporated. A well regulated current of vapor conducted over the flame of bituminous coal by Dr. Fyfe, raised the evaporative effect produced 37 per cent. above what was obtained from the unassisted coal. This increase is due to the decomposition of the steam by the hot fuel, and the consequent effect of the pure oxygen on the carbon. Well seasoned wood (beech or oak), still containing about 20 per cent. of water, and well dried peat, have about equal evaporating power, and are only about two-fifths as effective as an equal weight of ordinary bituminous coal.

Welter has observed that those quantities of a combustible body which require an equal amount of oxygen for combustion, evolve also equal quantities of heat; although later researches show this conclusion not to be strictly true, it is supported by many facts. In all cases of combustion, the action is reciprocal; the oxygen is burned in the fuel as truly as the fuel by the oxygen, and, therefore, the same amount of heat is generated by a given amount of oxygen, whether in converting carbon into carbonic acid, or hydrogen into water. To burn one part of carbon, requires 2·66 parts of oxygen ($CO_2 = 16 \div 6 = 2{\cdot}66$), and to burn one part of hydrogen, requires 8 parts of oxygen. It has been proved experimentally (by Rumford) that 78 parts of water are raised from 32° to 212° by burning one part of carbon, while one part of hydrogen so burned will raise 236·4 parts of water through the same degrees. It therefore follows, that one part of oxygen, burning carbon, will heat $78 \div 2{\cdot}66 = 29{\cdot}25$ parts of water from 32° to 212°; and also that the same quantity of oxygen, in burning hydrogen, will heat $236{\cdot}4 \div 8 = 29{\cdot}56$ parts of water through the same degrees. The heating effect of oxygen may, therefore, be assumed to be 30, or, in units of heating power, 5400.

If the heating effect of pure carbon is taken at unity the relative heating effects of combustibles will range as follows for equal weights:—hydrogen, 3; vegetable oil, 1·15—1·22; ether, 1·02; carbon, 1; wood charcoal, 0·96; alcohol, 0·86; good coal, 0·77; dry wood, 0·46; wood (with 20 per cent. water), 0·35; peat, 0·33—0·38.—(Knapp.) Compare § 753.

The best expansive steam-engines, it is calculated, give back, in the form of mechanical work, only about 18 per cent. of the heat generated by the fuel burned in driving them.

Prof. W. R. Johnson ("experiments on coals") and others, argue, as the result of experiment, that the total amount of carbon in a fuel, is the measure of its practical evaporative power. His results very nearly sustain this view. He found, also, that about 86 per cent. of the total heating power were expended in evaporating water, and about 14 per cent. were lost in the products of combustion. Of the total heating power, by calculation, about 26 per cent. were lost in practice,—as deduced from the experimental effects stated in his tables.

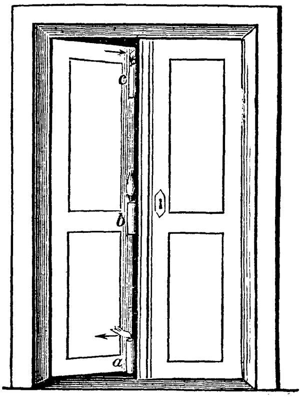
§ 11. Ventilation and Warming.

I. VENTILATION.

716. **Currents in air and gases** depend upon principles which have already been fully explained,—but the subjects of ventilation and artificial heating are of such great importance in daily life, that they demand a brief and separate consideration.

Currents arise in air from differences of temperature and variations of pressure. The perfect freedom of movement in air, renders its fluctuations from these causes incessant. If the air was visible, every candle, gas-light, stove, furnace-flue, and human body, would be seen to be the centre of an ascending column of heated air, whose place was constantly supplied by other and colder particles.

506

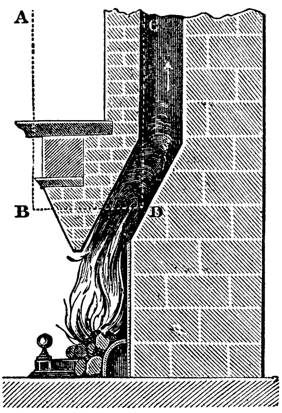


On the law of the equilibrium of fluids, the ascending currents must induce others, descending and horizontal, and thus a circulatory motion is imparted, even by a single lighted candle, to the whole gaseous contents of a quiet apartment. These currents are made visible whenever the candle smokes. If the door of a heated apartment stands ajar, and a candle is held near the top crack, *c*, fig. 506, the warm air of the room is seen to draw outwards, carrying the flame with it, and a corresponding cold current, *a*, flows in at the bottom,—while a point, *b*, will be found, midway its height, where the candle flame is undisturbed. So a window, partly open, will occasion a draught of cool air, blowing in at the bottom of the opening, and a compensating warm current will escape outwards, above.

This constant interchange of motion in unequally heated masses of air, while it soon poisons the confined atmosphere of a close apartment, where many persons, with or without lights, are assembled, also supplies the easy means of curing one of the greatest evils of civilized communities.

717. **Draught in chimneys.**—Chimneys draw because the products of combustion discharged into them, are specifically lighter than the outer air. The column of heated air, C D, fig. 507, rises with a velocity proportionate to the excess of weight in a column of the outer air, A B, of the same area and height. The laws of falling bodies (71) apply to this case in every particular.

507



Suppose, for example, a chimney is 18 feet high, and the gases in it are heated to 100° F., the outer air being 70°. The contained column would, therefore (607), expand $\frac{30}{491}$, or about $\frac{1}{18}$th of its original bulk at 70°. A column of 19 feet of such air would, therefore, be required to counterbalance one of 18 feet high in the external air at 70°, and of the same area. The heated air will, therefore, rise (for the same reason that a balloon rises), with a velocity equal to that acquired by a body falling through one foot, *i. e.*, a space equal to the difference in height of the two columns—of *equal weight.* The laws of gravity, therefore, supply the means of calculating the theoretical velocity of the ascending column, and of course that, with the area of the cross section of the flue, will determine the quantity of air passing through the chimney in a given time. But the friction of the air against the sides of the flue, and the varying density of the products of combustion compared with air, diminish the theoretical velocity, and it is usual to allow a deduction of one-fourth for these causes. The following rule will be found to give a sufficiently exact practical expression of the velocity of air in chimneys and ventilating flues.

Multiply the square root of the difference in height of the two columns of air (deduced as above) expressed in feet and decimals of a foot, by eight; deduct one-fourth, and the product of the remainder, multiplied by sixty, will give the velocity of efflux per minute; and the area of the flue in feet, or decimals of a foot, multiplied by the velocity, will give the number of cubic feet discharged per minute.—(Hood.)

This rule, in the case supposed, would be $\frac{3}{4}\left(8\sqrt{1}\right) \times 60 = 360$ cubic feet of gases discharged per minute, by a flue 18 feet high and one foot area, whose temperature is 30° above the outer air.

Chimneys, in the sense we mean, were not known to the ancients. Holes in the roof, and windows allowed the escape of smoke from the kitchens of the luxurious Romans. But the mild climate of the Mediterranean shores did not require much attention to means of artificial warmth. In the houses of ancient Herculaneum and Pompeii, exposed in modern times, there are no chimneys. But even in England and France, where fires in winter are necessary, chimneys were first introduced only in the middle of the 14th century. The Curfew Bell (*couvre-feu,* fire cover) was needed as a precaution against the danger of fires, without chimneys.

Reversed draughts and smoky chimneys occur, 1st, when the flue or fire-place is badly constructed; 2d, when two flues open into one apartment, or two connecting apartments, and there is a fire in only one flue; 3d, when a powerful fire exists in one part of the house, as the kitchen, for instance, without an adequate supply of air from without, it will draw the needed supply through the smaller flues in all parts of the house, reversing the draught in them; 4th, when (as in many old houses) the flue is so large that cold currents may descend in the angles, while a heated one ascends the axis; 5th, when a neighboring higher house or eminence directs, in certain states of the wind, a cold current down the flue.

The remedy for reversed draughts is best found in one commanding central stack, into which all the minor flues discharge, while exhausting cowls, like fig. 511, are the best cure of smoky chimneys.

718. **Products of respiration and combustion, and necessity for ventilation.**—By contact with the lungs, and with burning fuel, the air is contaminated, chiefly with carbonic acid, water, effete nitrogen, oxyd of carbon, and animal odors. Every full grown individual consumes, in every minute of quiet respiration, about 500 cubic inches of air. About 14 ounces of carbon are burned by the air out of the body of a man in twenty-four hours, and all this is returned in the form of carbonic acid to the air.

Such air cannot be breathed again without danger. Mixed with the surrounding air, it contaminates that also. Headache, languor, uneasy respiration, nausea, faintness, and syncope are results which always follow from breathing air contaminated with these poisonous exhalations, even in very moderate quantity. Even two per cent. of carbonic acid, derived from respiration or combustion, may produce all the symptoms above named. The full chemical and physiological evidence upon this important subject cannot be here given, but the evils arising from the slow and insidious effects of the poison of bad ventilation, can hardly be over estimated. In ordinary combustion, especially with slow fires and an imperfect supply of air, carbonic oxyd is also produced, and this is one of the gases most likely to leak from hot-air furnaces when the joints are not tight. It is much more destructive of life than carbonic acid, or those gases of sulphur, whose presence is at once declared by their odor.

719. **The quantity of vapor given off by the body**, in sensible and insensible perspiration, and by the lungs, is very considerable, being not less than ten or twelve grains each minute, or about three pounds per day, which, with the quantity of carbonic acid expired, makes about three and a third pounds, besides other excrementitious matter, given off in twenty-four hours.

If the air of a crowded apartment is conducted through water, so much animal matter is collected in the water as to occasion a speedy putrefactive fermentation, with a disgusting odor. The blast of air escaping at the upper ventilator of a crowded assembly room, is so oppressive as to produce immediately the most distressing symptoms. While we instinctively shun all contact with unclean persons, and what we call dirt, even refusing a cup that has pressed the lips of another, and esteem all water not transparent as foul, it is marvellous with what thoughtlessness we resort to crowded and ill-ventilated public places, and drink in the subtle poison exhaled from the lungs, skin, and clothing of every individual in the assembly. Especially when we remember that while the digestive apparatus can select and assimilate nutriment from food of questionable quality, the lungs have no such power of selection, and can discharge their duty to the blood only by a full supply of pure air. If the transparency of air was troubled by the exhalations of the lungs as water is by the washings of the body, no argument would be needed to secure attention to the importance of ventilation; and yet it is quite true that the bodily health suffers more from inhaling effete air, than it could from drinking the wash alluded to.

720. **The quantity of air required for good ventilation**, is very variously stated by different authorities. Enough fresh air must be supplied, obviously, to replace all that is contaminated by the lungs, the body, and sources of illumination. But to determine exactly how much these several sources of deterioration demand, is not so easy. The amount of air needed to remove the products of respiration, is very much less than is required to absorb the vapor of water given off from the lungs and the skin. The quantity of vapor the air can take up, will depend on its dew-point and temperature. Hood estimates three and one-quarter cubic feet of air per minute, for each individual, in winter, with an external temperature of 20° or 25°, and a quarter of a cubic foot per minute to supply the waste from the lungs, making three and a half cubic feet per minute, or two hundred and ten cubic feet per hour in winter, and five hundred in summer. Peclet estimates it at two hundred and twelve cubic feet per hour. Dr. Reid estimates the quantity, as high even as thirty cubic feet per minute per individual. Brenan puts it at 10·25 cubic feet.

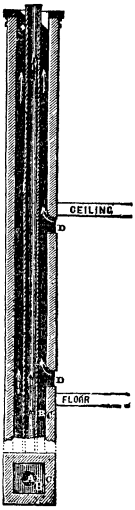
721. **Products of gas illumination.**—Every cubic foot of gas, of average quality, requires the oxygen of about twenty cubic feet of air (viz. 4·25 cubic feet oxygen) to burn it, and produces rather over a cubic foot of carbonic acid, still more water, and, if the gas is impure, sulphurous acid and compounds of ammonia will be added, which,

dissolving in the watery vapor, condense upon and corrode furniture, books, metallic articles, &c. Every pound of coal gas burned produces 2·7 lbs. of water, and 2·56 lbs. of carbonic acid; and, as a cubic foot of coal gas weighs about 290 grains, twenty cubic feet will weigh a pound, a quantity which four common fish-tail burners consume in an hour. The capacity of air for moisture at 68°, is 7·31 grs. per cubic foot. It would, therefore, require 2373 cubic feet of air at 68° to retain the water from twenty feet of gas, and nearly five times as much at 20° F., not to name the amount required to dilute the carbonic acid and free nitrogen produced.

It is needless to add, that the ventilation of gas burners is an important matter. Fortunately, a gas chandelier affords one of the best means of producing an upward current in an assembly room. Candles and oil consume more air, and, of course, produce more effete products for an equal amount of light than gas.

722. **The actual ventilation of buildings is a practical problem,** to be wrought out in each case, with careful regard to the principles and facts just stated. The supply of air required may be obtained in two ways:—1st, by the ascending column of heated air in a shaft, drawing after it the effete air to be removed, and supplying its place by fresh air, warmed in its progress to the apartments. This is called *thermal* ventilation; or, 2d, mechanical force may be employed, by means of revolving fan-wheels driven by a steam-engine, or otherwise, forcing the air through the apartments to be warmed and ventilated. This is called *mechanical ventilation.*

508



By the first method, Dr. Reid ventilated the House of Commons in England. By the second, Mr. Rice ventilated the House of Lords with a fan-wheel, over thirty feet in diameter.

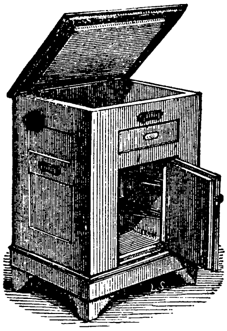
723. **Stone's ventilating shaft.**—An excellent combination of the thermal ventilation, with the plan of hot-air furnaces, so generally used in the United States, has been devised by S. M. Stone, Architect, which has been found efficient in the New Haven City Prison, the State Reform School, and other similar buildings. Fig. 508 shows a plan and section of this system.

A ventilating shaft of brick, C, rises in the centre of the house, through the axis of which passes a cast-iron smoke flue, A, carrying off the waste products of the furnace. The radiant heat of this iron flue heats the air in the shaft B. Openings, D and D, are pierced from the various apartments into this shaft, and allow the air of the rooms free opportunity of escape, solicited by the powerful ascending draught of the vertical shaft. Distant apartments are connected with the shaft by horizontal pipes of wood or tin. The openings, D, should be covered with wire gauze, and fitted with Dr. Arnott's self-acting noiseless valve, which allows the passage of an upward current only. The apartments are supposed to receive their supply of fresh and warm air through hot air flues, ascending in the walls. In summer it would be found needful to establish a current in the shaft by an occasional fire in the furnace, or by a special furnace for that purpose in the top of the house. In cities, the air taken into buildings may be strained through fine wire gauze and spray of water, as was accomplished by Dr. Reid in the House of Commons. By the rule (717), the power of such a shaft to discharge air can be calculated.

724 **Cold currents produced by ice.—Refrigerators.**—Air, in contact with ice, acquires, of course, a low temperature, and parts with a large part of its moisture. Thus, snow-clad mountains and glaciers naturally send down to the valleys below a current of cold air, flowing like water over the surface, especially at night, when the absence of the sun prevents the accumulation of heat on the earth's surface.

Adroit use has been made of this cold dry current, in the construction of refrigerators for preserving food in warm weather; and the same principle has been applied, on a large scale, to the cooling of large apartments. Figs. 509 and 510 show a section and elevation of Winship's Refrigerator. The ice, A,

509



510

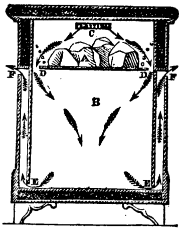
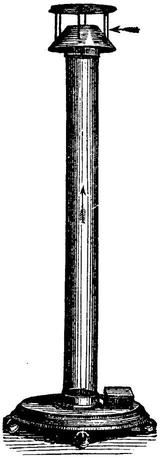


fig. 510, is sustained upon a shelf in the upper part of the box, surrounded with double charcoal linings. The air enters by the register openings, C, and coming in contact with the ice, is cooled, and falls to the bottom, as indicated by the arrows, where it finds its egress at E, between hollow walls, and finally escapes at F, as in an inverted syphon. In this way a gentle current of about 45° F. is steadily maintained as long as the ice lasts, and, being dry, articles of food are preserved sweet and free from mould for a long time. A similar device has been used for large apartments.

725. **Cowls.—Emerson's ventilators.**—Advantage is taken of the currents in the external air, to aid in establishing ventilation in houses, and draught in chimneys, by the use of ventilating cowls.

These contrivances are often conical, and hung with a vane, to turn against the wind. One of the most generally approved, however, in common use, appears to be the ventilator of Emerson, fig. 511, which is simply a cone of metal, surrounding the flue, over whose vent, and a short distance above it, is sustained a disc of metal. If the wind blows from any point, its effect, on striking this conical surface, is to pass upwards and across the open flue, with an increased velocity. The result is to solicit an upward current in the shaft, as shown by the arrow, in the figure.

511



If it is desired to direct a current of fresh air *into* the shaft, an *injecting ventilator* is used, which is simply the above cone *inverted*, or two or three such placed one over the other. These are found very efficient in projecting fresh air into the cabins of ships, and other similar situations.

726. **The supply of fresh air in dwellings** is derived, in the winter, almost exclusively from the cracks and joints of doors, windows, and other openings. In all good systems of general warming, this supply is derived from the open air, or a free basement, and is warmed in its progress through the heating apparatus. When it is requisite to introduce cold air into a house, it is important to do so in such a manner as to avoid local and sharp currents; for this purpose, perforated cornices, or openings covered by wire gauze, are provided. A too rapid current is both inconvenient and costly, from the needless waste of fuel.

In public buildings, the supply is best obtained through a trench, or horizontal tube, opening into a clean area, and protected against powerful winds. The injecting cowl may be advantageously used to cover the opening of such a supply. The distribution of the ascending warm current is best made through a hollow or double floor, perforated with numerous openings, while the spent air is taken off above, as already described.

II. WARMING.

727. **The artificial temperatures demanded in cold climates** are produced, 1st, either by radiant heat solely, as in the common open fire-place, 2d, by convection only, as in hot-air furnaces of every description, in which the air is warmed by its passage through a heating chamber, and then introduced into the apartments to be warmed,—or 3d, by radiant heat and convection united, as in stoves, and steam or hot water pipes.

728. **The open fire** contained in a simple brick fire-place, fig. 507, whether coal or wood is burned, warms the air of the room solely by radiant heat. The burning fuel solicits the air of the apartment to be warmed towards the chimney, where, coming in contact with the fire, it parts with a portion of its oxygen to sustain combustion, is intensely heated, and rising, escapes at the flue, with the heated products of combustion. Hence only the heat radiated from the burning fuel and hot walls, is effectual in warming the apartment, while much the largest part of the heat (three-fourths to four-fifths of the whole) escapes up the chimney.

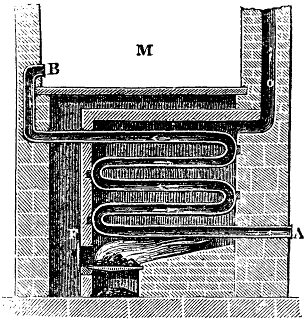
The genial effect and cheerful aspect of an open fire, combined with the efficient means of ventilation it affords, render this old system very popular, when combined with some competent general plan of warming the whole house.

Dr. Franklin improved the common fire-place by introducing iron stoves, of the same general form, and connecting them with the chimney by a circuitous pipe, by which means a much better economical effect was attained. Rumford improved the form of the fire-place very much, and especially with reference to the throat of the chimney and angle of the jambs. He also combined it with a circulation of hot air behind and at the sides of the fire, so as to obtain the effect of a stove.

Stoves of iron, standing in the apartment to be warmed, offer, perhaps, the most economical mode for burning fuel—but when, as is too often the case, they are closed tight, except a very small opening for draft, they are among the vilest contrivances in use for the ruin of the public health. The atmosphere of the room unavoidably becomes over-heated and corrupted by the products of respiration, in the almost universal absence of any mode of ventilation.

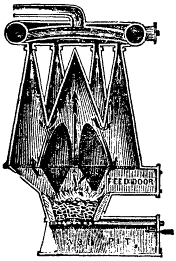
729. **Hot air furnaces.**—Large buildings and dwelling-houses are frequently warmed by air heated in its passage through a structure in the lower part of the house. One of the simplest forms of apparatus for this purpose is seen in fig. 512, being a sectional view of a hot air furnace, in which the cold air entering at A, passes, as indicated by the arrows, through an extended system of iron passages set in brick work and heated by the products of combustion, and the direct action of the fire, F. The heated air gains the apartment, *m*, by openings, B, in the floor or sides of the wall, while the gases of combustion escape by the flue, O. Such an apparatus serves only to illustrate the general principle, and would prove valueless in practice.

512

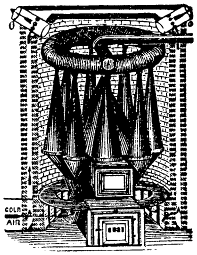


Very numerous forms of hot-air furnaces are in use in the United States, chiefly for the combustion of anthracite coal. They are essentially alike in principle, but very unlike in construction. All take cool air from without, or from an airy basement, and after heating it in a brick chamber, by contact with surfaces of hot iron surrounding the fire, and conveying away the products of combustion, distribute it by flues in the wall to the several apartments. Fig. 513 presents a sectional view of one of the best hot-air furnaces at present in use (Chilson's). The fire of anthracite is contained in a large shallow pot of cast-iron, with soap-stone, or iron staves, and the heated products of combustion are expanded in an extensive system of chambers of cast-iron, all communicating with an annular cast-iron pipe, leading to the chimney. This arrangement affords a very extended radiating surface, with few joints, to allow the escape of noxious gases into the surrounding hot-air chamber. The arrows indicate the direction of the current.

513



514



In fig. 514 is seen the furnace, surrounded in the brick work, which is hollow. The cold air enters at the bottom, and being gently heated by contact with the hot-air surfaces within the chamber, as well as by the radiant heat from the same source, it escapes by the openings, *o o*, to the various apartments. The extended iron surface in this apparatus prevents any part of the furnace from becoming very hot, usually the chief causes of complaint against this mode of heating. Air is materially injured for purposes of respiration by contact with over-heated surfaces, owing to the charring of the particles of dust and dirt always floating in it. The chief objection resting against this and similar modes of heating is the entire absence of *radiant heat in the apartments*, whose occupants are, so to speak, immersed in a warm air bath, and require, consequently, several degrees more heat, by the thermometer, for comfort, than when radiant heat forms a part of the means of an artificial temperature.

Hot-air furnaces are commended on account of their economy of construction, and ease of management, and when combined with a good system of ventilation, such as is secured by an open fire in one or more apartments, the objections to them are in great measure removed.

730. **Heating by hot water**, distributed in pipes, offers many advantages for the salutary and economical distribution of heat. The high specific heat of water (653), enables it to heat over three thousand times its own bulk of air in cooling through a single degree of temperature. That is, one cubic foot of water, by cooling one degree, will raise the temperature of 3419 cubic feet of air a like amount; for $0{\cdot}2379 : 1 = 813{\cdot}435 : 3419$. In this proportion the specific heat of air is the first term, and the third term is the bulk of air equal to a unit weight of water. As hot water is usually distributed in cast-iron pipes, experiments have been made upon the rate of cooling of these pipes, which show that one foot in length of pipe four inches in diameter, will heat 222 cubic feet of air one degree per minute, when the difference between the temperature of the air and the pipe is 125°. The advantage of hot water as a means of heating, depends much on its high capacity for heat, and its slow rate of cooling, by which the temperature declines very slowly, after the fire is extinguished.

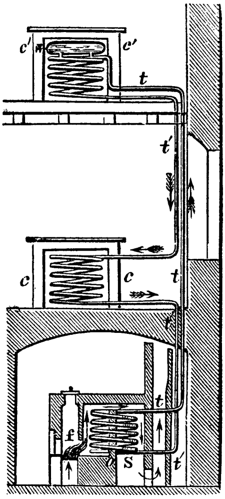
For horticultural and manufacturing structures, and other buildings where large pipes are not an objection, it has prominent claims. In private houses, where the hot water pipes occupy a chamber in the basement, and the air is heated by passing among them, all advantage of the radiant heat is lost, and the apparatus becomes comparatively inefficient, and very costly, if a sufficient number of pipes are laid in to do the work.

731. **Perkins' high pressure hot water apparatus.**—The system just named uses water at a very low pressure, never over six lbs. to the

square inch. Mr. Perkins has, however, patented a system, in which the hot water is distributed in very small iron pipes, under enormous pressure.

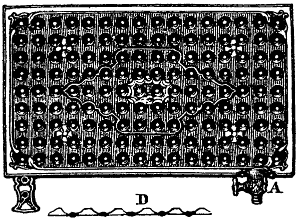
The plan of this system is seen in fig. 515. A coil of pipe, S (1 inch outside and ½ inch inside), is heated by the fire, *f*. The rising pipe, *t t t*, is carried to the top of the circulation, and in each story or apartment, a coil, *c′ c′*, *c c*, distributes the heat, the water returning by the descending pipe, *t′ t′ t′*, as indicated by the arrows. At the highest point of the circulation is placed a ten inch pipe, called the "expansion-pipe," fitted with a cock for the escape of air, and the admission of water. A sufficient void is left to accommodate the expansion of the water, which is about one-twelfth the whole bulk. Thus arranged, the temperature of the pipes can be raised to any required degree—and in practice it varies from 300° to 550°—*i. e.*, from about 75 lbs. to about 675 lbs. to the square inch. No safety valve is used on this apparatus, and numerous explosions of the fire coil have happened with its use. The high temperature of the pipes endangers buildings, and gives to the air heated by it the empyreumatic, burnt odor, which is so objectionable from cast-iron stoves. It is undoubtedly more efficient than the low pressure hot water system. The system of high pressure *steam* pipes is very similar to this, and equally open to the objection of over-heating the air, and endangering buildings from fire.

515



732. **Gold's steam heaters.—The radiators.**—In this system, the heat is radiated from surfaces of japanned sheet-iron, fastened together by rivets at the bottom of concave depressions in the outer sheet, as seen in fig. 516. This arrangement divides the whole steam space into numerous communicating cells, as seen in the cross section, D; the steam arrives from the boiler, fig. 517, under very low pressure (one pound to the inch), by the inlet cock, A, and the air escapes at an outlet cock in the opposite corner above. The water of condensation returns to the boiler by the same pipe that conveys the steam, which is made sufficiently large for that purpose.

516

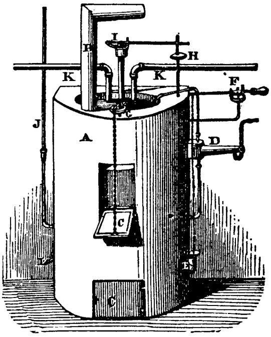


These radiators are placed in the apartments to be heated, either singly or in groups of three or four, concealed under an ornamental screen and covered with marble. The heat, in that case, is both radiant heat and heat of convection. The radiators may also be confined in a space below the apartments, and the air to be warmed passed through or among them, in which case only heat of convection reaches the apartments, as in the common hot-air furnaces. This system is economical of fuel, efficient for the most severe weather, and when combined with a proper system of ventilation, entirely unexceptionable. It has the great merit of securing exactly the desired degree of heat just where it is wanted, however remote from the boiler, and is, by means of the air-cock, adjustable to any temperature.

733. **The boiler of Gold's steam heater** is perfectly automatic, and is a beautiful illustration of the ease with which so powerful an agent as steam can be brought under entire self-control and rendered quite free from all danger.

Fig. 517 is an elevation of this boiler, set for use in its masonry, A. The water rises in the tube, J, which is open to the air, to counterpoise the pressure, which is adjusted to one pound on the inch. J is therefore a hydrostatic balance. The lower ash door, C, being closed, no air has access to the fire except through the side vent, E. This closes by a conical cover at the end of a chain, as soon as the limit of pressure is reached, for then the lever, F, rises, by reason of the water pressing up the elastic cover of F. A like arrangement, G, next opens the upper feed door, C; if the fire is not sufficiently held in check by F, C continues to open until sufficient cold air enters the flues to reduce the steam to its limit and hold it there. The safety valve, I, is likewise under the control of a similar arrangement, H, which comes into action after F and G, if needed. K, K, are the steam-pipes leading to the radiators, and the smoke reaches the chimney by the pipe, R. Such nice adjustments secure great economy of fuel, as the combustion cannot proceed faster than the demands of the radiators require.

517



§ 12. Sources of Heat.

734. **Sources of heat.**—Four great sources of heat may be named:—

1. *Mechanical sources.*—The principal of these are friction, compression, and percussion.

2. *Physical sources*, of which the chief are solar radiation, stellar radiation, terrestrial radiation, and atmospheric electricity.

3. *Chemical sources*, comprising chemical combinations, the chief of these being combustion.

4. *Physiological sources*, comprising the production of heat in living beings. This, according to views now generally received, is only an extension of the third head.

I. MECHANICAL SOURCES OF HEAT.

735. **Friction.**—When two bodies are rubbed together, heat is generated by the friction of their surfaces. The supply of heat from this source is apparently unlimited. As the generation of heat is

unaccompanied by any change in the calorific capacity of the bodies, and generally by no chemical action, it must be attributed to a molecular movement of the bodies excited by friction.

736. **Quantity of heat produced by friction.**—The experiments of Joule show that the amount of heat developed by friction depends only on the amount of force exerted, and not upon the nature of the substances rubbed together.

Count Rumford published in the Royal Philosophical Transactions, 1798, the results of some of his experiments upon this subject. A brass cannon, weighing 113 lbs., was revolved horizontally, at the rate of 32 revolutions per minute, against a blunt steel borer with a pressure of 10,000 lbs. In half an hour the temperature of the metal had risen from 60° to 130° F. This heat would have been sufficient to raise the temperature of 5 lbs. of water from 32° to 212°. In another experiment, the cannon was placed in a vessel of water, and friction applied as before. In two hours and a half, 18¾ lbs. of water actually boiled. The heat generated in this case was calculated by Rumford to be at least equal to that given out, during the same time, by the burning of 9 wax candles, ¾ inch in diameter, and each 245 grains in weight. A remarkable instance of the excitation of heat by friction is afforded by an experiment of Sir Humphrey Davy, in which two pieces of ice rubbed together in vacuo at a temperature below 32° were melted by the heat developed at the surfaces of contact.

737. **Circumstances which vary the quantity of heat developed by friction.**—The quantity of heat developed by friction depends, 1st, on the nature and state of the surfaces (138); 2d, on the pressure; 3d, on the velocity.

738. **Illustrations and application of the heat developed by friction** are of frequent occurrence in common life.

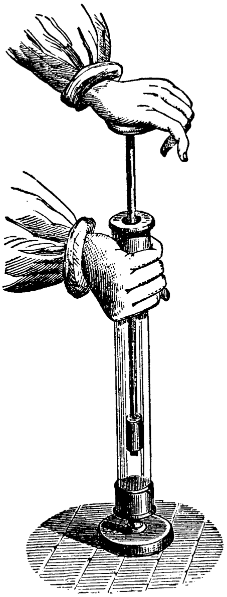
When a piece of steel is struck by a flint, particles of the metal are torn off, and are so intensely heated as to ignite in the air. These heated particles falling upon tinder or gunpowder cause it to burn. Similar sparks often fly off from the iron shoes of horses as they strike a stone. In grinding knives and other instruments upon a dry grindstone, or upon an emery wheel, a brilliant train of sparks is produced. Among uncivilized nations fire is frequently produced by the friction of pieces of wood against each other. Seneca relates the same fact, and adds that it is necessary to employ particular species of wood; as, laurel and ivy.

Sufficient heat is caused by rubbing a match on a rough surface to ignite the phosphorus on its end. The axles of car wheels, and other parts of machinery, when not well greased, are sometimes heated sufficiently hot to cause the ignition of the surrounding woodwork. It is by friction that the brown rings sometimes seen on wooden articles, turned in a lathe, are produced. A pointed piece of wood is held against the rapidly revolving article, the heat generated by the friction is sufficient to cause the wood to smoke and partially char it.

In a few instances in this country the fall of water has been used to produce friction, and thus develop heat. In the state of Vermont, plates of iron were rapidly revolved against each other, and, by the heat developed, the mill was warmed. The thermogenic apparatus of Messrs. Beaumont and Mayer (Am. Jour. Sci. [2] XX., 261) is a most successful contrivance for converting motion into heat by means of friction, and where there is abundant and cheap water-power, may be of economical importance as a source of heat.

739. **Compression.**—When any substance undergoes a diminution in volume, there is generally a development of heat. The evolution of heat by compression is most strikingly seen in gases which undergo a great diminution in volume by pressure.

518



The condensing syringe, fig. 518, is an admirable instrument for showing the phenomenon referred to. It consists of a metal or glass tube closed at one extremity. Into the other extremity fits tightly a piston which has a bit of tinder on its end. When the piston is forcibly driven into the cylinder, the compression of the air develops so much heat that the tinder becomes ignited.

Owing to the small compressibility of liquids, and their great capacity for heat, it is not easy to determine the heat developed in them by compression. Messrs. Colladon and Sturm have obtained, with certain liquids, at a pressure of 30 atmospheres, an elevation of temperature of as much as from 7° to 10° F. The heat generated by the compression of solids may better be considered as by percussion.

740. **Percussion** is a combination of friction and compression, and is an active mechanical source of heat. The amount of heat developed by percussion seems to depend to a great extent on the diminution in bulk which the body struck undergoes.

This is strikingly shown by an experiment of Berthollet, in which a piece of copper was submitted to the action of a stamping-press. The greatest development of heat occurred with the first blow, where the metal underwent the greatest diminution in bulk, and diminished with the succeeding blows as did the amount of condensation. The quantities of heat evolved at the first three strokes were 17°·3, 7°·5, and 1°·9 F.

741. **Capillarity.**—Pouillet has shown that the simple act of moistening any dry substance is attended with a slight, yet constant, disengagement of heat.

Pouillet operated on the powdered metals, the insoluble oxyds, glass, brick, clay, &c. The liquids used were water, alcohol, ether, acetic acid, turpentine, &c. The rise in temperature was only from 5° to 2° F. It appears to be independent of the nature of the body. Organic bodies of various kinds were operated upon; as, flax, wool, silk, starch, wood, sponge, ivory, horn, &c.: with these there is a rise in temperature of from 3° to 18° F.

These results cannot be attributed to chemical action, for the different liquids produced the same heat when they were absorbed by the same porous body.

The development of heat in these cases is attributed to the condensation of the liquid on the surface of the solid which it moistens. It may also be due in part to the effect of the friction of the liquid molecules upon those of the solid, as they move into their position of equilibrium.

II. PHYSICAL SOURCES OF HEAT.

742. **The sun** is the most abundant source of heat to our globe. Its distance from the earth is 95,000,000 of miles. The diameter of the sun is about 888,000 miles, or about 111 times that of the earth, consequently its volume is 1,400,000 times the earth's volume. The sun turns on its axis once in about 25 days. Philosophers are divided as to the cause of the immense amount of heat which escapes from this body.

It is conjectured that there are three atmospheric strata about the sun. That nearest his surface is called the cloudy stratum. It is incapable of reflecting light, and is heavily loaded with vapors. The next in elevation is thought to consist of an intensely luminous medium. To this is attributed the diffusion of light and heat. Beyond this there probably exists a third envelope of a transparent gaseous nature.

Dark spots are often seen on the sun's surface (by the aid of a telescope). These are of immense size, and often rapidly change their form. One noted by Sir John Herschel contained an area of 400,000,000 square miles. These spots are supposed to be formed by the opening and dispersion of the stratum of luminous clouds, revealing the dark mass within.

743. **Quantity of heat emitted by the sun.**—Pouillet, by means of an instrument called a pyrheliometer, has made observations from which he estimated that the amount of heat annually received by the earth from the sun, would be sufficient to melt a crust of ice surrounding the earth 101 feet thick. The atmosphere absorbs nearly 40 per cent. of the heat of the sun's rays.

From the size of the earth, and its distance from the sun, it has been determined that the entire amount of heat emitted by the sun, is 2,381,000,000 times as great as the heat which it gives to the earth; and it is calculated that the intensity of heat at the surface of the sun, is seven times as great as the heat of an ordinary blast furnace.

The fixed stars, the suns of other systems, notwithstanding their great distance, exert a very important influence upon the temperature of the earth. It has been estimated that they furnish to our earth four-fifths as much heat as the sun; and that, without this addition to the sun's heat, neither animal nor vegetable life could exist upon the earth.

744. **Extremes of natural temperature.**—Captain Parry, in 1819, found at Melville Island, a temperature of —59° F., and Captain Black, at Fort Reliance, at 60° 46′ N. latitude, observed a temperature of —70° F. Dr. Azariah Smith records the extreme heat at Mosul, Western Asia, in 1844, as 114° F. (Am. Jour. Sci. [2] ii., 75.) At Bagdad, in 1819, the thermometer rose to 120° F. in the shade. In the sun, at

Mosul, near the site of the ancient Nineveh, Dr. Smith records the summer temperature at 146° F. This is probably the highest natural temperature authentically recorded. Thus, the extreme range of natural temperature observed is 206°·46 F. In this latitude, between summer and winter, there is often a difference of 110° F., and in the shade, comparing the temperature in the sun of summer, there would be an increase of at least 30°.

745. **Terrestrial radiation.**—The heat which the earth receives from the sun, does not penetrate more than from 50 to 100 feet. At Paris, this stratum (called the first stratum of invariable temperature) is found at a depth of 86 feet. Descending into the earth, below the point of constant temperature, there is a gradual and regular increase of temperature. The amount of this increase is about 1°·8 for every hundred feet of descent.

Observations on this point have been extensively made in deep mines and Artesian wells, and always with a nearly constant result. The variations undoubtedly arising from the nature of the soil, and its conductibility for heat. Water from Artesian wells has always a higher temperature than surface water. Thus, the water arising in the Grenelle Artesian Well near Paris, from a depth of about 2100 feet, has a temperature of 86° F. At Neusalzwerke, in Westphalia, is a well 2200 feet deep. The water rising from it has a temperature of 91° F. Compare § 204.

Assuming the ratio given above for the increase in temperature as we descend into the earth, at the depth of two miles water would boil; at about 23 miles, or only $\frac{1}{160}$ of the earth's radius, there would be a temperature of 2200° F. At this temperature cast-iron melts in the open air, and trap, basalt, obsidian, and some other rocks, become perfectly fluid. But, as Pouillet observes, the enormous compression produced by the weight of the upper strata resting upon the lower portions of the earth's crust, raises the point of fusion, so that the point of perfect or partial fluidity is carried far lower than a direct ratio would give; but, with due allowance for the effect of pressure upon the temperature of fusion, the thickness of the earth's crust cannot be supposed to exceed one hundred miles.

746. **Origin of terrestrial heat.**—Numerous theories have been advanced to account for terrestrial heat. Some attribute the heat to local chemical action. Thus, Boyle explained it by the decomposition of pyrites—a view no longer esteemed tenable. The belief in a central fire within the earth, now generally entertained, is found in the mythology of many nations, originating, most likely, in observation of volcanic fires. For evidence that the earth was once a fluid mass, see §§ 91 and 102. This question is of the highest geological interest, and its discussion must be referred to treatises on that science.

747. **Atmospheric electricity.**—Another source of heat is atmospheric electricity, the origin of which is, at present, shrouded in mystery. The more usual form in which its calorific powers are presented

to us, is seen in the effects of a powerful flash of lightning, which not unfrequently fuses metals and earthy matter with which it comes in contact.

III. CHEMICAL SOURCES OF HEAT.

748. **Chemical combination.**—When two substances enter into chemical combination, there is generally an elevation of temperature, but sometimes also a depression.

Where there is a slow and gradual chemical combination, the development of heat cannot always be appreciated. The same amount of heat is developed as if the combination took place quickly, but, being extended over a greater time, it is inappreciable at any single moment, and cannot be accurately measured.

Chemical combination sometimes takes place at the ordinary temperature; but it is often necessary to heat the bodies before they will unite. An example of the first class, is the mixture of sulphuric acid with water, or the slaking of burnt lime,—in both cases a large amount of heat is developed. As examples of the second class, are wood, sulphur, and phosphorus, which do not inflame at the ordinary temperature.

749. **Combustion.**—When the heat developed by the chemical combination of two bodies produces luminosity, the bodies are said to *burn*, and the phenomenon is called *combustion.* If one of the bodies burning is solid, it is called *fire;* if gaseous, *flame.* As bodies are usually burned in the atmospheric air, the term combustion has come to be restricted, in a popular sense, to the union of bodies with oxygen, developing light and heat.

In a chemical sense, however, the term combustion has a wider range, and refers, generally, to chemical union, even when the bodies combining together do not evolve either light or sensible heat. Thus, iron slowly rusts or oxydizes in the air, wood gradually decays; these, to the chemist, are as truly cases of combustion, as those more rapid combinations with oxygen, which are accompanied by the splendid evolution of light and heat.

750. **On the cause of the heat generated by combustion**, there is a great diversity of opinion. According to the dynamical theory of heat, it is the vibratory motion of the constituent atoms of the bodies, as they combine together, that produces the rise in temperature.

When the state of aggregation of one or both of the bodies combining is changed, the heat which was latent becomes sensible; or if there is a depression of temperature, as is sometimes the case, a portion of the sensible heat becomes latent. When there is no change in the state of aggregation of the bodies combining, it is explained by the specific heat of the compound being less or greater, according as there is a depression or elevation of temperature.

751. **The amount of heat developed by chemical action**, is of great practical importance, and has, for a long time, engaged the attention of physicists. The first experiments upon this subject were made

in 1790, by Lavoisier and Laplace, by means of their ice calorimeter Count Rumford, in 1814, Welter, in 1822, and Despretz, in 1823, are among those who have contributed valuable researches upon this subject. Compare § 715. The most elaborate series of experiments upon this subject, was made by Favre and Silbermann, in 1844; a portion of their results is found below.

The thermal unit is the heat necessary to raise a weight of water, equal to that of the combustible, one degree of the scale of Fahrenheit's thermometer.

HEAT DEVELOPED BY BURNING DIFFERENT SUBSTANCES WITH OXYGEN.

Names of substances.	Formulæ.	Quantity of heat emitted by one of the combustibles.
Hydrogen		62031
Oxyd of carbon	CO	4325
Marsh gas	C_2H_4	23513
Wood charcoal		14544
Natural graphite . . .		14006
Diamond		14184
Sulphur		4032
Olefiant gas	C_4H_4	21344
Good coal		10800
Sulphuric ether	C_4H_6O	16248
Wood spirit	$C_2H_4O_2$	9552
Alcohol	$C_4H_6O_2$	12931
Stearic acid	$C_{38}H_{38}O_4$	17676
Acetic ether	$C_8H_8O_4$	11326
Beeswax		18892
Essence of turpentine . .	$C_{20}H_{16}$	19533
Dry wood		6480
Peat		4860

The quantity of heat disengaged during the combustion of an elementary body, is the same, whether it attains at once its maximum of oxydation, or at a number of times.

Thus, carbon disengages a certain amount of heat during its conversion into carbonic acid. The same amount of heat is evolved when it is first converted into carbonic oxyd and afterwards burned to convert it into carbonic acid.

752. **The pyrometrical heating effect of a substance,** is the intensity of the heat evolved during its combustion. This varies much with different substances, and depends not only on their composition, but also on their state of aggregation. The conclusions of an economic character derived from this subject, are as follows:—

1st. *The pyrometrical heating power of carbon is greater, and that of hydrogen smaller, than those of any other combustible.*

2d. *The pyrometrical heating power of the ordinary fuels, composed* *of carbon and hydrogen, is greater in proportion to the amount of carbon they contain.*

3d. *The pyrometrical heating powers of different fuels are much greater in oxygen than in air. Thus, between carbon and hydrogen burned in oxygen, there is a difference of* 12,000° F., *in air only* 1500° F.

753. **Relative value of fuel.**—In the following table is given the absolute, specific, and pyrometrical heating effects of different combustibles:—

TABLE SHOWING THE HEATING EFFECTS OF DIFFERENT BODIES BURNED IN AIR.

	Heating effect.		
	Absolute.	Specific.	Pyrometrical.
Hydrogen	0·23	0·0077	2900°
Gaseous combustibles . .	0·80—0·25	0.00010—0·00027	1850—1150°
Vegetable oils, &c. . . .	1·15—1·22	0·30	
Sulphuric ether	1·02	0·21	
Wood	0·36—0·47	0·14—0·28	1575—1750°
Peat	0·37—0·65		1575—2000°
Lignite	0·43—0·85		1800—2200°
Bituminous coal (5 p. c. water, 5 p. c. ash.) . .	0·79—0·96	1·06—1.44	2200—2350°
Peat charcoal	0.33—0·85		2050—2350°
Wood charcoal	0·64—0·97	0·10—0·20	2100—2450°
Coke (not more than 5 p. c. ash.)	0·84—0·97	0·38—0·46	2350—2450°

The lighter woods burn more quickly, with a greater flame, and more intense heat than the more dense woods. The amount and intensity of the heat from the different fuels depends, to a great extent, on their state of dryness.

754. **Combinations in the humid way.**—Messrs. Hess, Andrews, and Graham have made important researches upon the heat evolved in combinations in the humid way. Their principal results may be summed up as follows:—

1st. *Equivalents of the different acids combining with the same base, produce the same quantity of heat.*

2d. *Equivalents of the different bases combining with the same acid, produce different quantities of heat: generally the more energetic bases disengaging the most heat.*

3d. *When a neutral salt is converted into an acid salt, there is no disengagement of heat.*

4th. *When a neutral salt is converted into a basic salt, there is a disengagement of heat.*

755. **Animal heat; warm and cold-blooded animals.**—The temperature of organized beings is seldom that of the medium which sur

rounds them. There is within the living body a series of chemical actions taking place, and these are sources of heat.

In warm-blooded animals, as the mammifers and birds, the heat produced at each instant compensates for that lost from the exterior, and thus the body is kept at a uniform temperature. In cold-blooded animals, as reptiles, fishes, and mollusca, heat is also generated, but so slowly, that their temperature is but a very few degrees above that of the surrounding medium, and often is only equal to it.

756. **The cause of animal heat**, it has long been conjectured, was the chemical action taking place within the body.

Crawford appears to have been the first to advance the doctrine that respiration was the cause of animal heat. Lavoisier supposed that the air underwent in the lungs a real combustion; its oxygen combining with the carbon and hydrogen of the blood, forming carbonic acid and the vapor of water. The lungs, according to this view, were the furnaces, the arterial blood carrying the heat developed by the combustion into all parts of the body. This view of Lavoisier has been essentially modified. A new theory, founded upon the researches of Spallanzani and Magnus, is now generally received. They determined that the arterial blood contained a large quantity of oxygen, and the venous blood a large quantity of carbonic acid. It has been concluded that the venous blood reaches the lungs loaded with carbonic acid, which, by endosmose, traverses the humid walls of the pulmonary cells, and passes into the air. Carbonic acid is thus exhaled, and oxygen is absorbed. Becoming then arterial blood, the blood is forced through the arteries into the capillaries of the different organs, where a more or less considerable combustion of carbon takes place.

It has not as yet been demonstrated, that the hydrogen of the blood combines with the oxygen of the air. Indeed, most physiologists think that the vapor of water exhaled in respiration, is simply an evaporation from the lungs.

757. **Temperature of vegetables.**—As the plant is the seat of numerous chemical actions, so also it is a source of heat. The temperature of plants is, in general, from 0°·9 to 1°·1 higher than that of the surrounding air. In a few exceptional cases, it is much higher. Thus, the *Arum cardifolium* of the Isle of France, at the time of blossoming, reaches a temperature of 120°·2 F., while that of the air is about 67°. Plants attain their highest and lowest temperatures some hours later than the maxima and minima of daily temperature.

§ 13. Correlation of Physical Forces.

I. MECHANICAL EQUIVALENT OF HEAT.

758. **Relations of heat and force.**—It is well known that there is an intimate relation between heat and mechanical force, and that one may be exchanged for the other. A given quantity of one may be converted into a determinate quantity of the other, as is shown in the case of the steam-engine, and in the production of heat by mechanical means (§§ 735-741).

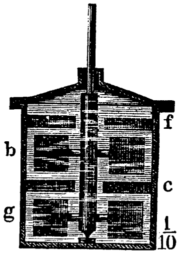
759. **Unit of measurement, the foot-pound.**—In the experiments upon the mechanical equivalent of heat, the unit adopted in England and in this country, is the foot-pound, or the mechanical force expended in raising a pound weight, one foot high (714). In France and other European countries, the unit adopted is the mechanical force expended in raising one kilogramme (2·2046 lbs.) one metre (39·37 in.) high.

760. **Determination of the mechanical equivalent of heat.**—According to the Dynamical Theory of Heat, the mechanical equivalent of heat is independent of the nature of the body by whose agency the transformation of mechanical force into heat is effected; hence the same result should be arrived at, whatever course of experiment is adopted. Mr. J. P. Joule, of Manchester, England, has made the most exact determination of the mechanical equivalent of heat in a series of very careful and elaborate experiments, conducted between the years 1840 and 1843.[*] He determined the mechanical equivalent of heat in a number of ways, reversing the question, and determining the amount of heat produced by a certain expenditure of mechanical force.

One method was by the compression of gases; compressing air with a great force in a copper receiver; in one series of experiments filled with air only, and in another with water. The whole apparatus was placed in the water of a calorimeter, whose temperature, before and after the experiment, was carefully determined.

The heat developed by the friction of water and of oil, was determined in an apparatus consisting of a brass paddle-wheel, fig. 519, having revolving arms, *b*, *g*, working between stationary vanes, *c*, *f*. This wheel was made to revolve by the descent of a known weight, and thus the mechanical force exerted was determined. A similar apparatus, of smaller size, and made of iron, was used for experiments on mercury. In all cases, the apparatus was placed in a metallic vessel filled with the liquid, and the temperature noted before and after the experiment.

519



In his experiments on the friction of solids, Mr. Joule used an apparatus consisting of a vertical axis, which carried a beveled cast-iron wheel, against which a fixed cast-iron wheel was pressed by a lever. The whole was plunged in an iron vessel filled with mercury, the axis passing through a hole in the lid. In all of these experiments, the temperatures were noted by thermometers, which indicated a variation of temperature of the one two-hundredth of a degree F.

761. **Results of Joule's experiments.**—In the following table are given the most important results obtained by Mr. Joule. The second column gives the results obtained in air, the third column, the same results corrected for a vacuum.

---

[*] Phil. Trans. 1850, p. 61.

FORCE REQUIRED TO HEAT ONE POUND OF WATER 1° F.

Material employed.	Equivalent (in foot-pounds) in air.	Equivalent in vacuo.	Mean.
Water, . . .	773·640	772·692	772·692
Mercury, . .	773·762 776·303	772·814 775·352	774·083
Cast-iron, . .	776·997 774·880	776 045 773 930	774·987

Conclusions deduced from Joule's experiments.—1. *The quantity of heat produced by the friction of bodies is always proportional to the force employed.*

2. *The quantity of heat capable of increasing the temperature of one pound of water* (*weighed in vacuo, and between* 55° *and* 60°) *by* 1° F., *requires, for its evolution, the expenditure of a mechanical force represented by the fall of* 772 *lbs. through the space of one foot.*

Consequently a force of one horse power (714) would raise 42·7 lbs. of water 1° F. each minute, and would bring it to boil from 60° in two and a half hours. Prof. Thomson (Phil. Mag., Feb. 1854) says, it is mathematically demonstrated from the dynamical theory of heat, that any substance may be heated 30° F. above the atmospheric temperature, by means of a properly contrived machine driven by an agent, spending not more than one thirty-fifth of the energy of the heat communicated, and that a corresponding machine, or the same machine worked backwards, may be employed to produce cooling effects.

When a body is heated by such means, $\frac{34}{35}$ of the heat is drawn from surrounding objects, and $\frac{1}{35}$ is produced by the action of the agent.

II. DYNAMICAL THEORY OF HEAT.

762. **The dynamical theory of heat**, which rests upon the supposition that heat is motion, or the result of motion, is founded upon the constant relation which exists between heat and mechanical force.

763. **Motions of the molecules.**—In this theory it is assumed that the particles of all bodies are in constant motion, and it is this motion which constitutes heat; the kind and quantity of the motion varying with the solid, liquid, or gaseous state of the body.

Thus in *solids*, it may be assumed that the molecules are continually oscillating about their position of equilibrium. This motion may be vibration of the constituent atoms of a molecule, or of the entire molecule, and may be rectilinear or rotary.

In *liquids*, the molecules have no constant position of equilibrium, the repulsive and attractive forces being nearly equalized. The movements of the liquid molecules may therefore be either vibratory, rotary, or progressive.

In *gases*, the repulsive force predominating, the molecules move onward in straight lines.

764. **Changes in the state or volume of bodies.**—This view

explains the production and consumption of heat, which accompany changes of state or volume in bodies. The work performed is partly internal and partly external.

Thus when a solid is melted, there is an internal work, employed in changing the relative position of the molecules, and in consequence, an absorption of heat proportional to the work accomplished. In evaporation there is an internal work, employed in separating the molecules, and an external work in overcoming the forces which oppose themselves to the expansion of the vapor.

When, on the contrary, a gas or vapor is liquefied by compression, the external work is supplied, and the internal work due to the cohesive force which draws the atmosphere together, is transformed into heat. Again, when a liquid solidifies, the internal work which unites the molecules is transformed into heat, and appears as sensible heat.

It is evident that this theory would modify the ideas generally received of the amount of heat in bodies. Thus the heat which is rendered latent, when a solid is liquefied, cannot be regarded simply as being insensible; it must be considered as being converted into motion.

III. ANALOGY OF LIGHT AND HEAT.

765. **Vibrations producing heat and light.**—A careful consideration of the phenomena and laws of heat has led many able physicists to conclude that heat is not, as was formerly supposed, a fine imponderable substance, but that, like light, it is a peculiar vibratory motion of the ultimate particles of bodies. The exact nature of the vibratory motion of atoms which constitutes heat is more difficult to determine.

The polarization of heat is best explained, like the polarization of light, by the theory of transverse vibrations. On this theory:—*Heat and light are different effects produced by one and the same cause, and they differ physically only in the rapidity and amplitude of their vibrations.* While the phenomena of light are due to vibrations whose utmost range of velocity is comprehended within the limit of an octave in music (531), vibrations of less rapidity and greater amplitude produce heat, while the vibrations which produce light, also in their turn produce the phenomena of heat.

766. **Impressions of light and heat.**—It is natural to admit that the more rapid vibrations of ether are generally those which have the least amplitude. In fact this result is deduced from an examination of the spectrum, which presents a more feeble illumination in the blue and violet. It is the same with sounds. The more acute sounds have generally the least intensity, while the bass notes are more prolonged. As grave sounds have little intensity, because their amplitude is great, so with vibrations of the luminiferous ether, we observe that the extreme red of the spectrum has but little brilliancy.

Vibrations impressed upon the air by sonorous bodies may produce upon us two sorts of sensations; the one perceived by a special organ, the ear, when the vibrations are sufficiently rapid, the other affecting the entire surface of our bodies, producing that general trembling which results from energetic vibrations, as in the case of thunder or the roar of cannon. Only grave sounds correspond to vibrations of sufficient amplitude to produce this general effect. On the other hand, vibrations of too great rapidity, and consequently too little amplitude, may fail to affect even the ear, as is the case with vibrations exceeding 36,500 per second, § 378. We may consider the same vibrations communicated to the ether by the molecules of luminous bodies as giving rise to two sorts of impressions; the one peculiar to the organ of vision, the other affecting the whole surface of the body; the former constituting the impression of light, the other giving the impression of heat when the amplitude of the vibrations is sufficiently great.

But the colored rays which pertain to the extreme violet of the spectrum are produced by vibrations which are very rapid, and which consequently have very little amplitude. Such vibrations are not suited to produce the general effect which is denominated heat. But when the vibratory energy is feeble, as in the spectrum obtained from the electric light, there are more evident signs of heat in the violet portion of the spectrum. In general, the less rapid vibrations found in the yellow, orange, and red, produce heat, and even beyond the extreme red, where the vibrations are too slow to produce light, the greater amplitude of the vibrations gives them great power to produce the phenomena of heat. Compare § 463.

Aërial vibrations of great amplitude, and a moderate degree of rapidity, affect the entire system (386), and when less than 32 per second, they seldom produce the sensation of sound. So also if the vibrations exceed 36,500 per second, their amplitude is so small that no audible sound is produced.

The gradual weakening of the violet tint of the spectrum, and the existence of invisible rays beyond the extreme violet, as attested by chemical action and fluorescence, §§ 463, 533, prove the same thing in regard to light. Heat and light may therefore be regarded as different effects of the same cause.

767. **Bodies become luminous by incandescence.**—When a body is heated, the source of heat first communicates vibrations to the ether, and then to the molecules of the body. The vibrating molecules in turn react upon the ether, and excite undulations of different lengths; the longer vibrations corresponding to the calorific rays of least refrangibility, will have a greater amplitude, and will be the first to become sensible as light.

Melloni discovered that the heat rays, emitted by bodies of low temperature, are but little refracted by a prism of rock-salt, but as the heat of the body becomes more intense, and the amplitude of all the vibrations may be considered greater, the rays of heat are more refracted, the more refrangible rays appear as light, and the body becomes luminous. This result takes place at the temperature of about 947° Fahrenheit, whatever be the nature of the luminous substance. Draper formed a spectrum by means of light from a narrow opening, and examined with a lens and micrometer the positions of the dark lines of Fraunhofer (461). He afterwards employed, instead of the narrow opening, a platinum wire, the temperature of which he caused to vary by means of an electric current, more or less intense, and he found that the red part of the spectrum appeared first, and as the heat and brilliancy of the wire increased, the other colors of the spectrum successively appeared up to the violet. This result is in beautiful harmony with the theory stated above. Common observation also shows that a heated body becomes first red, then yellow, purple, and at length a full white heat. Compare ∂ 178.

768. **Heat and light produced by chemical and mechanical action.**—It is easy to understand that, in the molecular conflict which constitutes chemical action, the ether around the molecules will be violently agitated, and become the seat of undulations of different rapidity. If the chemical action is weak, the vibrations will be slow, and they will have only sufficient amplitude to be sensible, and it has been observed that the heat thus produced furnishes rays more and more refrangible as the chemical action becomes more active. When this action becomes sufficiently energetic to give to the more rapid vibrations a sufficient amplitude, light accompanies the heat.

Experiments show that the color of the luminous rays depends upon the nature of the substance from which they proceed, and it is also probable that the temperature at which light begins to appear, depends also on the nature of the substance or the color which it gives forth. We can easily understand that the nature of the molecules will affect the rapidity of the vibrations, and we may presume that, if it were possible to augment gradually the energy of the chemical action, we should find the temperature at which light begins to appear is more elevated in proportion as the color of the light which the substance affords approaches to white or violet. This conjecture is confirmed by the fact that the incandescence due to chemical action, when it is feeble, gives forth red light.

In *mechanical action*, the vibrating molecules impress upon the ether vibrations of different rapidity, and when the action is sufficiently violent, as in the shock of two flints, or in the sudden compression of a gas, light is emitted in connection with the heat. Here, again, if we could graduate the intensity of the action, we ought to obtain a color approaching more nearly to white as the mechanical action is more energetic.

We thus see how the effects which heat and light exercise upon bodies can be explained by the theory of undulations.

The phenomena of heat in the interior of bodies are more difficult to comprehend, and it is impossible to explain them by the system of emission; but by comparing them with other effects in elastic bodies, they are readily explained by the theory of undulations.

769. **Dilatation and change of state.**—The heat received by a body agitates the ether; this agitation is communicated to the molecules, and the volume of the body is increased in proportion as the amplitude of the oscillation of the molecules becomes greater. It is thus that bodies which vibrate longitudinally appear larger, and a vibrating cord appears swollen. In the same manner, obstacles opposed to vibrating parts become repelled, if they are so light as not to arrest the vibrations

This explanation leads us to a very simple and clear definition of temperature:—*Temperature consists in the vibratory state of the ether* *within the body, and its intensity depends upon the amplitude of the vibrations.*

The theory of changes of temperature is naturally explained by the tendency to establish an equilibrium between the amplitude of the vibrations of bodies near each other, through the medium of the ether which fills the space that separates them. The molecules of bodies should, therefore, be considered as in a perpetual state of agitation. There can, then, be no *absolute zero* only where there is a state of *perfect repose.*

The only difficulty in admitting the existence of such a state, is the fact that celestial space is certainly filled with agitation by the transmission, in every possible direction, of the different radiations which emanate from the multitude of stars which people space.

Change of state produced by heat.—If the motion of the molecules is sufficiently energetic, they leap out, as it were, from each other, and become independent, as a glass rod vibrating rapidly in the direction of its length, is divided into many pieces. We thus explain the phenomena of fusion.

If we revert to the theory of the mechanical equivalent of heat, we can understand how the conversion of heat into mechanical work, and *vice versa,* is a direct consequence of the preceding; for, according to this theory, heat is a species of motion, and the work which produces this motion of the ether, ought to be changed into vibrations of this latter sort; that is, it should be transformed into heat.

It is the same with the mechanical work developed by a vibrating body. The work represented is that which has been expended in putting it into vibration. The heat developed in moving bodies, by electro-dynamic induction, and the work which it represents, are all related to the same theory.

770. **Quality of heat changed by absorption and radiation.**—In all experiments upon radiant heat, it has been observed, that heat, once absorbed, retains none of the peculiarities of the source from which it was derived; but its refrangibility and other properties, when again radiated, depend only on its temperature, and the nature of the body from which it is again emitted.

Heat, transmitted through diathermanous bodies, appears to be sifted, or to leave behind some of those rays which are transmitted with difficulty through that substance; so that a larger percentage of the remaining heat will be transmitted through another similar screen.

Even rock-salt, generally considered colorless for heat (646), has been found, by the later researches of Prof. Forbes, to transmit a somewhat greater proportion of heat of high temperature than of heat of low temperature.

It is well known that heat of great refrangibility, or small wave-length, passes more readily through glass and mica than heat having the opposite qualities. The difficulty with which heat radiated by rock-salt penetrates these substances, as compared with ordinary heat, would lead us to infer that heat from rock-salt has a greater wave-length than ordinary heat radiated from lampblack.*

---

* See an able article on radiant heat, by B. Stewart, Esq., in the Trans. of Royal Soc. of Edinburgh, Vol. XXII., part I.

771. **Difference between quantity and intensity of heat.**—Another curious fact connected with this subject is, that no amount of heat of low temperature can be so applied to an object as to raise it to a higher temperature than that of the source from which the heat emanated. Thus, the heat of the sun, when absorbed by a blackened wall, and radiated, cannot be again raised to the intensity requisite to ignite ordinary combustible substances, which are readily ignited by the direct rays of the sun concentrated by a burning-glass.

The same degradation of heat, or loss of intensity, is observed in condensing steam in distillation. The whole heat of the steam, both latent and sensible, is transferred without loss to perhaps fifteen times as much condensing water; but the intensity of the heat is reduced from 212° to perhaps 100° F. The heat is not lost; for the fifteen parts of water at 100° are capable of melting as much ice as the original steam. But by no quantity of this heat at 100° can temperature be raised above that degree; no means are known of giving it intensity.

If heat of low, is ever changed into heat of high intensity, it is by mechanical means, as by the compression of gases or vapors to a smaller volume, when the temperature is elevated; but this is rather the conversion of mechanical force into heat, than the elevation of the intensity of heat previously existing as such. Graham's Chemistry, Vol. I., p. 100.

It is stated that Dr. Wollaston received the beam of the full moon, concentrated by a powerful lens, in his eye, without feeling the least heat. Melloni obtained only an extremely feeble indication of heat, by concentrating the rays of the moon by a lens over three feet in diameter, and directing the brilliant focus of light upon the face of a very sensitive thermo-multiplier. This may merely show that the heat reflected or radiated by the moon, has become heat of too low intensity to pass through a glass lens, or to warm bodies at the ordinary terrestrial temperature.

All these phenomena are more readily explained on the undulatory theory, than by the theory of emission.

772. **Conclusion.**—We conclude, from what has been stated, that the theory of undulations, which so completely explains the phenomena of heat and light, as well as the different sensations produced upon our organs by the two sorts of radiations, may also enable us to compute, with a little uncertainty in some cases, the different effects which heat and light exercise upon bodies. We see that heat and light are due to the same cause, *to ethereal vibrations;* and that the same vibrations also produce the two sorts of effects when their amplitude is sufficient, and their rapidity comprised between certain limits.

It remains only to explain, by these movements of the ether, the numerous and complex phenomena which are presented to us by electricity.

It is possible that these effects are produced by either *longitudinal* or *rotary* vibrations, which accompany the transverse vibrations corresponding to light and heat.

But, while it is very easy to understand the facts relative to the propagation of electricity, it is somewhat difficult to conceive how vibratory movements produce attraction and repulsion. We ought not to regard this difficulty as insurmountable, especially when we remember that polarization was, for a long time, considered inconsistent with ethereal vibrations, until the idea of transverse vibrations dissipated the objection, and gave new clearness to the whole series of phenomena.

If this difficulty were once conquered, there would appear a possibility of uniting to the system of ethereal vibrations, the grand phenomena of universal gravitation, which has been attempted hitherto without success.

But, when all the phenomena of nature, in their infinite variety, are reduced to one and the same cause, wonderful simplicity will be joined to the idea which we form of the power and majesty of the GREAT AUTHOR of all things.

To bring the detailed study and interpretation of facts to prove this grand unity of cause, is the mission which science should propose to herself at the present day.

This close correlation of physical forces, is in harmony with recent philosophical views entertained by many of the first Physicists of our time, but by no one more felicitously expounded than by Prof. Grove.*

A full and satisfactory discussion of this subject will be found in the excellent Traité de Physique of Daguin (Vol. III., 1859), from which the foregoing is condensed.

Problems on Heat.

Thermometers.

209. What number of Centigrade and Reaumur degrees correspond to the following temperatures in Fahrenheit's degrees?

Melting-point of mercury,	—40° F.
“ “ bromine,	— 4
“ “ white wax,	+158
“ “ sodium,	194
“ “ tin,	442·4
“ “ antimony	771·8
Incipient red heat,	977
Clear cherry-red heat	1,832
Dazzling white heat,	9,732

210. How many Fahrenheit and Reaumur degrees correspond to the following temperatures in Centigrade degrees?

Temperature of maximum density of water,	+3°·87 C.
Boiling-point of liquid ammonia,	—40
“ “ sulphurous acid,	—10
“ “ alcohol,	+75
“ “ phosphorus,	290
“ “ mercury,	360

211. How many times must the capacity of the bulb of a thermometer exceed the capacity of the tube, in order that the thermometer may measure temperatures from 40° below zero to 500° F.?

---

* The Correlation of Physical Forces: pp. 229. London, 1855.

Expansion.

212. If rods of the following substances, iron, brass, copper, glass, platinum, silver, measure each 3 feet 2 inches in length at the temperature of 50° F., what will be their respective lengths at temperatures of 10°, 25°, 75°, and 100° F.?

213. If a glass globe holds exactly one gallon at 60° F., what will be its capacity if measured at the temperature of boiling water?

214. If a railroad is constructed in winter, when the average temperature is 25° F., how far apart must the ends of the iron rails, 18 feet long, be laid to allow sufficient room for expansion at the temperature of 120° F.?

215. What change of temperature is required to produce an elongation of 3 inches in a portion of the Britannia tubular bridge (§ 172), 917 feet in length?

216. Gas-pipes, laid 3 feet below the surface of the earth, are exposed to a change of temperature of 60° F., from summer to winter; what is the extent to which the joints (10 feet apart) will be opened in winter, if the strain is equally divided among the several joints?

217. Calculate the lengths of the steel and brass rods required to adapt Harrison's gridiron pendulum to vibrate seconds at the following places: London, Paris, New York, and St. Petersburgh.

218. Reduce the following heights of the barometer, observed at the annexed temperatures, to the equivalent heights at the freezing-point:—

1.	30·1 in.	$t = 40^\circ$ F.	5.	23·2 in.	$t = 50^\circ$ F.
2.	29·4 "	$t = 25^\circ$	6.	24·7 "	$t = 80^\circ$
3.	27·9 "	$t = 65^\circ$	7.	17·4 "	$t = 19^\circ$
4.	28·3 "	$t = 75^\circ$	8.	15·8 "	$t = 10^\circ$

219. Reduce the following barometric observations made at 8° C., to the temperatures indicated by the values of t, given below:—

1.	24 in.	reduce to $t = 30^\circ$ C.	3.	28·5 in.	reduce to $t = 55^\circ$ C.
2.	27·5 "	" " $t = 25^\circ$	4.	19·5 "	" " $t = 19^\circ$

220. A sphere of brass, 3 inches in diameter, immersed in water, is suspended from the pan of a hydrostatic balance, and counterpoised at the temperature of 60° F. What weight will be required to restore the equilibrium when the temperature of the water and globe is raised to 200° F.?

221. To what temperatures must an open vessel be heated, the pressure remaining constant, that $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the air it originally contained, may be successively driven out of it?

222. A balloon containing 1000 cubic feet of gas at 80° F., and 29 inches barometric pressure, rises to a position where the thermometer stands at 40°, and the barometer at 22 inches. Calculate the volume of the gas, supposing the capacity of the balloon to allow it to expand freely.

Specific Heat.

223. How much heat is required to raise the temperature of

50 lbs. of water	from 40° F.	to 150°?
24 " " sulphur,	" 63°	" 212°?
45 " " charcoal,	" 45°	" 930°?
25 " " alcohol,	" 35°	" 65°?
11 " " ether,	" 5°	" 132°?

224. The following quantities of water were mixed together,—2 lbs. of water at 40° F.; 5 lbs. at 65°; 7 lbs. at 70°; and 3 lbs. at 90°. What was the temperature of the mixture?

225. How much water at 200° F., and how much water at 50°, must be mixed together in order to obtain 20 lbs. of water at 85°?

226. Equal volumes of mercury at 212° F., and water at 32°, are mixed together. What is the temperature of the mixture?

227. What temperature will be produced by mixing equal volumes of mercury at 32° F., and water at 212°?

228. Five pounds of ice at 32°, are mixed with 7 lbs. of water at 200° F. What will be the temperature of the mixture after the ice is melted?

229. How much ice at 32°, must be mixed with 100 lbs. of water at 50° F., in order to reduce the temperature of the mixture to 35° F.?

230. How much ice at 32°, is required to cool 10 lbs. of mercury at 300°, to the freezing-point of water?

231. In order to determine the heat of fusion of lead, 200 ounces of melted lead at the melting-point were poured into 1850 ounces of water at 50° F. After the lead had cooled, the water was found at 20°·76 Centigrade. Required the heat of fusion of lead in degrees Fahrenheit.

232. How much heat is required to raise the temperature of a cubic foot each of air, oxygen, carbonic acid, and hydrogen from 32° F. to 75°, if the gas is allowed to expand freely, and the barometer remains stationary at 30 inches?

233. In a room 20 by 30 feet, and 10 feet high, the barometer standing at 30 inches, how many units of heat are required to raise the temperature of the air from 40° F. to 75°?

234. In the last example, how many units of heat are expended in expanding the air of the room?

Tension of Vapors.

235. Before filling a barometer with mercury, a small quantity of water was poured into the tube. How high will the mercury stand in the barometer when the temperature is 75° F., and the pressure of the air is 29 inches in an accurate barometer?

236. Solve the last problem, assuming, first, that alcohol, secondly, that sulphuric acid, and thirdly, that oil of turpentine were used instead of water.

237. Calculate the tension of the vapor of water at the following temperatures: 50°, 75°, 110°, 175°, 220°, 265°, and 300° F.

238. Determine the boiling-point of water, ether, and alcohol at the following pressures: 31 in., 29·75 in., 29·21 in., 28 in., 27·4 in., 23·7 in.

239. A cylinder is filled with steam at a temperature of 250° F., and a pressure of two atmospheres. What will be the tension of the vapor if its volume is diminished one-half by pushing down the piston? What will be the tension of the vapor if it is allowed to expand to twice its former volume?

240. If a cubic inch of water is hermetically sealed in a bomb-shell, capable of holding 200 cubic inches, and strong enough to sustain a pressure of 450 lbs. to the square inch; what temperature is required to burst the bomb-shell?

Ventilation and Warming.

241. How many flues, each six by twelve inches, and fifty feet high, are required to ventilate a lecture-room seating 1200 persons, when the temperature of the room is 70° F., and the external air at 30°, allowing each person three and a half cubic feet of fresh air per minute?

242. Repeat the calculations of the last problem, on the supposition that 1500 persons are in the room, and make additional allowance for illumination by 50 gas burners, consuming each 3½ cubic feet of gas per hour, at an expenditure of 20 feet of air for every cubic foot of gas consumed.

CHAPTER III.

ELECTRICITY.

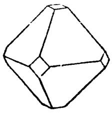
773. **General statement.**—Electricity is conveniently subdivided into, 1. Magnetic electricity or magnetism; 2. Statical or frictional electricity; and, 3. Dynamical or Voltaic electricity. We will consider the subject in this order.

§ 1. Magnetic Electricity.

I. PROPERTIES OF MAGNETS.

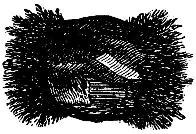
774. **Lodestone—natural magnets.**—There is found in nature an ore of iron, called by mineralogists *magnetite,* or magnetic iron, some specimens of which possess the power of attracting to themselves small fragments of a like kind, or of metallic iron. This power has been called *magnetism,* from the name of the ancient city of Magnesia, in Lydia (Asia Minor), near which the ore spoken of was first found. It crystallizes in forms of the monometric system, often modified octohedra, like fig. 520, and is a compound of one equivalent of peroxyd of iron with one of protoxyd. ($FeO + Fe_2O_3 = Fe_3O_4$.) It is one of the best ores of this valuable metal.

520

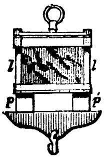


Formerly all magnets were lodestones, or natural magnets. A fragment of this ore rolled in iron filings or magnetic sand, becomes tufted, as in fig. 521, not alike in all parts, but chiefly at the ends. Fig. 522 shows a similar mass mounted in a frame, *l l*, with poles, *p p′*, of soft iron. Thus mounted, the lodestone gains in strength, by sustaining a weight from the hook below, on a soft iron cross-bar.

521



522

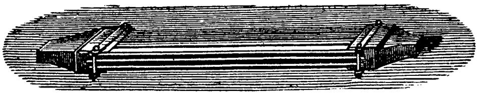


775. **Artificial magnets** are made by touch or influence from a lodestone, or from another magnet, or by an electrical current. Hardened steel is found to retain this influence permanently, while masses of soft iron become magnets only when in contact with, or within a certain distance of a permanent magnet. Artificial magnets are more powerful than the lodestone, and possess properties

entirely identical with it. Magnets attract at all distances, but their power increases, like all forces acting from a centre, inversely as the square of the distance. Heat diminishes the power of magnets, but if not heated beyond a certain degree (full redness), this power returns on cooling, and is increased at lower temperatures. Above that point, the coercitive force is destroyed, and they lose all magnetic power.

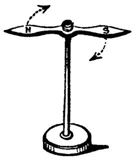
Various forms are given to magnets. The *bar magnet* is a simple straight bar of hardened steel. If curved so as to bring the ends near together, it is called a *horse shoe* magnet, and if several bars, straight or curved, are bound together into one, fig. 523, it is called a compound magnet, or magnetic battery. The most powerful artificial magnets can sustain only about twenty-eight or thirty times their own weight. Usually they sustain very much less than this.

523

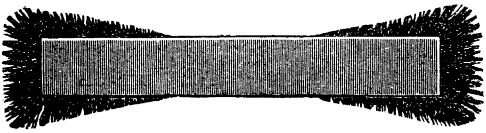


Magnetic needles are light bars, fig. 524, suspended on a central point so as to move in obedience to terrestrial or artificial attractions. The mode of making magnets, and the circumstances influencing their power, are noticed hereafter.

524



525

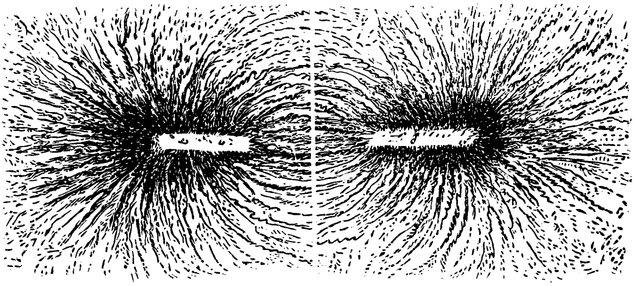


776. **Distribution of the magnetic force—polarity.**—The magnetic force is not equally distributed in all parts of a magnet, but is found concentrated chiefly about the ends, and diminishing toward the centre, which is neutral. The points of greatest attraction are called *poles*. When a magnet is rolled in iron filings or magnetic sand, the position of the poles is seen as in the bar magnet, fig. 525, whose centre is found to be quite devoid of the attracted particles which cluster about the ends. The point of no attraction is called the neutral point—line of magnetic indifference, or equator of magnetism. Every magnet has at least two poles, and one neutral point. The magnetic poles are distinguished as N or S, Austral or Boreal (A and B), or by the signs, plus (+) and minus (—), all these signs having reference to the earth's attraction, and to the antagonism between the poles of unlike name. The law regulating the distribution of magnetic force in a bar,

was determined by Coulomb, by means of the torsion balance, § 820, to be very nearly as the squares of the distance of any given point, from the magnetic equator or neutral point.

777. **Magnetic phantom—magnetic curves.**—The distribution of the magnetic force about the poles of a magnet is beautifully shown by placing a sheet of stiff paper over the poles of a horse-shoe magnet, and scattering fine iron filings or magnetic sand from a sieve or gauze bag over the paper. As they touch the surface of the paper, each filing assumes a certain position, marking the exact place of the magnetic poles and of the neutral line, as seen in fig. 526. The magnet may be laid horizontally, or a series of magnetic bars may be placed as in fig. 532, producing very pleasing and instructive results. Tapping the edge of the paper gently with the nail, or a pen-stick, facilitates the adjustment of the filings. The curves exhibited by the magnetic phantom have been mathematically investigated by De Haldat, who for that purpose transferred them to a glued paper.

526



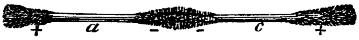
To fix the curves, Nicklés uses a waxed paper, and when the figures are produced, they may be fixed in position by holding a heated plate of iron near the surface of the paper. As soon as the wax is fused, which is easily perceived by its shining appearance, the source of heat is withdrawn, and as the wax cools the filings become fixed in position and in full relief. (Am. Jour. Sci. [2] XXX. 62.) The curves may then be more conveniently studied.

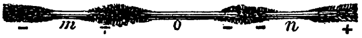
778. **Magnetic figures** may be produced on the surface of a thin steel plate, by marking on it with one pole of a bar magnet. Magnetism is thus produced in the steel along the line of contact, which is afterwards made evident by magnetic sand, or iron filings sprinkled on the plate. These lines may be varied or multiplied at pleasure, with pleasing effects; their polarity is always the reverse of that carried by the bar. They may be made even through paper or card-board, and will remain for a long time. Blows, or heat, will remove them. Hard

plate steel is best for this purpose, about one-twentieth to one-eighth of an inch thick, and six inches to twelve inches square.

779. **Anomalous magnets** are such as have more than *two* poles. Thus the bar seen in fig. 527 has a pair of similar poles (—), at the centre, and its ends are consequently similar (+), while it has two neutral points at *a* and *c*. Fig. 528 shows a bar with three sets of poles, arranged alternately — and +, with three neutral points at *m*, *o*, and *n*. Broken at these neutral points, every magnet becomes two or more separate magnets, with corresponding polarity.

527





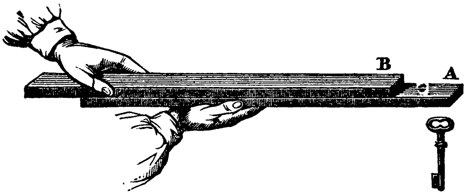
528

780. **Attraction and repulsion.**—The law of magnetic attraction and repulsion is, that *like poles repel, and unlike poles attract each other.*

If a piece of soft iron is presented to either pole of a magnetic needle, fig. 524, there is attraction, which is reciprocal between the needle and the iron; for if the iron is suspended, and the needle approached to it, the iron is attracted by either end of the needle. If, however, a magnet is approached to the needle, + to —, there is attraction; if — to — or + to +, there is repulsion.

If the unlike poles of two equal magnetic bars, tufted with iron filings, are approached, the tufts join in a festoon; but if the poles are of the same name, most of the filings fall. For the same reason, if a magnetic bar, B, fig. 529, is slid upon another bar, A, of equal power to B, as the two opposite ends approach each other, the key, previously suspended, falls, because the two bars mutually neutralize each other by the opposing action of the austral and boreal magnetism.

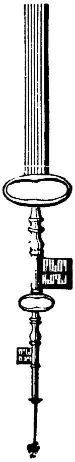
529



781. **Magnetism by contact.**—When a mass of iron, or of any magnetizable body, is placed in contact with a magnet, it receives magnetism throughout its mass, and of the same name as the pole with which it is in contact. Thus, in fig. 530, the soft iron key is sustained by the north pole of a magnetic bar; a second key, a nail, a tack, and some iron filings, are, in succession, also sustained by the magnetism imparted by contact from the bar magnet through the soft iron. The series of soft iron rings, in fig. 531, is sustained from the bar magnet under the same conditions of polarity. Tested by a delicate needle,

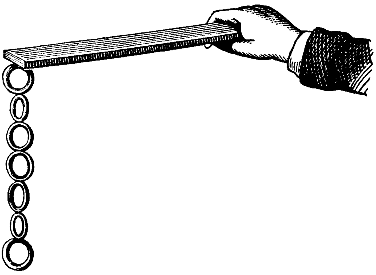
every part of the sustained masses will manifest only north polarity, and we may regard them as only prolongations of the original pole. This is analogous to electrical conduction.

530



Pure soft iron receives magnetism sooner and more powerfully than steel or cast iron, and also parts with it sooner. Hardened steel and hard cast iron retain more or less of the magnetic force permanently. No other metals beside iron, nickel, cobalt, and possibly manganese, can receive and retain magnetism by contact. These are, therefore, called the magnetic metals.

531



782. **Magnetism in bodies not ferruginous.**—Beside the magnetic metals, so called, Cavallo has shown that the alloy, brass, becomes magnetic (slightly) by hammering, but loses that property again by heat. Some minerals are magnetic, particularly when they have been heated. The pure earths, and even silica, are found to have the same property. In the case of silica, and some other minerals containing oxyd of iron in combination, this is not so surprising. M. Biot determined in the case of two specimens of mica, one from Siberia (muscovite), and the other from Zinnwald (lithia mica), that their magnetic powers were (by the method of oscillations) as 6·8 to 20, and he remarked, if the oxyd of iron be the cause of their magnetic virtue, it should exist in the minerals in the above proportion ; and curiously enough, the result of Vauquelin's analyses (then unknown to M. Biot) corresponded, almost exactly, to these numbers.

Some states of chemical combination, however, appear to destroy, or cloak, the magnetic virtues of iron ; *e. g.* an alloy of iron, one part, with antimony four parts, was found by Seebeck to be utterly devoid of magnetic action ; and the magnetic power of nickel is entirely concealed in the alloy called German silver.

The researches of Faraday have shown matter of all kinds to be subject to a certain modified degree of influence by magnetism (§ 799. *Diamagnetism*).

II. MAGNETIC INDUCTION OR INFLUENCE.

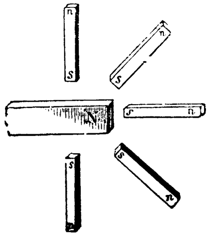
783. **Induction.**—Every magnet is surrounded by a sphere of magnetic influence, which has been called its magnetic atmosphere. Every magnetizable substance within this influence becomes magnetic also (without contact), the parts contiguous to the magnet pole, having an

opposite, and those remote from it, a similar name. This influence is called *induction*.

Thus, in fig. 532, the north end of a bar magnet induces south polarity in the contiguous ends of the five bars surrounding it, and north polarity in their remote ends. If these bars are of hardened steel, they will retain a small portion of the magnetic force induced from a powerful bar, but if they are of soft iron, they will part with their magnetism as soon as the source of excitation is withdrawn. In this case, the magnetized bars have a tendency to move up to the magnet, and are prevented from doing so only by friction and gravity. The attraction is reciprocal, and we hence infer that there is induction in every case of magnetic attraction.

In the iron filings, arranged in magnetic curves, fig. 526, on a glass plate, or card-board, the same tendency is observed.

532



Small pieces of soft iron wire suspended from the ends of a thread near, and parallel to each other, when approached by a bar magnet, receive induced magnetism, the farther ends diverging by mutual repulsion. Two sewing-needles thus suspended and influenced, become permanent magnets.

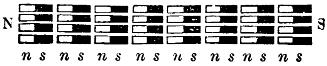
The ingenuity of the teacher will furnish many pleasing and instructive illustrations of magnetic induction.

784. **Theoretical considerations.**—The real nature of the magnetic force is unknown to us; but the analogies offered by electro-magnetism and magneto-electricity, lead to the conviction that it is one mode of electrical excitement. Unlike light, heat, and statical electricity, magnetism affords no phenomena immediately addressed to the senses. It is distinguished from statical electricity chiefly by its permanent character when once excited, and by the very limited number of substances capable of receiving and manifesting it.

785. **Theory of two fluids.**—It may be assumed that there are two magnetic or electrical fluids (the Boreal or positive, and the Austral or negative), which are in a state of equilibrium or combination in all bodies; that in iron, nickel, &c., these two forces are capable of separation, by virtue of the inductive influence of the earth, or of another magnet, while, in other bodies, this permanent separation cannot be effected. The two magnetic forces are never seen isolated from each other, but are always united in one bar. Hence, we cannot have a boreal magnet, or an austral magnet, as we may in statical electricity produce, at pleasure, vitreous or resinous excitement over the whole surface of a body. Both poles must coexist in every magnet. If we break a magnetic bar at its neutral point, we have two magnets of diminished force, but each half has its two poles like the original bar, and its neutral point also. The *anomalous* magnets, figs. 527, 528, will render this statement intelligible. Every magnet must, in this view,

be regarded as an assemblage of numberless small magnets, every molecule of steel having its own poles antagonistic to those of the next contiguous particle. This conception is rendered clearer to the senses by fig. 533. Here the N and S poles of the several particles are each represented as pointing one way respectively, and towards the N and S ends of the bar. These opposing forces, therefore, constantly increase from the centre or neutral point, where they are in equilibrium, to the ends, where they find their maximum. This arbitrary illustration enables us to conceive how such a body may excite similar manifestations of power in another, without itself being weakened, and how each part becomes a perfect magnet, if the bar is broken. The experiment shown in fig. 529, illustrates well the reunion of the two fluids to form the neutral state of the undecomposed influence.

533



De Haldat has shown that a brass tube, filled with iron filings, confined by screwed caps of brass, can be magnetized by any of the modes used for bars, and have its poles and neutral point like a bar magnet; but if, by concussion, the particles of iron are disarranged, the magnetic force diminishes, and finally disappears.

The magnetic pastes of Dr. Knight and of Ingenhausz, also illustrate the fact, that little particles of magnetic iron, or of pulverized lodestone, may determine the existence of the magnetic poles, and a neutral line, when they are compacted into a mass, by drying oils, or by the use of some gummy substance.

Even so small a quantity as one-sixth of ferruginous particles, in five-sixths of sand or earthy matter, can be magnetized as a bar, showing clearly the decomposition of the neutral fluid in each particle.

786. **Coercitive force.**—The resistance which most substances show to the induction of magnetism, has been distinguished by the term *coercitive force.* In soft iron, this force may be regarded as at a minimum, since this substance will receive magnetic influence even from being placed in the line of magnetic dip, while in steel which has been hardened, a peculiar manipulation is required to induce any permanent magnetism. Soft iron parts with its induced magnetism as readily as it receives it; but, if it is hardened by blows, or violent twisting, or by small portions of phosphorus, arsenic, or carbon combined with it, a portion of magnetism is permanently retained by it from induction.

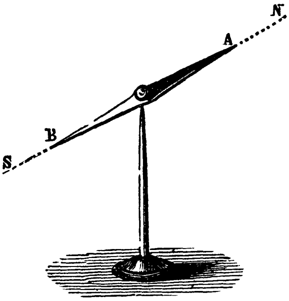
As blows, by hardening, may induce permanent magnetism in soft iron, so, in steel, the coercitive force may, by simple vibration, as by blows on a magnetic bar, or by an accidental fall, destroy a large part of the force developed, by giving opportunity to the coercitive force to resume its supremacy. In general, whatever cause induces hardness, increases the coercitive force; and, conversely, it is diminished by annealing, or any cause which results in softening the mass.

III. TERRESTRIAL MAGNETISM.

787. **Magnetic needle.—Directive tendency.**—A magnetic nee-

dle, suspended over the poles of a horse-shoe magnet, comes to rest in the plane of the poles; and, in obedience to the fundamental law of magnetic attractions, its A and B poles will be opposite to the B and A poles of the attracting magnet. The suspended needle, in fig. 534, assumes its position by reason of the same law, and comes to rest with its A pole toward the N pole of the earth, and its B pole towards the south. All bar magnets, having a free motion in a horizontal plane, arrange themselves in this manner in every part of the earth.

534

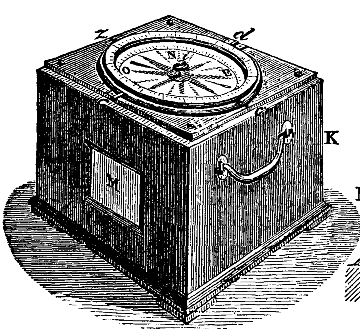


This directive tendency of the magnet has been known to European nations since the twelfth century; but was known, it is said, to the Chinese, 2000 B. C. The earliest mariner's compass, used by Syrian navigators in 1242, was a common sewing-needle, rendered magnetic, thrust through a reed or cork, and allowed to float on water. (Klaproth.) This directive power renders the compass invaluable to the explorer of a pathless wilderness, to the surveyor and the miner; the mineralogist and the physicist also find it indispensable in many researches.

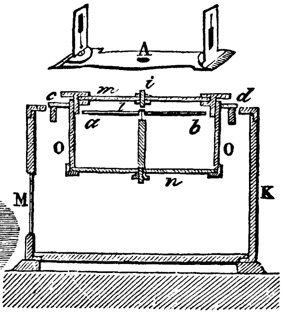
The terms *Austral* and *Boreal* have been applied to the polarity of the magnetic needle, in allusion to the free Austral and Boreal magnetism assumed to exist respectively in the southern and northern regions of the earth. In accordance with magnetic law, the end of the needle pointing north is called *Austral*, and that pointing south, *Boreal*. For greater simplicity, the mariner's compass is marked N on that point which turns to the north, and conversely; but the terms austral and boreal may be used interchangeably with positive and negative, or north and south polarity.

The mariner's compass is arranged in a box (K, fig. 535) called a binnacle, illuminated at night through the glass, M. The magnetic

535



536

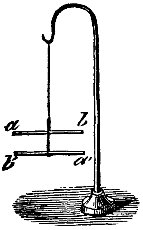


needle, *a b*, fig. 536, delicately poised on a socket of agate, is attached to the lower side of a card or plate of mica, *t*, on which is printed the star of thirty-two points,—seven between each two of the cardinal points, N., E., S., and W. The compass-box, *o o*, is hung on points called gimbals, *c d c z* (pronounced *gimbles*), which allow it to remain always horizontal, however the ship may roll. The transom or cross-sights, A, may be placed at pleasure on the face, *m*, of the compass, when the object is to measure points on the coast. Both parts of the figure are similarly lettered.

The astatic needle is an instrument in which the directive tendency of the earth's magnetism is neutralized, by placing two equal needles, *a b*, *b′ a′*, fig. 537, parallel, one above the other, with their unlike poles opposed to each other. This system is suspended by a fibre of raw silk, and is a most sensitive test for feeble magnetic currents. Such is the construction adopted in the galvaniscope, to be hereafter described. The two needles must be of exactly equal force, or *a b* and *a′ b′* will not neutralize each other, and the system will have a directive tendency, equal to any difference of force in the two needles.

The most simple astatic needle is made by touching a steel sewing-needle, at its centre of weight, by the N. pole of a powerful magnet; the point touched develops two S. poles, and the two ends are N. Such a needle is very nearly astatic.

537



788. **Magnetic meridian—declination or variation.**—There are but few places in the world where the magnetic needle points to the true, or astronomical North; and in all other places, a plane passing through the axis of the magnetic needle (the magnetic meridian), fails to coincide with the geographical meridian. Moreover, the magnetic meridian in any given place is not constant, but changes slowly from year to year (called *secular variation*), being now on the E., and again on the W. side of the true North. This is called the *declination* or *variation* of the magnetic needle. The declination is called Eastern, or Western, according as it may be to the East or to the West of the astronomical meridian. The angle formed by the meeting of the true and the magnetic meridians is called the *angle of declination.* Thus, at Washington City,* the angle of declination in 1855-6, was 2° 36′ W., and at New Haven it was 6° 37′·9 W., August 12, 1848. Js. Ruth, observer.

Columbus, in his first voyage to America, found the needle to have, as he sailed westwards, an increasing variation from the true North, a circumstance not before observed, and which caused the greatest consternation in his superstitious crew, "who thought the laws of nature were changing, and that the compass was about to lose its mysterious power." (Irving's Columbus.) Notwithstanding these and other similar observations, it was not until the middle of the seventeenth century, that the variation of the compass was an established fact in magnetic science. The observations on the declination of the compass in England. date from 1580. The following table, from Harris, contains the declination with the mean rate of motion, as referred to certain periods of observation in London, between 1580 and 1850, or about two hundred and seventy years. Eastern declination being distinguished by the negative sign, and western by the positive sign.

---

* U. S. Coast Survey Report, 1858, 196. C. A. Schott, Observer.

	Eastern Declination.		Zero.			Western Declination.		
Years,	1580.	1622	1660	1692.	1730.	1765	1818	1850
Declination,	—11° 15′	—6°	0	+6°	+13°	+20°	+24° 41′	+22° 30′
Rate per year,	7′	8′	10′	11′	11′·5	9′	0′	5′

Thus, in a period of eighty years from the first observation, the needle gradually reached the true meridian, and then, for a period of one hundred and fifty-eight years, it moved Westward, reaching its maximum Westerly declination in 1818, and it is now again slowly moving Eastwards. The rate of this movement is not uniform, but is greater near the minimum, and least near the maximum, point of declination.

Observations since 1700 establish the same facts in the United States, at a great number of places. Thus, at Burlington, Vt., in 1790, the declination was +7°·8; in 1830, +8°·30; in 1840, +9°·07; and, in 1860, +10°·30. In Cambridge, Mass., in 1700, it was +9°·9, and steadily diminished to 1790, when it was +6°·9, and has since regularly increased to the present time, being, in 1855, +10°·90. At Hatborough, Pa., in A. D. 1680, the declination was +8°·5; in 1800 it had, by a regular rate, decreased to +1°·8, and, in 1860, was +5°·32. At Washington, D. C., it was +0·6 in A. D. 1800, and in 1860 had increased to +2°·9.

South of Washington, the declination is uniformly Easterly, ranging, at Charleston, S. C., from —3°·7 in A. D. 1770, to —1°·7 in 1860. On the Western Coast of North America, it is also Easterly; being, for example, at San Francisco, in 1790, —13°·6, and in 1860, —15·°8. The annual change (increasing E. declination) being, in 1840, —1′·6; in 1850, —1′·2; and in 1860, —0′·8.

For a full discussion of Magnetic Declination in the United States, the student will refer to the Reports of the United States Coast Survey; and for an able extract of all the results of secular change on the Atlantic, Gulf, and Pacific Coasts of the United States, refer to a "Report by Assistant Charles A. Schott," in Am. Jour. Sci. [2] XXIX., p. 335.

The first attempt to systematize the variations of the magnetic needle, and to connect by lines, called *isogonic* lines, all those places on the earth where the declination was similar, was made by Halley, about 1700. He thus discovered two distinct lines of no inclination, called *agonic* lines, one of which ran obliquely over North America and across the Atlantic Ocean, and another descended through the middle of China and across New Holland; and he inferred that these lines communicated near both poles of the earth.

789. **Variation chart.—Isogonal lines.**—In fig. 538, is seen a projection of the lines of equal and no declination, on a Mercator's chart of the earth, embracing observations down to 1835. The Ameri-

can line of no variation, or *agone*, crosses the eastern point of South America, in latitude 20° S., skirts the Windward Antilles, enters North Carolina near Cape Lookout, and passing through Staunton, in Virginia, crosses Lake Erie midway on its course to Hudson's Bay. The chief Asiatic agone (for, in fact, there are two lines of no variation), after traversing the Indian Ocean in a southerly direction, crosses the western part of New Holland near 120° E. All the entire lines on this chart indicate western declination, while the dotted lines mark eastern declination. According to the theory of Gauss, the eminent German astronomer, no lines of equal variation can form diverging branches, or be tangents to each other; but when there is a space within which the declination is less than outside any portion of its limiting line, that line must form a loop, the two branches intersecting at right angles. The observed line of 8° 40′ in the Pacific, beautifully illustrates and confirms this theoretical position, as shown on the chart, fig. 538.

538

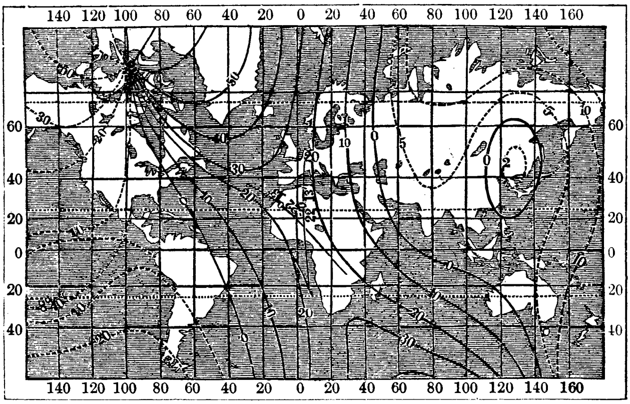
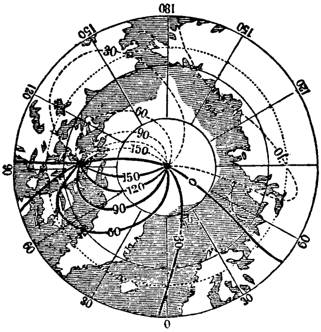


Figure 539 illustrates the circumpolar relations of the corresponding lines of equal variation in the northern hemisphere. It will be seen that much the larger number of the isogonal lines, converge on the Mercator's projection at a point near Baffin's Bay, in lat. 73°·0 N., long. 70°·0 W., its opposite pole is to the southward of New Holland.

539



Halley's original chart assumes the existence of two magnetic poles in each hemisphere, one fixed, and the other revolving about it in a certain period. Hansteen, in 1828, in his well-known chart, accepts the same view. By Gauss's theory of terrestrial magnetism, only one magnetic pole in each hemisphere is required, and thus far observation has shown a wonderful conformity between the theory of Gauss and the facts.

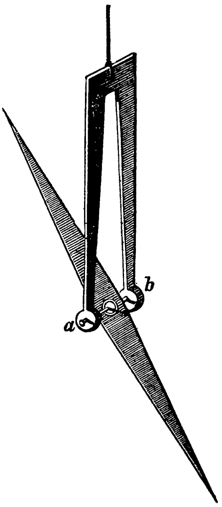
790. **Daily variations of the magnetic needle.**—Besides the great secular movements of the magnetic needle already noticed (788), it is found to vary sensibly from day to day, and even with the different periods of the same day. The most refined means have been in our time applied to the exact investigation of this phenomenon, first noticed by Graham, a London optician, in 1722. It has been shown that the north pole of the needle begins between seven and eight A. M. to move westward, and this movement continues until one P. M., when it becomes stationary. Soon after one o'clock it slowly returns eastward, and at about ten P. M., the needle again becomes stationary at the point from which it started. During the night, a small oscillation occurs, the north pole moving west until three A. M., and returning again as before. The mean daily change, as observed by Capt. Beaufoy, is not quite one degree. This daily disturbance of the magnetic needle is undoubtedly due to the action of the sun, and it will therefore vary in different latitudes. In the Southern hemisphere, the daily oscillations are of course reversed in direction to those of the Northern hemisphere.

The annual variation of the needle was discovered by Cassini, in 1786 We have, therefore, 1st, the great *secular variations,* continued through long periods of time; 2d, *annual variations,* conforming to the movement of the sun in the solstices; 3d, *daily variations,* conforming nearly to the periods of maximum and minimum temperature in each day, and lastly, *irregular variations,* connected with the aurora borealis, or other cosmical phenomena, which Humboldt has called *magnetic storms.*

791. **Dip or inclination.**—A needle, hung as in fig. 540, within a stirrup upon the points *a b*, the whole system being suspended by a thread, will, before magnetizing, if carefully adjusted, stand in any position in which it may be placed. If now the needle be magnetized, it forthwith assumes the position seen in the figure, its pole dipping toward the North pole of the earth. In this latitude (41° 18′), the dip was, in 1848, 73° 31′·9. Such a needle is called a *dipping needle,* and if constructed as in the figure, it shows both the declination and dip, or inclination, of terrestrial magnetism for any given locality. As the whole system is free to move, it will obviously arrange itself in the magnetic meridian, and its position of equilibrium will be the resultant of the two forces of declination and dip. Approaching the equator, the dipping needle becomes constantly less and less inclined, until at

last a point is found where it is quite horizontal, and this point will be in the *magnetic equator;* an imaginary plane near, but not coincident with, the equator of the earth.

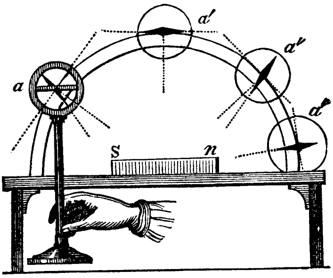
540



The discovery of the magnetic dip or inclination, was made in 1576, by Robert Norman, a practical optician of London, who constructed the first dipping needle, by which he determined the dip at London at that time to be nearly 72°. The magnetic dip, like the declination, is subject to continual and progressive changes, both secular and periodical, and it is at this moment rapidly decreasing. Thus at London in 1576 it was 71° 50′, in 1676 it had become 73° 30′, and in 1723 it was 74° 42′, having then reached its maximum. In 1790 it had decreased to 71° 3′, and in 1800 to 70° 35′. Sabine, in 1821, fixed it at 70° 3′, and Kater, in 1830, at 69° 88′. It is now, in England, about 68° 30′, having decreased in 128 years about 6° 12′, or at the rate of nearly 3′ yearly, the mean annual movement from 1830 to 1850 being at the rate of more than 4′ yearly, while between 1723 and 1790 it was about 2·5′ yearly, showing an accelerated and retarded movement in the secular changes of the dipping needle, or magnetic inclination.

792. **The action of the earth's magnetism** on the dipping needle is neatly illustrated by the simple arrangement seen in fig. 541, where the magnetic bar *s n*, is placed horizontally on the diameter of a semicircle, representing an arc of the meridian, on which a small dipping needle is made to occupy successively the position seen at *a*, *a′*, *a′′*, *a′′′*.

541

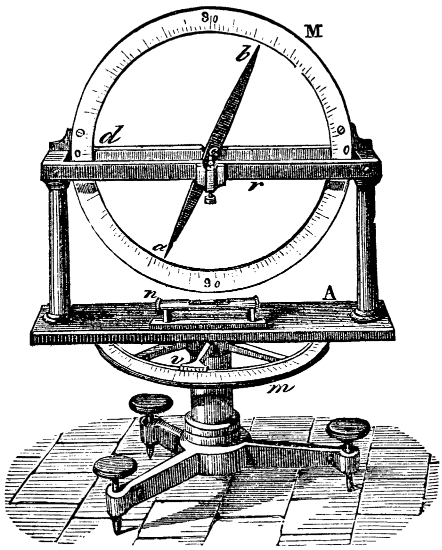


At *a′*, the needle is horizontal, being at the magnetic equator, and equally acted on by both poles. In every other position, the influence of one pole must predominate, to a greater or less extent, over the other. Several sewing-needles, suspended over a magnetic bar at equal distances, one over each end, one over the centre, and one intermediate, will illustrate the same point satisfactorily.

793. **Dipping needle.**—The dipping needle of Biot, shown in fig. 542, is wholly of brass, and embraces two graduated circles, *m* and M, one horizontal and one vertical. The circle, M, with its supporting frame, A, moves in azimuth over *m*, by which it is placed in the magnetic meridian It is leveled by the level, *n*, adjusted by three

milled heads in the feet. The needle, *a b*, is suspended on the bars, *r*. To fix the magnetic meridian by this instrument, the circle, *m*, is revolved until the needle, *a b*, stands vertical and points to 90°, it is then in the magnetic equator, a position of course exactly 90° from the magnetic meridian, which is then obtained by revolving the frame, A, 90° backwards. The angle, *a c d*, is the angle of inclination (or dip), and is read on the arc M.

542



Two small errors of observation exist in this instrument; 1st, from the fact that the magnetic axis of the needle does not coincide with the axis of its form, and 2d, from the circumstance that the centre of gravity of the needle does not lie in the points of suspension, and that therefore the angle, *d c a*, is greater or less than the true angle of inclination, by a very small quantity. The first is corrected by reversing the plane of the instrument, by a revolution of 180°, and taking the mean of the two readings; the second, by reversing the polarity of the needle by touch on the opposite poles of two bar magnets, provided for the purpose. By this means, the centre of gravity is brought, first above, and then below the point of suspension, and the mean of the two readings is the true angle sought.

794. **Inclination map, or isoclinal lines.**—In fig. 543, is presented a Mercator's projection of the line of no dip, or magnetic equator, and the position of the isoclinal lines of 30°, 50°, 70°, 80°, and 85° north, and 30°, 50°, and 70° south. It will be noticed that the magnetic is below the terrestrial equator, in all the western hemisphere, and is above it in the eastern, crossing it near the island of St. Thomas, in longitude 3° E., and again in the Pacific ocean. These points of intersection of course vary with the progressive changes of the magnetic dip. The greatest declination of the magnetic equator from the equinoctial line, amounts to about 20° N., near 53° E. longitude, and its greatest southern declination is 13°, in about 40° W. longitude, near the bay of Bahia, on the East coast of South America

The inclination of the needle at any place is, approximately, twice its magnetic latitude. (Kraft.)

543

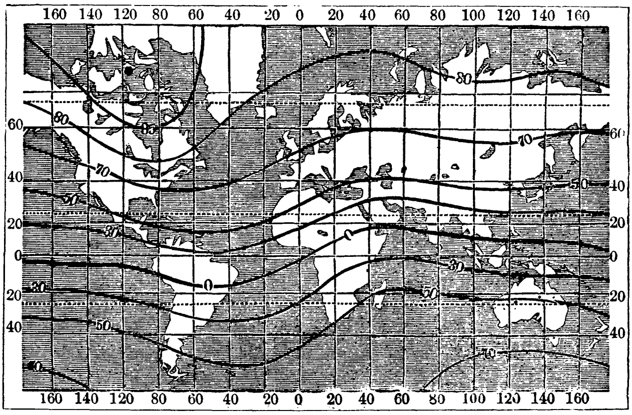
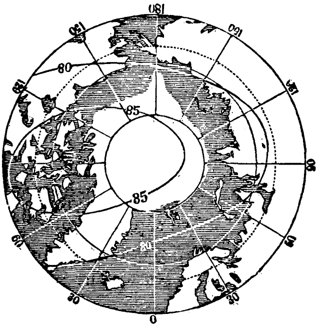


Figure 544 shows the relation of the isoclinal lines of 80° and 85° in the northern hemisphere, to the lines of latitude, and to the N. magnetic pole, near Baffin's Bay. Sir James Ross, in 1832, found the needle to dip near Prince Regent's Inlet, lat. 70° N., longitude 96° N., within one minute of 90°.

544



It is to be observed, that the lines of equal magnetic inclination (isoclinal lines), are found to approach in position, with very considerable conformity, to the isothermal lines, or lines of equal temperature, thus indicating a close relation between the earth's magnetism and the distribution of the terrestrial heat.

795. **Magnetic intensity.**—It is plain, from the phenomena of the magnetic declination and dip already considered, that the distribution of magnetic force over the earth is unequal, although in general it is most active about the poles, and least so about the equator. The question arises, how may the magnetic intensity at any given point of the earth be determined? This question is answered by the use of the *needle of oscillation.* A large number of facts serve to show, that a freely suspended needle in a state of oscillation, is influenced by the

magnetic force of the earth, in a way analogous to that of a common pendulum, oscillating by the influence of gravity; and that hence by means of such a needle, we may determine the ratio of the intensity of terrestrial magnetic force throughout the whole extent of the earth's surface.

This mode of determining the magnetic intensity in different regions of the earth, was first suggested by Graham, in 1775, and was afterwards more fully perfected and employed by Coulomb, Humboldt, Hansteen, and Gauss. Humboldt carefully determined the time of a given number of oscillations of a small magnetic needle, first at Paris, and afterward in Peru. At Paris, the needle made two hundred and forty-five oscillations in ten minutes: in Peru, it made only two hundred and eleven in the same time. The relative intensities were therefore as the square of these two numbers, or as 1 : 1·3482, which, assuming the point on the magnetic equator in Peru as unity, will give the magnetic intensity at Paris as 1·3482. This kind of observation has since been extended to nearly every known part of the globe, and full tables have been published, giving the results, Thus the intensity at Rio de Janeiro is 0·887; Cape of Good Hope, 0·945; Peru, 1·; Naples, 1·274; Paris, 1·348; Berlin, 1·364; London, 1·369; St. Petersburg, 1·403; Baffin's Bay, 1·707.

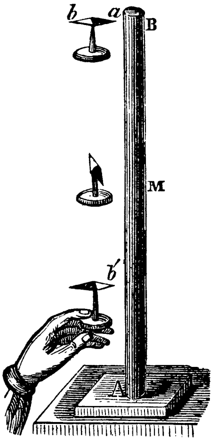
The most complete statement of the results of American observations on the magnetic elements has lately been published by Dr. A. D. Bache, in Am. Jour. Sci. [2] XXIV., p. 1, where all the earlier observations are collated, with the more extended results of the Coast Survey, with maps.

796. **Isodynamic lines**, or lines of equal power, are such as connect places in which observations show the magnetic intensity to be equal. These lines are not always parallel to the isoclinal lines, although nearly so, and the points of greatest and least intensity are not exactly identical with the points of greatest and least inclination. Hence the intensity of the magnetic equator may not be everywhere the same. These lines are probably curves of double curvature returning into themselves, implying the existence of two intensity poles, the western, near Hudson's Bay, in lat. 50° N., lon. 90° W; and the eastern or Siberian pole, about 70° N., and lon. 120° E. The two southern poles have been placed, one to the south of New Holland, in lat. 60° S., lon. 140° E.; the other, in the South Pacific, also in lat. 60° S., but lon. 120° W. These four poles are not therefore diametrically opposite to each other.

The terrestrial magnetic force increases toward the south pole, nearly in the ratio of 1 : 3, and as both the maximum and minimum magnetic intensity on the globe are found in the southern hemisphere, it would appear that the ratio of 1 : 3 expresses very nearly the maximum and minimum magnetic force of the whole earth. From the profound inquiries of Gauss, it appears that the absolute terrestrial magnetic force, considering the earth as a magnet, is equal to six magnetic steel bars of a pound weight each, magnetized to saturation, for every cubic yard of surface. Compared with one such bar, the total magnetism of the earth is as 8,864,000,000,000,000,000,000 : 1, a most inconceivable proportion. (Harris.)

797. **The inductive power of the earth's magnetism** is manifested by the polarity developed in any bar of soft iron, or of steel, placed in an erect position, as in fig. 545, or better, in the angle of the dip of the place. The end of the bar toward the earth is always Austral, Boreal magnetism existing at the upper end, B, and a neutral point at the centre, M. These facts are demonstrated by the action of a small needle, held in the hand at the three positions, shown in the figure. If the experiment were made in the southern hemisphere, the polarity would be reversed.

545



For this reason, all masses of iron standing in a vertical position become magnetic. In soft iron this magnetism is transient, but in steel tools, especially such as are subject to vibration, as drills, the magnetism developed is permanent.

Barlow found that globes of iron, like bomb shells, a foot or more in diameter, become miniature copies of the earth by virtue of the inductive force exerted upon them by the earth's magnetism; having a magnetic axis in the line of dip at the place of experiment, and an equator at right angles to their axis. Delicate needles, poised on the equatorial line of such globes, suffered no disturbance, while in any other position on the sphere, both declination and dip were manifest.

Barlow further discovered, that such a sphere of iron, placed in a certain relation to a compass needle on board a ship, united, and harmonized the local attractions of the ship's iron, so as to free the compass from the effects of such disturbing causes.

798. **System of simultaneous magnetic observations.**—The distinguished Prussian philosopher, Alex. v. Humboldt, in 1836, proposed to the scientific world to set on foot a series of connected and simultaneous observations, to be made over as large a portion of the earth's surface as possible, for the purpose of establishing the laws relating to the magnetic forces.

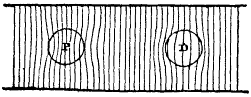
In accordance with this suggestion, the leading governments of Europe (France excepted), and many of the scientific societies both in the old and new world, commenced such observations, with instruments specially contrived for the purpose, and in buildings made without iron, both on and beneath the earth's surface. Expeditions were sent to the Arctic and Antarctic circles, to Africa, to South and North America, and to the Pacific Ocean, while at numerous stations in India, Russia, Europe, and North and South America, hourly and simultaneous observations have been carried on for a long period, and in many places are still continued. In this way a great mass of facts has been accumulated, from a careful comparison of which the laws of terrestrial magnetism already announced have been educed or confirmed.

Perhaps the most remarkable result of these observations is the fact, first established by them, that not only the greater variations in the earth's magnetism, but the most minute and irregular disturbances occur at the same instant in places the most distant from each other, showing a wonderful connection and coincidence in the causes of these phenomena throughout the world.

799. **Lines of magnetic force.**—The illustrious English philosopher, Faraday, has demonstrated that all matter is subject to magnetic influence.

As the evidence on which this important induction rests is chiefly derived from the use of electro-magnetism, its particular consideration is more conveniently referred to that subject. His general views, connected with terrestrial magnetism, may be thus stated. All space both above and within the limits of our atmosphere may be regarded as traversed *by lines of force*, among which are the lines of magnetic force. The condition of the space surrounding a magnet, or between its poles (777), may be taken as an illustration of this assumption. It is not more difficult to conceive of force existing without matter, than the converse, and it is certain that we know matter chiefly by the effects it produces on certain forces in nature. The lines of magnetic force are assumed to traverse void space without change, but when they come in contact with matter of any kind, they are either concentrated upon it, or dispersed, according to the nature of the matter. Thus we know that a suspended needle is attracted *axially* by a magnet, while a bar of bismuth, and many other solid, liquid, or gaseous bodies, similarly placed between the poles of a magnet, are held in a place at right angles to the axis, or *equatorially*. Hence all substances may be classified either as those which, like iron, point axially, and are called PARAMAGNETIC substances, and those which point equatorially, and termed DIAMAGNETIC. The force which urges bodies to the axial or equatorial lines is not a central force, but a force differing in character in the axial or radial directions. If a liquid paramagnetic body were introduced into the field of force, it would dilate axially, and form a prolate spheroid; while a liquid diamagnetic body would dilate equatorially, and form an oblate spheroid.

546



The diagram, fig. 546, will serve to render more clear the action of diamagnetic and paramagnetic substances, upon the lines of magnetic force. Thus a diamagnetic substance, D, expands the lines of force, and causes them to open outwards, while a paramagnetic body, P, concentrates these lines upon itself. Bodies of the first class swing into the equator of force, or lie at right angles to the lines of force, while those of the paramagnetic class become axially arranged, parallel to the lines of force.

800. **Atmospheric magnetism.**—The discovery, by Faraday, of the highly paramagnetic character of oxygen gas, and of the neutral character of nitrogen, the two chief constituents of the atmosphere, is justly esteemed a fact of great importance in studying the phenomena of terrestrial magnetism. We thus see two-ninths of the atmosphere, by weight, consisting of a substance of eminent magnetic capacity, after the manner of iron, and liable to great physical changes of density, temperature, &c., and entirely independent of the solid earth. In this medium hang suspended the magnetic bars, which are used as

tests, and this magnetic medium is daily heated and cooled by the sun's rays, and its power of transmitting the lines of magnetic force is thus affected, influencing, undoubtedly, those diurnal changes already considered.

801. **Notions of the origin of the earth's magnetism.**—Two hypotheses have hitherto divided the opinions of philosophers in explaining the phenomena of terrestrial magnetism.

The older of these views (Hansteen's) assumes the existence of an independent magnetism in the earth, with its focus, or seat, near the earth's centre. This internal power manifests itself chiefly at four points near the surface, two of which, at the opposite ends of the supposed magnetic axis, are the most energetic, and are known as the magnetic poles. The minor poles have their own independent axis, and move around the principal axis from west to east in the western hemisphere, and the reverse in the southern, giving origin to the well-known phenomena of the secular variation of the needle. However well this hypothesis met the facts of terrestrial magnetism some years since, the rapid progress of our knowledge of magnetic phenomena, both terrestrial and general, within a short period, has materially changed scientific opinion. The diurnal and irregular variations in the magnetic forces, cannot be explained upon Hansteen's hypothesis, and especially the simultaneous occurrence of these disturbances at different points of observation. Nearly all bodies are now known to be susceptible to magnetic influence, while the maximum and minimum magnetic intensity are found in those regions of the globe where the minimum and maximum of superficial heat exist.

It is hence now argued, that the crust, or surface, and not the interior of the earth, is the seat of the magnetic force. That this force is manifested with least energy at the equator of magnetism, and with increasing power toward the poles, where, as in an artificial magnet, it attains its maximum development, because there we find the most perfect separation of the magnetic fluids : that the coercitive force (786) of the materials of the earth's surface is resolved by the solar heat, and that the depth to which this separation occurs is closely connected with the mean heat of the earth's crust, if not absolutely dependent upon it. Axes and poles have, therefore, in view of this hypothesis, no existence in fact, but are merely convenient mathematical terms for expressing our ideas of magnetic phenomena more closely, just as in crystallography we employ the same terms for the same reasons.

In conformity to this view, the manifestation of the magnetic forces will vary with all the diurnal changes of temperature, giving the relation of cause and effect between these changes, and the magnetic perturbations. The annual fluctuations in the mean temperature of the earth's surface will, therefore, be reproduced in corresponding movements in magnetic declination and dip. Hence, the magnetic meridian, and the system of isoclinal and isogonic curves ought to correspond closely, as they do with isothermal lines, and the peculiar distribution of temperature in both hemispheres. Indeed, we may assume, should this hypothesis prevail, that the differences now noticed between the isothermes and isogones (due, probably, to imperfect observations), will vanish under new and more extended researches.

IV. PRODUCTION OF MAGNETS.

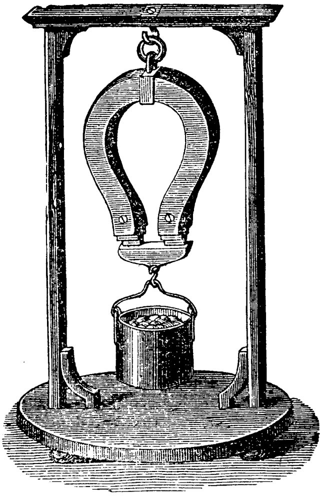
802. **Artificial magnets** are produced (1.) by touch, or friction from

another magnet; (2.) by induction; (3.) by electrical currents; and (4.) by the solar rays.

The method by touch is accomplished by very various modes of manipulation, of which we shall describe only one or two, referring the reader to larger treatises on magnetism for fuller details. Since the introduction of the method by electro-magnetism, the old methods of producing magnets by touch are far less important than formerly.

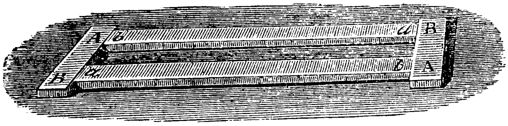
The circumstances affecting the value of magnets, are chiefly the nature and hardness of the steel, the form and proportion of its parts, and the mode of keeping. The most uniform and fine-grained cast-steel, wrought with as little disturbance of its particles as possible, forms the best magnets.

547



This is tempered as high as possible, and the temper is then drawn by heat to a violet straw color, at which hardness it has been found to receive and retain a maximum of magnetism. The proportions of a bar magnet should be, for width, about one-twentieth the length; and the thickness, one-third to one-fourth the width. In a horse-shoe, the distance between the poles ought not to be greater than the width of one of the poles. The faces should be smooth and level, and the whole surface be highly polished. It is quite essential for preserving the power of a magnet, that its poles should be joined by a keeper or armature of soft iron, made to fit its level ends, and be suspended, as seen in fig. 547. Thus armed, a magnet gains power; but if left unarmed, it suffers material loss. Bar magnets are arranged as in fig. 548, either four magnets with their opposite poles in contact, or two magnetic bars, side by side, with two pieces of soft iron joining their opposite poles.

548



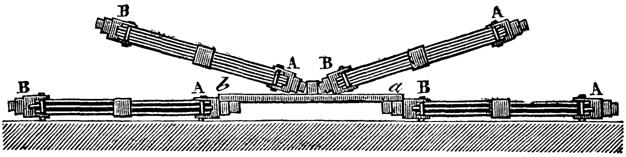
803. **Magnets by touch.**—Touch one pole of a powerful magnet with one end of a sewing-needle, or the point of a pen-knife, and it becomes instantly a magnet, attracting iron filings, and repelling or attracting the magnetic needle. The coercitive force has, in this case,

been decomposed by simple touch. If the magnet is very powerful, a near approach of the needle to it without contact will develop a feeble magnetism by induction.

More powerful magnetism is, however, developed by drawing the bar to be magnetized, from its centre to the end, several times over one pole of a magnet, returning it each time through the air, and repeating the stroke in the same direction. Then place the other pole in the middle of the bar, and stroke the opposite end as before.

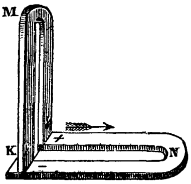
Two magnets may be placed together, with their dissimilar poles in the middle of the bar, as in fig. 549, and then be moved in opposite directions, at a low angle, to the extremities of the bar. The impregnation of the bar will be more powerful and speedy if it rests by its ends on the two opposite ends of two other magnets, as practiced by Coulomb. By inspecting the letters in fig. 549, this arrangement will be quite clear. Care is taken to prevent the ends of the two inclined bars from touching, by placing a bit of dry wood between them. This is called *single touch,* and is to be explained in accordance with § 785.

549



To magnetize a bar by means of the *double touch,* two bars, or horse-shoe magnets are fastened together, with a wedge of dry wood between them, so that their dissimilar poles may be about a quarter of an inch asunder; or a horse-shoe magnet may be used if its poles are quite near together. The magnet, in this mode, is placed upright, on the middle of the bar, and is then rapidly drawn towards its end, taking care that neither of its poles glides over the end of the bar. The magnet is then passed over the opposite end of the bar as before. The poles will be dissimilar to those of the touching magnet.

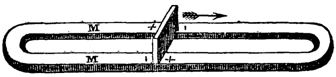
550



804. **Horse-shoe magnets** are easily magnetized by connecting the open ends by a soft iron keeper, while another horse-shoe magnet of the same size is passed from the poles to the bend, in the direction of the arrow in fig. 550; the poles being arranged as indicated by the figure.

The easiest mode of obtaining a maximum magnetic effect in a bar, by touch, is that of Jacobi, viz.: to rest its ends against the poles of another magnet, and then to draw a piece of soft iron, called a feeder, from it several times along the bar. This mode is applied to horse-shoe magnets, as seen in fig. 551. The dissimilar poles are placed together, and the feeder is drawn over the horse-shoe, in the direction of the arrow; when it reaches the curve, it is to be

551



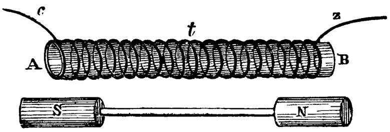
replaced, and the process repeated; turn the whole over without separating the poles, and treat the other side in like manner.

A horse-shoe of one pound weight may be thus charged, so that it will sustain 28·5 pounds. By the best method of touch before known, fig. 550, 21 lbs. 9 oz. was the highest attainable result. (Peschel.)

805. **Magnets by electro-magnetism.**—The mode of producing electro-magnetic currents will be hereafter described. By their means, powerful magnets of soft iron are easily produced, and, from these, by the methods of touch just described, very powerful artificial magnets may be made.

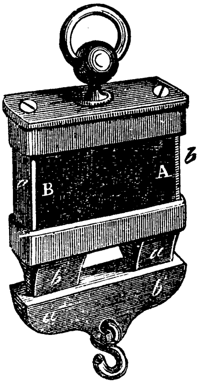
Logemann, of Haarlem, in Holland, has in this way produced the most powerful magnets ever made. One in possession of the author, sustained 28½ lbs.; its own weight being 1 lb. The mode of producing these powerful magnets will be understood from fig. 552. A spiral of insulated copper wire, *t*, is wound on a paste-board tube, A B, in the manner of the electro-magnetic helix. The bar to be magnetized is armed with two heavy cores, or cylinders of soft iron, S N, just fitting the inside of the spiral; when in its place, the ends of the spiral, *c z*, are connected with a few cells of Grove's or Bunsen's battery, and the powerful temporary magnetism induced in the masses of soft iron, reacts, to induce an uncommonly strong permanent magnetism in the bar of steel. A horse-shoe magnet is charged in a similar way, by encircling it with a helix of proper form, with similar armatures of soft iron. The close analogy of this mode to that of Jacobi, in the last section, will be noticed.

552



806. **Compound magnets** are made of several plates of steel, separately magnetized, as in fig. 523 and 549. As the coercitive power of steel appears to be overcome, chiefly, on its surfaces, there is an advantage in multiplying the number of plates, but as each plate serves to neutralize a portion of the polarity of its neighbor (similar poles, of necessity, being brought into contact), there is soon found a limit beyond which there is no advantage in extending these batteries.

553



Large magnets are not as powerful, in proportion to their weight, as small ones. Sir Isaac Newton is said to have worn in his finger-ring a magnet (lodestone) weighing three grains, and capable of sustaining over 250 times its own weight (760 grains). A lodestone of three or four pounds weight, mounted as in fig. 534, can rarely sustain over two or three times its own weight.

The most powerful artificial magnet on record, was that made by Dr. G. Knight, of London, and now in possession of the Royal Society. It consisted of two prismatic bundles, each of 240 powerful bar magnets five feet in length, nounted on wheels; between the end plates of this combination, the poles of the most energetic single magnet were reversed or powerfully reinforced.

807. **Magnetism of steel by the sun's rays.**—Although the fact is doubted by some experimenters, the weight of testimony appears to support the conclusion, that the sun's violet rays possess the power of inducing permanent magnetism, when concentrated by a lens, on steel needles.

808. **To deprive a magnet of its power**, it is only necessary to reverse the order adopted to impart magnetism to it, stroking it from the ends to the centre with poles of the same name opposed. In this way the magnetic virtue may be wholly or very nearly destroyed.

The approach of a feeble magnet to a strong one may reverse its polarity. Leaving a magnet without its keeper greatly impairs its power. Suddenly jerking it off the keeper, or striking it with a hammer, in a way to make it vibrate, does the same. Heat accomplishes the total destruction of magnetism, and in short, anything which weakens its coercitive power. Conversely, hanging an armed magnet in the position it would assume if free to obey the solicitation of the forces of terrestrial magnetism, is the best position to favor its greatest development. Every magnet which has been charged while its poles are connected by a keeper, possesses more power before the keeper is removed than after. It is indeed overcharged, and the excess may be likened to that residual force which retains the keeper of an electro-magnet in its place after the circuit which excited it is broken, or to the residual charge of a Leyden jar. Every time the keeper of a magnet is moved suddenly, a loss of power is sustained, and hence the keeper should be removed by sliding it gradually off endways, and only when it is required for the performance of an experiment.

§ 2. Statical or Frictional Electricity.

I. ELECTRICAL PHENOMENA.

809. **Definitions.**—Electricity is the ethereal or imponderable power which in one or another of its forms affects all our senses. In this respect it is unlike all other ethereal influences. It appears, as far as our knowledge goes, to extend throughout nature, and is probably connected inseparably with matter in every form. Bodies in their natural state give no evidence of its presence, but by different means it may be evoked from all. Hence *statical electricity* implies that condition of this subtle ether existing in all bodies in a state of *electrical quiescence.* Statical electricity is the opposite of that state of excitement following friction, chemical action, &c., which is called *dynamic electricity*, or electricity in motion.

An arbitrary meaning has, however, attached itself to the terms statical and dynamical electricity, materially different from the exact meaning of those terms as used in mechanics. *Statical* or frictional electricity means only that form of electrical excitement produced by friction, while *dynamical* electricity is a term confined to the electrical excitement produced by chemical action, or voltaic electricity. Strictly speaking, all quiescent electricity is *static*, and all electricity in motion, from whatever source, is *dynamic*. Such, however, is not the established use of these terms.

Electricity is a term derived from the Greek word for amber (ἤλεκτρον).

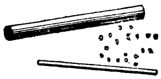
The ancients knew this resin to be capable of what we now call electrical excitement, when it was rubbed.

810. **The chief sources of electrical excitement are**:—1st, *Friction* of dry substances, as of glass, by cat's fur or silk, and of sulphur or resin by flannel: this is ordinary or statical electricity; that of the atmosphere and of common electrical machines; 2d, *Chemical action*, or the contact of dissimilar substances, under circumstances favorable to chemical change; 3d, Magnetism, producing magneto-electricity; 4th, *Heat*, or thermo-electricity; 5th, Animal-electricity.

The electricity from all these several modes of excitement differs in degree and intensity, according to its source, but not in kind, and each may, in turn, be cause or effect. Each will be the subject of separate consideration.

811. **Electrical effects.**—A dry and warm glass rod, rubbed with a cat's fur or silk handkerchief, is excited in such a manner as to attract to itself bits of paper, shreds of silk or cotton, metallic leaf, pith, feathers, and a variety of light substances, holding them for an instant, and then repelling them again, to the table or support, as in fig. 554.

554



In the dark, a feeble bluish light is seen in the path of the rubber. If the excited glass is presented to the knuckle, or to a metallic body, a bright purple spark will dart off from the glass, with crackling sound, to the object presented. Brought near to the face, a creeping sensation is felt, as if a delicate cobweb was in contact with the skin. These effects are produced by the rubber, as well as by the body rubbed, and may be evolved from a number of substances as well as from glass. A peculiar odor always accompanies electrical excitement, thus completing the list of the effects of this subtle agent on our senses, if we add the taste from voltaic electricity.

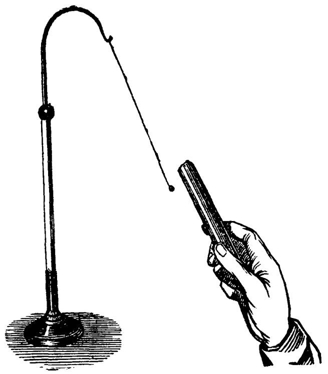
Bodies thus excited are said to be *electrified;* a condition which is only transient.

These very simple experiments, which can be repeated anywhere and with the simplest means, contain the germ of electrical science.

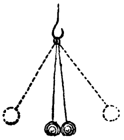
812. **Attraction and repulsion.**—In the electrical pendulum, fig. 555, the pith-ball is first attracted to the excited glass or resin, and at the next instant is repelled, until, by touching some body in connection

with the earth, or in some other way, it has parted with its excitement The two balls in fig. 556, when thus excited, mutually repel each other, because they are similarly excited. The light bodies in fig. 554, oscillate between the table and the rod, first by attraction, and then by repulsion; when, losing their excitement by contact with the table, they are again attracted, and so on. So with the balls in fig. 556. We recognise in these simple experiments the similarity between these actions and the law of magnetic attractions and repulsions. *Bodies similarly excited repel, and those which are of unlike excitement attract each other.*

555



556



The phenomena of attraction and repulsion are not however so simple as might at first appear, since for their correct explanation a knowledge of the phenomena of *induction* is required, and these remain to be explained further on.

813. **Vitreous and resinous, or positive and negative electricities.**—The species of electrical excitement depends upon the kind of material which is subjected to friction. If the pith-balls, fig. 556, are repelled by the excitement from glass rubbed by silk, they will be attracted by a stick of wax, gum lac, or sulphur rubbed by flannel; or *vice versa.*

This difference of action is due to an inherent difference in the two substances, and the kind of electrical excitement which the two respectively produce, is entirely opposite and antagonistic each to the other. The one is *vitreous* or positive, the other *resinous* or negative. This fundamental distinction in the kind of excitement produced by friction in various substances, was first recognised by the French philosopher, Du Fay, in 1733, and was re-discovered by Franklin in 1747. Glass and resin are but types of two large classes of substances, which possess more or less perfectly this characteristic difference in respect to the sort of electricity which they are capable of developing.

Electroscopes serve to distinguish the two sorts of electrical excitement from each other. The pith-balls, fig. 556, form a convenient electroscope—two silk ribbons, or the electrical pendulum, fig. 555, answer the same purpose. Much more delicate instruments of this kind will be described shortly.

It is only requisite to excite the balls, fig. 556, with known vitreous or resinous electricity, when the approach of any excited body whose electrical state is unknown, will, if of the same kind, cause a farther repulsion, and if of a different sort, will occasion an attraction of the balls.

814. **Conductors of electricity.**—Bodies electrically excited part with their excitement variously,—some instantly, others very slowly,—depending both on the nature of the substance excited, and of those with which it is brought in contact. The pith-balls of the electroscope lose their excitement very slowly, the electricity being removed only by the surrounding air. Touched by the finger, or a metallic body in connection with the earth, they are instantly discharged, and return to their natural unexcited condition. The electricity is removed by *conduction* over the touching body. And as bodies vary very much in their power to conduct electricity, they are called *good* and *bad conductors*, or *conductors* and *non-conductors.*

Good conductors propagate the excitement to all parts of their surface, and when in connection with the earth, part with it as quickly as they receive it.

The following are among the good conducting bodies, placed in the order of their conducting power. The metals, as a class (silver and copper standing first, and lead and quicksilver last), well-burnt charcoal, plumbago, coke, hard anthracite, acids, saline solutions, numerous fluids, metallic ores, sea, spring, and rain water, ice above 13° F., snow, living things, flame, smoke, vacuum, vapor of alcohol and ether, earths and moist rocks, powdered glass, and flowers of sulphur.

Bad conductors receive and part with electricity very slowly. If touched by an electrified body, they receive excitement only at the point touched; or if, when excited over their whole surface, they are touched by a good conductor, the excitement is removed only from the part touched. They retain free electricity for a long time, and obstruct its motion.

Insulation.—Good conductors are capable of manifesting electrical excitement only when their communication with the earth is cut off by some bad conductor. So situated, they are said to be *insulated,* and the poor conductors used for this purpose (glass, resin, or dry wood), are called *insulators.* Fig. 557 shows a brass tube thus insulated by a handle of glass. Among the chief insulating bodies are the following, placed in the reverse order of their insulating power, viz.: dry metallic oxyds, oils, ashes ice below 13° F., many crystalline bodies, lime and chalk, lycopodium, native caoutchouc, camphor, porcelain, dry vegetables, baked wood, dry air, and gases, steam above 212°, leather, parch-

557

ment, paper, hair, dyed silk, white silk, diamond and precious stones, mica, glass, jet, wax, sulphur, the resins, amber, gum lac. Gutta percha, and whalebone rubber, are among the best insulators known; probably better than gum lac.

Some bodies which, when solid, are non-conductors, become conductors when liquefied by fusion, viz.: metallic chlorids, glass, wax, sulphur, resin, &c. Heat diminishes the electric conducting power of metals. Length of conductor retards electrical motion, while an increase in other dimensions favors the rapid transmission of electricity. Every body has a certain *electrical retarding power* (813), which is inverse to its conducting power. Tables of electrical conducting powers will be found in larger works; but, in general, this power is very nearly the same as any given body has for conducting heat.

815. **The earth is the great common reservoir** or receptacle into which all electrical excitements are returned, and, regarded as a whole, is a good conductor. The air, even in its ordinary condition, is a very poor conductor, and, in view of its immense extent, is by far the most important of non-conductors. It serves to insulate the earth in a non-conducting envelope, more or less perfect, in proportion to its density, and the absence of aqueous vapor. Except for this property of the air, all electrical phenomena would have remained invisible and unknown to us. The earth is always negatively excited.

In a vacuum, all electrified bodies speedily lose their excitement, while in dry, dense air, they retain it longest. Nevertheless, slight electrical excitement can be produced in a vacuum by friction.

816. **Theories of electricity, or electrical hypotheses.**—Philophers generally agree in attributing the phenomena of electricity to the existence of an assumed *electrical fluid.* This supposed fluid is sc subtle and ethereal as to escape detection by all the means used to recognise matter, being imponderable, and manifesting itself only by its effects. It is assumed to pervade all nature, and to exist in a state of combination or electrical quiescence in all bodies in their natural state. This quiescence is disturbed by friction, and various physical and chemical causes. All electrical phenomena are supposed to be due to the efforts of the electrical fluid to regain its previous condition of static equilibrium. Two principal hypotheses have been devised to explain the phenomena of electricity, namely: 1st, that of Franklin and Æpinus; 2d, that of Symmer, sometimes attributed to Du Fay.

Franklin's single-fluid hypothesis is recommended by its simplicity, and was, for a long time, the view generally adopted, both in England and America. It assumes a single electrical fluid, whose particles are self-repellent, but attracted by matter of all kinds, combining therewith, and when so combined, losing this self-repellent tendency. This fluid is present in all bodies, but in varying proportion, each substance possessing a certain capacity of saturation peculiar to itself. In its natural state, every substance has exactly its own quantity of the electric fluid, and is consequently in a state of electrical indifference. If any cause of electrical excitement exists, this state of quiescence is disturbed, and the body becomes *negatively* electrical, if its natural charge is diminished, and *positively*, if it is in excess. By this hypothesis, bodies become electrical either by addition to, subtraction from, or disturbance in the equal distribution of, the normal quantity of the electric fluid proper to them. In those bodies which manifest positive electricity, the equilibrium is restored by parting with the excess, and in those whose excitement is negative, by receiving from surrounding bodies enough to satisfy their deficiency.

This hypothesis will be recognised as strikingly like that commonly received in explanation of the equilibrium of heat.

Æpinus found, that, in order to account mathematically for the mutual repulsion of two negatively electrified bodies on the single-fluid hypothesis, it was necessary to assume that the particles of matter were mutually repulsive instead of attractive, according to the Newtonian law of universal attraction. This *reductio ad absurdem* has led to the almost universal rejection of the Franklinian hypothesis.

The hypothesis of Symmer, or Du Fay, assumes the existence of two fluids, extremely tenuous, imponderable, in the highest degree expansive, mutually repellent (as a consequence of this expansive nature), and yet possessing a strong mutual attraction when not opposed by any obstacle. They therefore combine, when favorably situated so to do; and when equally combined, their expansive and repellent forces are neutralized, and electrical quiescence results. Each of these kinds of electricity may exist separately; they are then in a state of antagonism, and exhibit polarity, and other electrical effects. Every substance becomes thus excited whenever any part of its natural electricity is decomposed by friction or otherwise. If a plate, it may possess the two electricities on its opposite sides, one being vitreous, and the other resinous; if a rod, the decomposition of a part of its natural electricity will make the rod vitreous at one end and resinous at the other. When the cause of excitement ceases, the two fluids reunite, and quiescence is restored. By this hypothesis, all electrical phenomena arise from the tendency of the two fluids when separated to reunite and neutralize each other.

Either of these two views is capable of explaining most electrical phenomena, but the weight of scientific opinion is now in favor of the last. Neither view can be actually true, since the term *fluid* is only a convenient expression for an unknown cause, and there is no reason why we should assume the existence of a separate fluid or ether, as a medium for light, heat, or magnetic electricity, when it is more in accordance with a sound philosophy to assume that these separate manifestations are but functions of the ethereal medium which fills the universe, and from whose correlations to the particles of matter, all physical phenomena proceed. Compare §§ 765–772.

817. **Electrical tension.**—This term is employed to express that condition of bodies in which the electricity is free—a condition the reverse of electrical quiescence. This condition is well illustrated in the phenomena of the Leyden jar, § 847, where there is perfect equilibrium between the excitement of the outer and inner surfaces, due to their antagonism. The energy with which the decomposed electricities reunite, when communication is made between them, shows the state of tension in which they existed. This may be regarded as analogous to the tension of a bent spring, in which equilibrium is regained by a reaction equal to the compressing force. Electrical tension is a condition of constrained equilibrium, and when the free electricities to which it is due, reunite, an *electrical current* is produced from the reaction of the opposing fluids, analogous to mechanical motion from the recoil of a spring. From this state of electrical tension are derived the *primary* effects of electricity, and from electrical currents arise its *secondary* effects. All electrified bodies manifest electrical tension; they attract other bodies, decomposing their natural electricity, deriving from them a portion of the opposite fluid. If this is insufficient to satisfy the antagonism of the excited electric, the attracted bodies are next repelled (812). Hence, two bodies equally excited, but of opposite names, attract each other, and reunion of the two fluids with electrical indifference results. If one contained an excess of either fluid, both remain excited after contact, with that description of electricity which was in excess; the excess being divided in the ratio of their surfaces.

Electrical currents are either momentary or permanent.—The first occur when contact is formed between substances oppositely excited by friction or otherwise, and their effects are instantaneous and transient.

Permanent electric currents arise only from the sustained action of some continuous cause; as, from the continued motion of the electrical machine, or, more simply, from the chemical action of unlike substances, as in the voltaic battery, in which the electrical current is kept up as long as any chemical action exists.

818. **Path and velocity of electric currents.**—If several conducting paths are open to an electric current, it will always choose the shortest, and that in which it meets the least resistance. If the current is powerful, and the conductor inadequately small, its passage will be marked by light, and perhaps by the combustion and deflagration of the conductor. The velocity of static electricity, by Wheatstone's

experiments, over a copper wire, was found to be 288,000 miles in a second—nearly half again more than the velocity of light (§ 404).

It appears from Dr. Gould's discussion (Am. Jour. Sci. [2] XI., 161), of the very numerous telegraphic observations in the United States, made under the direction of Prof. Bache, for the Coast Survey, and by other astronomers, that the velocity of a voltaic current, when the earth forms part of the circuit, does not exceed 16,000 miles per second, and it has been measured as low as 11,000 miles per second; showing a great retarding force in a conductor of 1500 miles circuit. In the famous Atlantic cable, the electrical retardation was much greater than this, being mixed with the accompanying phenomena of induction.

II. LAWS OF ELECTRICAL FORCES AND DISTRIBUTION OF ELECTRICITY UPON THE SURFACE OF BODIES.

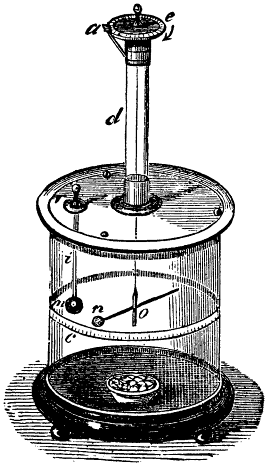
819. **Coulomb's laws.**—Coulomb (died 1806), a distinguished French physicist, by the use of the torsion balance, first demonstrated the following laws of electrical attractions and repulsions:—

1st. *Two excited bodies attract and repel each other with a force proportional to the inverse square of their distances from each other.*

2d. *The distances remaining the same, the attractions and repulsions are directly as the quantities of electricity possessed by the two bodies.*

Coulomb's laws of torsion have already been demonstrated (266). He happily applied these principles, first established by himself, to the measurement of electric forces in his torsion electrometer.

558



820. **Torsion electrometer.**—This instrument, fig. 558, consists of an exterior glass cage, protecting a slender needle, *n o*, of gum lac, suspended by a fine wire of silver or platinum, centrally attached to the under side of the cap, *e*, upon the tube *d*.

This cap is graduated, and turns like the cover of a box. The graduation is read at the vernier, *a*. A small weight, *o*, of brass, keeps the wire tense, while through it the gum lac needle passes. At one end of the needle, *n*, is a small gilded ball of pith, or a disk of tinsel paper. The cover of the glass case is perforated for the free passage of a glass insulating rod, *i*, carrying a polished brass ball at *m*. The glass cage is graduated in a zone at C, into 360 degrees, to measure the angular spaces traversed by the needle. The zero of the graduation, and of the arc on the cap, are both made to correspond (by revolving the tube, *d*,) with the normal position of the needle when at rest, and unexcited. To avoid the loss of electricity, the air in the cage is kept dry by a little quick-lime, placed in a dish for that purpose, on the bottom.

821. **Demonstration of the first law.**—The apparatus being thus arranged, the insulated rod, *i*, is withdrawn, and the ball, *m*, placed in contact with some excited surface—as the electrical machine. Thus excited, *m* is immediately returned to its place by the insulating handle, taking care that it touches nothing. Forthwith the disk, *n*, is attracted to *m*, is oppositely electrified, and then repelled with a force proportioned to the intensity of *m*. After a few oscillations, *n* comes to rest say at 30 degrees on the graduated circle. This angle then represents the repellent force of the electricity on *m*, since the torsion of a wire is directly as the twisting force. But what would be the force requisite to hold the disk, *n*, in equilibrium at half this angular distance, or 15°? Revolving the movable circle, *e*, in the direction of the arrow, we find it is necessary to carry it from 0 to 105°, in order that the needle may point to 15°. The wire is then twisted at top with a force of 105°, and at bottom with a force of 15°, giving 120° as the angle representing the force with which the two electrified bodies repel each other, at the distance of 15°—or, at half the distance, we have quadruple the force; at one-third the distance, or $7\frac{1}{2}°$, the force would be $472°{\cdot}5 + 7°{\cdot}5 = 480°$, according to the law of inverse squares.

In like manner, reversing the electricities, we prove that the force with which two electrified bodies attract each other, is inversely proportional to the square of the distance by which they are separated.

822. **Demonstration of the second law.**—Having repelled the needle, *n o*, by the excited ball, *m*, withdraw the latter, and touch it to a second metallic ball of the same size, insulated on a glass handle. The ball, *m*, parts with half its electricity to the second ball (827). Now return it to the torsion balance; it will be found that the needle, *n o*, is repelled to a distance equal to its former distance multiplied by the square root of one-half, $D' = D\sqrt{\frac{1}{2}}$. Touch *m* again to the second ball, as before, and it will then repel the needle to a distance equal to the first distance multiplied by the square root of one-fourth, $D'' = D\sqrt{\frac{1}{4}}$, and so on.

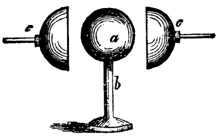
Sir Wm. S. Harris, of England, by the use of a bifilar electrometer, which substitutes the force of gravity for that of torsion, has shown that the two laws of Coulomb are not strictly accurate, unless the two excited bodies have the same size and form, or unless the sections of the opposing parts are equal. The result of his determinations is, that the attraction is directly proportional to the number of points immediately opposed to each other, and inversely to the square of their respective distances.

823. **Proof-plane.**—For the purpose of determining the relative quantities of electricity that are found on the different parts of the surface of an electrified conductor, a contrivance called a *proof-plane* is used. It is nothing but a small disk of tinsel, or metal, insulated, as in

the ball, *m*, of the torsion balance, fig. 558. This is touched to the surface whose electricity is to be examined, and receives therefrom a quantity of electricity equal to the sum of both of its own surfaces. It may then be inserted in the balance of torsion, or used on any other electroscope. The electricity on the body touched is diminished to the same extent, but when the proof-plane is small, compared with the area of the excited conductor, no sensible error can arise from this loss. The most important source of error to be guarded against in the use of this instrument, arises from the effects of induction, presently to be explained.

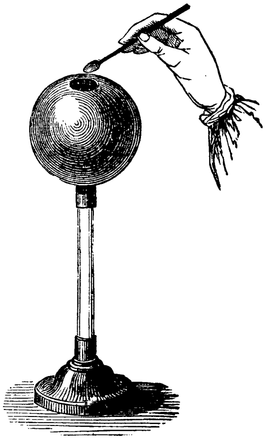
824. **Electricity resides only on the outer surfaces of excited bodies**, and not in their substance, or on their interior surfaces. This fact is attributed in part to the repulsive power of the electric fluid acting upon the particles of matter interiorly, thus driving the excitement to the outer surface, where it meets the non-conducting air, and is arrested. It is also due to the inductive influence of the electricity of surrounding bodies, and of the walls of the room. The following experiments will illustrate this law.

559



A. Electrify the metal sphere, *a*, fig. 559, on an insulating stand, *b*, and approach to it the two hollow hemispheres of brass, *c c*, insulated, and made accurately to cover the sphere. On removing them, *a* will be found without the least trace of electrical excitement, as may be proved by a delicate electroscope, while the two hemispheres are fully excited. To remove the enveloping hemispheres is to remove the surface of the sphere, and with them its electricity.

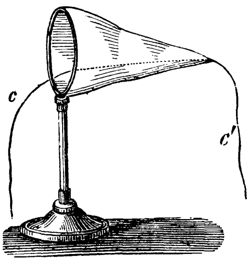
560



B. Fig. 560 shows an insulated hollow sphere, with a hole in the top. When this is electrified, the proof-plane may be introduced by the opening, and placed in contact with its inner surface, without acquiring any excitement (provided care be taken to avoid the inducing effect of the edges of the opening, which may otherwise decompose the neutral electricity of the gum-lac handle), while from contact with any point of the outside, the proof-plane acquires abundant excitement.

C. Faraday has described a muslin bag in the form of a net, fig. 561, sustained on an insulated ring of wire, and provided at the point of the cone with two insulated silk strings, *c c'*, so that it may be turned inside out at pleasure, without touching it. When this is electrized *exteriorly*, it may be turned inside out by means of the strings, without a trace of electricity being found on the inside (which an instant before was the outside), and this may be repeated several times before the electricity is dissipated. He is in the habit of covering his most delicate electroscopes with muslin bags, to protect them from the influence of excited electrical machines, with entire success.

561



562

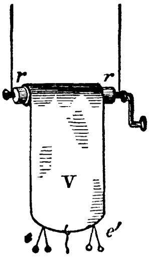


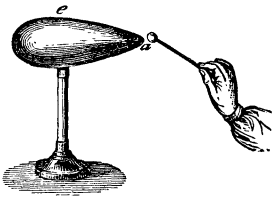
Fig. 562 shows a ribbon of metallic paper wound around a metallic axis, insulated by the silk threads *r r*; two pith-balls, *e e'*, are suspended by linen threads, at one end of the ribbon. When the ribbon is wound up, and the whole is electrized, the balls of the electroscopes diverge powerfully. If the ribbon is now unwound by drawing the insulating string below, the electroscope balls gradually fall, and finally come almost in contact; but when the ribbon is again wound up, the balls diverge as before. This may be repeated several times. This beautifully illustrates the relation of surface and intensity. As the surface is increased, the same quantity of electricity is spread out over a larger surface, and its energy declines, but is increased again as the surface is diminished by re-winding the ribbon.

D. It appears from these experiments that a ball of wood or pith, covered with tin foil or gold leaf, can accumulate on its surface as much electricity as if it was of solid metal.

It is thus proved that all the electricity with which a conducting body is charged, is disposed on its surface.

825. **Distribution of electricity.**—The form of conductors influences the distribution of electricity on their surface. In a sphere, the distribution is uniform, as would be anticipated from the known properties of the solid. The proof-plane, applied to any part of an excited sphere, acquires, as tested by the balance, the same power. In an ellipsoid of revolution, like fig. 563, the proof-plane applied at *a*, gives a much larger angle of torsion in the balance than at any other point, while the minimum is in the vicinity of *e*; showing a tendency in electrical excitement to accumulate about the extremities of any solid having unequal axes.

563



In cylinders, the concentration of force is within about two inches from each end, and is feeble at the middle. So in plates, the maximum of accumulation is about an inch from the edge. The same is true of the edges and solid angles of prismatic bodies.

826. **The power of points** (first investigated by Franklin) in concentrating electricity is such that the excitement passes off, as rapidly as it accumulates, to the nearest bodies, or is diffused into the ambient air in an electrical brush or pencil, visible in the dark (842 (6)). This

fact follows as a consequence of the tendency of electricity to accumulate at the smaller end of an ellipsoid. The ellipsoid may be so elongated that the electricity escapes—it then becomes a blunt point. These facts are of the greatest importance in the construction of electrical machines.

827. **The loss of electricity in excited bodies**, even when insulated in the best manner, is constant, chiefly from two causes, viz.: 1st, the moisture of the air; and 2d, the imperfection of the insulation. The loss from the first cause, in still air of average dryness, is proportioned to the state of electrical tension. Bodies feebly excited, and perfectly well insulated in dry air, retain their state of tension for weeks or months, while those highly excited, and not carefully preserved, are soon deprived of all electrical excitement. The rate of loss by imperfect conduction is, of course, dependent on the non-conducting material used, the perfection of workmanship, and care of the apparatus.

The loss of electricity by an excited conductor, when placed in contact with an unexcited body, insulated from the earth, is in proportion to the relative surfaces of the two bodies. One gains, the other loses. Two equal spheres will divide the whole quantity between them. If they remain in contact, the distribution is unequal, being least at the point of contact, and increasing to a maximum at 20° to 30° from the point of contact. The proof-plane determines exactly all such questions.

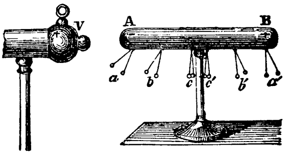
In a vacuum, a high state of electrical tension is impossible, since the restraining power of the air is wanting. But a feeble tension can be preserved in an exhausted receiver for a long time. The movement of the mercury against the walls of the tube of a barometer, excites electrical tension, and even luminous electricity, as was shown by Cavendish.

III. INDUCTION OF ELECTRICITY.

828. **Electrical influence or induction.**—Every electrified body is surrounded, so to speak, by an atmosphere of influence, analogous to that surrounding a magnet (783), within which every insulated conductor becomes also excited. Bodies so affected are said to be *electrified by induction*, having their neutral electricity decomposed by the tension of the excited conductor, exercised through the intervening air.

Let the conductor, V, fig. 564, of an electrical machine, be approached within say six inches of an insulated metallic conductor, A B. The small electroscopes, *a a'*, suspended beneath its ends, instantly diverge, and at the same time are respectively attracted to V, at A, and repelled from it at B. If V is + A is —, and the remote end, B, is plus. The intermediate electroscopes, *b b'*, also diverge, but in a less degree, while those near the middle, *c c'*, do not diverge at all. If, while thus excited, A B is withdrawn from V (care being taken not to touch the

564



conducting surface), the electroscopes will presently cease to indicate any excitement. The explanation of these facts is, that the neutral fluid of A B has been decomposed by the influence of V, the + fluid being repelled to B, and the − attracted to A, while, near the centre (never exactly in the centre), a neutral point is found, where no decomposition exists. When V, the disturbing cause, is withdrawn, the two electricities of A B unite again, and leave the conductor entirely passive. If, however, the conductor, A B, is made in two parts, joined at the neutral point, and each on a separate glass leg, when it is inductively excited, the two parts may be separated, and each part will then be found, when removed from influence, to possess the same excitement developed in it, under the inductive power of V.

By means of a glass tube, or stick of resin gently excited, and approached to one of the electroscopes, it is, of course, easy to determine the description of excitement in V, which we now assume to be +.

829. **The laws of induction** may be thus stated:—1st. A body electrized by induction, possesses no more electricity than before. This is shown by the fact, that as soon as the inducing influence ceases, the two fluids reunite in A B, and no trace of excitement remains.

2d. If a conductor, excited by induction, is touched, or made to communicate with the earth in any part of its surface, it parts with a portion of electricity, always of the same name with the inducing body, and it retains the fluid of opposite name. If the inducing cause is then withdrawn, the insulated conductor remains excited, with the fluid of opposite name to that of the inducing body.

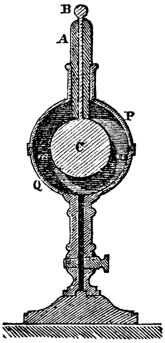
Thus, we note the important distinctions between a body electrized by induction and by conduction. Induction occasions no transmission of free electricity to the other body; but only a decomposition of the ± electricities of the insulated conductor. Induction produces dissimilar conduction, similar electricity to that of the exciting body; and the distance to which electricity of induction extends, greatly exceeds that to which it can be propagated by conduction, where absolute contact or very close proximity is required. A strong analogy exists between electric and magnetic induction. Both magnetism and electricity by *contact*, are of the same name with the body touched. By influence, the neutral fluid of the excited body is decomposed, and the polarities are in accordance with laws already stated.

830. **Induction is an act of contiguous particles.**—Dr. Faraday has modified the usual view of induction just stated, by showing that induction never takes place at a distance, without polarizing the molecules of the intervening non-conductor, causing them to assume a constrained position, which they retain as long as they are under the influence of the inductive body.

Because air and other non-conductors permit the passage of electrical influence in this manner, Faraday calls them *dielectrics*, in distinction from *electrics*, or conductors which can become polarized only when insulated by some dielectric. Dielectrics differ in their specific inductive capacity air being the lowest in the scale, as follows, viz.: air, 1 resin, 1·77; pitch, 1·80; wax, 1·86; glass, 1·90; sulphur, 1·93; shellac, 1·95.

The apparatus used by Faraday in determining the relative inductive capacity of air and other gases, is seen in fig. 565, consisting, essentially, of two metallic spheres, C and P Q, of unequal diameter, the smaller placed in the centre of the larger, and insulated from it by a stem of shellac or gutta percha, A. The two halves of the outer sphere join in an air-tight joint, like the Magdeburg hemispheres (257). The space, *m n*, may be emptied of air by an air-pump, controlled by a cock in the foot, and filled with any other gas or fluid. This apparatus resembles the Leyden jar (847), with the advantage of changing the intervening dielectric at pleasure. The balls, C and B, constitute the charged conductor, upon the surface of which all the electric force is resident by virtue of induction. As the medium in *m n* may be changed at pleasure, while all other things remain the same, then any changes manifest by the proof-plane and torsion balance, will depend on changes made in the interior. The same end would be reached by having two exactly similar inductive apparatus, with different insulating media. When one was charged and measured, the charge being divided with the other, the ultimate conditions of both indicate by the torsion balance whether or not the media had any specific differences. (For further details, see Faraday's Exp. Res. 1197.)

565



831. **The attractions and repulsions of light bodies** (811) can be explained only in view of the phenomena of induction. The excited tube or resin, decomposes the neutral electricity in the pith-balls or bits of paper, repelling the electricity of opposite name, and being thus left of an opposite excitement to the rod or resin, they become attracted to the exciting body, in obedience to electrical laws. All cases of electrical repulsion are equally referable to attraction under inductive influence. Thus the apparent repulsion of the two pith-balls in an electroscope, is really the effect of the attraction of surrounding bodies, whose electrical equilibrium is disturbed by the inductive influence of the exciting cause.

The following experiment illustrates, in an interesting manner, the development of electricity, and the attractions and repulsions of light bodies by induction. Support by its edges a pane of dry and warm window-glass, about an inch from the table, on two pieces of dry wood, and place beneath it several pieces of paper or pith-balls. Excite the upper surface by friction with a silk handkerchief, the electricity of the glass becomes decomposed, its negative fluid adheres to the silk, and its positive to the upper surface of the glass plate; this, by induction, acts on the lower surface of the glass, repelling its positive electricity, and attracting its negative, the intervening dielectric being polarized as explained, and the lower surface of the glass electrified by induction through its substance, attracts and repels alternately, the light bodies, like the excited tube (811). (Bird.) Numerous experiments, illustrative of induction, are given under the electrical machine.

832. **Electrometers.—Cavallo's, Volta's, and Bennett's.**—The electroscopes mentioned in section 813, serve to indicate the presence and name of the electricity. Electrometers are designed to give approximate measures of the quantity of electricity.

566

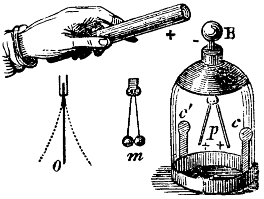


Fig. 566 shows Cavallo's electrometer—a bell-jar with a metallic rod and button, B, sustaining two pith-balls, *m*, at the ends of two wires. Volta substituted for the two pith-balls, two delicate blades of straw, *p*. Bennett replaced these by two strips of gold leaf, *o*, placed face to face. When the knob, B, receives electricity, the pith-balls, straws, or gold leaves, diverge, and by the degree of their divergence, measured on a graduated arc, the intensity of the electricity is judged of. Two strips of tin foil, *c c'*, are pasted to the inside of the glass bell, to discharge the diverging leaves, when they are repelled, so as touch the sides. Otherwise the inside of the glass jar would be electrified by induction, and render the apparatus useless; and to avoid dampness, the top of the bell is varnished, and the air within, dried by quick lime. Approaching an excited body towards B, the gold leaves diverge, because the positive fluid, if the excitant is positive, is driven into them, while the negative is attracted to B. Touching B with the finger, the positive fluid passes off to the earth, but on withdrawing the finger, the leaves diverge under the influence of the negative electricity remaining in the apparatus.

567

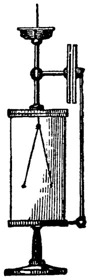


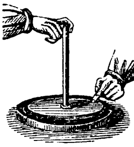
Fig. 567 is a sensitive form of gold leaf electrometer, with brass condensing plates, ᶳ 846, and a cup at top to illustrate the effect of evaporation in producing electrical excitement.

Hare's single leaf electrometer, and the condensing electrometer, are mentioned in section 846, and Bohnenberger's under the dry pile, ᶳ 873.

IV. ELECTRICAL MACHINES.

833. **The electrophorus.**—Any apparatus by which electrical phenomena may be obtained at pleasure, is an electrical machine. The simplest apparatus of this sort is Volta's *electrophorus*, or carrier of electricity, invented in 1775.

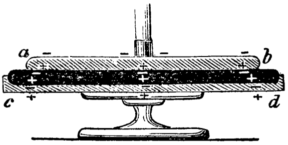
568



569

A cake of resin, or a disk of whalebone-india-rubber, or gutta percha, eight or ten inches in diameter, is excited by a fur or warm flannel, and a smaller disk of brass, or tin plate (with rounded edges), is placed on it by an insulating handle. Touch the upper surface of the metallic disk with the finger (fig. 568), in order to allow the escape to the common reservoir (815) of the negative electricity, resulting from the decomposition of the neutral fluid in the metallic plate by the excited resin. Now removing the finger, raise the disk by th insulating handle, and approach its edge to any conductor, as the knuckle, fig. 569; immediately a strong spark is seen, due to the free positive electricity existing in A. Place A on B again, touch it as before, and the same result may be obtained as often as desired. If A is left in repose upon B, it will remain charged a long time, even for weeks, and a Leyden jar may be charged with it at any time: on the table of the laboratory it may be more conveniently used than an ordinary electrical machine for exploding gases. The section of the electrophorus, seen in fig. 570, shows how the inductive action of the excited resin acts, in affecting the electrical nomenclature of each surface, as indicated by the signs + and —.

570

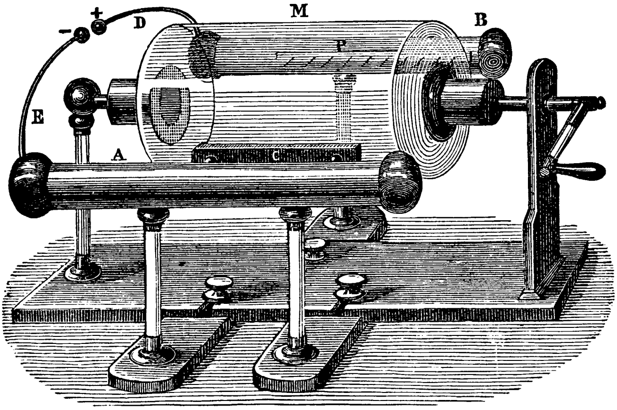


The phenomena involved in the electrophorus are identical with those explained in § 829. Of course, if the plate, A, were raised without touching, it would manifest no electrical excitement; the two fluids re-combining as in the insulated conductor, fig. 563.

834. **The cylinder electrical machine.**—When larger quantities of electricity are required than can be obtained from the means already described, resort is had to machines of larger size, and more power, all of which, however various in form, consist principally of three parts, viz.: 1st, a non-conductor, usually of glass, revolving on a horizontal axis, and producing friction; 2d, a *rubber*, against which the non-conductor presses. The rubber is a soft, elastic non-conducting body, as a cushion of leather, usually armed with amalgam, to be described hereafter. 3d, two *conductors*, usually of brass, mounted on glass supports, one to receive the + and the other the — electricity.

In the cylinder machine, fig. 571, a smooth cylinder, M, of glass, insulated

571

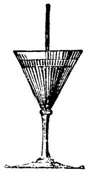


and filled with perfectly dry air, is revolved by the winch before the rubber C, sustained on the insulated prime conductor, A, and covering about one eighth of its surface: an apron of silk is usually attached to the upper edge of the rubber, and extends as far as the points, P, on the second conductor, B, designed to receive the + electricity excited at C. If the connecting rods, E D, are approached as in the figure, and the cylinder is revolved, and there is no connection with the earth, then the + electricity accumulated on the positive conductor, will reunite with a spark with the — electricity of the negative conductor, A, again to be decomposed as before at C. If the negative conductor, A, is connected with the earth by a chain or metallic thread, then, when the machine is worked, a continuous flow of sparks of positive electricity will pass from the positive conductor, B, to any conductor near enough to receive them. But if A is insulated, and B is connected with the earth, then from E, a continuous flow of negative electricity is obtained. In this case a flow of positive electricity takes place from the cushion, C, through the conductor, B, to the earth, thus leaving the conductor A charged with negative electricity. This form of cylinder machine was designed by an Englishman named Nairne.

Amalgam.—No considerable quantity of electricity can be evolved from an electrical machine of glass, unless the rubber is excited with an amalgam composed usually of four parts mercury, eight zinc, and two tin, mixed with some unctuous matter and spread on silk or leather. The zinc is first melted; the tin is added, and the mixture stirred, and poured, not too hot, into a wooden box, coated inside with chalk, and into which the heated mercury has been first poured. The lid is put on, and the box violently shaken, until the amalgam becomes cool. It is then finely pulverized in a mortar, and becomes as soft as butter.

835. **Ramsden's plate machine**, as its name indicates, has a plate or plates of glass substituted for the cylinder. This form of apparatus is seen in fig. 572. The plate, F F, is revolved by the winch, M, sustained in a frame, O O, of baked wood. Two pairs of spring cushions, *a c*, armed with amalgam, produce friction. The conductors, C C, collect the electricity from the glass by the points seen on the inside of their curved branches, placed near the surface of the plate. Each of the cushions is connected with the earth by the conductor, D; strips of tin foil pasted upon the edges of the frame, O, and shown in the figure unshaded, communicate between the four sets of cushions and the chain D.

573

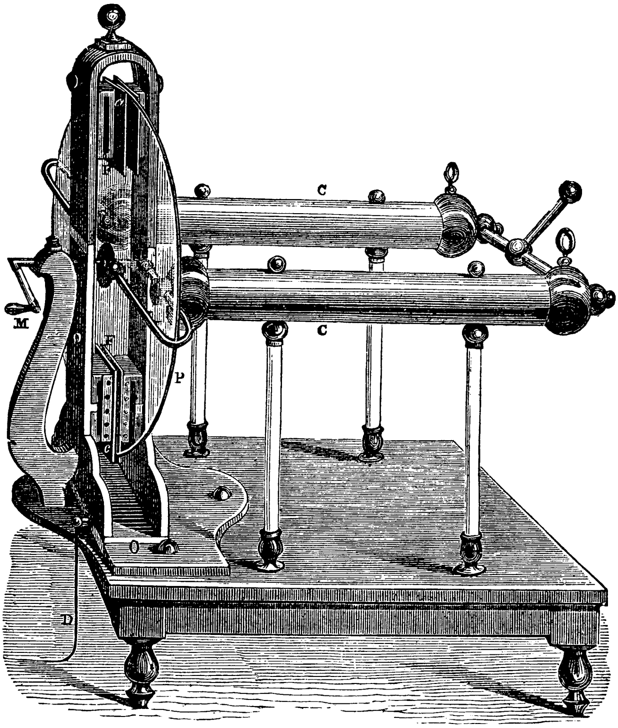


Ramsden's apparatus originally gave only positive electricity. It was so modified by Von Marum as to obviate this defect. This form of electrical machine was contrived by Ramsden, of London, in 1776.

The earliest electrical machine was made by Otto V. Guericke (who invented the air-pump), and was a globe of sulphur or resin, driven by a motor wheel, the hand being used for friction. A cone of sulphur, fig. 573, cast in a wine-glass, and provided with a glass rod as a handle, serves to illustrate this early electrical apparatus. But hard india-rubber is a more convenient and certain source of negative electrical excitement.

In the United States, our mechanicians have brought all apparatus for electrical illustrations to a degree of perfection leaving little to be desired.

572



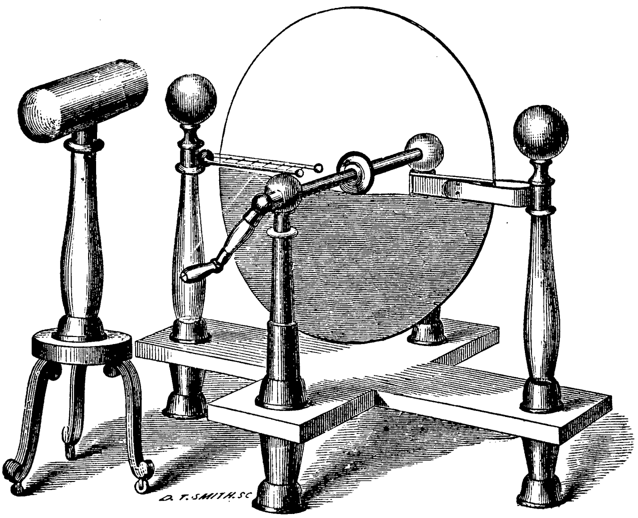
836. **The American plate electrical machine.**—The form of plate machine commonly adopted in the United States is seen in fig. 574 (from *Ritchie*). It is arranged for the exhibition of both electricities at pleasure, and has a prime conductor on a movable stand.

Dr. Hare has very ingeniously met the difficulty of obtaining both electricities from the plate machine by making the plate revolve horizontally, and thus allowing the positive and negative conductors to stand like arches in two vertical planes at right angles to each other above the plate. Am. Jour. Sci. [1] VII. 108.) Dr. Hare was an ardent supporter of the Franklinian hypothesis, and this apparatus was contrived to sustain his arguments in favor of that view.

837. **Large electrical machine —Ritchie's double plate ma-**

hine.—The largest electrical machine hitherto constructed, so far as we are advised, is that made for the University of Mississippi, at Oxford, under the

574



direction of President Barnard, by Ritchie, of Boston. The general construction of this gigantic electrical apparatus may be understood from fig. 575 (frontispiece), in which, however, the prime conductors are not shown. These, for greater convenience of manipulation, are made movable on separate supports.

This machine has two plates of French glass, each six feet in diameter, sustained by an insulated steel axle, upon eight cut-glass supports, on a frame of rosewood. The plates are excited by four pairs of rubbers, made of brass, and lined with fine wool felt three-eighths of an inch thick, such as is used for the dampers of the grand piano. These are covered with firm India silk, upon which the amalgam is spread. Rubbers of this construction are found to be far more efficient than those in general use. The prime conductors of this machine expose fifty square feet of polished brass cylinders in three sections, about one foot in diameter by seven in length, sustained also on cut-glass insulating pillars. One turn of this machine fills the apartment with an overpowering odor of ozone. It is so arranged as to afford negative electricity from four rubbers. One battery for this machine contains one hundred and twenty glass bells, arranged in detachments, whose coated surfaces expose about ninety square feet of area. No detailed description of the performance of this superb machine has yet been made public. It cost over three thousand dollars without its batteries.

The Tylerian machine.—The largest and most famous plate machine mentioned in the books before that of Ritchie, of Boston, both on account of its size and performance, was made by Cuthbertson for Von Marum in 1755, and was placed in the Tylerian Museum, at Haarlem, in Holland. It was a double plate machine, each plate sixty-five inches in diameter, with eight cushions, nearly sixteen inches in length, and twenty-three and a half feet surface in the conductor. It gave three hundred sparks twenty-four inches long in a minute forked like lightning, and with rays six or eight inches long, branching from the angles of the flash. It deflected a thread six feet long, six inches from a perpendicular, at a distance of thirty-eight feet, and the balls of Cavallo's electroscope (832) diverged half an inch asunder when forty feet distant from it. The prime conductor was supported on three glass pillars sending out collecting branches between the plates. Two, and sometimes four men, were employed to turn it. When in full force, a single spark from the conductor sufficed to burn and dissipate a strip of gold-leaf twenty inches long, and one and a half lines wide. A pointed wire exhibited the appearance of a luminous star when held twenty-eight feet from the conductor.

All glass is not equally fit for electrical plates; that which is white, hard, and free from bubbles, is most esteemed. If too much alkali is used in the composition of the glass, its surface attracts moisture, and soon becomes damp and rough. Such a plate is worthless. The preference given to old plates is due, probably, to the fact, that their composition has enabled them to preserve their properties uninjured.

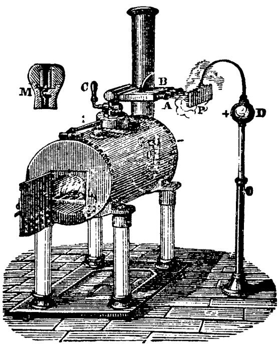
838. **Care and management of electrical machines.**—Perfect insulation and freedom from dust and roughnesses, are essential to the good condition of all electrical machines. For this end, the glass columns are varnished, to avoid the deposition of moisture, and all the polished surfaces of metal, as well as the glass, must be kept quite clean, and free of dust. If the surface of the cylinder or plate becomes streaked with amalgam, it must be wholly removed. It is better not to put any amalgam into immediate contact with the glass, but to spread it upon the cushion pretty thickly, and then to cover it with a piece of silk; a sufficient quantity will pass through the silk, as the machine is worked.

If the glass becomes greasy, it is best washed with rectified camphene, burning fluid, or ether. It is indispensable that the surface of the amalgam should be in good metallic communication with the earth, which is accomplished by the use of tin foil, or tinsel. Cushions stuffed with metal filings are preferred by some, chiefly for this reason. A cushion or rubber made of two or three folds of cotton-flannel, between which is laid a continuous strip of tin foil, of the same size with the rubber, works exceedingly well. Ritchie prefers piano felt. *Aurum mussivum* (the bisulphuret of tin), a yellow bronzy powder, is an excellent substitute for amalgam. It is supposed to suffer chemical decomposition when in use, and so to quicken the activity of the machine. Finally, a dry winter air is indispensable for the best working of an electrical machine; hence, radiant heat, falling on the machine, or an apartment heated by a dry furnace air, is especially favorable. In carpeted rooms, it is desirable to connect the rubber with a gas-fixture, to secure a good communication with the common reservoir.

839. **Electricity from steam.**—Armstrong's hydro-electric machine, fig. 576, is designed to illustrate the development of powerful electrical effects from high steam; a fact well known to all concerned in the

management of locomotive steam-engines, but first scientifically noticed in 1840, by Mr. Armstrong, at Newcastle-on-Tyne. The apparatus which he contrived to show these effects, is a common high pressure steam boiler, about three feet long and twenty inches in diameter, mounted on insulating pillars, and strong enough for a pressure of 200 lbs. to the inch. The steam is suffered to escape by jets, A, of a peculiar form, on the side of the box, B, into which it is admitted by the cock, C. Faraday, in investigating the electricity of steam, found that *dry* steam gave no excitement, and that the electricity resulted from the friction of vesicles of water against the sides of the orifice. Hence, B contains a little water, over which the steam escapes, and is partially condensed. The jet has an interrupted passage, seen at M, to produce friction, and its nozzle is lined with dry box or partridge wood. The vapor escapes against a plate, P, covered with metallic points, to collect the electricity, and ending in a brass ball, D, insulated from the earth. The boiler is negative, and positive electricity is collected at D, provided the water is pure and free from grease. Turpentine, and other volatile essences, reverse the polarity, while grease, or steam from acid, or saline water, destroys all excitement. If the nozzle of the jet ends in ivory or metal, there is also no excitement. A boiler, such as is described, will develop in a given time, as much electricity as four plate machines forty inches in diameter, making sixty turns a minute; a truly surprising result.

576



840. **Other sources of electrical excitement.**—1. The bands of leather, India-rubber, or gutta-percha, used to drive machinery, sometimes become powerful sources of resinous electrical excitement, giving sparks of negative electricity over twenty inches in length.

In cotton-mills, so much electricity is thus set free, that it becomes necessary to let steam into the carding and roving rooms, to avoid inconvenience from the repulsions and attractions of the cotton. A leathern band, mentioned by Mr. Bachelder (Am. Jour. Sci. [2] III., 250), gave sparks to the finger at three feet, and a luminous brush, to a steel point, at seven feet. The discharge from leather, as from all bad conductors, is local, or danger would attend it.

Dr. Franklin, in a letter to Bowdoin, suggested, for a portable electrical machine, a crossed band of stuffed leather, moved by a winch over drums.

2. The friction of shoe-leather on woollen carpets, in houses warmed by hot-air furnaces, or steam, in cold weather, is a fertile and curious source of negative electrical excitement.

The young people in the author's house find an unfailing source of amusement in cold weather, in giving electrical shocks, by kisses and otherwise, to unwary people, or in lighting the gas by a spark from the finger, or a key-handle, after running briskly over the carpet. Prof. Loomis has noticed these effects in detail in the Am. Jour. Sci. [2] X., 821, and XXVI., 586. They appear to be unknown in Europe, owing probably to the fact that European houses are seldom warmed and dried by hot-air furnaces.

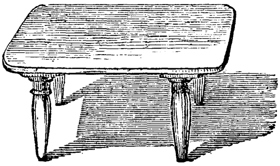
841. **Theory of the electrical machine.**—The phenomena of the electrical machine may be explained, either on the theory of one or two fluids. The explanations of induction (828), already given, apply equally to the development of free electricity, upon the prime conductors of electrical machines. When the machine is turned, the neutral electricity of the rubber is decomposed, the positive fluid follows the glass, until coming opposite the points on the prime conductor, the negative electricity of the conductor flows out, to unite with the positive of the glass, while the positive fluid of the conductor is repelled to the other end, thus leaving the prime conductor powerfully positive. Reaching the rubber, the neutral fluid of the glass is there decomposed, its negative portion seeks the common reservoir, and the positive follows the revolving glass to the points as before. The conductor does not acquire positive electricity from the plate, but gives its negative thereto, thus becoming itself positive.

It is still an open question whether the action of the amalgam is chemical or mechanical (834). It is certain that an amalgam of silver, or gold, does not act to excite electricity, like amalgams of oxydizable metals; and Dr. Wollaston demonstrated, that the latter did not act in an atmosphere of carbonic acid or nitrogen, free of oxygen.

In all cases, the discharge of an electrical conductor, by a spark or otherwise, is accompanied by the induction of an opposite excitement in the body receiving the shock, whose opposite electricity, uniting with that of the conductor by a forcible disruption of the intervening dielectric, produces the sound and flash of the electric discharge.

842. **Experimental illustrations of electrical attractions and repulsions.**—A multitude of instructive and amusing experiments may be made with the electrical machine, illustrating the law of attraction. A few must suffice.

577

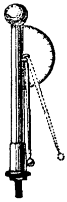


1. **The insulating stool** is a bench with glass legs, fig. 577 (a board on four black bottles answers perfectly), on which a person may stand or sit, while in communication with the electrical machine. Being thus insulated, the free electricity can escape only through the surrounding air,—approaching the knuckle to any part of the person or dress of one so situated, a strong spark is received. Except for the hair being repelled, the person charged is not conscious of any change from an ordinary state. A doll's head, with paper hair, set upon one of the conductors, is a common electrical toy.

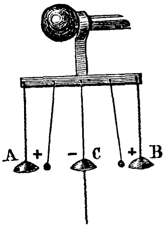
2. **Henley's electroscope**, fig. 578, serves to mark the degree of tension in the machine by the repulsion of a pith-ball at the end of a straw: it is mounted on one of the conductors, and in dry weather remains extended a long time, but in damp weather falls immediately, when the machine stops.

3. **Electrical bells**, fig. 579; the bells, A and B, are suspended by a metallic thread, from the ends of a cross-bar of metal hung on the machine; the bell, C, and the two clappers, are hung by insulating threads. C is connected with the earth; and when the machine is worked, A and B become positive, and by induction C becomes negative, and the little clappers being alternately attracted and repelled, a constant ringing is kept up as long as the excitement lasts. If the machine is too active, luminous sparks pass, and the bells remain still.

578



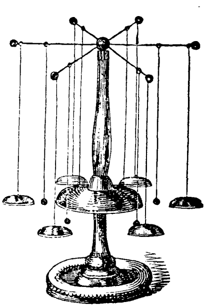
579



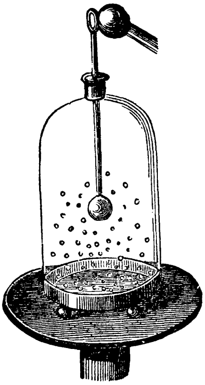
The bells may be conveniently arranged on an independent foot, as in fig. 580.

4. **Volta's hail-storm** is a contrivance designed to show how (in Volta's view) hail might be produced. It is a larger way of showing the same facts already explained in § 811 A glass bell communicates with the machine, fig. 581, above, and rests on a metal plate in communication with the earth. When the machine is worked, the pith-balls on the plate are violently agitated, being drawn up and repelled again actively, while the excitement lasts. A simple bell-glass, or large tumbler, electrized by contact of its interior surface with the conductor of an electrical machine, answers quite as well, and may be placed over a heap of pith-balls on the table; they are violently thrown about as long as the excitement continues. The dance of puppets, fig. 582, is only a substitution of little figures of dancing peasants, made of cork or pith, and placed between two metallic plates.

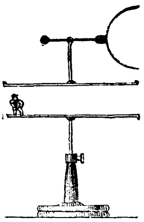
580



581



582

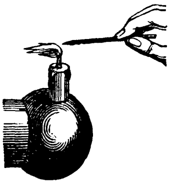


5. **The electrical wheel** is composed of several points fixed in a centre, so balanced as to rest on an upright, sustained on one of the conductors, fig. 583; as the machine is worked, the escape of the electricity from the points reacts on the air with sufficient force to revolve the wheel with activity. The existence of such a current of air, caused by the escape of electricity from points, is further shown :—

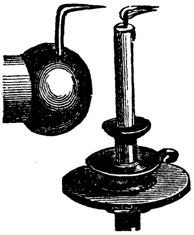
6. **By a candle flame**; a candle, fig. 584, held before the point, has its flame blown aside by the rush of air accompanying the electricity. If the candle is placed as a conductor, and a point is held out to it, the direction of the flame is altered by the reverse fluid induced on the point, fig. 585.

This experiment has been called *the electrical blow-pipe.* The rush of air from the points may be so energetic from an active machine, as to extinguish the flame. In the dark, all points on an electrical machine emit a stream of light, called the electrical brush. Of course no sparks can be drawn from points, but a Leyden jar may be silently charged from them. Masts and yards of ships are often seen thus tipped with fire (called St. Elmo's fire) in a thunder storm. If the point is covered with a ball an inch or two in diameter, its peculiar action ceases, and the ball emits sparks.

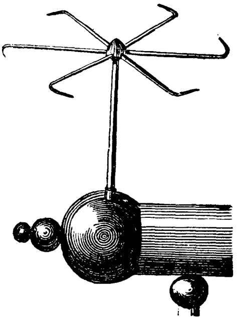
585



584



583



7. Franklin's spider, Ellicott's electrical water-pot, the inclined plane, and the electrical planisphere, are other well-known forms of apparatus, designed to show the same principle. The catalogues of all leading instrument-makers, contain numerous additional illustrations to the same end.

V. ACCUMULATED ELECTRICITY AND ITS EFFECTS.

843. **Disguised or latent electricity.**—The phenomena of induction, already explained, have a curious and most important extension in the subject of this chapter. When two equal and insulated conductors, equally excited by the two opposite electricities, are separated from each other by only a thin plate of glass, or other dielectric material, no signs whatever of any electrical excitement are communicated by either to an electroscope connected with them. The dielectric prevents the union of the opposing electricities, but not their mutual inductive action, whereby their presence is entirely masked to surrounding bodies.

Removed to some distance from each other, each manifests free electricity, by the divergence of its electroscope. But if they are once more brought together, this evidence of excitement again disappears, and so on, until the imperfect insulation of the air gradually neutralizes all free electricity.

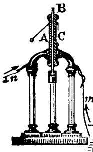
When so situated, the electricities are said to be *latent or disguised,*—paralyzed by their mutual attractions.

844. **The condenser of Æpinus.**—The phenomena of disguised electricity are illustrated by the use of various *condensers,* consisting essentially of two extended metallic surfaces, and an insulating me-

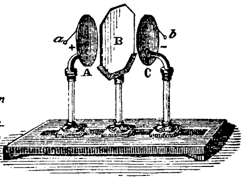
dium. They are sometimes adapted to accumulate electricity of high tension, and sometimes their aid is invoked to render sensible, quantities of electricity otherwise insensible.

The condenser of Æpinus, figs. 586, 587, is designed for the former purpose. Two polished metallic surfaces, A C, with electroscopes, *a b*, and an intermediate thin glass plate, B, fig. 587, are all mounted on insulating glass pillars, and slide in a groove cut in the base. In fig. 586, the two disks are placed in close contact with the intervening dielectric, B, while, by the chain, *n*, positive electricity flows into A, from the excited conductor of an electrical machine.

586



587



Did A stand alone, it could only receive so much electricity as would raise its surface to the same tension with the prime conductor of the machine. But this condition is wholly changed by the presence of the second plate, C, cut off from actual contact with A, by the dielectric, B, but entirely within its inductive influence. A part of the natural fluid of C is at once decomposed by this influence of A, attracting its negative fluid to the inner surface of C, and holding it there, while the corresponding positive fluid from C, is expelled by the conductor, *m*, to the earth. No free electricity would remain if it were possible for B to exist and act as a dielectric without thickness: but, as this is evidently impossible, it happens that a little less negative fluid is drawn to the surface of C, than exists of positive on A, by reason of the thickness of B. Consequently, the electroscope on A remains slightly elevated (residual charge), even after some time, while that on C continues wholly passive.

But the neutralization of A's positive fluid by the decomposition of an equivalent of natural electricity in C, results in diminishing the tension of A, to the low degree corresponding to its residual free electricity Hence, A can receive a fresh charge from the machine, raising its tension to its first condition, and inducing the decomposition of a fresh portion of neutral electricity in C as before, and thus the action proceeds, until the whole of the natural fluid of both plates is decomposed and disguised, or rendered latent, excepting that small portion which at each instant constitutes the *free* electricity, equivalent to the difference due to the thickness of B, and which, as we have seen, would be null, if B could be conceived of as having no thickness. It is this small residue which constitutes the residual charge in the Leyden jar.

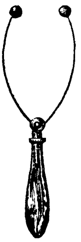
In performing this experiment, the knuckle may serve as a conductor to the earth, in place of *m*, when a rapid series of sparks will be received (positive electricity), constantly diminishing, and ceasing with the maximum charge of A and C. This point of maximum charge is dependent on, 1st, the extent of surface in A B, 2d, on the tension of the prime conductor; and 3d, on the thickness of B.

When the point of saturation in A and B is reached, and all the electricities possible are disguised in the condenser, the pendulum on A still shows only a feeble excitement, although both A and B are in a state of extreme tension. Withdraw, now, A and C from B, as shown in fig. 587; now, the electroscopes, A and B, both show high excitement: restore the plate again as at first, and *b* becomes again entirely passive, while *a* shows the same feeble excitement as before. The opposing fluids in A and C are wholly occupied with their mutual attractions, and only the small excess of + fluid is free, as already explained, to affect the electroscope, B. The plates, A and C, are now fully charged with disguised electricity, rendered latent by mutual inductions, and the polarization of the dielectric B. Although apparently passive, they are actually in a state of high tension, as may be proved by their discharge.

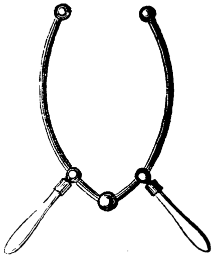
845. **The discharge of the condenser** may happen in three ways:—1st, *insensibly;* by the imperfect insulation of the supports, especially if the air is damp; but always gradually.

2d, by the *disruptive discharge.* If the plate, B, is too thin in reference to the extent of surface in A and C, the tension of the opposing fluids will overcome the cohesion of the glass, B, and it will be shivered in pieces, with a loud explosion, and brilliant spark. The same spark and explosion may take place without destroying the dielectric, if we use a *discharging rod,* to effect communication between A and C. This apparatus is either single, as in fig. 588, or double, as in fig. 589. If this rod is provided with glass insulating handles (as in the figures), the experimenter feels no sensation; but otherwise, or if A and C are brought into connection by the naked hands, then a powerful *shock* is experienced, convulsing the whole frame. The same sensation, in a feeble degree, is felt when the knuckle receives the sparks of an excited machine.

588



589

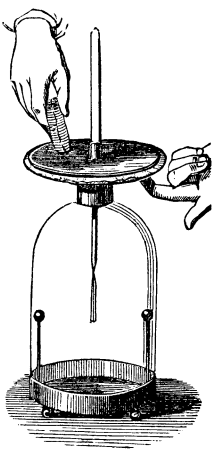


3d, and lastly, the charged plates may be *slowly* discharged by *alternate connection with the earth.* While the condenser is in the condition indicated by fig. 586, touch C with the finger; no effect follows; touch A, and a feeble spark is received; the electroscope, *a*, falls, while, at the same instant, that on C is raised to the same degree, showing that what A has lost in free positive electricity, C has gained in free negative fluid. Touch C; a slight shock, a feeble spark, and the fall of the electroscope, B, ensue, while the electroscope on A again manifests its original excitement. Thus, by the alternate discharge of A and C, the whole of their disguised fluids are gradually set free and discharged.

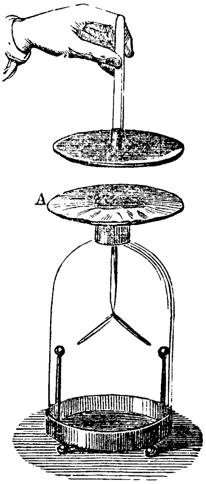
846. **Volta's condensing electroscope.**—This instrument depends on the principles just explained, and is used to render sensible by condensation, electricity of too feeble a tension to affect the ordinary gold-leaf electrometer.

The plate A, fig. 590, is covered with waxed silk, slightly larger than the disks; this takes the place of the dielectric, B, fig. 586. When the instrument is used the upper plate is placed in the position shown in fig. 591, and its surface is touched with the body whose excitement we would measure; *e. g.* a crystal of tourmaline. At the same time, the under surface of the lower plate (in connection with the gold leaves), is touched by the moistened finger of the other hand The influence of the excited electric acts in this case exactly as has been already explained in the condenser of Æpinus. No divergence is seen in the gold leaves, until the upper plate of the condenser is raised, as in fig. 590, when the gold leaves promptly diverge: this action being heightened by the inductive influence of two little balls of polished brass, rising within the glass as high as the lower edge of the gold leaves.

591

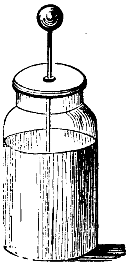


590



Dr. Hare's single gold-leaf electrometer.—In this instrument, a single gold leaf, about three inches long, by one-third of an inch broad, is hung by a brass rod from the top of a bell or globe, as in the last instrument. Immediately opposite to the lower end of the leaf, a similar rod of metal passes through an opening in the side of the vessel carrying a gilded disk of wood or paper half an inch in diameter. This lateral rod is graduated to measure small distances. To use this instrument, the lateral rod is put in communication with the earth, and an electric is brought in contact with the upper disk, when, if the distance between the leaf and the lower disk is small, the most minute attractive force is detected. In the original instrument, Dr. Hare employed a plate of zinc, on an insulating handle, and one of copper on the instrument arranged as in Volta's electrometer, when the simple separation of these two disks would evince a tenfold delicacy of action compared to Volta's condenser.*

592



847. **The Leyden jar.**—Accident led to the discovery of this remarkable piece of apparatus, long before its principles were made clear by the condenser of Æpinus, and the explanations of Franklin. It consists of a thin glass jar, fig. 592, coated inside and outside with tin foil, as far as the bend of the neck. The inner surface is extended by the

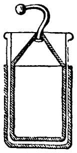
---

* Hare's Mechanical Electricity, Philadelphia, 1835, p. 29.

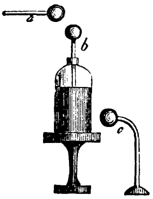
wire, carrying a brass knob at top, touching the inner coating by a piece of chain or wire, and sustained in its place by passing through a non-conducting cover of dry wood.

The relations of the jar and the two metallic coatings, will be seen by fig. 593, showing in section a Leyden jar, the parts of which are separable, and its wire bent conveniently into a hook, to suspend it on the machine. It will be seen at a glance that this arrangement is identical in principle with the condenser of Æpinus, and the electrical plates of Franklin. If the knob, *b*, of the Leyden jar, insulated upon a stand, fig. 594, is presented to the conductor, *a*, of the electrical machine in action, only a single spark or so, will enter it, unless a way is provided, as by the conductor, *c*, for the escape of the similar electricity from the exterior coating, while its opposite is then fixed as in C, fig. 587. The charging of the jar then proceeds, and for every spark which darts from *a* to *b*, a corresponding one of similar electricity, is seen to escape from the outer coating to *c*. When it is held in the hand, the same effect follows through the arm, accompanied by a slight twinge in the nerves. Presently the point of saturation is reached, and the two decomposed electricities are latent. Either coating may be fearlessly touched alone, but as soon as by the discharger or otherwise, communication is made between them, a loud snap and brilliant spark follow with a violent shock.

593



594

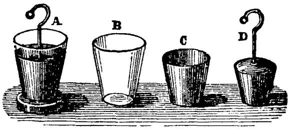


The invention of the Leyden jar, or vial, is commonly attributed to Cunæus or Muschenbroek of Leyden, in 1746. Von Kleist, dean of the chapter at Comin, in Pomerania, also independently discovered this important instrument by a similar accident.

With a view to fix electricity in some insulated substance, Cunæus, in 1746, led electricity into a small vial containing water, by a bent nail thrust through the cork, and hung upon the prime conductor. Endeavoring, in one of these trials, to detach the vial and nail from the electrical machine, Cunæus, to his great amazement, received a violent shock. Von Kleist, in the course of a valuable series of experiments (1745) on electricity, led the fluid by a brass wire or pin into a bottle containing mercury. "As soon," he says, "as this little glass, with the pin, is removed from the electrical machine, a flaming pencil issues from it so long, that I have been able to walk sixty paces in the room with this little burning machine; and if the finger or a piece of money be held against the electrified pin, the stroke coming out is so strong that both arms and shoulders are shaken thereby."

This discovery of so wonderful a power in nature, before unsuspected, created immense excitement over the civilized world, and it was precisely at this time that Franklin immortalized himself by his contributions to the new science. He explains the action of the Leyden vial by his single fluid hypothesis, in his "Observations and Experiments on Electricity," in a manner which must ever win for him the reputation of a profound philosopher.

595

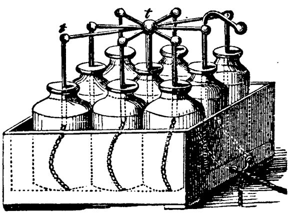


848. **Electricity in the Leyden jar resides on the glass.**—In fig. 595, the jar, A, is composed of the three separable pieces; B, the glass, C, its outer, and D, its inner metallic coatings. When this jar is charged, and set on an

insulating surface, it may be separated into its three parts without being discharged; but C and D will then be found by the electroscope entirely free from excitement, while B remains strongly excited. Putting the parts together again as in A, the jar will be found charged as at first, if the air is dry, and too much time has not passed.

849. **The electric battery.**—As the charge of the Leyden jar is, other things being equal, directly as its surface, large jars are plainly of more power than small ones. But a limit of size is soon reached, which the thickness of glass required for strength, and other circumstances, render it unprofitable to pass. Hence several coated jars, of moderate size, are united by joining all their inner coatings by metallic rods, and all their outer coatings by a common conducting base, as shown in fig. 596. Such an arrangement is called an *electrical battery*. When charged from a common source, and discharged in the usual way, they all act as one great jar, the result being not quite in the ratio of the number of jars, but nearly so. Hence, the smaller the number, the thinner the glass, and greater the size of the jars, the better, and several batteries of seven and nine jars, united to the charging rods of the central jars, are preferable to more extended single series. They are charged by connecting the interior with the prime conductor by *t*, and the exterior with the earth. If the battery is extensive, and the machine powerful, great caution is requisite to avoid receiving its shock; an accident which might be serious in its consequences.

596



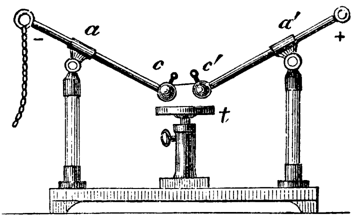
The battery used by Von Marum, with the machine already noticed (837), embraced one hundred jars, each thirteen inches in diameter and two feet high. The coated surface was five hundred and fifty square feet (five and a half feet to each jar). When fully charged, its force was irresistible. A bar of steel nine inches long, half an inch wide, and one-twelfth of an inch thick, was rendered powerfully magnetic by the discharge. A small iron wire, twenty-five feet long, was deflagrated, and various metals were dissipated and raised in vapor, when placed in the circuit of discharge. A book of 200 pages was pierced by it, and blocks of hard wood, four inches square, split into fragments.

850. **Discharge in cascade.**—A series of two or three Leyden jars may be placed horizontally upon insulating stands, so that the interior of each succeeding one may receive the spark from the outer coatings of the one preceding.

This mode of charging cannot be carried beyond two or three jars, owing to the accumulated resistance soon vitiating the result. But Mr. Baggs, of London, has very ingeniously contrived an electric battery, the jars of which are charged together, but are discharged consecutively. Each jar is supported in a horizontal position on a vertical spindle, their knobs, while being charged, pointing outward, like the radii of a circle, and when the battery is to be discharged, the knobs, by a quarter revolution, are brought opposite, each to the bottom of the next jar. In this way the disruptive power or intensity of the spark is multiplied as the jars, the *quantity* remaining the same. Mr. Boggs is said to have discharged his battery of twelve jars through a space of three feet. (Am. Jour. Sci. [2] VII. 418.)

851. **The universal discharger.**—Various contrivances are in use for regulating or measuring the discharge of the electric battery, and the single jar. Of these, Henley's universal discharger, fig. 597, is, perhaps, the most useful. By means of this simple apparatus, the electrical fluid may be made to pass through any substance placed upon the table, *t*. Two rods, sliding in the joints *a a′*, end in balls, *c c′*, covering points which can be exposed by their removal. The rod, *a′*, connects with the positive side of the battery, for example, while by the discharging rod, fig. 589, communication can be made at pleasure between *a* and the negative side of the battery, by a chain or metallic thread.

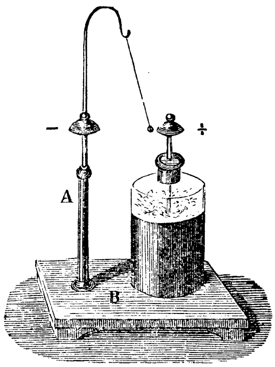
597



The charge of the battery may be prevented from passing a given limit, by using the discharging electrometers of Lane or Cuthbertson, in which a ball is sustained at such a distance from the discharging knob of the battery, that when its charge reaches the proper tension, it discharges itself.

A beautiful illustration of the slow discharge of a charged jar is seen in fig. 598, where a charged Leyden jar, with a small bell in place of the knob, is set upon a board, near to a little brass ball, hung from a silken thread, upon a wire, carrying a second bell in connection with the earth by A B. The effect is, that the + electricity of the jar attracts the little ball, but after striking the bell, the ball is repelled, until, coming in contact with the other bell, it is discharged, and so on for many hours, this little chime is rung by the electrical pendulum.

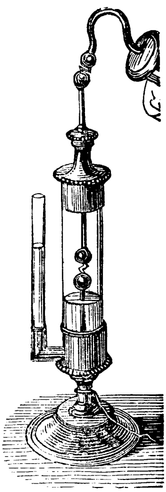
598



852. **The electric spark.**—The electric light and spark result from the reunion of the two electricities in the air. In a vacuum there is no *spark* (fig. 601). The zigzag path of the spark and of lightning is due to the resistance of the air Every electrical discharge produces expansion of the air,

and the form and color of the spark are materially influenced by the density and chemical composition of the gaseous medium through which it passes. The character of the sparks depends also on the form, area, and electrical intensity of the discharging surfaces, likewise on the kind of electricity on the conductor in which the spark originates; from the negative conductor the sparks are far less dense and powerful than those from positive electricity.

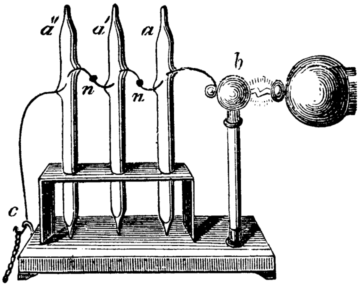
599



Kinnersley's thermometer, fig. 599, shows the agitation and expansion of the air following an electrical explosion. A portion of water in the larger vessel, which is air-tight, communicates freely with the small open tube, attached to the foot, and ending in a narrow glass tube. When an electrical discharge takes place through the apparatus, the consequent expansion of the air violently raises the column in the smaller tube, but, after the commotion is over, the fluid gradually regains its original level, as the air in the larger vessel cools. *The electrical mortar* discharges its ball by the force of air expanded at the moment of the electrical discharge.

The color of the electrical spark.—Faraday observed that in air, oxygen, and dry chlorohydric acid gas, the spark was white, with a light bluish shade, especially in air. In the heavy thunderstorms common in an American summer, the light of a powerful flash of lightning is distinctly purple, and sometimes violet. In nitrogen it is blue or purple, and gives a remarkable sound; in hydrogen it is crimson, and disappears when the gas is rarefied; in carbonic acid the color is green, and the form of spark very irregular; in oxyd of carbon it is sometimes green and sometimes red; in chlorine it is green.

600

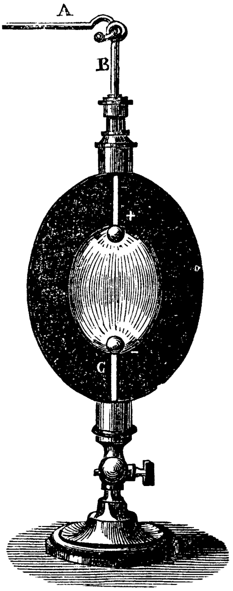


The little apparatus, fig. 600, is well calculated to show these effects by contrast at one view. The three tubes, *a a' a''*, are respectively filled with various gases and sealed. Each tube has two short platinum wires, *n n*, soldered into its sides, through which the electric spark from *b* must pass on its way to the ground by *c*.

The electrical discharge in a vacuum, becomes an ovoidal tuft of light, uniting the conductors. The apparatus, fig. 601, is designed to show these effects. A large egg-shaped glass vessel is mounted at the lower extremity with a stop-cock, for attaching it to the air-pump, in order to remove the whole or a part of the air, or to replace it by vapor of alcohol, ether, or any other gas not acting on brass. By the rod, A, connection is established with the electrical machine, while the distance between the electrical poles, B C, may be adjusted by sliding the upper rod in its air-tight socket. This apparatus is called the electrical or philosophical egg. The rarer the air, the more globular becomes the spheroid, and, at the same time, less brilliant. The *auroral tube* is only a modification of the same apparatus.

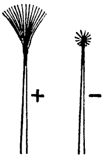
This apparatus is also used with splendid effect with the Induction coil.

601



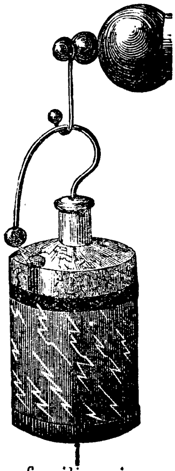
Difference between the positive and negative spark.—The tuft of light from positive electricity is far more beautiful than that from negative, as seen from the ends of two points. Thus, while positive electricity gives an opening sheaf of light, negative electricity gives only a small star, fig. 602. In rarefied air, these differences are much less apparent. Faraday suggests that they are due, chiefly, to the greater facility with which negative electricity escapes in air, than positive, as conductors negatively charged, lose their excitement sooner than those positively charged.

602

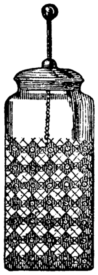


The diamond jar.—To show that the coatings of the jar convey the electricity collected on the glass to the point where it meets the cause of discharge, a jar may be coated with metallic filings, fig. 603, or patches of tin-foil, fig. 604, cut in lozenges (a diamond jar). The wire of the jar is bent over, as in fig. 603, so as to bring the ball near the outer coating, which connects by a chain with the earth. When the machine (on whose arm this jar is hung) is worked, a brilliant spark is seen at intervals to dart from the knob to the outer coating, and thence to spread in zigzag courses over the whole surface.

603

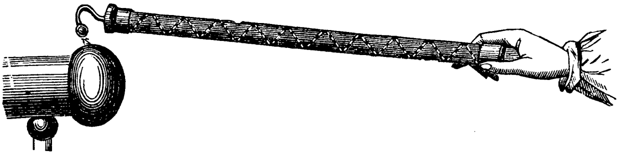


604

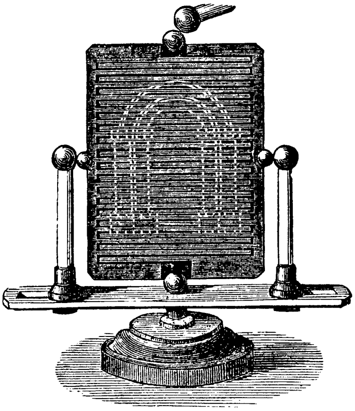


Scintillating tube and magic squares.—Every collection of electrical apparatus contains these familiar pieces of apparatus, illustrative of the phenomena of the electric spark. The scintillating tube, fig. 605, like the jar, fig. 604, has rows of lozenge-shaped pieces of tin-foil pasted on its interior, usually in a spiral, and, when held by the hand, as shown in the figure, the electricity flashes from point to point at the same apparent instant, producing a most agreeable effect.

605



606



The magic squares are panes of glass on which are interrupted strips of tin foil, cut to represent some design, to be made visible only when a spark passes. These squares are mounted on a foot, in connection with the earth, and are set near the ball of the prime conductor. By scattering metallic filings over a varnished surface of glass, the same effect is produced as upon the jar, fig. 603.

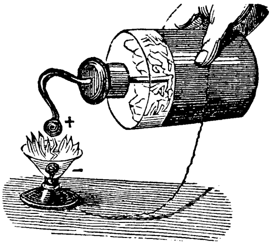
853. **Effects of the electric discharge.**—The effects of the electric discharge are chiefly, 1st, physiological; 2d, physical; 3d, mechanical; 4th, chemical. The passage of the electricities through bodies, is sometimes impeded by their bad conducting power, or by want of proper dimensions; and, in either case, a powerful electric discharge manifests itself in one of those modes.

854. **Physiological effects.**—These are seen in the shock experienced by all living beings, in the passage of electricity through any of their members. Any number of persons, joined hand to hand, will receive, at the same instant, the shock of an electric battery. Abbé Nollet imparted it to over six hundred persons in his convent at one time—those in the middle of the chain being little less affected than those near the conductors.

A person charged on the insulating stool, feels a prickly heat and glow of the skin, resulting in perspiration. Many useful applications have been devised of this agent in medicine.* It needs hardly to be said, that the full shock of a powerful battery will destroy life in man. Sparks, fifteen or eighteen inches long, begin to be unsafe, if from large surfaces. Small animals, as birds, are easily killed by a moderate discharge, on the table of the universal discharger. Fig. 597.

855. **Inflammation of combustibles.**—Although no sense of heat is felt when the knuckle receives strong sparks from an active machine, yet the smallest spark serves to inflame ether, whether from a Leyden jar, from the finger, or, more strikingly, from an icicle held in the fingers of one mounted on an insulating stool.

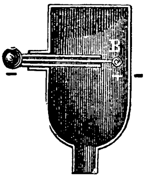
607



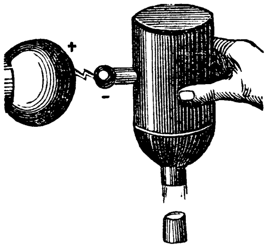
The ether is placed in a metallic cup, and the spark should be drawn on its edge moistened with ether, fig. 607. Gunpowder placed on the table of the universal discharger, over the *points* of the rods, *a a'*, fig. 597, is simply thrown about, without being fired; but if a wet string, in place of one of the conducting wires, forms part of the connection, its retarding power is such as to fire the powder. The lighting of gas from the finger of one charged by running on a carpet, has already been mentioned (840, (2).) Lycopodium, alcohol, a newly extinguished candle, and many other combustibles, are also easily inflamed by the spark. Gold leaf confined between two glass plates with the edges hanging out, will burn with the explosion of the glass, and, if held between cards, will stain them with purple oxyd of gold. Silhouette likenesses of Franklin are thus printed: a powerful current from a battery is needed for this.

856. **Chemical union effected by electricity.**—A mixture of hydrogen two volumes, and of oxygen one volume, or of hydrogen with seven or eight times its volume of common air, is exploded by a spark passing through the containing vessel, *e. g.* the tin air-pistol, called "Volta's pistol," fig. 608, is provided with an insulated conductor, ending near the inner surface of the pistol at B. Its mouth is closed tightly by a cork, and the spark caused to pass by holding it near the prime conductor, fig. 609, or to the electrophorus. The cork is then violently expelled, by the expansion of steam, with a loud explosion.

608



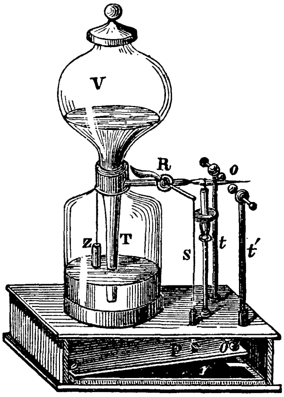
609



---

* Consult Garratt's Electro-physiology, Boston, 1860, 8vo., pp. 708; or Channing's Medical Electricity.

610



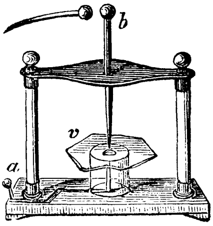
Volta's electrical lamp.—A self-regulating hydrogen apparatus is seen in fig. 610, similar in its action to the common hydrogen lamp, with platinum sponge. In its base drawer is an electrophorus, *r* P, the plate, P, of which is always charged. A silk cord connects the upper plate P, with the gas cock, R, in such a way that when the gas in T is drawn, the communication is effected at *o*, with the insulated wire, *t′*, and the electricity thus finds its way in a spark between the buttons at O, and escapes to the earth by *t*. As the hydrogen is flowing at that moment from the jet, it is inflamed, and kindles a little candle standing in its path. Every time the cock, R, is moved, the plate, P, rises, and communicates a spark. With care, this instrument remains in action for weeks, from a single excitement.

857. **The mechanical effects of the electrical discharge.**—Any thin non-conducting substance placed between the balls of the universal discharger, is either pierced or broken where the fluid passes. The phenomena attending these experiments are curious and instructive in point of theory.

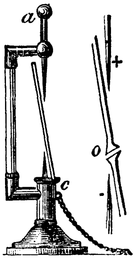
When a thin piece of glass, *v*, is placed in the position seen in fig. 611, between the points of the conductors, *a b*, a small hole will be made through the glass, as if with a drill, provided the effect of the fluid is concentrated by placing a drop of oil at the point to be pierced. The hole is circular, starred, and its edges smooth, and sometimes it remains filled with the powdered glass in fine dust, easily removed. It requires a powerful battery to pierce glass one-twelfth of an inch thick.

If a card is placed in the path of the fluid, it is pierced with a raised edge (burr) on both sides of the hole. When the card is placed obliquely, as seen in fig. 612, between the points, *a c*, of the insulating holder, the hole is made in the place and direction seen at *o* in the section; that is, nearer the negative pole, its edges being raised, or thickened, a circumstance due, probably, to the decomposition of the neutral fluid in the card, occasioning a rush of electricity in both directions. This has been esteemed a fact inexplicable on the single fluid hypothesis, while the position of the hole, always near to the negative pole, indicates that the negative fluid passes less readily through the air than the positive. Many other examples of the frac-

611



612



ture, or dispersion of non-conducting bodies, may be gathered from the larger treatises.

858. **The chemical effects** of statical electricity are generally feeble. Besides those before alluded to (856), Wollaston, with very fine points of gold wire immersed in water, decomposed water in very small quantity. A paper moistened with iodid or bromid of potassium, is stained brown by the electrical discharge when it is laid upon the scintillating square (fig. 606). Olefiant gas, sulphuric acid, chlorohydric acid, ammonia, and nitrous oxyd, are decomposed by the electric discharge, with the separation of their constituent elements, and carbonic acid is decomposed into oxygen, and oxyd of carbon. The elements of the air unite under a prolonged series of sparks (Priestley), to form nitric acid (Cavendish), and lightning in the atmosphere forms the same compound, as the analysis of rain-water has shown (Liebig). Numerous other evidences of the chemical effects of electricity have been recorded; perhaps the most important of these, is that atmospheric effect, called *ozone.*

859. **Ozone.**—This term is derived from the Greek, in allusion to the peculiar odor which is always perceived after an electrical discharge or excitation of a machine, and sometimes improperly compared to the odor of sulphur, which it does not at all resemble. It is due to a remarkable state or condition induced in oxygen gas by electricity (and by several other causes also). Mr. Schonbein, of Basle, has devoted himself to the study of the curious properties of this singular product, the record of which belongs rather to Chemistry than to Physics.

VI. ATMOSPHERIC ELECTRICITY.

860. **Franklin's kite.**—We owe to Dr. Franklin the demonstration that the phenomena of a thunder-storm are due to electricity, identical with that excited in electrical experiments. He proposed two modes, in 1749, by which he supposed electricity might be drawn from the clouds. Dalibard, at his suggestion, erected in the open air near Paris, in 1752, a pointed and insulated iron rod, 40 feet long. On the 10th of May, 1752, electrical sparks were obtained from this rod, with the usual snapping sound. In June of the same year, Franklin, tired of waiting for the erection of a tall spire in Philadelphia on which to place his pointed conductor, conceived the idea of reaching the higher regions of the air by a kite. This he formed of a silk handkerchief stretched over two light cedar sticks. It had a pointed wire at top, and a silken cord insulated the hempen string, at the lower end of which he tied an iron key.

Watching the approach of a thunder-storm, he raised the kite, and soon had the satisfaction of seeing the fibres of the hempen string bristle and repel each other, and finally when the rain had rendered the string sufficiently a conductor, he enjoyed the unspeakable satisfaction of seeing long electrical sparks dart from the iron key. Thus was realized by actual experiment one of the boldest conceptions and most interesting discoveries in the history of science.

Efforts have been made to rob Franklin of the honor of this discovery, but it is one thing to suggest that two phenomena may be identical, and quite another thing to prove it. Dalibard's experiments were undertaken at Franklin's suggestions and hardly preceded his own in date.

These experiments were everywhere repeated, and it soon became evident that they were far from being free from danger. Romas, in June, 1753, during a thunder-storm in France, drew flashes of electrical fire ten feet long, from a kite raised by a string 550 feet long. The experiment was accompanied by every evidence of intense electrical tension in the attraction of straws, the sensation of spiders' webs over the faces of the spectators, and in the loud reports and roaring sounds, similar to the noise of a large bellows. In August, 1753, Prof. Richmann, of St. Petersburgh, lost his life while engaged in similar experiments. Cavallo, in London, in 1777, obtained enormous quantities of atmospheric electricity by an electrical kite, and noticed that it frequently changed its character as the kite passed through different layers of the air. In telegraph offices during a thunder-storm, vivid sparks, often very inconvenient and not without danger, are constantly flowing from the receiving instruments, being induced on the telepraph wires from the atmosphere, during thunder-storms. (Henry, Am. Jour. Sci. [2] III. 25.)

861. **Free electricity in the atmosphere.**—That the atmosphere, besides the combined electricity proper to it, contains also at all times free electricity, is proved by raising an insulated conductor a few feet into the air, as by a long fishing-rod, and connecting it with the condenser of the electrometer, the leaves of which will diverge sensibly when there is no sign of any thunder-storm. Near the earth (say within three or four feet), no evidence of free electricity can be detected, but as we rise in the air, its force constantly increases. Becquerel and Breschet, sent up arrows, attached to a tinsel cord ninety yards long, from the top of the Great St. Bernard, while the other end was connected with the condenser of an electrometer; they found that the gold leaves diverged in proportion as the arrow rose higher.

It appears from experiments like these and others made chiefly by Ronalds, of Kew, that the atmospheric electricity increases and decreases daily, twice in twenty-four hours, and the following general results are established.

1st. The electricity of the air is always positive—is fullest at night—increases after sunrise—diminishes towards noon—increases again towards sunset, and then decreases towards night, after which it again increases.

2d. The electrical state of the apparatus is disturbed by fogs, rain, hail, sleet, or snow. It is negative when these approach, and then changes frequently to positive, with subsequent continued changes every three or four minutes.

3d. Clouds also, as they approach, disturb the apparatus in a similar way, and produce sparks from the insulated conductor in rapid succession, so that an explosive stream of electricity rushes to the receiving pole, which should be passed off to the earth. Similarly powerful effects frequently attend a driving fog and heavy rain.

Crosse, of England, had over a mile of insulated wire sustained on poles one hundred feet high above the tall trees of his park, connecting by pointed conductors with his laboratory, where he has frequently collected, during a heavy fog, electricity enough to charge and discharge a battery of fifty jars, and seventy-three square feet of coated surface, twenty times in a minute, with a report as loud as that of a cannon.

Numerous hypotheses have been put forward to account for what has been considered the free electricity of the atmosphere. Prof. Henry, after an attentive study of the whole subject, feels compelled to reject them all as insufficient except that of Peltier, which refers these phenomena not to the excitement of the air, but to the inductive action of the earth on its non-conducting aërial envelope. This view involves the assumption that the earth was in some way primarily electrified. It is a fact that the earth is always in a state of negative excitement, as was shown by Volta, who for this purpose received the spray from a cascade on the balls of a sensitive electrometer, when the leaves diverged with negative electricity. (See an able article on Atmospheric Electricity by Prof. Henry, Patent Office Report, Agriculture, 1859, p. 485.)

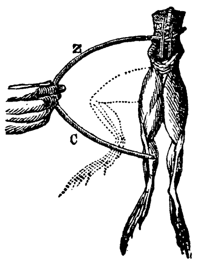
The subject of atmospheric electricity, especially the description of electric meteors, is more properly referred to Meteorology.

§ 3. Dynamical Electricity.

I. GALVANISM AND VOLTAISM.

862. **Discovery of galvanism.**—In 1786, Luigi Galvani, professor of anatomy in the University of Bologna, while engaged upon a long series of observations on the effects of atmospheric electricity upon animal organisms, noticed that the legs of some frogs, prepared for experiments, became convulsed, although dead and mutilated—when the vertebræ, with portions of the lumbar nerves, were pressed against the iron railing of the window balcony where they were placed, awaiting the use for which they had been designed. Repeating this novel and curious observation in various ways, he soon found that the convulsions were strongest when he made connection by means of *two metals* between the lumbar nerves, and the exterior muscles denuded of the skin, as shown in fig. 613, where rods of copper and zinc, being thus held, convulse the leg into the position shown by the dotted line. But contact of metals with the animal tissues he found not to be *essential* to produce these convulsions, since they occur also by contact of the exterior mucous with the interior nervous surface.

613



To repeat Galvani's experiment, strip the skin from the legs of a vigorous frog and cut the animal in two, an inch above the thighs. Expose the lumbar nerves within and on either side of the backbone, by pushing aside the muscles with the finger, so that the point of an arc of the two metals may touch the nerves; then bring the other metal rod into contact with any portion of the outer surface, and strong twitchings will be developed as if the animal was alive, both on touching and removing the rod, even some hours after death.

The galvanic fluid.—Galvani regarded the convulsions of the frog as excited by a nervous or vital fluid, which passed from the nerves to the muscles by way of the exterior communication established between them: this fluid, in his view, existed in the nerves, it traversed the metallic arc, and falling on the muscles, it contracted them, like the electric discharge.

Galvani was an anatomist and physiologist, and not a chemist or physicist. He did not work out all the teachings of his own discovery, being more interested in demonstrating, as he did, the existence of a true animal electricity, developed between the outer surface and the nerves. The physical branch of the subject he left to others, and chiefly to VOLTA, devoting the few remaining years of his life to the study of animal electricity. Volta's doctrines Galvani never accepted, and died in 1798, before the Voltaic Pile was given to the world. In the department of vital electricity, Galvani's labors have been justly appreciated only in our time, having been naturally eclipsed in his own by the splendid discovery of the Voltaic Pile, and the crowd of wonders following in its train.

The story usually found in text books, of the accidental discovery, in 1790, of the new science by the twitching of frogs' legs, prepared for the repast of Madam Galvani, is a fabrication of Alibert, an Italian writer of no repute. Galvani had then been for eleven years engaged upon a laborious series of electro-physiological experiments, using frogs' legs *as electroscropes*. No great truth was ever discovered by *accident*, and years of laborious research had prepared the way for this discovery. It is undoubtedly true that what we find is often more important than what we seek, but it is *research* and not *accident* which makes the discovery. Every hypothesis is good which bears fruit in discovery; but to accept the discovery and reject the hypothesis when no longer fruitful, requires all the self-denial of the highest philosophy, and is a noble attribute of the greatest minds.

863. **Origin of Volta's discovery.—Contact theory.**—Adopting at the outset with the greatest enthusiasm the vitalist hypothesis of Galvani, Volta came, after no long time, to the conviction that the electrical effects attributed by Galvani to the animal electricity of the frog, were really due to *the contact of dissimilar substances*, and that the frogs' limbs were only the sensitive electroscope, adapted to indicate the electrical current developed by the two unlike metals. Each discoverer saw but half the truth. Thus originated his celebrated "*contact theory;*" a view of the source of dynamic electricity, that long held almost universal sway over scientific opinion until gradually supplanted by the *electro-chemical theory*, which refers these phenomena to *chemical action*.

By the use of his condensing electrometer (846), Volta sought to establish the contact theory by a great number of well-devised experiments. Being assured of the passive state of the electrometer, he established communication between the earth and the upper plate by the moistened fingers, while at the same time a bit of zinc plate held also in the moistened fingers of the other hand is placed in contact with the lower plate; after a single instant, contact is broken, and on raising the upper plate, the gold leaves diverge. Whence the electricity? Volta replied, "from the contact of the two unlike substances," overlooking the fact that there was a chemical action, due to the effect of the moist fingers on the zinc. As the plate touched by the zinc became positive, and the copper negative, he assumed that there was an "*electromotive force*" capable of developing these electrical states in the two metals as a result of *simple contact*. This experiment was repeated with conductors of every sort, and always, when one of them was an alterable substance, with the same results. He divided conducting bodies into two classes; the first class, including the metals, metallic ores, and carbon, he calls *electromotors;* the second class contains liquids, saline solutions, animal tissues, &c. He found that a double combination of three elements, so arranged that their order was reversed, neutralized each other, and produced no spasm in the frog's legs, which he uniformly used as an electroscope. This was in 1796, four years in advance of the date usually assigned as that of the invention of the Voltaic pile.

Passing over the long controversy between Volta and his cotemporaries, we come to the essential fundamental fact of Volta's discovery, viz.: *that certain metals, and particularly the oxydizable metals, disengage electricity and charge the condenser, when placed in the conditions just described.*

This discovery immediately led to the second, and by far the most celebrated of Volta's discoveries, viz., the *Voltaic pile, or battery.*

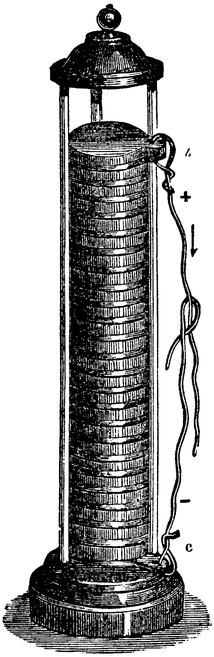
864. **Volta's pile, or the Voltaic battery.**—Every form of apparatus designed to produce a current of dynamic electricity is called a *battery* or *pile.* Volta's original apparatus was, as its name implies, *a pile* of alternate silver and zinc disks, laid up as in fig. 614, with disks of paper or cloth between them, moistened with brine, or acid water. This arrangement was more commonly made with alternate disks of copper (C) and zinc (Z), care being taken always to observe the order, copper—cloth—zinc. The terminal disks were provided with ears for the convenient attachment of wires. Thus arranged, the following characteristic results are observed. 1st. The pile being insulated by glass or resin, touch Z with the plate of the condenser (covered with silk), while the finger rests on C, and then apply the plate to the condenser; the gold leaves will indicate strong vitreous electricity. 2d. Reverse this order, touching C with the plate while the finger is on Z, and a strong charge of resinous electricity is received.

The pile may be regarded as a Leyden jar, or electrical battery, perpetually charged, and capable of re-charging itself as long as the given conditions are maintained.

These results may be repeated an indefinite number of times, as long as the cloths remain moist, and the intensity of the action is directly as the number of plates in the pile.

Each touching couplet of copper and zinc may be soldered together, and is then called a *couple, pair*, or *voltaic element*. Any two metals of unlike properties may be substituted for the zinc and copper, with the same results.

614



The end of the pile which yields vitreous electricity is called its *positive pole*, and that which yields resinous electricity is called the *negative pole;* a name also applied to the wires or conductors connecting the two poles.

Arranged as in fig. 614, the pile, when its poles are joined, gives a decided shock, similar to, but less intense than, that from, statical electricity. On breaking contact between the poles, a brilliant spark of voltaic electricity is seen; and if the wires end in points of gold or platinum, inserted in water, without mutual contact, a flow of gas bubbles from them, announces the decomposition of the water. We thus classify the effects of the pile into *physiological, physical*, and *chemical* phenomena.

The discovery of the pile, Volta announced in March, 1800, to Sir Joseph Banks, both in the form just described and also the crown of cups (*Couronne des tasses*), a series of twenty glass goblets arranged in a circle, with wires connecting the + and — elements of each cup to the opposite of the next.

This last is the type of all modern batteries with separate cells. He classified its effects, but made no mention of its power of chemical decomposition, a property he had not then observed. This last power was immediately discovered by Nicholson and Carlisle, in London, on the 2d of May, 1800. Aside from Volta's theoretical notions, history will ever assign him a high place as a philosopher, and as having by his genius blessed the world by one of the greatest and most fruitful discoveries in science.

Distinction between Voltaism and Galvanism.—It will be seen that *Voltaism* and the *Voltaic pile* are terms properly applied only to the discoveries of Volta, and that the term *galvanic battery* is a misnomer, Galvani never having seen such an instrument. The term *Galvanic fluid* is justly applied to *animal electricity*, which Galvani was the first to discover.

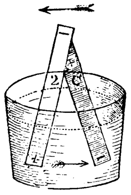
865. **Quantity and intensity.**—There is a very marked difference between the tension of the electricity from the Voltaic pile, and that of friction. No sensation follows the touch of either pole of a Voltaic battery singly: both poles must be touched simultaneously in order to perceive the shock. The projectile force in Voltaic electricity is so nearly null, that in the most energetic and extensive series of cells, the terminal points must be brought indefinitely near, or into actual contact, before any current is established, unless in vacuo. The intensity of the battery is however, under some circumstances, increased by reduplicating the number of couples of a given size (see § 881). The *quantity* of electricity set in motion in the Voltaic battery depends not on the number of the series, but on the *extent of surface* brought into action in each pair, the conducting power of the interposed liquid, and also upon the external resistance.

The views formerly expressed by most authors on the subject of quantity and intensity have been modified in important respects by the application of the "*law of Ohm*;" for a discussion of which compare § 881.

866. **Simple Voltaic couple.**—Whenever two unlike substances, moistened by, or immersed in, an acid or saline fluid are brought into contact, a Voltaic circuit is established. The earliest recorded observation on this subject (Sulzer's), was the familiar experiment of a silver and copper coin, or bit of zinc, placed on the opposite sides of the tongue, and the edges brought together, when a sharp, prickly sensation and twinge is felt, and if the eyes are closed, a mild flash of light is also seen. In this case, the saliva is the saline fluid, exciting a Voltaic current due to its chemical effect on the zinc or copper; and the nerves of sense are the electroscope. The action depends on contact, and ceases, or is renewed, as often as this is broken or made.

In fig. 615, we have the simplest form of Voltaic battery, a slip of amalgamated zinc, Z, and another of copper, C, immersed in a glass of water, acidulated by sulphuric acid. When these strips touch (either within or without the fluid), an electrical current is set up, passing from the zinc to the copper in the fluid, and from the copper to the zinc in the air, as shown by the arrows. The polarity of the ends in the air is the reverse of that in the acid, as shown by the signs plus and minus. This is analogous to the decomposition of neutral electricity in a rod of glass or of wax. While contact is maintained, either directly or by conducting wires, evidence of chemical action is seen in the constant flow of gas bubbles (hydrogen) from the zinc to the copper, from the surface of which they are given off. This action ceases at any moment when contact ceases, and if the separation of the metals takes place in the dark, a minute spark is seen at the moment of breaking contact in the air.

615



The direction of the Voltaic current depends entirely on the nature of the chemical action producing it. Thus if in the arrangement just described, strong ammonia water was used in place of the dilute acid, all the electrical relations of the metals and the fluid would be reversed: since the action would then be on the side of the copper, and the zinc would be relatively the electro-negative metal.

867. **Electro-positive and electro-negative** are relative terms, designed to express the mutual relations of two or more elements in reference to each other. Thus zinc, being a metal very easily acted on by all acid and many saline solutions, becomes electro-positive to whatever other element it may be associated with, unless, as in the last section, the other element is acted on, and the zinc is not, when it becomes electro-negative. Oxygen is an element which acts upon every other, and is therefore the type of electro-negative substances; gold, platinum, and silver, being among the least easily oxydized metals, become electro-negative substances to all others more easily acted on than themselves, and therefore these are fit substances for the negative element of Voltaic couples. In chemical works, tables will be found in which all the elements are grouped in this relative order of electro-positive and electro-negative power.

868. **Amalgamation.**—Commercial zinc is seldom or never pure and the foreign substances which it contains are such as to stand in an electro-negative relation to the zinc. A slip of common rolled zinc, immersed in dilute sulphuric acid, is actively corroded with the escape of abundance of hydrogen, while if a strip of chemically pure zinc was used, no action would happen. (De la Rive.) This action of common zinc is called a *local action*, implying the existence of as many small local Voltaic circuits as there are particles of foreign electro-negative substances on its surface; each of which constitutes, with the contiguous particles of zinc, a minute battery, and thus the whole surface is presently corroded and roughened, and the power of the whole couple reduced just in proportion to the extent of this local action.

Rub the freshly corroded surface of such a piece of commercial zinc with a little mercury, when instantly it combines with and brightens the whole surface, covering it with a uniform coating of *zinc amalgam.* This perfectly protects the zinc from local action by covering up the electro-negative points, and makes the whole surface of one electrical name. Zinc, thus *amalgamated,* may be left indefinitely long in acid water, without injury, and when brought into contact with the electro-negative element of a Voltaic couple, it becomes a much more energetic source of electricity than before.

The discovery of this property (due to Mr. Kempt) is hardly less important than the discovery of the battery, for without it, sustained and manageable batteries are impossible.

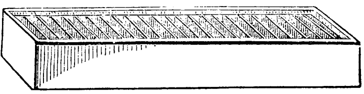
II. BATTERIES WITH ONE FLUID.

869. **Voltaic batteries** are constructed for use either with one or with two fluids.

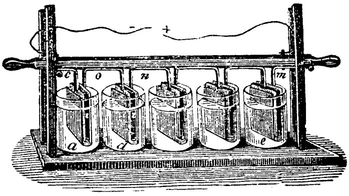
The first embraces the original crown of cups (864), and all batteries with one fluid and a single cell. The batteries with two fluids and two cells, of whatever name, involve a double chemical decomposition, and are, hence, more complicated, but also, generally, more efficient; we will consider these separately, remarking, that the interest attached to the first class, with a single exception, is now chiefly historical.

870. **Trough batteries.**—The inconvenience of Volta's original form of the pile, fig. 614, led to placing the elements in a trough, as seen in fig. 616, called, from the inventor, Cruickshank's trough. Each compound couple of zinc and copper was cemented water tight into a groove, all the zincs facing in one direction. The filling of these cells with dilute acid was a tedious operation, with extended series, and as the zincs were not amalgamated, the best force of the apparatus was spent before it could be filled. Davy and Nicholson greatly improved the trough by attaching the couples to a bar of wood by straps, *c o n m*, as in fig. 617, and Dr. Wollaston surrounded each zinc, *z*, with the copper, on both sides, thus doubling the effective surface. Thus arranged, the whole series could be plunged at one movement into glass cells, *a a*, or into a porcelain trough divided into cells. The famous battery of the London Royal Institution, (first used in May or June, 1810,) was a series of 2000 couples of this construction, arranged in 200 glass or porcelain troughs, ten couples in each trough, each plate having an effective surface of twenty-two square inches. This battery was placed in the vaults under the Royal Institution, where its hydrogen and acid vapors did not annoy the experimenter, and its power was led up by conductors to the laboratory above.

616



617



The battery with which Davy made his immortal discovery of the metallic bases of the alkalies (October, 1807), contained 250 pairs of plates, made in 1803. [See Am. Jour. Sci. [2], XXVIII., 279.]

Hare's calorimotor consisted of twenty plates each, of copper and zinc, nineteen inches square, and so combined in a cubical box as to form but two large elements of fifty square feet each, or two hundred square feet of active surface in both members, all plunged by one movement in a vat of acid.

The deflagrators of Dr. Hare, as originally constructed, were formed of spirals of copper and zinc, rolled with a narrow space between them, and the opposing metals held from contact by wooden strips. Each zinc was 9 × 6 in., and each copper 14 × 6 in.; so that every part of the zinc was opposed to the copper surface; eighty of these coils were so arranged on bars of wood as to plunge by an easy mechanism into glass cylinders containing the acid. The facility of immersion and removal of these coils in contact of the acid liquor, made Hare's deflagrators as much superior to the early trough batteries as the batteries of two fluids are superior to Hare's. In a very efficient form of Hare's deflagrator, the members were connected in a box, suspended, to revolve on an axle having another box placed at right angles to the first, so that a quarter revolution of the apparatus turned on or off the exciting acid at pleasure, without deranging the connections, which were established through the axis of revolution.

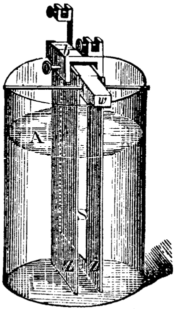
A battery constructed for Prof. Silliman, in Boston, in 1836, on the plan of Wollaston and Hare combined, contained nine hundred couples of copper and zinc (10 × 4 in. each), exposing five hundred and six square feet of available surface, arranged in twelve parallel series, capable of being used consecutively as nine hundred couples, or in three series of three hundred each. One plunge immersed the whole battery, and when this instrument was new, the arch of flame between its poles measured over six inches.

Mr. Crosse, and also Mr. Gassiot, have constructed very extended series of trough batteries for physiological experiments; the former had twenty-four hundred pairs of plates, the cells well insulated; the latter put up three thousand five hundred and twenty cylindrical pairs, placed in cells of varnished glass, and insulated by glass pillars varnished. The batteries were excited by water only. Except for the purposes of low intensity and long-continued action, batteries of this description are now no longer constructed.

The want of sustained and regular action in all batteries of the original form, has led to the contrivance of other and more scientific batteries; some of the most valuable of which we will now describe.

871. **Smee's battery** is formed of silver and amagalmated zinc, and needs but one cell and one fluid to excite it. The silver plate, S, fig. 618, is prepared by washing in nitric acid to roughen it, and then coating its surface with platinum, thrown down on it by a voltaic current, in that state of fine division, known as platinum-black. This is to prevent the adhesion of the liberated hydrogen to the polished silver. Any surface of polished metal retains a film of gas with singular obstinacy, thus preventing, in a measure, the contact between the fluid and the plate. The roughened surface produced from the deposit of platinum-black, entirely prevents this. The zinc plates, *z z*, in this battery, are well amalgamated, and face both sides of the silver. The three plates are held in position by a clamp, *b*, at top, while the interposition of a bar of dry wood, *w*, prevents the passage

618



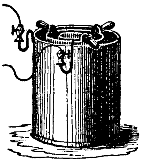
of a current from plate to plate. Water acidulated with one-seventh its bulk of oil of vitriol, or, for less activity, with one-sixteenth, is the exciting fluid.

The surface of the negative plate is kept clean in daily use by occasional immersion in chlorid of iron, which removes all foreign substances deposited on it. For large sized batteries, the silver plates are made by electro casting, to secure entirely plane surfaces. (Mathiot.)

The quantity of electricity excited in this battery is very great, but the intensity is not so great as in the compound batteries presently to be described. This battery is nearly constant, does not act until the poles are joined, and, without any attention, will maintain a uniform flow of power for days together. A thick plate of lead, well silvered, and then coated with platinum-black, will answer equally well, and, indeed, better than a thin plate of pure silver. This battery is recommended over every other for the student, as comprising the great requisites of cheapness, simplicity, and constancy. This is the battery generally employed in electro-metallurgy. Chester has patented an improved form of this battery, much used by the telegraph companies. It is the only single fluid battery now much used in physical experiments.

Mathiot has described the form of Smee's batteries used in the large electrotyping operations of the United States Coast Survey Office. See Am. Jour. Sci. [2], XV., 303.

619



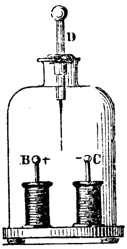
872. **The sulphate of copper battery** is designed to use a solution of sulphate of copper in dilute sulphuric acid, the copper element being made to contain the exciting fluid. This battery, fig. 619, is used for electro-magnetic experiments, and although, it soon becomes encumbered with a pulp of metallic copper thrown down on the zinc, its cheapness and constancy will always render it a valuable instrument.

III. DRY PILES.

873. **Dry piles of Zamboni and DeLuc.**—These piles are constructed of disks of metallic paper, as of copper and zinc (called gold and silver papers), placed back to back, and alternating, as in the pile of Volta, fig. 614, all the coppers facing in one direction. Sometimes zinc paper gilded on one side; or zinc paper smeared with black oxyd of manganese and honey on the other side, is used, and with more marked effects. Some hundreds, and even thousands of these disks, as large as a quarter dollar, are crowded into a glass tube, just large enough to receive them, varnished within and without. Screw caps of metal compress and retain the disks, forming at the same time metallic connections with the outer pairs to propagate the electrical effect. A feeble current is thus set up, which may last for years;

but, if the paper has been artificially dried, so as to free it from all absorbed moisture, no current exists.

620



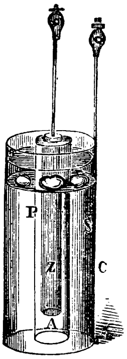
Zamboni (1812), and DeLuc (1810), who first constructed piles of this sort, arranged them in pairs to ring bells by the vibration of a small electric pendulum (fig. 598), alternately attracted and repelled between the columns, which are in the condition of a perpetually-charged Leyden jar of low tension. A set of these bells rang in Yale College laboratory for six or eight years unceasingly.

Bohnenberger's electroscope is constructed on this principle. B and C, fig. 620, are the poles of two dry piles, between which hangs a single gold leaf, ending in the knob, D. When any feebly electric body is approached to D, the gold leaf at once declares its electrical name, by being attracted to its opposite. This is undoubtedly one of the most delicate electroscopes known.

IV. BATTERIES WITH TWO FLUIDS.

874. **Daniell's constant battery.**—This truly philosophical instrument was invented in 1836; up to which time the improvements in the original Voltaic pile had been only mechanical. Prof. J. F. Daniell, of London, first discovered and applied an effectual means of preserving the power and continuing the action of the apparatus for a length of time. All other batteries with two fluids are only modifications of his original instrument.

621

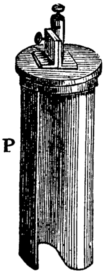


It consists of an exterior circular cell of copper, C, fig. 621, which serves both as a containing vessel and as a negative element; a porous cylindrical cup of earthenware, P (or the gullet of an ox tied into a bag), is placed within the copper cell, and a solid cylinder of amalgamated zinc, Z, within the porous cup. The outer cell, C, is charged by a mixture of eight parts of water and one of oil of vitriol, saturated with blue vitriol (sulphate of copper). Some of the solid sulphate is also suspended on a perforated shelf, or in a gauze bag, to keep up the saturation. The inner cell is filled with the same acid water, but without the copper salt. For the most constant results, he used a saturated solution of blue vitriol, made slightly acid for the outer cell, and for the zinc, twenty parts water to one of sulphuric acid. Eight or ten hours is about the limit of its constancy. Any number of cells so arranged are easily connected together by binding screws, the C of one pair to the Z of the next, and so on.

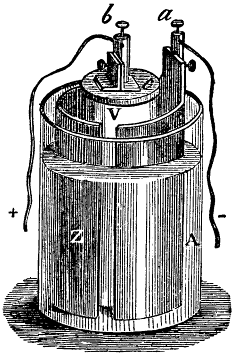
The hydrogen from the decomposed water in this instrument is not given off in bubbles on the copper side, as it is in all forms of the simple circuit of zinc and copper; but the sulphate of copper there present is decomposed in the circuit, atom for atom, with the decomposed water, and the hydrogen takes the atom of oxyd of copper, appropriating its oxygen to form water again, while metallic copper is deposited on the outer cell. If the zinc is well amalgamated, no action of any sort results in this battery until the poles are joined, and it gives off no fumes. Ten or twelve such cells form a very active, constant, and economical battery, and twenty or thirty are ample for most uses. Hot solutions increase its power, while the extent of zinc surface, and not the diameter of the copper, limits the amount of electrical effect.

875. **Grove's nitric acid battery.**—Mr. Grove, of London, has successfully applied the principle of Daniell's battery, to produce the most powerful and intense sustaining battery known. The fluids used are strong nitric acid and dilute sulphuric acid, kept apart by a porous jar. The metals are amalgamated zinc, placed in the sulphuric acid of the outer vessel, and platinum in the porous vessel: fig. 623 shows this arrangement complete, while the platinum element, P, is seen isolated in fig. 622. The cover, *c*, upon the vase, V, fig. 623, tends to keep down the strong vapors of nitrous acid evolved when the battery is in action. The binding screws, *a b*, serve to unite the elements of separate pairs. The zinc here surrounds the platinum, because both that metal and the nitric acid are to be economized as much as possible, being the costly parts of the arrangement. From six to ten parts of water are used in A, to one of acid.

622



623



The action of this battery is intense and splendid. The hydrogen is immediately engaged by the nitric acid, which it decomposes very readily. There is therefore a double chemical action, and an increased flow of electricity, since no part of the power is lost in combination. The fumes of nitrous acid are partly absorbed by the nitric acid, turning it at last intensely green; but enough are evolved to render it important to set the apparatus in a clear space, or good draught. Four cells, with platinum three inches long by half inch wide, decompose water rapidly; and twenty such cells form a battery giving intense effects of light.

Platinum, in the nitric acid battery, is estimated as sixteen or eighteen times more powerful than copper in Daniell's battery; that is, six square inches of platinum is as efficacious as one hundred square inches of copper; and Peschel found that three hundred and forty times as much surface of copper was needed in a spiral battery on Hare's construction, as of platinum in Grove's, to insure equal effects.

A Grove's battery, constructed by Jacobi, of St. Petersburgh, contains sixty-four platinum plates, each thirty-six square inches surface, or combined, sixteen square feet. This would be, by comparison, equal to a Daniell's battery of two hundred and sixty-six square feet, or a Hare's battery of about five thousand five hundred square feet. Grove's battery is rather costly, and very troublesome to manage, as are all batteries with double cells and porous cups, although the trouble involved in their use is not to be compared with the vexation involved in the earlier single fluid batteries.

876. **Carbon battery.**—The great cost of large members and extensive series of Grove's platinum battery, led Prof. Bunsen, of Marburg, to use the carbon of gas coke as a substitute for the platinum. Prof. Silliman, Jr., in 1842, described a battery (see Am. Jour. Sci. [1], XLIII., 393, and XLIV., 180), in which natural plumbago was used in place of the platinum of Grove's arrangement. This was before Bunsen's apparatus was known of in this country.

624

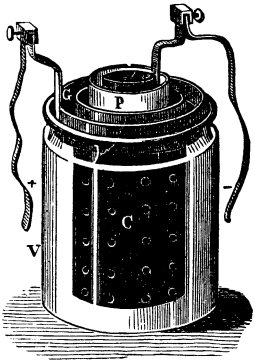
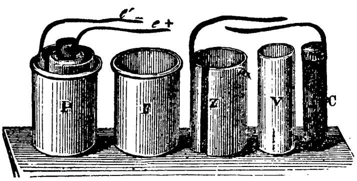
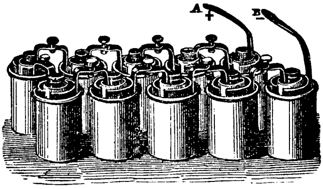


Fig. 624 shows the original form of Bunsen's cells. Where the carbon, C, is contained in an exterior vase, V, of nitric acid, the amalgamated zinc is in a porous cup, P, of dilute sulphuric acid. The objection to this arrangement is the large consumption of nitric acid and smallness of the zinc. In the Author's plan, afterwards adopted essentially by M. Deleuil, the carbon was in the porous cup, surrounded by the zinc. In fig. 625, this arrangement is shown in detail. P, is the pile complete. F, is the jar of hard pottery to contain the zinc, Z, and the dilute sulphuric acid; V is the porous vase, to contain the carbon, C, with its nitric acid. The attachment of a conductor to the carbon is accomplished by a conical hole in the centre, into which a plug of hammered copper is crowded with a wrenching motion. If prisms of the hard carbon of the gas retorts are used (and this kind of carbon is unquestionably the best), a copper band is attached to the top by electro-galvanic soldering. The carbon of Bunsen's cells is prepared by pulverizing, and baking in moulds, the coke of bituminous coal. Fig. 626 shows a series of ten cups of the carbon battery arranged for use, the alternate members being joined by binding screws, as made by Deleuil, of Paris, each zinc being twenty-two centimetres (eight and three-quarter inches) high. As the electro-motive energy of the battery depends on size as well as number, these large members have great

625



626



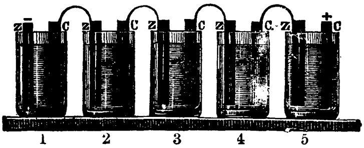
advantages. The Author demonstrated, in 1842 (*loc. cit.*), that carbon was nearly if not quite as good as platinum, surface for surface. A battery of fifty cells, like fig. 626, costs fifty-five dollars in Paris, and, with such a series, all the most splendid effects of the electric light, deflagrations, and chemical decompositions can be very satisfactorily shown.

877. **Other forms of Voltaic battery** exist in great variety, but involving no principle not already explained. Some have special adaptation to a particular use, like Chester's form of Smee's battery for telegraphic use; Farmer's copper battery; the battery of Bagration, of zinc and copper in moist earth; or Grove's oxygen and hydrogen gas battery, so instructive theoretically. But further descriptions are excluded by want of space.

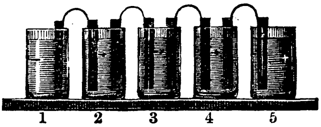
V. POLARITY, RETARDING POWER, AND NOMENCLATURE OF THE VOLTAIC PILE.

878. **Polarity of the compound circuit.**—In batteries of two or more couples, connection is formed, not as in the single couple, between members of the same cell, but between those of different names in contiguous cells, as in fig. 627, where the copper of 1 joins the zinc of 2, and so on. The current flows from the zinc to the copper in the fluid, but from the copper to the zinc in the air (fig. 628), both in simple and compound circuits. This is important to be remembered, since the zinc is called the electro-positive element of the series, although out of the fluid it is negative. Consequently, in voltaic decompositions, the element which goes to the zinc pole is called the *electro-positive*, and, for the same reason, that which goes to the copper, is the *electro-negative element.* The terminal plates, Z and C, in 1 and 5, fig. 627, are not concerned in the electrical effect, being in fact only conductors of the electricity, and hence they may be removed as in fig. 628, without altering the power or nature of the battery. They serve, in fact, merely as a convenient mode of joining the poles, as in fig. 629. The apparent polarity of the simple circuit is, therefore, the reverse of that of the compound circuit; but,

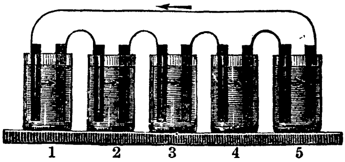
627



628



629

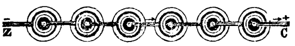


an attentive observation of these explanations, and of the figures, will prevent all confusion on this point.

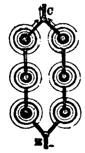
879. **Grouping the elements of a pile**, in various numerical relations, is an important means of modifying its power, and the character of its effects, already explained.

Take, for example, six cups, as in fig. 630, arranged in consecutive order, and we have, owing to the resistance to the electric flow, the maximum intense effects possible with such number. Changed to two groups of three each, the quantity is doubled, with half of the intensity, fig. 631. In fig. 632, are three groups of two cups each, so arranged as to present three times the surface in 630, with a proportionate loss of intensity. Lastly, in fig. 633, each zinc, and each copper, joins one common conductor, each on its own side, throwing the six couples into one surface of six-fold extent to fig. 630. The arrangement may be expressed, assuming the resistance of a single cup as unity, thus: 1. $\frac{3}{2} = 1·5$. $\frac{2}{3} = 0·666$. $\frac{1}{6} = 0·166$, and so for any number of couples.

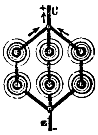
630



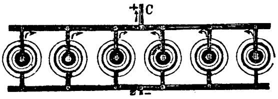
631



632



633

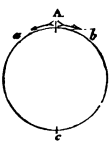


880. **Electrical retarding power of the battery.—Ohm's law.**—A certain resistance to the passage of a voltaic current is offered by every additional element placed in the circuit as well as by increased length of conductor. The new properties thus acquired by the compound circuit have been already alluded to (865).

Ohm, of Berlin, in 1827, first demonstrated mathematically the law regulating the flow of electricity in the compound battery. As the apparatus is composed solely of conductors of different retarding power, the electric current must proceed, not only along the connecting wire, from pole to pole, but also through the whole apparatus; the resistance offered to the passage of the current consists therefore of two parts, one exterior to, and one within, the apparatus.

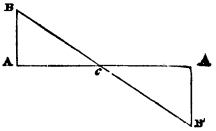
Let the ring, *a b c*, in fig. 634, represent a homogeneous conductor, and let a source of electricity exist at A. From this source the electricity will diffuse itself over both halves of the ring, the positive passing in the direction *a*, the negative in *b*, and both fluids meeting at *c*. Now it follows, if the ring is homogeneous, that equal quantities of electricity pass through all sections of the ring in the same time. Assuming that the passage of the fluid from one cross section of the ring to another, is due to the difference of electrical tension at these points, and that the quantity which passes is proportional to this difference of tension, the consequence is, that the two fluids proceeding from A, must decrease in tension the farther they recede from the starting point.

634



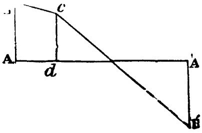
This decreasing tension may be represented by a diagram. Suppose the ring in fig. 635, to be stretched out to the line A A′. Let the ordinate A B represent the tension of positive electricity at A, and A′ B′ the negative tension at A′, then the line B B′ will express the tension for all parts of the circuit by the varying lengths of the ordinates A B, A′ B′, at every point of A *c* or *c* A′. Hence Ohm's formula $F = \frac{E}{R}$, where F represents the strength of the current, E the electro-motive force of the battery, or the attraction of zinc for oxygen, and R the resistance. Therefore, the greater the length of the circuit, the less will be the amount of electricity which passes through any cross section in a given time. In exact terms, this law states that *the strength of the current is inversely proportional to the resistance of the circuit, and directly as the electro-motive force.*

635



In the simplest Voltaic current, we have not a homogeneous conductor, but several of various powers. To illustrate this, let the conductor, A A′, fig. 636, consist of two portions having different cross sections. For example, let the cross section A *d* be *n* times that of *d* A′; then, if equal quantities pass through all sections in equal times, and if through a given length of the thicker wire no more fluid passes than through the thinner wire, the difference of tension at both ends of this unit of length of the thicker wire must be only $\frac{1}{n}$th of what it is in the latter. Thus, "the electric fall," as Ohm calls it, will be less in the case of the thick wire than of the thinner, as shown by the line B *c* in the figure. The result is expressed in the law that the "*electric fall" is directly as the specific resistances of the conductors, and inversely as their cross sections.* Hence, the greater the resistance offered by the conductor, the greater the fall. The very simplest circuit must therefore present a series of gradients expressive of the tension of its various points—as one for the connecting wire, one for the zinc, one for the fluid, and one for the copper. The electro-motive force of a voltaic couple ("E" of Ohm's formula) may be experimentally determined, and is proportional to the electric tension at the ends of the newly broken circuit.

636



881. **Formulæ of electric piles.**—It follows, from what has been stated, that the intensity, I, of a current, united by a homogeneous wire whose length is L, may be represented by the formula

$$I = \frac{E}{L + r},$$

in which r designates a length of wire representing the resistance of the pile or its *reduced length,* and E the electro-motive force measured by the tension at the poles when the circuit is broken.

If the resistance of the pile is nothing, or an insensible quantity, as in the case of a thermo-electric couple of great surface, the formula becomes $I = \frac{E}{L}$. That is, the intensity of the current is in the inverse ratio of the length of the homogeneous wire joining the poles of the battery. If, however, the pile itself offers sensible resistance, as is the case with hydro-electric piles composed of many couples, the formula shows that *the intensity of the current is in the inverse ratio of the length of the connecting wire increased by a constant quantity*, r, which represents the resistance of the pile itself.

In the case of many different sources of resistance, interposed in the circuit of the connecting wire, L represents the sum of the *reduced lengths* equivalent to these resistances.

Intensity given by many couples.—In the formula, $I = \frac{E}{L + r}$, let E be the electro-motive force, and r the resistance of a single couple, L and r being always reckoned as lengths of the same kind of wire taken as a standard of comparison; we may then consider many couples united one to another in a series as shown in fig. 630. Ohm considers that each couple produces an electric current which traverses the pile as if that couple was alone, so that the electro-motive force of the series, or the tension at the poles which measures it, will be the sum of the electro-motive forces, $E + E' + E'' + \ldots$, of all the couples in the series. In the same manner the current, produced by each couple traversing all the others, meets with a resistance equal to the sum, $r + r' + r'' + \ldots$, of the resistances of all the couples; hence,

$$I = \frac{E + E' + E'' + \ldots}{L + r + r' + r'' + \ldots}.$$

If the couples are all equal to each other, and n represents their number, the formula becomes $I = \frac{nE}{L + nr}$. This shows that the intensity of the current, from a series arranged one by one, is proportional to the sum of the electro-motive forces of all the couples, and inversely as the total resistance of the circuit including the pile itself.

If we designate by c, l, s the conductibility, the length, and the section of a wire; and by c', l', s' the same quantities for a second wire, it follows from the principles already established, and from the fact that the resistance of a wire is in the inverse ratio of its conducting power, that two wires will produce equal diminution of the intensity of the same electric current when:—

$$\frac{l}{sc} = \frac{l'}{s'c'}, \text{ or, } ls'c' = l'sc, \text{ or, } l = l'\,\frac{sc}{s'c'}.$$

The third equation expresses the length of a wire whose section is s, and conductibility c, that will produce the same effect upon the current as another wire whose length is l', section s', and conductibility c'.

This value of l is called the *reduced length* of the first wire as compared with the second.

If we have a series of wires united together by their ends as a compound conductor, the equivalent length of the first wire will be expressed by the formula:—

$$l = l'\frac{sc}{s'c'} + l''\frac{sc}{s''c''} + l'''\frac{sc}{s'''c'''} + \ldots = sc\left(\frac{l'}{s'c'} + \frac{l''}{s''c''} + \frac{l'''}{s'''c'''}\right).$$

Effect of increasing the number of couples in a battery.—The consideration of the formulæ given above shows that,

1. The intensity of the current increases with the number of couples. Dividing both terms of the fraction by n, it becomes $I = \frac{E}{\frac{L}{n} + r}$ this shows that the value of I increases with increase of n, or the number of couples.

2. The increased intensity of the current, by increasing the number of couples, is more evident when L is great in comparison with r, or when the external resistance to be overcome is much greater than the resistance of the battery. If on the contrary L is very small, $\frac{L}{n}$ is also very small, and the intensity of the current changes but very little with any increase of the number of couples.

3. If there is no exterior resistance, or, $L = 0$, $I = \frac{nE}{nr} = \frac{E}{r}$. In this case the intensity is not varied by varying the number of couples, or one couple gives as great intensity as any number of couples.

4. The intensity is not increased by increasing the number of couples when each couple added is accompanied by an exterior resistance equal to L; or, in other words, when the exterior resistance increases in the same ratio as the number of couples, since on that supposition the formula becomes $$I = \frac{nE}{nL + nr} = \frac{E}{L + r}.$$

5. If the exterior resistance, L, is very great, I is very small, unless n is made very great. This shows that it is necessary to employ a great number of couples when a great amount of resistance is to be overcome, as in the *voltaic arch*, and in the electrolysis of bodies that are imperfect conductors, or in sending an impulse through a long telegraphic circuit.

Effect of enlarging the plates of a battery.—If, instead of uniting many couples in a series, we unite a number of couples, m, by poles of the same name as in fig. 633, it will be equivalent to enlarging the dimensions of the plates of a single couple, the resistance of the battery will be $\frac{r}{m}$, and the formula becomes $I = \frac{E}{L + \frac{r}{m}}$. We thus see that if L, the exterior resistance, is very great in proportion to r, the resistance of the battery, the intensity will be but little increased by uniting the couples by poles of the same name. If L is very small in proportion to r, the intensity will be much increased by this method, and if L is so small that it may be neglected, the intensity will be proportional to the extent of surface acting as a single couple. We know indeed that, when chemical action is exerted over a large surface, the quantity of electricity which traverses the connecting wire is also very great.

Effect of enlarging the couples and increasing their number.—If the number and dimensions of the couples are both increased at the same time, the formula becomes $I = \frac{nE}{L + \frac{nr}{m}} = \frac{E}{\frac{L}{n} + \frac{r}{m}}$. This shows that increasing the number of couples produces the same effect as diminishing in the same proportion the resistance of the exterior circuit, and increasing the surface of each couple has the same effect as diminishing the resistance of the pile. Hence:—

If L, the resistance of the exterior circuit, is great, it will be most advantageous to unite many couples in a single series: But if the resistance of the exterior circuit is small, greater advantage may be obtained by uniting the couples by poles of the same name, in such a manner as to form couples of large extent of surface.

A most interesting application of these principles to the practical construction and use of batteries will be found in a paper by Mr. G. Mathiot, Electrotypist of the United States Coast Survey. (Am. Jour. Sci. [2] XV. 305.)

882. **Faraday's nomenclature.**—Faraday has introduced certain terms into the language of electrical science, which are generally adopted for their convenience, and their absence of assumed or theoretical notions.

Electrode is used in place of *pole*, to which latter term, meaning the terminal wires of a battery, Davy and others seemed to attach a sense as if it possessed a certain attractive force, like the pole of a magnet.—Electrode (from ἤλεκτρον, and ὁδος, a way), means simply the way or door by which a Voltaic current enters or leaves a substance.

Anode is that surface of a body receiving the current, or the positive side of the series, from ανα, upwards (as the sun rises), and ὁδος, a way.

Cathode is that surface of a body from which a current flows out towards the negative side of the series, from κατά, downwards, as the sun sets, and όδος, a way). The observer is supposed to face the north, with the positive of the battery on his right hand, and its negative on his left.

Electrolyte is any substance capable of separation into its constituents by the influence of a Voltaic series (from ἤλεχτρον, and λύο, to set loose). *Electrolysis*, the act of decomposition. *Electrolyzed*, and *electrolyzable*, are obvious derivatives from the same words.

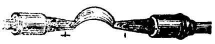
Ions (from ἰόν, neuter of εἶμι, to go), are the elements into which an electrolyte is resolved by the current. These are either *anions*, elements found at the anode, or *catons*, ions found at the cathode. Hereafter we shall employ these terms when they are appropriate.

VI. THE EFFECTS OF THE VOLTAIC PILE.

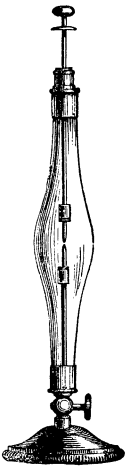
1. *Physical effects.*

883. **The Voltaic spark and arch.**—In 1809, Davy, with the extensive series of two thousand couples at the Royal Institution, first demonstrated the full splendors of the Voltaic arch between electrodes of well-burned charcoal. However powerful the series may be, no effect is seen, in the air, until the points of the carbon electrodes are brought into actual contact, or at least insensibly near. Herschel noticed that an electrical spark from a Leyden jar, sent through the carbon points, when near each other, established the flow of the Voltaic current, by projection no doubt of material particles. When the spark passes, then the electrodes may be withdrawn, as in fig. 637, and the arch of electric flame connects them with a white and violet light of intolerable brightness; several inches in length if the pile is very powerful. This arch of seeming flame is not produced by the combustion of the carbon electrodes, since it exists, with even greater brilliancy, in a vacuum, or in an atmosphere of nitrogen or carbonic acid. Despretz states that *in vacuo* with a powerful pile, the Voltaic arch may be formed at some centimetres distance, without contact. Fig. 638, shows a convenient apparatus for this experiment, *in vacuo*, or in various gases, as in Davy's original experiments. The Voltaic arch is accompanied by a loud hissing or rushing sound, due to the mechanical removal and transportation of particles of carbon from the positive to the negative electrode, by which the former is diminished in length, or made cup-shaped, while the latter is sensibly elongated, as first noticed and described by Prof. Silliman, in 1822 (Am. Jour. Sci. [1] V. 108), in the use of a powerful deflagrator constructed by Dr. Hare.

637

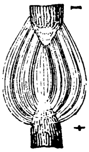


638

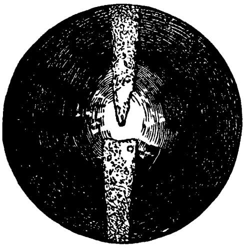


Through colored glasses, these particles of carbon can be conveniently observed apparently moving slowly from pole to pole, and giving unquestionably that oval form to the arch, seen in fig 639, when the electrodes are vertical, and the negative carbon is uppermost. There is also distinctly to be seen, a certain structure in zones, or bands of different brilliancy. When the image of the carbon electrodes is projected on a screen, fig. 640, as was first done by Foucault with the electric lantern, the growth of the negative and the decrease of the positive electrode is easily observed, without injury to the eyes. The negative carbon is seen to glow first, as if the light originated there, but as the experiment advances, the positive carbon becomes the most briliant, and maintains this superiority during the experiment; becoming at the same time cup-shaped.

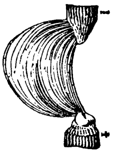
639



640

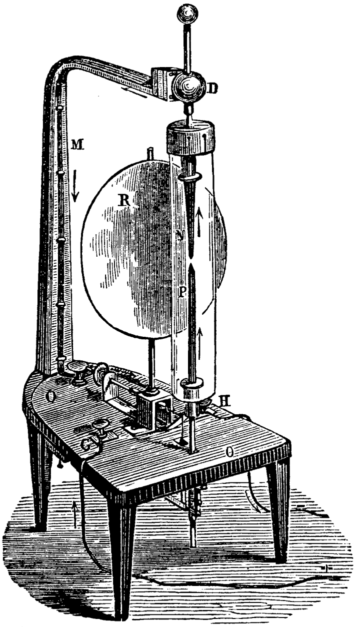


641



The Voltaic arch is magnetic, or capable of influencing the magnet, by the approach of which it is deflected, as seen in fig. 641, or it is made to revolve with a loud hissing noise; a fact first observed by Davy, but since carefully studied by De la Rive, Quet, and Despretz. This fact is far more conspicuous in the arc from the induction coil, § 933.

642

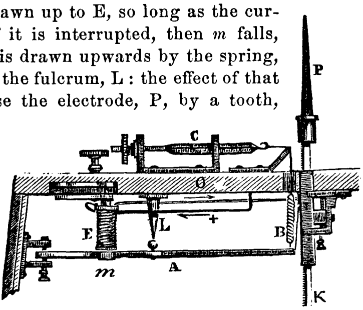


884. Regulators of the electric light.

—Since the introduction of powerful constant batteries, it has been possible to use the electric light for scientific and economical purposes. For this purpose *regulators* have been devised to render the light constant, by approaching the electrodes in proportion as they are consumed.

In fig. 642, is shown that of Deleuil, of Paris, and its details, in fig. 643. The two carbon points, P and N, are held in position by two vertical rods, of which the lower one, P, is moved upwards by the mechanism in fig. 643; while the upper one, N, passes through the ball, D, with friction. The flow of the current is shown by the arrows arriving at G, and departing at H. The frame of the apparatus is of cast-iron. The slightly concave mirror, R, is for certain purposes replaced by a large parabolic mirror. When the zinc electrode is connected with G, and the carbon with H, communication is established by depressing N with the hand. As the current in its circuit passes through a coil of wire surrounding the electro-magnet, E, fig. 643, the soft iron armature, *m*, on the lever A, is drawn up to E, so long as the current flows, but if it is interrupted, then *m* falls, and the lever A is drawn upwards by the spring, B, acting against the fulcrum, L : the effect of that motion is to raise the electrode, P, by a tooth, I, catching in notches on the upright, K. In this way connection is again established with the battery; and when this simple mechanism is once adjusted, it will act for hours with great certainty, maintaining the light constant.

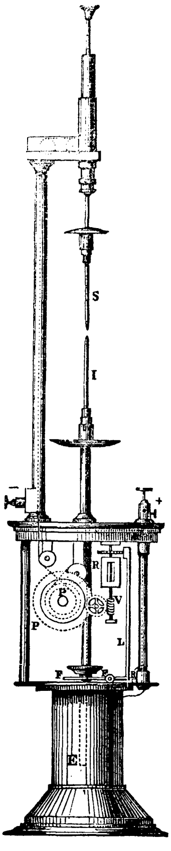
643



Duboscq's photo-electric lantern is seen in fig. 644. This instrument is used to replace the sun in all optical experiments requiring a strong white light.

The poles S and I are preserved at the same distance by the action of an electro-magnet in the foot E, upon a soft iron bar F F in connection with an endless screw V, moving the pulleys P P′, which are connected by cords with the poles S and I. The contact of S and I induces magnetism in the electro-magnet E, while the springs R L regulate the motion of the machinery. The apparatus is simple and portable, and its effect is to make the electrical light so steady and constant that it may be used for all optical experiments. The positive pole consumes much more rapidly than the negative, both from a more intense action upon it and because its particles are carried over and deposited on the negative pole, elongating the point of the latter. To provide for this difference, the pulley P′ is variable, and carries the pole I up proportionably faster, so that the focal position of the light remains unchanged.

644

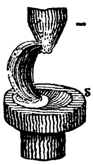


885. **Properties of the electric light.**—Like the solar light, it is unpolarized. It explodes a mixture of hydrogen and chlorine, and acts on chloride of silver, and other photographic preparations, like the sun. Bodies made phosphorescent by the sun, are similarly affected by the electric light. In 1842, Silliman took daguerreotypes with it (Am. Jour. Sci. [1] XLIII. 185), and it is now used in preference to solar light, for the purpose of taking microscopic photographs. (Duboscq.)

Fizeau and Foucault, have compared, by photometric measurement, the light from ninety-two carbon couples, arranged in two series of forty-six (879), with the solar beam, and also with the oxyhydrogen or Drummond light. In a clear August day, with the sun two hours high, the electric light, assuming the sun as unity, bore to it the ratio of 2·59 : 1, *i. e.*, the sun was twice and a half more powerful: while the Drummond light was only the one one hundred and forty sixth that of the sun. Bunsen found the light from forty-eight elements of carbon, equal to five hundred and seventy-two candles. The intensity of the electric light depends far more on the size of the individual members of the pile than on their number. The effect from forty large sized couples was found by Fizeau and Foucault to be about the same as that from double the number, when the eighty were arranged consecutively, as in fig. 630, while, with the same elements in two parallel series, there was a very great increase of effect. Fraunhofer showed that the spectrum of the electric light was distinguished from that of the sun by a very bright line in the green, and a somewhat less luminous one in the orange (461). Dove has lately shown (Poggendorff's *Annalen*, 1857, No. 6) that this light has two distinct sources: 1st, the ignition or incandescence of the translated particles, passing in the course of the discharge: 2d, the proper electric light itself. On the contrary, Draper has shown that the spectrum from a glowing platinum wire heated by the battery, contains no dark lines, so that, unlike the electric light, it is strictly white (Am. Jour. Sci. [2] VIII., 340). It is not only particles of carbon which pass in the Voltaic arch, but of whatever conductor may form the positive electrode, as platinum, or any metal, and the light varies in its optical properties with every change of the electrode. (Wheatstone.)

886. **Heat of the Voltaic arch.—Deflagration.**—When the positive electrode is fashioned into a small crucible of carbon, as in fig. 645, gold, silver, platinum, mercury, and other substances, are speedily fused, deflagrated, or volatilized, with various-colored lights.

645



The fusion of platinum (like wax in a candle) before the Voltaic arch is significant of its intense heat, and still more, the volatilization and fusion of carbon, a result first announced by Prof. Silliman in 1822, and since confirmed by Despretz, who, by the union of the heat of six hundred carbon couples arranged in numerous parallel series, and conjoined with the jet of an oxyhydrogen blow-pipe, and the heat of the midday sun, focalized by a powerful burning-glass, succeeded in volatilizing the diamond, fusing magnesia and silica, and softening anthracite. The diamond is also softened, and converted into a black spongy mass resembling coke, or, more nearly, the black diamond found in the Brazilian mines.

A delicate stream of mercury being allowed to flow from a narrow elongated funnel (the negative electrode), upon a surface of mercury in a glass vase forming the positive electrode, is deflagrated with transcendent splendor. Many yards of number twenty platinum wire, held between the electrodes, may be kept in the full glow of white heat for a long time. The teacher can devise many pleasing additional experiments, as drawing the arch beneath water, oil, and other liquids, from points of carbon, or from platinum and steel wires.

When a fine platinum wire is made the positive electrode, and a solution of chlorid of calcium, or any other metallic chlorid, is made the negative electrode, on touching the surface of the liquid with the point of the fine wire, if the series is powerful the wire is fused on the surface of the liquid, evolving a light of surpassing beauty, whose color is that appropriate to the metal in solution; *e. g.*, from calcium salts, violet-red; from sodium, yellow; from barium, reddish-yellow; from potassium, violet; from strontium, red, &c. These beautiful facts were first noticed by Dr. Hare.

Dr. Page has described a singular motion imparted by the current to globules of pure mercury, placed in a shallow dish, and covered by acidulated water: the globules elongate to ovoids and move actively about, one end, that towards the + pole, being clouded by escaping gas-bubbles. If the mercury contains zinc, the position of the clouded end is reversed. (Am. Jour. Sci. [2], XI., 192.)

887. **Measurement of the heat of the Voltaic current.**—By means of a long wire coiled into a close spiral, and enclosed in a calorimeter of glass, containing water, Becquerel and others have established the laws regulating the flow of heat in the electric current, by its effect in elevating the temperature of the water. A coil of platinum wire contained in the bulb of a Sanctorio's thermometer, becomes a means of estimating the heat of currents too feeble to be otherwise measured. The results are, that when a Voltaic current traverses a homogeneous wire, the quantity of heat in a unit of time is proportional:—

1. *To the resistance which the wire opposes to the passage of the electricity:*

2. *To the square of the intensity of the current.* The intensity of a current is measured by the quantity of water which it will decompose in a given time.

For a given quantity of electricity, the elevation of temperature at different points on a conducting wire, is in the inverse ratio of the fourth power of its diameter.

Draper has applied the coefficient of expansion to determine the degree of heat corresponding to a particular color (585).

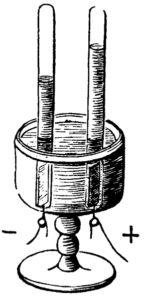
2. *Chemical effects of the pile.*

888. **Historical.**—The chemical effects of the pile are most wonderful, and the present advanced state of chemical science is largely attributable to the flood of light shed by the researches of Davy and Faraday upon the electrical relations of the elements and the decomposition of compounds by the Voltaic circuit.

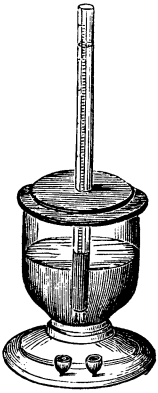
In 1800, immediately after Volta's announcement to Sir Joseph Banks of his discovery of the pile, Messrs. Nicholson and Carlisle constructed the first pile in England, consisting of thirty-six half crowns, with as many discs of zinc and pasteboard soaked in salt water (864). Observing gas-bubbles arise when the wires of this pile were immersed in water, Nicholson covered them with a glass tube filled with water, and, on the 2d of May, 1800, completed the splendid discovery, that the Voltaic current had the power to decompose water and other chemical compounds. Stimulated by so fine a result, chemists and physicists everywhere repeated the experiment, perfecting the methods of obtaining the oxygen and hydrogen gases in a separate condition. The chemical theory of the pile, originally advanced by Fabbroni, a countryman of Volta's, some years before, was taken up and ardently advocated by Davy, who, in 1801, had succeeded to a place in the laboratory of the Royal Institution: where, on the 6th of October, 1807, he made, by the Voltaic pile, the memorable discovery of potassium, the metallic base of potassa, before regarded as a simple substance; and soon after established the startling truth, that all the earths and alkalies, until then esteemed simple substances—the whole crust of the globe, in fact—were oxyds of metals, whose existence had hitherto been unsuspected.

889. **Electrolysis of water.—Voltameter.**—The Voltaic decomposition, or electrolysis of water, is the finest possible illustration of the chemical power of the pile. Water is a compound of oxygen and hydrogen gases, in the proportions of one measure of the former to two of the latter. When two gold or platinum wires are connected with the opposite ends of the battery, and held a short distance asunder in a cup of water, a train of gas-bubbles will be seen rising from each, and escaping at the surface. If the electrodes are not of gold or platinum, the oxygen combines with one of them, and only hydrogen escapes, as in Nicholson's original experiment. With two glass tubes placed over the platinum poles, fig. 646, we can collect these bubbles as they rise. The gas (hydrogen) given off from the negative electrode is twice the volume of that obtained from the positive. When the tubes are of the same size, this difference becomes at once evident to the eye. By examining these gases, we shall find them, respectively, pure hydrogen and oxygen, in the proportion of two volumes of the former to one of the latter. Agreeably to principles already explained, the oxygen (electro-negative) appears at the + electrode, and the hydrogen (electro-positive) appears at the — electrode. The rapidity of the decomposition is greater when the water is made a better conductor, by adding a few drops of sulphuric acid; and for rapid electrolysis the number of couples in the series should be increased to overcome, by superior tension, the low conducting power and chemical affinity of the electrolyte. If a single tube only covers both electrodes, as in fig. 647, the total electrical effect is easily measured by the graduation of the tube, the quantity of gases given off in a unit of time being directly as the current. The contents of this tube will explode if a lighted match is applied to them, or if an electric spark passes through them. Such an instrument is a *Voltameter*.

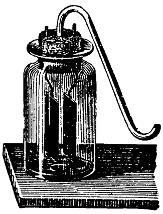
646



647



648



A convenient form of this instrument is seen in fig. 648, made of a common bottle, filled with acid water; the platinum electrodes pass through the cork and end in two plates of platinum, while a bent gas tube of glass conveys off the accumulating gases as fast as they are evolved by the electrolysis.

890. **Laws of electrolysis.**—From a great number of elaborate experiments, the accuracy of which remains unshaken, Faraday has deduced the following general laws of electrolysis.

1st. The quantity of any given electrolyte, resolved into its constituents by a current of electricity, depends solely on the amount of electricity passing through it, and is independent of the form of apparatus used, the size or dimensions of the electrodes, the strength of the solution, or any other circumstance. Hence, the amount of water decomposed in a given time in the Voltameter, is an exact measure of the quantity of electricity set in motion.

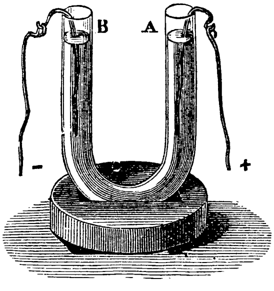
2d. In every case of electrolysis, the elements are separated in equivalent or atomic proportions, and when the same current passes in succession through several electrolytes in the same circuit, the whole series of elements set free are also in atomic proportions to each other. It follows, therefore, that the amount of electricity required to resolve a chemical combination, is in constant proportion to the force of chemical affinity by which its elements are united.

3d. The oxydation of an atom of zinc in the battery, generates exactly so much electricity as is required to resolve an atom of water into its elements. Thus, 8·45 grains of zinc dissolved in the battery, occasions the electrolysis of 2·35 grains of water. But these numbers are in the ratio of 32·5 : 9 the equivalents, respectively, of zinc and of water. Hence follow these corollaries:—First, *The source of Voltaic electricity in the pile is chemical action solely.* Second, *The forces termed chemical affinity and electricity, are one and the same.*

One or two additional illustrations of these laws will suffice in this place, referring the student to chemical treatises for a fuller discussion of this very important topic.

891. **Electrolysis of salts.**—In the bent tube, B A, fig. 649, put a solution of any neutral salt; *i. e.*, sulphate of soda, and diffuse the blue solution from a purple cabbage in the liquid. Let the current of a Voltaic pile communicate with this saline solution by two platinum wires, dipping into the legs of the tube—presently the blue color of the solution is changed on the positive side for red, and on the negative for green, indicating the presence of an acid set free in A, and of an alkali in B

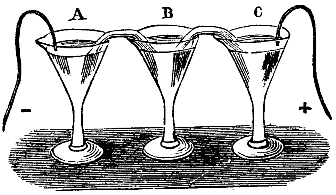
649



If the action is kept up, the whole of the blue liquid is changed to red and green. Transpose, then, the + and — wires, so as to reverse the direction of the current; presently, the red and green change back to blue, and, in a short time, that which was red becomes green, and *vice versa.* This is a case of electrolysis in which the electrolyte (sulphate of soda) is changed, not into its ultimate elements, but only into the acid and alkali, which may be called its proximate constituents; any other saline fluid may be substituted with similar results. If an alkaline chloride is used, *i. e.*, common salt, the free chlorine evolved on the + side, discharges all color, while the soda produces on the — side its appropriate green tint. If a metallic salt, *e. g.*, sulphate of copper, or acetate of lead, is used in A B, then, on the — side, metallic copper or lead is evolved; while, on the + side, is the free acid before in combination in the salt.

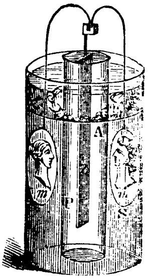
A more surprising example of the apparent transfer of elements under the power of the Voltaic current, is illustrated in fig. 650, where in B, the centre glass, of the three wine-glasses, A B C, is a solution of sulphate of soda, while A and C contain only pure water, blued with cabbage solution. Filaments of moist cotton wick connect the three glasses, and the electrodes are introduced into A and C, when the same series of changes, already described in fig. 649, takes place, with the same reversals when the electrodes are transferred. B remains apparently unchanged, while C is reddened, and A becomes green, or *vice versa*. There is, in fact, nothing more wonderful in this case than in the last, only the dissection of the process into three parts, makes the result still more striking. In place of A B C, any number of glasses, with different salts and compounds, may, with a powerful series of Bunsen, be substituted, with results conformable to the law in § 890.

650



892. **Electro-metallurgy.—The electrotype.**—The cold casting of metals by the Voltaic current, is a fine example of the rich gifts made by abstract science to the practical arts of life. Every Daniell's battery is, in fact, an electro-metallic bath, in which metallic copper of a firm and flexible texture is constantly thrown down from solution.

651

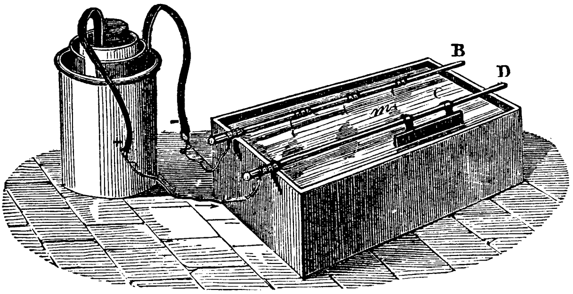


The very simple apparatus required to show these results experimentally, is represented in the fig. 651. It is nothing, in fact, but a single cell of Daniell's batttery. A glass tumbler, S, a common lamp-chimney, P, with a bladder-skin tied over the lower end and filled with dilute sulphuric acid, is all the apparatus required. A strong solution of sulphate of copper is put into the tumbler, and a zinc rod, Z, is inserted in P; the moulds, or casts, *m m*, are suspended by wires attached to the binding screw of Z. Thus arranged, the copper solution is slowly decomposed, and the metal is evenly and firmly deposited on *m m*. A perfect reverse copy of *m* is thus obtained in solid malleable copper. The back of *m* is protected by varnish, to prevent the adhesion of the metallic copper to it. In this manner the most elaborate and costly medals are easily multiplied, and in the most accurate manner. In practice, reverse casts of the object to be copied are first made in fusible metal or wax. The art is now extensively applied to plating in gold and silver from their solutions; the metals thus deposited adhering perfectly to the metallic surface on which they are deposited, provided these be quite clean and bright.

Even alloys, as bronze, brass, and German silver, may be deposited according to electrolytic law.

The positive electrodes should be of the same metal as that in solution, and as large as the surfaces to be coated, and these should not be larger than the plates of the battery furnishing the current. The arrangement of apparatus commonly used in this art, is seen in fig. 652, where the metallic solution is held in a separate bath, over which are extended two stout rods; B, carrying the objects, *m*, in connection with the negative side of the battery; and with the positive side, the rod D, on which is suspended a plate of the metal proposed to be deposited, to maintain the uniform strength of the solution, which is preferably kept at a somewhat higher temperature than that of the air. Wood-cuts and printers' types are thus copied in copper, the moulds taken in wax from them being made conductors by dusting over the surface with extremely fine plumbago. All the copper-plates for the charts of the United States Coast Survey, are reproduced by the electrotype—the originals never being used in the press, but only the copies; and any required number of these may be produced at small expense. For an instructive account of these extensive electrotype operations, the student is referred to a paper by the Electrotypist of the Coast Survey, Mr. G. Mathiot (Amer. Jour. Sci. [2], XV., 305).

652



893. **Crystallization from the action of feeble currents.**—It was known to the alchemists, very early in chemical history, that certain metals, as gold, silver, copper, lead, tin, &c., were deposited in a pure, or "*reguline*" condition, from their solutions, when another metal was present, or even sometimes without that condition. Thus the lead tree (*arbor Saturnæ*), the tin tree (*arbor Jovis*), the silver tree (*arbor Dianæ*), were so called by the alchemists, from the apparent growth

of these metals out of their solutions, and in tree-like forms. This growth we now know to be due to Voltaic crystalline deposition.

Examples.—A solution of chlorid of gold in ether, by slow change, deposits spontaneously, crystals of fine gold, in elegant moss-like growths; and Liebig has shown us how to prepare a silver solution, which, by the aid of an essential oil as a reducing agent, will coat glass with a film of silver so thin as to be transparent, and still so brilliant as to reflect light more perfectly than the best mercurial mirrors.

A dilute solution of acetate of lead (half an ounce to a quart of rain water), surrenders all its lead to a strip of zinc hung in the containing bottle, in elegant crystalline plates (*the arbor Saturnæ*); this, and the next case, are true Voltaic circuits, while in the first two cases, hydrogen appears to supply the want of the second element of Voltaic couple. In like manner, a dilute solution of nitrate of silver, placed over mercury, soon deposits all its silver in an arborescent form (*arbor Dianæ*) on the mercury.

But the most instructive case of this kind is when a bar of pure tin is placed upright in a tall vessel, the lower half of which is filled with a saturated solution of protochlorid of tin, while above it rests a dilute solution of the same salt. The bar is therefore in two solutions chemically identical, but physically unlike The result is a Voltaic current, by which metallic tin, in beautiful brilliant plates, is deposited upon the upper part of the bar, while the lower part is correspondingly dissolved by the free electro-negative element of this electrolysis.

The earliest recorded experiments with this species of Voltaic circuit are those of Bucholz (1807), whence this slow-acting pile is sometimes called the "*Bucholz-pile.*" Becquerel has greatly extended our knowledge of the actions thus produced, forming thereby many non-metallic crystalline products. Cross thus formed crystals of carbonate of lime in two days in the light, or in six days in the dark. Mallett thus produced crystals of copper, and of red oxyd of copper, in a single night from the nitric solution. (Am. Jour. Sci. [2] XXX. 253.)

894. **Deposit of metallic oxyds and Nobili's rings.**—Becquerel has shown that oxyd of lead and oxyd of iron may be deposited in a thin film on the surface of oxydizable metals by using an alkaline solution of the metallic oxyd, and making the plate to be oxydized the negative electrode of a constant battery; a deep brown coating of the oxyd is thus deposited in a few minutes so firmly as to withstand the action of the burnisher, and perfectly protect the iron or steel from atmospheric action.

If the film of oxyd of lead is very thin, it presents, over a surface of polished silver or steel, a most pleasing exhibition of colored rings, analogous to the colored rings of Newton from thin plates (530). For this purpose the negative electrode is made of a thin platinum wire, protected from the solution by a glass tube, except at the extremity, where a mere point is presented. A rim of wax on the edges of the plate retains the solution of potassa, saturated with oxyd of lead, while it is connected on the positive pole, and the negative point is held for a few seconds within a line of the polished surface. These colored rings were first noticed by Mr. Nobili, whence their name.

3. *Physiological effects of the pile.*

895. **The physiological effects of the Voltaic pile.**—Galvani's original experiment, and the earlier observations of Swammerdam and Sulzer, of two metals on the tongue, deserve to be remembered as being our earliest knowledge of this subject. From a single cell, or even a small number of pairs, the dry hands, grasping the electrodes, receive no sensation; number, and not size of elements, is requisite for the physiological effect. Thus, from a column of fifty elements, or still more from fifty cups of Bunsen, or a Cruickshank's trough (870), a smart twinge is felt, reaching to the elbows, or if the hands are moistened with saline or acid water, the shock will be felt in the shoulders. This shock is unlike the sharp and sudden commotion from statical electricity, being a more continued sensation, accompanied, during the continuance of the current, by a sense of prickly heat on the surface. But it is only at the making and breaking of contact that a *shock* is felt. If the battery contains some hundreds of couples actively excited, the shock becomes painful, or even fatal. It may be passed through any number of persons whose moistened hands are firmly joined, but it is sensibly less acute at the middle of such a circuit than to those at the electrodes. Even after death, this power produces spasmodic muscular contractions, efforts to rise, and contortions of the features frightful to behold.* Persons in whom animation was suspended, have been restored by the influence of the hydro-electric current on the nervous system.

The senses of sight, hearing, and taste, are all affected by a Voltaic current; a flash of light, a roaring sound, and a sub-metallic savor being received when the shock of a small battery is passed, successively, through the eyes, the ears, and the tongue.

From the experiments of Becquerel, it appears that seeds subjected to a gentle electric current, germinate sooner than otherwise. Von Marum observed that plants with a milky juice, like the *Euphorbiaceæ*, do not bleed after a powerful electrical shock, owing, he suggests, to the loss of contractile power in the plant.

For a detailed account of the application of electricity to medical uses, consult the works of Dr. G. Bird (of London), W. F. Channing (of Boston), and the late elaborate volume of Dr. Garrett.

The magnetic effects of the pile belong to electro-dynamics, while its electrical effects have already been considered in §§ 863, 864.

VII. THEORY OF THE PILE.

896. **Three views.**—1. It has already been stated (863), that Volta and his school ascribed the effects of the pile to the simple contact of unlike metals, each decomposing the neutral electricity of the other.

---

* See a notice of Dr. Ure's experiments on a newly executed criminal, at Glasgow, in 1818. HARRIS, Galvanism, 123, J. Weale.

He argued that the chemical action of the battery was requisite only to afford conductors for the electricity, while the metallic substances remaining in every way unchanged, they are supposed to discharge into each other. According to this hypothesis, the two metals are in opposite electrical states, one being positive, the other negative; these states becoming at once destroyed by the intervening fluid. This theory assumed that the whole effect of the apparatus is but a disturbance and reproduction of electrical equilibrium. This view, however, cannot be maintained, since it involves an impossibility:—the production of a continual current, flowing on against a constant resistance, without any consumption of the generating force.

2. On the other hand, Fabbroni, Davy, Wollaston, and, above all, in our day, Faraday, De la Rive, and Becquerel have sought to establish that the Voltaic excitement was only the reciprocal of the chemical action; and as this was more intense, and properly directed, so was the pile more powerful. In addition to the statements and arguments already adduced, it is proper here to consider the ground of these two views, and somewhat more in detail.

3. A third view or theory of the pile has been advanced by Peschel, which he calls the *molecular theory*, and which rests on a sort of middle ground between the contact and the chemical theories.

897. **Volta's contact theory.**—The advocates of this mode of explaining the action of the pile (embracing nearly the whole body of the German physicists), contend that they have experimentally established the following points in support of Volta's theory, viz.: 1st, That Volta's original experiments demonstrate the fact beyond question, that the simple contact of heterogeneous metals does produce an electrical current (846). 2d, That in some cases, when a purely chemical action exists between a fluid and one of the two metals immersed in it, the contact of the metals arrests this action, and an opposite action commences. 3d, That there are even cases of hydro-electric combinations, in which electrical action exists, without any chemical action whatever on the electromotors. 4th, The advocates of this view further contend that chemical action is never the primitive cause of electrical excitement; although some do not question the influence of chemical action in promoting and increasing the excitement originally due to contact.

Since scarcely any chemical action, or none at all, occurs in a constant battery without contact, it is, with reason, urged that contact of the heterogeneous metals is the one indispensable prior cause of the Voltaic current. Hence the real difficulty seems to be, to decide what share chemical influence really has in exciting the electrical action. Want of space prevents our giving the evidence in detail upon which the advocates of the contact theory rely for the support of the above propositions.

898. **The chemical theory** assumes the electrical current to be the reciprocal of the chemical action in the cells of the battery, and that chemical action is essential to the production of such a current.

De la Rive demonstrated this latter point in the following manner: A pair, formed of two plates, one of gold, the other of platinum, was plunged into pure nitric acid, without the development of any current; by the addition to the nitric acid of a single drop of chlorohydric acid, a very decided current was obtained from the gold to the platinum through the liquid. In the first case there was no chemical action; in the second case, the gold was attacked, and the platinum was not, or more feebly.

The laws of electrolysis, first demonstrated by Faraday, as already stated (890), lend the evidence of mathematical certainty to the chemical theory of the pile. Since we thus reach the unavoidable conclusion that an equivalent of electricity is a chemical equivalent, and so bring the discussion down to the rigid test of the balance, the *ultima ratio* of chemists and physicists.

In addition to the laws of Faraday, already rehearsed, are the following:—

Laws of the disengagement of electricity by chemical action, first stated by M. Becquerel:—

1st. In the combination of oxygen with other bodies, the oxygen takes the electro-positive substance, and the combustible the electro-negative.

2d. In the combination of an acid with a base, or with bodies that act as such, the first takes the positive electricity, and the second the negative electricity.

3d. When an acid acts chemically on a metal, the acid is electrified positively, and the metal negatively: this is a consequence of the second law.

4th. In decompositions, the electrical effects are the reverse of the preceding.

5th. In double decompositions, the equilibrium of the electrical forces is not disturbed.

The quantity of electricity required to produce chemical action is enormous, compared with the amount of statical electricity disturbed by the common frictional machine. Faraday has, in his masterly way, demonstrated this fact by simple experiment.

He has shown that the quantity of Voltaic electricity requisite for decomposing one grain of water, would be sufficient to maintain at a red heat a wire of platinum about one one-hundredth of an inch ($\frac{1}{104}$) in diameter, during three minutes forty-five seconds, the time requisite to effect the perfect decomposition of the grain of water. The quantity of frictional electricity required to produce the same effect, would be that furnished by eight hundred thousand discharges of a battery of Leyden jars, exposing three thousand five hundred square inches of surface, charged with thirty turns of a powerful electrical machine.

Becquerel, by a different mode of experiment, arrived at nearly the same results. Therefore, to decompose a grain of water, requires an amount of electricity equal to that furnished by the discharge of an electric pane having a surface of thirty-two acres. "Equal to a very powerful flash of lightning." "This view of the subject gives an almost overwhelming idea of the extraordinary quantity of electric power which naturally belongs to the particles of matter." (Faraday Expt. Res., 853-861.)

899. **Polarization and transfer of the elements of a liquid.**—The electro-chemical theory has been much expanded by the researches of De la Rive; he explains the phenomena of polarization and the transfer of the elements of a liquid in the following manner:—

His theory assumes that every atom has two poles, contrary, but of the same force. The different kinds of atoms differ from each other in that some have a more powerful polarity than others. When two insulated atoms are brought near each other, they attract each other by their opposite poles; the positive pole of that which has the strongest polarity unites with the negative pole of that which has the feeblest polarity. A compound atom, when insulated, has therefore two contrary polarities between the poles of a pile; for example, the atom is so arranged that its + pole is turned to the platinum (or — side) of the pile, and the — pole is turned to the zinc (or + side) of the pile. This same action occurs with other atoms, so that there is produced a chain of polarized particles between the poles of the pile.

The oxygen of the particle of water nearest the zinc becomes negative, because of its affinity for the zinc, and the hydrogen becomes positive. The other particles of water become similarly electrified by induction, but the platinum has become negative by induction from the zinc, and therefore is in a condition to take up the positive electricity from the zinc of the contiguous hydrogen. The action now rises high enough for the zinc and the oxygen to combine chemically with each other. The oxyd of zinc thus formed dissolves in the liquid (dilute sulphuric acid), and is thus removed. But the particle of hydrogen nearest the zinc, now seizes the oppositely electrified oxygen of the adjacent particle, producing a fresh atom of water. The particle of hydrogen which terminates the flow is electrically neutralized by the platinum, to which it imparts its excess of positive electricity, and escapes in the form of gas; and other particles of water are continually produced, to supply the place of those decomposed, and thus continuous action is maintained. These changes, continually taking place, furnish an uninterrupted flow of electricity, which is conveniently termed a Voltaic current.

Other instances of electrolysis are explained in a similar way.

900. **Chemical affinity and molecular attraction distinguished.**—According to De la Rive, and in support of the view of the polarity of atoms, the distinction between chemical affinity and molecular attraction is as follows: chemical affinity is the attraction of atoms, operating by their contrary electric poles, which come into contact, while physical attraction results from the mutual attractive action that the atoms exercise over each other in virtue of their masses. This last attraction is never able to produce contact, because of the repulsive force of the ether which envelops the atom, and which increases in proportion as the sphere which separates the attracted atoms diminishes (146).

901. **Peschell's molecular theory of the pile.**—Resting upon the

opinion long held by many chemists, that those forces which lie at the basis of adhesion, and those which cause chemical affinity are not essentially different, Peschel holds that—*When electricity is generated in any Voltaic arrangement, it results from a molecular change, brought about in the touching bodies by the adhesive force which subsists between them.*

This theory possesses the advantage, that no new power need be assumed to exist, whereas the contact theory demands the existence of an "*electro-motive force*," of which we know nothing. It also accounts for the production of electricity, apart from any chemical action. In common with the chemical hypothesis, it deduces the phenomena of the single battery from the molecular forces; it considers the fluid not merely as a conductor of electricity, but as engaged in its production, and that the elements of the battery, by the physical changes which they undergo, are the actual sources of electricity; that their contact renders this change possible, and it is, therefore, the occasion, and not the generating cause, by which the electricity is produced. By this view, the chemical hypothesis is only a special case of the molecular. The simultaneous commencement of chemical action with the development of electricity, and the circumstance that the chemical intensity of a simple Voltaic arrangement increases and decreases as the chemical action on the fluid conductor, and on the elements of the battery is greater or less, fully accords with the statements of this theory. It follows, hence, that the electrical and molecular forces are one and the same, and that the latter appears as electricity whenever it passes from one mode of operation into the other, as, *e. g.*, when it ceases to hold the elements of the water, and so oxydizes the zinc.

§ 4. Electro-Dynamics.

I. ELECTRO-MAGNETISM.

902. **General laws.**—Electro-dynamics is that department of physics devoted to the mutual action of Volta-electric currents. These are distinct from the phenomena of static electricity. The phenomena of electro-dynamics may all be arranged under the following general propositions.

1. *Every conductor, conveying a current of electricity, affects a free needle as a magnet would do.*
2. *Electric currents affect each other like magnets.*
3. *A magnet acts upon an electric current as a second current would have done.*
4. *Electric currents in conductors excite similar currents in other conductors within their influence.*
5. *Magnets excite electric currents, and all the electrical effects depending upon them.*

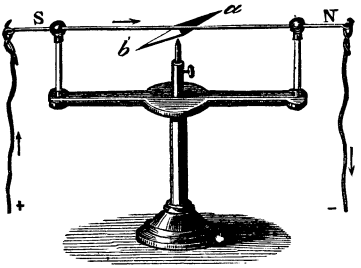
Hence, when magnetism is excited by electric currents, it is called *electro-magnetism:* and inversely, when electrical currents result from magnetism, they are called *magneto-electrical currents.*

It is impossible, in our narrow limits of space, to consider each of these propositions in full detail. We shall endeavor, however, to present those phenomena and their applications which are of most general interest.

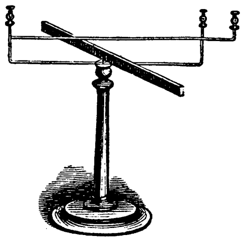
903. **Œrsted's discovery.**—In 1819–20, Prof. Hans Christian Œrsted, of Copenhagen, in a course of researches upon the relation of the Voltaic apparatus to the magnet, made the discovery of the fundamental fact of electro-magnetism, stated in the first of the foregoing propositions. Many physicists had before sought to evolve the phenomena of magnetism from the battery; but in vain, because they proceeded without connecting the poles by a conductor, in which case, of course (as we now clearly see), the power of the apparatus is dormant, like stagnant statical electricity in an unexcited conductor. *Œrsted closed the battery circuit by a conductor;* and therein rests his discovery. He found when such a conjunctive wire was approached to a free needle, that the needle was influenced by it, as if he had used a second magnet: in other words, the conducting wire, of whatsoever metal it might happen to be, had itself become a magnet.

If positive electricity flows from south to north over a horizontal conducting wire, placed in the magnetic meridian, then a free magnetic needle, *b a*, fig. 653, would have its north end, *b*, deflected to the *west*, if it is placed *below* the conducting wire, and to the *east* if it is placed *above* the wire. If the needle is placed on the east side of such a conductor, its *north end is depressed,* if on the *west side* of the wire, the north end of the needle *is raised.* Reversing the direction of the current, reverses all these movements.

653



654



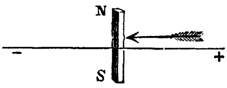
The rectangle, fig. 654, surrounding the magnetic needle, has three connections, by the use of which the current may, at pleasure, be sent above or below the needle.

Œrsted also found that only needles of steel or iron were thus affected, and not those of brass, lac, and other non-magnetic substances. He called the conductor a "*conjunctive wire,*" and he describes the effect of the electric current (or the "electric conflict," as he calls it), as *resembling a helix;* and that it is not confined to the wire, but radiates an influence at some distance.

The effect of Œrsted's discovery was remarkable. The scientific world was ripe for it, and the truth he thus struck out was instantly seized upon by Arago, Ampère, Davy, and a crowd of philosophers in all countries. The activity with which this new field of research has been cultivated, has never relaxed, even to this hour; while it has borne fruit in a multitude of important theoretical and practical truths, among which is the ELECTRO-MAGNETIC TELEGRAPH, one of the great features of this age.

904. **The electro-magnetic current moves at right angles to the course of the conjunctive wire.**—Let a current flow over a conductor in the direction of the arrow, fig. 655, from + to —; a small bar of soft iron, or a steel sewing-needle, held vertically before this wire, becomes instantly a magnet, with its N. pole toward the earth—place the rod of iron on the opposite side of the conjunctive wire, and its polarity is instantly reversed, as in the figure. Revolve it in either position in a vertical plane at right angles to the conjunctive wire, and the induced poles will retain their relation to the current in every position; *i. e.*, the end marked N. in the figure, will remain north at every point of the revolution. If a steel needle is used, it retains polarity after the current ceases to act on it. If the bar or needle be laid parallel to the conjunctive wire, then the *two sides* of the needle or bar have opposite polarities.

655



Hence, it follows, that a free magnetic needle tends to place itself at right angles to the path of an electro-magnetic current traversing a conjunctive wire, and were the needle free from the directive tendency of terrestrial magnetism, it would so place itself. The electro-magnetic current is, therefore, a *tangential* force, and acts tangentially upon a free needle.

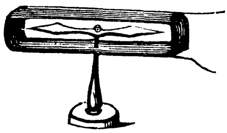
Simple as is the relation between the electric current on a wire, and the order of polarity induced by it in a needle, its correct expression is always difficult. To aid its exact statement by some simple formula, Ampère lays down the following rule:—

The north pole of a magnet is invariably deflected to the left of the current which passes between the needle and the observer, who is to have his face towards the needle, the electric current being supposed to enter from his feet and pass out of his head.

A verification of these cardinal principles by actual experiment, is the only way in which the student can obtain a vivid and lasting impression of them.

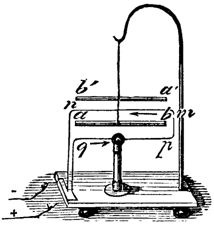
905. **Galvanometers or multipliers.**—If the conjunctive wire is bent into a rectangle, fig. 656, so as to carry the current once, or many times, around the needle, then the effect of the same force on the needle is multiplied in proportion to the number of convolutions. Thus Schweigger contrived his *multiplier*, fig. 656, composed of a flat spool of fine insulated copper wire within which the needle was

656

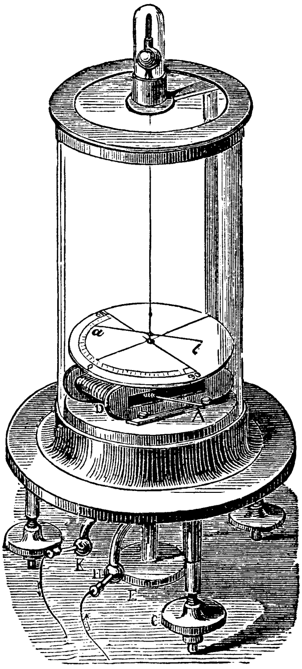


suspended. By this means a very feeble current became quite sensible. For ordinary purposes, a few turns, or, it may be, three hundred or four hundred convolutions suffice; but, for particular purposes, and where the current is very feeble, many thousand feet of very fine wire are used.

657

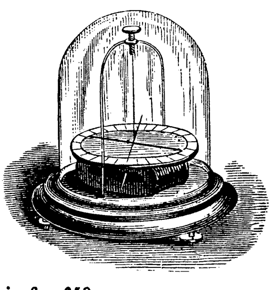


658



In *Nobili's double galvanometer*, an astatic needle (787), is used, in which the needles, *a b*, *b' a'*, fig. 657, are not quite equal, leaving a very slight directive force only. Fig. 658 shows this delicate instrument in its most perfect form, as used in determining the laws of transmission of heat, as well as for other purposes demanding a very sensitive instrument. Only the lower and stronger needle is enclosed in the helix, D, while the system is suspended by a fibre of raw silk, beneath a glass shade, leveled by three screw feet, C. The ends of the spool are seen at R K, while by the head, F, the whole instrument may be revolved so as to bring the wires of the spool parallel to the suspended needle at rest, which is the position of greatest sensitiveness. The sensitiveness of such an arrangement is very great. Suppose, for example, there are five hundred revolutions in the coil, then the lower needle is acted on one thousand times, and the upper one five hundred times by any given current; or the original force of the current is multiplied fifteen hundred times. But the directing force of the earth's magnetism on a given needle is proportional to the squares of the vibrations it makes (795). Now, assuming that the needles alone made sixty vibrations in a minute, and as astatic needles only ten, then we have 3600 : 100 as the numbers representing the effect of terrestrial magnetism in the two cases; or it is thirty-six times less in the astatic system than in the simple needles, and, consequently, the electric current will affect them thirty-six times more than if they were not astatic. The deflecting power of the current in question will, therefore, be increased by such a galvanometer, 1500 × 36 = 54,000 times.

659



A less expensive form of galvanometer is seen in fig. 659.

906. **The tangents or sine compass galvanometer.** — This instrument, invented by Pouillet, is designed to measure currents of greater intensity than can be measured by the common galvanometer. It depends on the established principle, that the intensity of a current is proportional to the sine of the angular deviation of the needle The angle of deviation being known, and consequently its sine, the intensity of the current is expressed in terms of the sine.

660

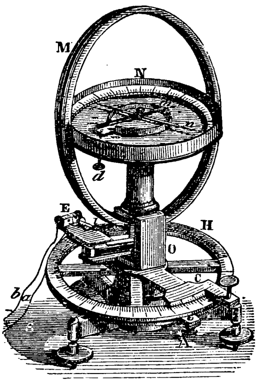
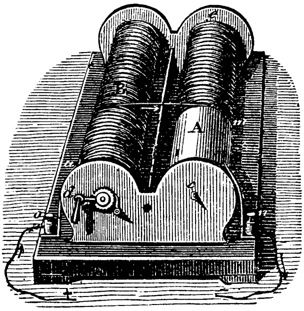


Fig. 660 shows the arrangement of this instrument, in which the current, entering by the conductors, *b a*, through the ivory piece, E, circulates a few times only, sometimes only once, over the vertical circle, M, placed in the magnetic meridian. The magnetic needle, *m*, is deflected upon the horizontal circle, N, in proportion to the force of the current in M, and a silver index needle, *n*, serves to record the angular deviation of *m* from its neutral point. When the needle is at rest, the vertical circle, M, is revolved upon the standard, O, by the button, A, until its plane coincides with the plane of deviation of *m*, and this angular distance is then read off by the vernier, C, upon the lower graduated circle, H. This galvanometer, or a simpler modification of it, is the form of instrument generally used in electro-magnetic researches.

907. **Rheostat.**—This simple contrivance of Wheatstone's serves to introduce a longer or shorter conducting wire into any circuit, the intensity of which it is proposed to measure by the galvanometer.

661



Since the intensity of the current is inversely as the length of the circuit (880), we may, by increasing or diminishing that length, produce from any current a determinate deviation (say 30°), on the galvanometer. Fig. 661 shows this arrangement, composed of two equal and parallel cylinders, one of wood, B, and the other of brass, A, supported in a frame-work and revolving on their centres. B is provided with a spiral groove, in which the turns of a copper conducting wire may be laid. One end of this wire is at *a*, in connection with the current pole, *o*. The wire may all be wound on B, in which case the current passes through its whole length, and escapes at *n*, through the metallic connection of its end, *e*, with A. If it is desired to shorten the conductor, the handle, *d*, is put on the axis, *c*, and A is revolved from left to right, until, as in the cut, one-half, for example, of the conductor, is wound on A. But A, being a metallic conductor, the current passes to *n* by the shortest course, and the only part of the wire in action is what remains wound on B, and this quantity is read off by an index and graduation engraved on the farther ends of the cylinders. This apparatus is indispensable in exact observations.

908. **Ampere's electro-magnetic discoveries and theory.**—Immediately after the first announcement of Œrsted's discovery of the magnetic powers of a conjunctive wire, Ampère, one of the most renowned of the French physicists (born 1755—died 1836), commenced a series of experiments (September, 1820) to determine the laws concerned in these curious phenomena. Of three principal hypothesis which he framed to this end, he finally accepted and demonstrated the following, viz.:—

A magnet is composed of independent elements or molecules, which act as if a closed electric circuit existed within each of them: in other words, each of these magnetic molecules may be replaced by a conjunctive wire bent on itself, in which a constant current of electricity is maintained, as from a Voltaic circuit.

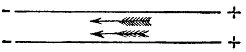
This hypothesis he maintained by singularly ingenious experiments, many of which were the direct suggestion of the hypothesis itself, and he brought all, by his power of mathematical analysis, into exact conformity with his theory. This theory recognises only such forces as are common to mechanical physics, and often called "*push and pull*" forces. These forces are mutual, and belong to all electric currents. In permanent magnets, the minute circular and parallel currents, pertaining, by this theory, to each magnetic molecule, all act at right angles to the magnetic axis or line of force. Hence, as in Œrsted's experiment (903), the magnetic needle strives to place itself at right angles to the path of the current on the conjunctive wire, it follows, that currents in the magnet seek a parallelism to that in the conjunctive wire. Granting this to be true, it follows, as a corollary from the premises,—

1st. *That two free conducting wires must attract or repel each other, according to the direction of the currents in them.*

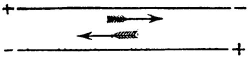
2d. *That a conjunctive wire may be made in all respects to simulate a magnet.*

909. **Mutual action of electric currents.**—*Parallel currents attract each other when they flow in the same direction.* Thus, in fig. 662, where the arrows and the signs + and — indicate the flow of the currents to

662

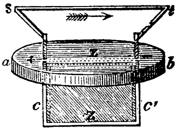


663



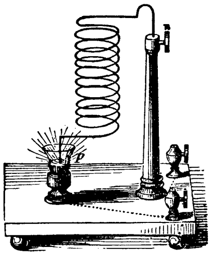
be identical, there is attraction, while, in fig. 663, the same signs show the currents to be reversed, in conformity to the law that:—*Parallel currents repel each other when their directions are opposite.* To illustrate these laws experimentally, one of the conductors should be fixed, and the other movable. The following simple apparatus also illustrates these laws, and several other points of interest presently to be noticed

664



De La Rive's floating current, fig. 664, is a little battery of amalgamated zinc, *z*, and copper, *c*, or zinc and platinum, set afloat by a disc of cork, *a b*, whose poles + and — are connected by a conjunctive wire, *s t*. When this little float is placed in a vessel of acidulated water (water with one-twentieth sulphuric acdi), an electric current flows in the direction of the arrow. Then join the poles of a single cell of Grove's or Smee's battery by a conjunctive wire of convenient length, and stretching the wire between the two hands, approach it parallel to *s t*; if the current is flowing in the same direction, the float will be attracted to the wire in the hands; if otherwise, repulsion is seen. If the two wires are not parallel to each other, then the movable current seeks to take up a position of parallelism, or one in which the two currents have a similar direction. A little rectangular frame of wood 3 × 6 in., may be wound with ten or twelve turns of fine copper wire, covered by silk in the manner of a galvanometer, and its free ends connected with a battery will give a stronger current. By simply turning the frame in the hand, the direction of the current is reversed.

665



Roget's oscillating spiral, fig. 665, also illustrates the law of attraction of parallel conductors. Here the conductor is coiled into a spiral, which is suspended from the top of an upright metallic standard in connection with one pole of a battery, while the other end dips into mercury in the glass, in connection with the other pole, K. When the poles are joined, each turn of the spiral attracts the next turn, shortening the spiral, and breaking the mercurial connection, with a spark. The weight of the spiral then restores the connection, and thus a continuous oscillating movement is kept up.

We add the following general propositions on this subject.

1. Two currents following each other in the same direction, as also different parts of the same current, repel each other.

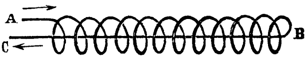
2. Two fixed currents of equal intensity, flowing near and parallel to each other in opposite directions (as when the same wire returns on itself without contact), exert no influence on a fixed current running near them: in other words, they exactly neutralize each other, and their effect is null.

The rotation of electric conductors about magnets, and the reverse; the rotation of a magnet on its own axis by an electric current, and the rotation of electrical conductors about each other, are all points most curious and instructive to trace, did space permit. The student will find these principles very neatly illustrated by appropriate apparatus in Davis's Manual of Magnetism. The researches of Henry, Page, and other American physicists, have made very important additions to this department of physics.

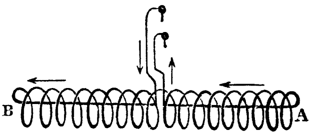
910. **Helix, solenoid, or electro-dynamic spiral.**—By winding

the conjunctive wire into a helix, as in fig. 666, and carrying the wire back again through the axis of this spiral, C B, the effects of the current from A to B, will be neutralized by its return from B to C, and there will remain only the effect due to its spiral revolution about C B. Ampère called this form of the wire a *solenoid.* The effect of the helix thus wound, is reduced solely to the influence of a series of equal and parallel circular currents. By winding the silk-covered wire in the manner shown in fig. 667, the two ends of the coil are returned to the centre of gravity, and being pointed with steel, the whole system can be conveniently suspended, as in fig. 668, upon what is called an Ampère's frame, in which the arrows show the course of the current from the battery to the helix or solenoid thus suspended. When the current is established, the axis of the solenoid, A B, swings into the magnetic meridian, while its several spires are in the plane of the magnetic equator. This position it assumes in obedience to the solicitation of terrestrial magnetism; consequently it simulates in all respects the character of a magnetic needle, although possessing not a particle of iron or steel in its structure. If a second helix, *b*, through which also a current passes, is now presented to the first, as in fig. 668, all the phenomena of attraction and repulsion will be seen, the action of the two helices or solenoids being to each other exactly like those of two magnets.

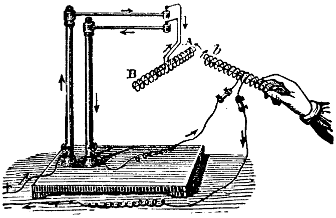
666



667

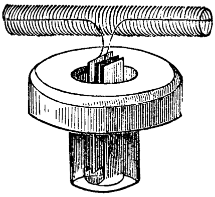


668



De La Rive's floating current, already explained in § 909, is also well adapted to illustrate the attractive and repulsive influence of a magnet on a free conjunctive wire, as well also as its obedience to the solicitations of terrestrial magnetism. For this purpose the conjunctive wire is wound, as in fig. 669, into a helix. Left to itself, this apparatus will act just as the solenoid on the frame, fig. 668, and will obey the impulses of a magnetic bar, or of another solenoid.

669



911. **Directive action of the earth.**—These effects are expressed in the following law:—

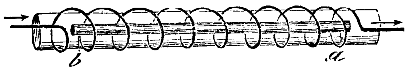
Terrestrial magnetism acts upon electric currents just as if the entire globe was encircled with electric currents from E. to W. in lines parallel to the magnetic equator.

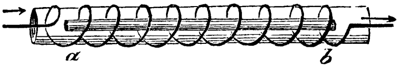
The direction in which these currents are supposed to move is the same with the apparent motion of the sun, and the one in which the earth's surface receives its advancing rays; and since it is now known that electrical currents generated by heat exert precisely the same influence on the magnetic needle as Voltaic currents do, therefore it has been inferred that the thermal action of the sun is the generating and maintaining cause of the currents of terrestrial magnetism (801).

912. **Magnetizing by the helix.**—We have already (805) described a mode of producing magnets from an electrical current. The explanation of this, after all that has been said, is easy. As each volute of the helix, carrying an electric current, is itself an active magnet, it is easy to conceive that under the united influence of a great number of such circular and parallel currents, the coercitive force of a steel bar, or bar of soft iron, should be decomposed, and active magnetism be thus induced, permanent or transient, according as steel or iron is the subject of experiment. Even a series of sparks from an excited electrical machine, passed through a helix, will magnetize a steel needle.

The position of the poles in a bar so situated will depend on the right-handed or left-handed twist of the spire. If the current flows from + to —, and the wire, as in fig. 670, turns from left to right (like the hands of a watch), then the north pole of the magnet is toward the left; but if the spire turns, as in fig. 671, from right to left, or opposite to the hands of a watch, then the poles are reversed. The following simple formula, by Faraday, will always enable the student to obtain definite notions of the polarity of the helix:—"Let a person," observes Faraday, "imagine that he is looking down upon the dipping needle, or north magnetic pole of the earth, and then let him think upon the direction of the motion of the hand of a watch, or of a screw moving direct; currents in that direction would create such a magnet as the dipping-needle."

670



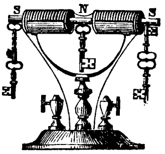


671

If the helix is wound on a tube of glass, paper, or wood, these substances offer no resistance to the passage of the power; but if a tube of copper or other metal were employed, the magnetizing power of the current on the enclosed bar would be destroyed.

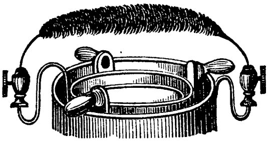
If the same helix is wound in two opposite directions, as in fig. 672, then, according to the direction of the current, there will be a pair of north poles at the point of reversal in the centre (or a pair of south ones), and the two ends will have the same name. A bar of steel placed in such a helix will remain permanently an anomalous magnet (779). Reversing the position of the bar in the helix, or reversing the position of the electrodes in the binding cups, will reverse its polarity.

672



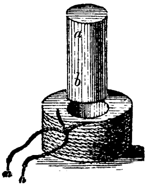
Arago's original experiment.—If a short conjunctive wire of copper, or any non-conducting metal, is strewn with iron filings, they will arrange themselves as seen in fig. 673, not bristling as in the magnetic phantom, with opposite polarities (777), but in close concentric rings, disposed over the whole length of the conductor. This fact was observed by Arago, in 1824, and by others, before the application of the helix to the induction of magnetism in soft iron.

673



When the helix is closely wound with many turns of insulated wire, and excited by a battery of considerable quantity, a cylinder of soft iron, as *a b*, in fig. 674, will be drawn into it from the position seen in the figure, with great power, and, after several oscillations, will come to rest in the middle of its length, in opposition to gravity, realizing the fable of Mahomet's coffin, suspended in mid air without visible support. This axial movement is availed of in the electro-magnetic engine.

674



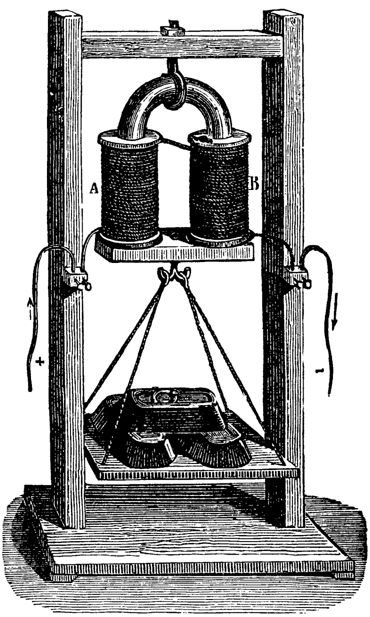
913. **Electro-magnets.**—Electro-magnets are masses of soft iron wound with coils of closely packed and insulated copper wire, varying in size and length, according to the use to be made of them. Fig. 675 shows the usual form of those designed to sustain great weights. The spools, A and B, are virtually continuations of one spool, the direction of the whorl being apparently reversed by the bend of the horseshoe. If a lever of the third order (113) is used as a steelyard, the lumber of heavy weights is avoided in the use of these instruments, and the power of the apparatus is easily tested.

Electro-magnets develop their surprising power only when the armature is in contact with the poles, a fact due to induction; without their armatures, they sustain not a tenth part of their maximum load. They are capable of over-saturation by an excess of battery power, and after that has been cut off, they retain a remarkable residual force so long as the keeper is in place, but as soon as the armature is detached, the whole of this residual magnetism is lost. Their polarity is instantaneously reversed by reversing the poles of the battery. This complete and immediate paralysis and reversal of power, renders these magnets of inestimable value in experimental researches.

Sturgeon, of England, in 1825, appears to have been the first to produce soft iron electro-magnets. Prof. Henry, and Dr. Ten Eyck, in 1830, produced the first electro-magnets of great power, by a new mode of winding the inducing coil. (Am. Jour. Sci. [1] XIX. 400.)

Prof. Henry, on a soft iron bar of fifty-nine lbs. weight, used twenty-six coils of wire, thirteen on each leg, all joined to a common conductor by their opposite ends, and having an aggregate length of seven hundred and twenty-eight feet. This apparatus, with a battery of four and seven-ninths feet of surface, sustained two thousand and sixty-three pounds avoirdupois: with a little larger battery surface it sustained twenty-five hundred lbs. This electro-magnet was constructed for Yale College Laboratory, in 1831, and is still among their instruments.

675

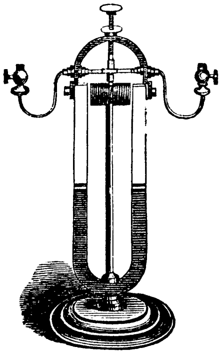


Mr. J. P. Joule (Annals of Electricity, V. 187), in 1840, constructed soft iron electro-magnets of peculiar form, being in fact tubes with very thick walls cut away on one side lengthwise, and wound in the direction of the length; one of which, weighing 15 lbs., held 2090 lbs., equal to nearly 140 times its own weight. It was wound with 4 covered copper wires, $\frac{1}{11}$ inch diameter, and each 23 feet long only, the length of the soft iron being 8 inches, and its outer diameter three inches. Another magnet weighing 1057 grains supported twelve pounds, or 1286 times its own weight; and a very minute one, which weighed only 63·3 grains, carried on one occasion 1417 grains, or 2834 times its own weight. The last is more than eleven times the proportionate load of the celebrated magnet of Sir Isaac Newton, § 806.

914. **Page's revolving electro-magnet**, fig. 676, affords satisfactory evidence of the great rapidity with which a mass of soft iron may receive and part with magnetism, having its polarity reversed also by a change of position. In this instrument, a permanent U-magnet has a vertical spindle in its axis, on the upper end of which is placed a mass of soft iron, destined to receive induced magnetism through the covered wire with which it is wound, and whose ends are represented by the two screw cups, one on each side. By a simple contrivance

called an interrupter, or break-piece (formed by sawing a silver ferrule on the axis into two parts by vertical slits), the continuity of the current is interrupted twice in every revolution, when the position of the armature is as seen in the figure. The effect of this arrangement is to paralyze the magnetic force at the right instant to permit the momentum of the mass to carry the armature by the poles of the fixed magnet, when the battery connection is again completed, new magnetism induced, and the motion continued as before. Such a little magnetic engine may revolve with a velocity of 2000 times a minute, equal to 4000 reversals of polarity,—each reversal being accompanied by a passive interval, when the soft iron is no magnet.

676



915. **Power of electro-magnets.**—The power of electro-magnets depends, 1st, on the intensity of the current; 2d, on the number of whorls in the helix; 3d, on the kind and shape of the iron bar; 4th, on the form and size of the keeper or armature. These points have been studied by Lenz and Jacobi, and many others, of whom the results of Dub are the most recent. Dub distinguishes between magnetism, attraction, and sustaining power, in electro-magnets, confining the term magnetism to the magnetic excitation due to the Voltaic current. Lenz and Jacobi measured this by means of the induced current excited by the vanishing of the magnetism to which it is proportional. When a second bar of soft iron is caused to approach the first, this also becomes magnetic (by induction), and by n-fold *magnetism*, n^2 times the *attraction* is produced; until actual contact happens, when this ratio is no longer maintained.

Dub gives the following summary of his results:—

1. The attraction of U-shaped electro-magnets, with an equal number of windings, is proportional to the squares of the magnetizing current force.

2. The attraction of U magnets is, with equal currents, proportional to the square of the number of windings of the magnetizing spirals.

3. The attraction of U magnets is proportional to the square of the current force multiplied by the square of the number of windings. [This is true alike for attraction and sustaining force, both in straight and in U magnets.]

4. The magnetism of massive cylinders of iron of equal length, magnetized by Voltaic currents of equal force, and by spirals of an equal number of windings, closely surrounding the core, is accurately proportional to the square roots of the diameters of these cylinders.

5. For the particular case in which the surface of contact does not

disturb the result, the attraction and sustaining force are, with equal magnetizing forces, proportional to the diameters of the bar or U-magnets.

6. The attraction of bar and U-shaped electro-magnets with equal magnetizing forces, increases the nearer the whole of the windings are to the poles.

7. The attraction, like the sustaining force of U electro-magnets—other things being equal—remains the same, whatever be the distance of the branches of the magnet.

8. The length of the branches of a U-shaped electro-magnet has no influence on its attractive or sustaining force, if the windings of the spiral surround its whole length.

In addition to these laws, the author has found that the attraction which a helix or spiral exerts upon a soft iron bar placed in its axis, follows the same law as an electro-magnet; hence it follows, that:—

9. The attraction of a spiral is proportional to the square of the magnetizing current, multiplied by the square of the number of windings.*

The sustaining power of an electro-magnet increases with the mass of the armature up to a certain point, not exceeding the mass of the electro-magnet itself; and, moreover, Liais has shown that an armature whose face of contact is not over one-third the breadth of the poles to which it is applied, gives a maximum effect.

For some curious results with circular and trifurcate electro-magnets, and the applications of this force to "break up" railway trains, consult the papers of Prof. Nicklès (Am. Jour. Sci. [2], XV., 104 and 380; and XVI., 110 and 337).

916. **Vibrations and musical tones from induced magnetism.**—Dr. Page, in 1837, noticed the production of a musical sound from a magnet, between the poles of which a flat spiral was placed. The sound was heard whenever contact was made or broken between the coil and the battery. Two notes were distinguished, one the proper musical tone of the magnet, and the other an octave higher. De la Rive, Delezenne, and others, have confirmed and extended these curious observations. The existence of molecular disturbance in receiving and parting with magnetic induction, has been farther illustrated by the same ingenious observer, by the vibrations imparted to Trevellyan's bars by the current from two or three cells of Grove's battery. (Am. Jour. Sci. [2], IX., 105.) Trevellyan's bars are prismatic bars of brass, hollow on one side, so as to rest by sharp edges on blocks of lead. When these are gently warmed, and then laid upon the leaden blocks, the unequal expansion and contraction of the two metals gives the brass bars a slight motion of vibration, due to molecular disturbance by heat. A Voltaic current, according to Dr. Page's observation, produces the same effect as heat, but more remarkably.

917. **Electro-magnetic motions and mechanical power.**—The

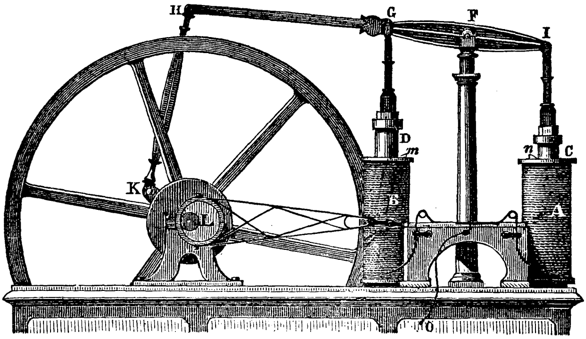
---

* Am. Jour. Sci. [2], XVII., 424.

facility with which masses of soft iron may be endued with enormous magnetic power by currents of Voltaic electricity, and again discharged, or reversed, in polarity, has led to numberless contrivances to use this power as a mechanical agent. A great variety of pleasing and instructive models of such machines, with the use both of permanent magnets and of electro-magnetic armatures, or of electro-magnets only, are described in Davis's Manual of Magnetism. The revolving armature, fig. 676, is one of these.

We annex a figure of an electro-magnetic engine, similar to one by which Dr. Page obtained a useful effect of ten horse-power, in driving machinery, and transporting a railway train. A and B, fig. 677, are two very powerful helices of insulated copper wire, within which are two heavy cylinders of soft iron, C D, counter-balanced on the ends of a beam, G F I, like the working beam of a steam-engine. By the movement of an eccentric, L, on the main shaft of the fly-wheel, the poles are changed, at the moment, to magnetize and de-magnetize, alternately, the two helices, drawing into them the two soft iron cylinders, by a force of many hundred pounds. Prof. W. R. Johnson tested the force of an engine of this kind built by Dr. Page, in 1850, and found it to give about six and a half horse-power. (Am. Jour. Sci. [2], X., 472.)

677

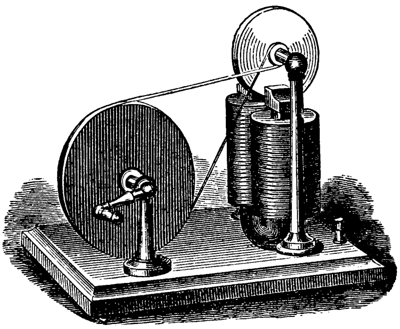


M. Jacobi, of St. Petersburgh, has studied this subject very carefully, and has contrived an effective form of rotating machine, very similar to that of Cook and Davenport, so well known in the United States in 1837. Froment, of Paris, has also constructed a powerful apparatus of this sort, in which armatures of soft iron on the periphery of a wheel are drawn towards electro-magnets placed radially.

In all these machines, it is heat developed by chemical action that is transformed, in the form of magnetic attraction, into mechanical work (761). As the result of a great many experiments, Mr. Joule has shown that the best theoretical result from the heat, equivalent to the solution of a grain of zinc in a battery, is eighty lbs. raised one foot high. But a grain of coal burned in a Cornish boiler, raises one hundred and forty-three lbs. one foot, and the price of the coal is to that of the zinc as 9d. per cwt. to 216d. per cwt. Therefore, under the best conditions (which are never reached in practice), the magnetic force is 25 times dearer than that of steam. Until, therefore, zinc is cheaper than coal, in the proportion of 80 to 143, coal will probably be burned in atmospheric air, preferably to the combustion of zinc in sulphuric acid, to produce mechanical work.

918. **Conversion of magnetism into heat.**—Foucault has shown that the magnetism induced in a disc of copper revolving between the poles of an electro-magnet, is converted into heat. For this purpose, a powerful electro-magnet is supported upon a basement, fig. 678; two pieces of soft iron are attached to the poles of the magnet, so that they concentrate, upon the two faces of a metallic disc, their magnetism of induction. This disc of copper receives, by means of pulleys, a rapid revolution, which will continue for a long time, if no current exists in the electro-magnet; but if a current from a battery of two or three cells is passed through the wire, the disc is almost immediately stopped; if, however, against this resistance the disc is forced to revolve, the expense of force is converted into *heat*, and the temperature of the disc is rapidly raised.

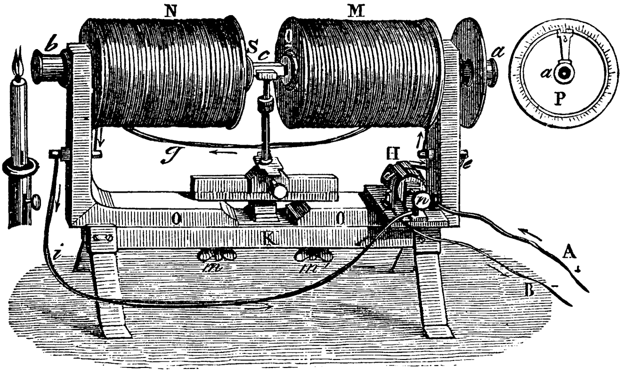
678



II. DIAMAGNETISM.

919. **Action of magnetism on light.**—Fig. 679 shows the apparatus designed by Ruhmkorff, in illustration of Faraday's magnetic

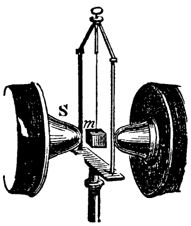
679



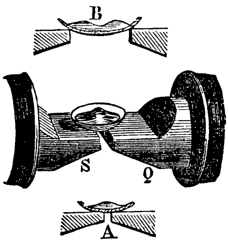
rotatory polarization of light already spoken of under Optics (560). Two powerful inducing coils, N and M, surround two hollow cylinders of soft iron, S and Q. The current enters the bobbins by A, and following the direction of the arrows, returns by B. The two coils slide in the groove in the base, K, on the two supports, O O, so that they may be approached or withdrawn at pleasure by turning the screws, *m m*. A commutator, or interrupter of the current, is arranged at H *n*. At *a* and *b* are two Nicol's prisms (553), of which *a* has a vernier or index, reading the degrees on the graduated circle, P. To make the experiment, a piece of heavy glass, or silicious-borate of lead, *c*, is placed on a support between the poles S and Q. A ray of light from the candle, polarized by the prism, *b*, is transmitted through the glass in the axis of the poles. When the current is applied, the ray of light appears to be revolved, similarly to the effect produced on polarized light by quartz, or oil of turpentine (556). A great number of other solids and liquids are found to act in a like manner, but to a less degree, than in the case of " heavy glass." As no rotation of the ray takes place unless there is *some medium* on which the magnetism may act, it has been argued with some force by Becquerel and others, that the action is wholly due to a molecular change in the solid under experiment. A reversal, however, of the direction in which the ray travels, reverses the direction of rotation in the polarized ray, a circumstance not found in bodies in the natural state. This apparatus also serves to illustrate the phenomena of diamagnetism.

920. **Diamagnetism.**—We have already (799) alluded to the action of magnetism upon all bodies, discovered by Dr. Faraday, in 1846, a discovery which alone would place its author in the highest rank of modern philosophers. By the use of the apparatus, fig. 680, he proved that every substance which he tried, solid, fluid, or gaseous, was subject to magnetic influence, assuming either the *equatorial* or *axial* position, according to its nature.

681



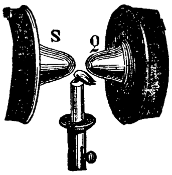
680



For solids, and some fluids, fig. 681 shows the arrangement. Two bluntly rounded polar pieces of soft iron are fitted into the openings of the spools, S and Q, while between them are suspended on a silk fibre cubes, *m*, or short bars of the various magnetic metals, bismuth, antimony, copper, lead, tin, &c. If the cube is spinning about when the current passes, the induced magnetism arrests its motion in whatever position it may be; and if the metal has the form of a little bar, it rests athwart the axis like a cross. If non-magnetic liquids, alcohol, water, and most saline solutions, are confined in little narrow bottles (like homœopathic vials), hung like *m*, fig. 681, these are similarly affected. If they are filled, however, with magnetic solutions, the salts of iron, nickel, or cobalt they then arrange themselves axially.

Plücker has shown that if these magnetic solutions are placed in watch-glasses upon the poles, S Q, as in fig. 680, according as the poles are nearer or farther asunder, these liquids are heaped up in one or two elevations, as in A and B.

682



The flame of a candle placed between the poles, S Q, fig. 682, is strongly repelled, a fact first observed by Father Bancalari, of Genoa, and the flames of combustible gas from various sources are differently affected, both by the nature of the combustible and by the nearness of the poles. The flame from turpentine is most curiously affected, being thrown into the form of a parabola, whose two arms stretch upward a great distance, and are each crowned by a spiral of smoke. Oxygen, which, in the air, is powerfully magnetic (799), becomes, when heated, diamagnetic. A coil of platinum wire, heated by a current of Voltaic electricity, and placed beneath the poles of Faraday's apparatus, occasions a powerful upward current of air, but, when magnetism is induced, the ascending current divides, and a descending current flows down between the upward currents. The following list expresses the order of some of the most common *paramagnetic* substances, viz.: iron, nickel, cobalt, manganese, palladium, crown-glass. platinum, osmium. The zero is *vacuum*. The *diamagnetics* are arranged in the inverse order, commencing with the most neutral: arsenic, ether, alcohol, gold, water, mercury, flint-glass, tin, "heavy glass," antimony, phosphorus, bismuth.

Plücker has further demonstrated the important fact, that the optic axis of Iceland spar is repelled by the magnet—a fact probably true of many crystals—in some of which the magnetic axis is parallel to the longer axis of crystallization. Thus, a piece of kyanite will, under the influence even of the earth's magnetism, arrange itself like a magnetic needle.

III. ELECTRIC TELEGRAPH.

921. **Historical.**—The thought of making telegraphic communications by electricity appears to have suggested itself as soon as it was known that an electrical current passed over a conducting wire without sensible loss of time. The following brief summary of well-known historical facts, will serve at once to show how impossible it is justly to bestow the exclusive merit of the electric telegraph upon any inventor, while at the same time it strikingly illustrates what is true of every important invention, that final success rescues from oblivion many schemes that had hardly vitality enough in their day to find a place in the records of history.

In 1747, Dr. J. Watson erected a telegraph from the rooms of the Royal Society, in London, for two miles or more, over the chimney tops, using frictional electricity on a single wire, with the earth for a return circuit. In 1748, Dr. Franklin set fire to spirits of wine by a current of electricity sent across the Schuylkill on a wire, and returning by the river and the earth. In 1774, LE SAGE, a Frenchman, established at Geneva an electric telegraph, in which he used twenty-four wires insulated in glass tubes buried in the earth, each wire communicating with an electroscope, and corresponding to a letter of the alphabet, and excited by an electrical machine. (MOGNIO *Traité*, 59.) In 1787, BETANCOURT, in Spain, made an effort to employ electricity for telegraphing by passing signals from a Leyden vial over wires connecting Madrid with Aranjuez, a distance of twenty-six miles. SALVA, in 1796, also presented to the Academy of Madrid, a plan of an electric telegraph of his own invention, which received the patronage of the Prince of Peace. In 1800, the public announcement of VOLTA's discovery of the pile supplied a new means for telegraphing, far more certain than frictional electricity, and accordingly we find, that in 1811, Prof. SOEMMERING, of Munich, proposed to the Academy in that city a complete plan, with details, for an *electro-chemical* telegraph, in which he used thirty-five wires (twenty-five for the German alphabet, and ten for the numerals), tipped with gold and covered by the same number of glass tubes filled with water, to be decomposed whenever the corresponding letter or numeral was touched by the battery wire on a key-board at the other end. This is the type of all electro-chemical telegraphs. Dr. J. REDMAN COXE, of Philadelphia, in 1816, in Thompson's Annals of Philosophy, apparently without knowledge of Soemmering's plan, proposes a similar one by the use of Voltaic electricity. In 1819–20, ŒRSTED's discovery of electro-magnetism, and AMPERE's development of the subject, opened the way to electro-magnetic telegraphy. Œrsted first, and then Ampère, proposed the plan of a telegraph, using the deflections of a magnetic needle for signals; the type of Wheatstone's needle telegraph; but their suggestions were never put in practice. In 1823, Dr. F. RONALDS, of England, published a volume detailing the plan upon which he had previously constructed eight miles of electric telegraph, and in which he used a movable disc, carrying the letters, the type of all dial telegraphs. In 1825, WILLIAM STURGEON, of Woolwich, England, made the first electro-magnet of soft iron, without which, further progress in the electro-magnetic telegraph was impossible. Prof. JOSEPH HENRY, in 1830, described a mode of giving greater power to electro-magnets, and the same philosopher, in 1831, devised the first reciprocating electro-magnet and vibrating armature, including also the principle of the relay magnet, so indispensable an auxiliary in the Morse system. (Am. Jour. Sci. [1] XX. 340.) In 1834, Messrs. WEBBER and GAUSS established an electro-magnetic telegraph at Göttingen, between the Observatory and the Physical Cabinet of the University, and used it for all th purposes of scientific communication.

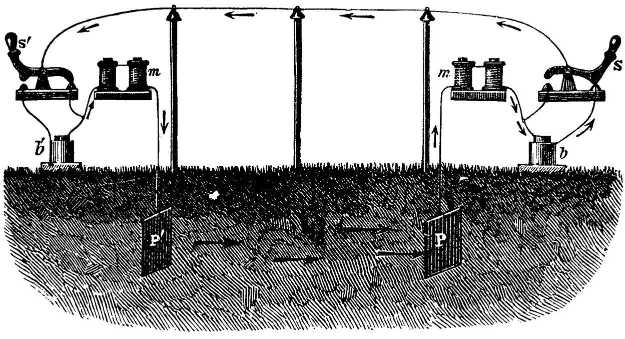
In 1836, Prof. J. F. DANIELL invented the constant battery (874), without which any mode of electric telegraph would have been futile.

In 1837—a year ever memorable in telegraphic history for the first general and successful introduction of the electro-magnetic telegraph—and almost at the same time appeared MORSE, in the U. S.; STEINHEIL, at Munich; and WHEATSTONE and COOKE, in England; as distinct and independent claimants for the honor of this discovery. Prof. J. D. Forbes, the able historian of the Physical Sciences, in the eighth edition of the Encyclopedia Brit. America, speaking of these inventions, says: "the telegraph of the two last (Steinheil and Wheatstone) resembles in principle Œrsted's and Gauss's: that of the first (MORSE) is entirely original, and consists in making a ribbon of paper move by clock-work, whilst interrupted marks are impressed upon it by a pen," &c. * * "The telegraphs of Morse have the inestimable advantage, that they preserve a permanent record of the despatches which they convey." This advantage, it is but just to say, they share with Bain's electro-chemical telegraph.

922. **The earth circuit.**—Although Drs. Watson and Franklin (1747–8) used the earth as the return circuit in their telegraphic experiments, it was considered essential in the use of Voltaic electricity to employ at least two wires, until Steinheil, in 1837, in the construction of his telegraph at Munich, dispensed with the whole resistance of the return wire by burying a large plate of copper at each station, with which the circuit wire communicated. This certainly must be esteemed one of the most important discoveries in connection with the telegraph; but from some cause or other it obtained for some years but little publicity, although described at length in the *Comptes-Rendus*, of Sept. 10, 1838. Bain re-discovered the same fact some years later, and Matteucci, of Pisa, in 1843, made experiments which convinced the most incredulous of the truth of this important fact.

Fig. 683 illustrates the mode of using the earth circuit, now universal in all telegraphs. S and S′ are two distant stations, with their batteries, *b b′*, and magnets *m m′*. A wire passing over insulating posts, through the air, connects S and S′. One pole of each battery is connected with the earth through the magnets, ending in plates of copper, P P′. Neither battery will, however, act, unless one of the breaks, or finger-keys, S or S′, is depressed. If the finger-key, S, is depressed, the circuit consequently is completed *through the earth*, for the battery, *b*, while that of *b′* remains open. The arrows show the course of the current, and this will be reversed when the circuit at S is closed. The explanation of this curious fact appears to be, not that the electricity is conducted back by the earth to its origin at the battery, but that the molecular disturbance in which the polarity of the circuit consists, is effectually relieved by communication with the common reservoir of neutral electricity (815), and so conduction proceeds without interruption. Any number of parallel currents may thus coexist without interference. This simple device saves not only half the expense

683



of constructing lines, but it more than doubles their power of electrical transmission. For the rapidity of the current, refer to § 818.

923. **Varieties of electro-telegraphic communication.**—There are essentially but two modes of electro-telegraphic communication, viz.: the *electro-mechanical* and the *electro-chemical.* Various and seemingly unlike as are the numerous ingenious contrivances for this purpose, they all fall under one of these two divisions.

The electro-mechanical form of telegraphic apparatus, embraces the *needle* telegraph, the *dial* telegraphs, and the *electro-magnetic,* or *recording* telegraphs: both those which, like Morse's, use a cipher, and those, like House's, which print in legible characters.

The electro-chemical telegraphs (having their type in Soemmering's original contrivance) depend on the production of a visible and permanent effect, as the result of some chemical decomposition at the remote station; of these, Bain's is the best known.

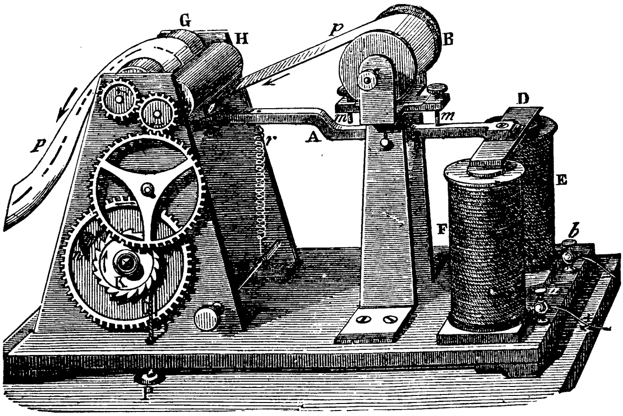
This is not the place, had we time, to give all the details of the well-known machines in use for telegraphic purposes. A few words, stating the principles on which they all depend, with a notice of two or three of those most used in the United States, must suffice.

As the needle telegraph of Messrs. Wheatstone and Cooke (depending on the deflection of a needle by a galvanometer coil) has never been used in this country, and cannot compete with either of the systems adopted here, it is needless to describe it. It requires one operator to read the movements of the needle, and another to record the message, and its average capacity is not over ten or twelve words per minute. The dial telegraph of Froment, and others, is open to the same objections.

924. **Morse's recording telegraph.**—Every electro-telegraphic apparatus implies the use of at least two instruments, one for recording, and one for transmitting the message. Besides these, in most cases there is need of a *relay magnet,* which receives the circuit current and acts to bring into use the power of a local battery, by which the work of recording is performed. This is requisite because the circuit current is usually too feeble to do more than establish a communication with the local battery. Every recording instrument has a clock-work, or some similar mechanical movement, to carry forward the paper fillet on which the record is impressed, at a regular rate of motion. Fig. 684 shows the Morse recording instrument.

It consists, essentially, of a simple lever, A, with a soft iron armature, D, over the electro-magnets, E F, by which the electrical impulses are propagated to the pen or stylus, *o*. A weight, P, gives motion to a train of wheels, K C, by which the fillet of paper, *p p*, is carried over the rollers, G H, in the direction of the arrows. A feeble spring, *r*, withdraws the point, *o*, and armature, D, when the electricity ceases, and the motion of the pen-lever is farther adjusted by two

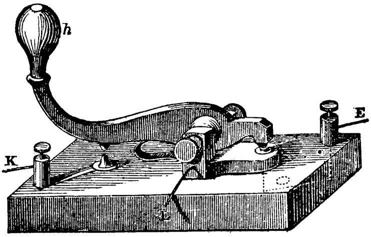
684



regulating screws, *m m*, that can be set at pleasure. The battery current enters the apparatus at the binding screws, *a b*.

The message is recorded by a cipher of dots and dashes, made on the moving fillet by the point of the pen-lever The lever moves in obedience to the impulses of the operator at the transmitting station, who presses the "*finger-key*," for a longer or shorter instant, according to what he would transmit. Every motion of the pen-lever gives a sound, corresponding to the letter communicated; and to a practiced operator, this sound becomes a definite language, which his ear interprets with unfailing certainty, so that he literally hears the message and translates it without the necessity of looking at the record. Fig. 685 shows the *spring finger-key*, by which messages are transmitted. The Morse instrument has the advantage of great mechanical simplicity, so that it requires but little skill to manage it, and its record being permanent and sufficiently rapid for all ordinary purposes, it has come into more general use in the United States than any other, and over the continent of Europe has also been very generally adopted. Mr. Morse conceived this plan of telegraphic transmission in 1833, but it was only in 1837 he applied for his first patent, and in 1844 the first line was built in the United States, from Washington to Baltimore.

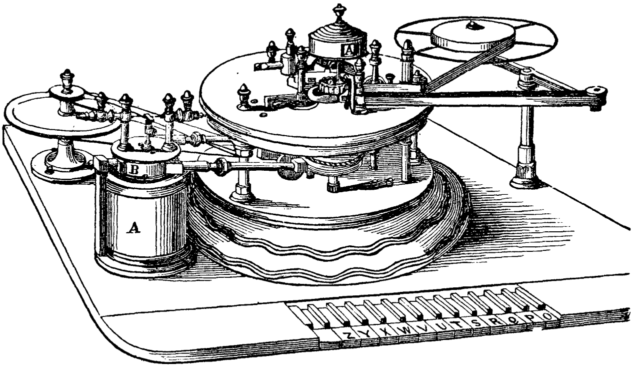
685



925. House's electro-printing telegraph.—This most ingenious instrument records its message in plain printed characters, and, as a mechanism, must be regarded as one of the most wonderful results of inventive genius. A drawing of its chief parts, some of the details being omitted, is seen in fig. 686.

Its chief parts are a key-board, marked with the letters of the alphabet; a type-wheel, *o*, on which the letters of the alphabet are engraved; a helical coil of fine wire in the cylinder, A, in connection with the circuit, and which operates to open a valve for the emission of a blast of air, compressed by a pump under the table into a reservoir, B. The purpose of this blast is to work the escapement regulating the motions of the type-wheel, *o*. This is the only function of the electricity in the recording machine; every other motion is a mechanical one. The electricity, by opening and closing the air-valve, regulates the motion of the type-wheel, arresting it at the pleasure of the operator at the distant station, who, by touching on his key-board the letter he would transmit, arrests the type-wheel of the recording instrument at that letter; a simple mechanism then presses the fillet of paper on the face of the type, and moves it forward to receive the next impression. Its actions are quicker than thought, and, owing to the exact duality of the two machines in every part, and the perfect equality of their motion, the operator transmitting is as conscious as him receiving, if there is any error, aided as he is by a tell-tale above the type-wheel, showing, in our design, the letter A. It is impossible, without many pages of detail, and minute drawings of the parts, to render this marvel of mechanical art perfectly intelligible. But the general thought of the inventor is clear enough, to place the recording apparatus at the control of the transmitting operator, through the agency of compressed air, controlled by the electric current, and controlling, in its turn, the escapements of the recording apparatus. It prints about one hundred letters per minute, on a circuit of one hundred and fifty miles.

686



926. **The electro-chemical telegraph** depends on the decomposition, by the electrical current, of a salt of iron with which the paper fillet is saturated, and the production of a blue or red stain upon it. The same clock-work movement used by Morse, carries forward the paper over a metallic cylinder, which is one pole of the circuit, while

a steel pen (if a blue mark is intended, or copper, if red is intended), in connection with the other pole, bears steadily upon the paper; the least transit of electric force decomposes the prussiate of potassa with which the paper is charged, producing a stain. To insure the dampness in the fillet requisite for electrical conduction, Maison-Neuve has proposed to charge it with a solution of nitrate of ammonia, a salt whose attraction for moisture is such that the paper remains always damp. To avoid errors, as well as to insure greater rapidity, Bain, who was the author of this system, proposed to prepare the messages, on fillets of paper, punched with holes by a machine called a *compositor* or *multiplier*. Humaston has lately so improved the mechanism of this compositor, that it is possible, by combining this apparatus with the Bain system of reading, to transmit not less than three thousand signals per minute, equal to six hundred letters, or one hundred and twenty-five words of five letters each. The punched fillets take the place of the finger-key as a circuit breaker for the transmission of the message.

Autograph telegraphic messages can be transmitted by the electro-chemical method, by writing upon the transmitting cylinder, with solution of hardened wax, and then causing a tracing point to traverse the cylinder with a close spiral from end to end. The result is, the interruption of the current where the wax is, and a corresponding blank space left on the paper at the receiving station. The union of these white spaces gives what was written in wax, as a white character on a dark ground.

927. **Submarine telegraphs—the Atlantic cable.**—The first submarine telegraphic cable was successfully sunk in August, 1851, connecting Dover, in England, with France, at Cape Griz Nez. Since that time, numerous other submarine cables have been laid, of which that through the Black Sea was the longest, until the placing of the Atlantic cable was accomplished, on the 5th of August, 1858. The failure of this great enterprise is now believed to be attributable to injuries received by the cable before submergence. Its failure was gradual,—over 400 messages being transmitted before it became totally inactive.

687

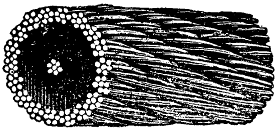


Fig. 687 shows the size and mode of construction of this cable. The conducting wire is formed of seven strands of No. 32 copper, twisted into a cord, and buried in refined gutta percha, laid on by machinery in three coatings, over which are placed several strands of tarred cord. The whole is encased in seventeen strands of iron wire, each strand formed of seven No. 30 iron wires. It weighs about two thousand lbs. to the nautical mile, and about two thousand miles of it lie submerged between Valentia Bay, Ireland, and Trinity Bay, Newfoundland. The shore end is formed of ten miles of much stronger cable, enclosing, however, the same conductor.

The problem of scientific as well as practical interest in long cables, is the possibility of transmitting signals through them with sufficient rapidity for useful purposes. Faraday has shown (Ept. Res. vol. 3d, p. 507—523 and 575), that a gutta-percha covered wire is, when submerged in water, in very different electrical conditions from what it is in air. In the water it simulates the character of the electrical condenser, or Leyden vial, and when thus charged by induction, must be discharged before a second wave can be transmitted through it; and when the electric pulses are frequent, as in telegraphic communications, the effect of the *electric conflict,* as Œrsted originally termed it, is to produce a tremor in place of sharp and decided beats. Those who would know the history of the telegraph more in detail, will consult Schaffner's Telegraphic Manual, and Prescott's History and Practice of the Electric Telegraph, Boston, 1860.

928. **Electrical clocks and astronomical records.**—If a clock pendulum is, by any mechanical device, made to open and close the circuit in a telegraphic arrangement, it is obvious, that if the clock beats seconds, these will appear recorded as dots at equal intervals upon the paper fillet. An astronomer, watching the transit of a star across the wires of his telescope, with his hand upon the finger-key of the same circuit, closes it at the exact instant of time, and the record of the passage of the star is fixed with unerring certainty between the beats of the clock and upon the same fillet which bears record of the time in seconds and their subdivisions. This beautiful system is wholly and peculiarly American, as the clear records of science show, and offers incomparably the best possible mode of determining longitude differences. The names of Bache, Bond, Gould, Locke, Mitchel, Saxton, Walker, Wilkes, and others, are inseparably connected with the history of this important application of the telegraph, for the details of which the student is referred to the American Journal of Science, the proceedings of the American Association for the advancement of Science, and the reports of the United States Coast Survey.

Bain, it is believed, constructed the first electrical clock (in 1842), which was moved by a current from a large copper and zinc plate buried in the earth, or, better, to a zinc plate buried in charcoal. By any simple mechanical arrangement, the motion of the pendulum reverses or breaks the current at every beat, and by the aid of a stationary magnet, the vibratory movement due to the electric current is strengthened and perpetuated. It is possible to transmit the same electric current to any number of clocks, in the same place, or in different places, and thus secure exact equality of time.

Fire-alarm.—Boston, Philadelphia, and some other cities are provided with a telegraphic system due to Dr. Channing and Mr. Farmer, by which a fire-alarm is sounded simultaneously in every district: a detailed description of which will be found in Am. Jour. Sci [2], XIII., 58.

§ 5. Electro-dynamic Induction.

I. INDUCED CURRENTS.

929. **Currents induced from other currents.—Volta-electric**

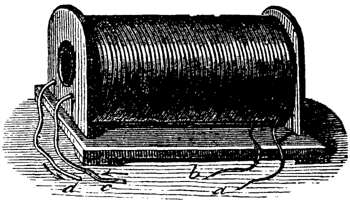
Induction.—The phenomena of electro-magnetism seem to point, as an almost necessary consequence, to the discovery made by Faraday, in 1831–2, of *induced currents*, as well as of *magneto-electricity*. Faraday argued thus:—

1st. *That as a wire carrying a current acts like a magnet, therefore it ought, by induction, to excite a current in another wire near it.*

2d. *That, as magnetism is induced by electric currents, so magnets ought also, under proper conditions, to excite electric currents.*

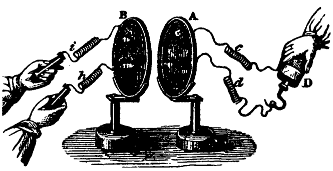
The first of these *theses* Faraday sustained thus: Let a double helix, or bobbin, be wound of two parallel silk-covered wires, about a cylinder of wood (which being withdrawn afterwards, leaves the helix hollow), in close contact, but perfectly insulated, so that the two wires run side by side through their whole course. Let the ends, *a b*, fig. 688, of one wire be connected with a galvanometer, or magnetizing spiral, while a battery current enters the other wire by *c*, and passes out by *d*. When contact is made between *c* and the battery, the galvanometer needle is deflected by a current moving in the *same direction* with the battery or *primary* current. This deflection, however, is only for a brief instant. After a few vibrations, the needle comes to rest, although the battery current still flows. Break now the contact between the wire, *c*, and the battery, and the galvanometer needle is again deflected by a secondary or induced current; but this time it moves in the *opposite direction* to the first. These are called *secondary* or *induced* currents. They are *momentary*, but are renewed with every interruption of the battery circuit, and their strength is always proportional to the strength of the primary or inducing current. If a mass of soft iron (or, better, a bundle of soft iron wires) is placed in the core of the helix, the force of the induced currents is greatly increased. This action of a current from a Voltaic battery, Faraday called *Volta-electric induction.*

688



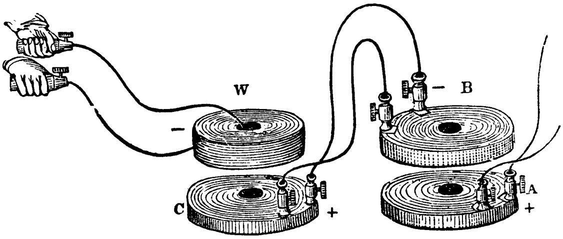
The phenomena of influence (828) in electricity present a strong analogy to these facts, and support the probability that the secondary currents in the case of Voltaic induction, are also due to decomposition of the natural electricity of the second wire, by the current on the first. In fact, a current of statical electricity may be substituted for the Voltaic current with similar results, as was shown by Henry, in 1838 (Trans. Am Phil. Soc., vol. 6, N. S., and Am. Jour. Sci. [1], XXXVIII., 209). Fig. 689 is a convenient form of apparatus designed by Matteucci, for this experiment. Two coils of insulated wire, A B are sustained on movable feet, admitting of near approach. When the charge of a Leyden jar, D, is passed through the coil *c d* on A, a person whose hands grasp the conductors, *i h*, of the coil, B, will receive **a** shock, the violence of which increases with the closer approach of A and B. The direction of the current in B, is the reverse of that in A. If a galvanometer is inserted in the circuit *i h*, its needle is deflected, or, if a magnetizing spiral is used, needles may be magnetized by it.

689



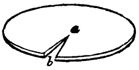
930. **Induced currents of different orders.**—By using a series of flat spirals of copper ribbon alternating with helices of fine insulated copper wire, arranged as in fig. 690, Prof. Henry (in 1838) demonstrated that secondary or induced currents produced other induced currents of the second, third, fourth, and so on, as far as the ninth order. Thus, the flat spiral, A, receiving the battery current, induces, at every rupture of that current, a secondary intense current of opposite name in B, while the second flat spiral, C, receives from B a quantity current, inducing a tertiary intense current in the second fine wire spiral, W, and so on. The signs + and — alternate after the first remove from the battery current, as is easily demonstrated by inserting magnetizing spirals in the conducting wires, and using steel sewing-needles as tests.

690



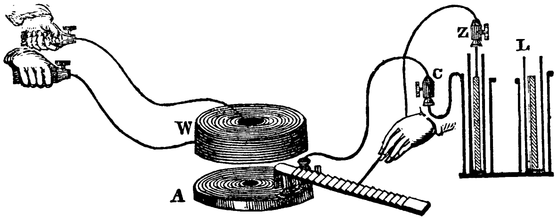
A screen or disc *of metal* introduced between any two of these coils, cuts off the inductive influence. But if the screen has a slit, *a b*, cut from the centre to the circumference, as in fig. 691, the induction is the same as if no screen were present. Discs or screens of wood, glass, paper, or other non-conductors, offer no impediment to this induction.

691



930. **Extra-current, or the induction of a current on itself.**—The effect of a long and stout conductor in giving a vivid spark and shocks from a single cell (which alone, or with a short conductor, gives neither sparks nor shocks), was first noticed in 1832 by Prof. Henry. (Am. Jour. Sci. [1], XXII., 404.) This fact was afterwards the subject of investigation by Faraday, in December, 1834, and also by Henry, in January, 1835. The arrangement used by Prof. Henry is seen in fig. 692. A small battery, L, is connected with the flat spiral of copper ribbon, A, by wires from the battery cups, Z and C; when this communication is broken by drawing the end of one of the battery wires, Z, over the rasp, a brilliant spark is seen at the instant of *breaking* contact. No spark is drawn on *making* contact. Moreover, if a fine wire coil, W, is placed in the relation to A shown in the figure, there is only a feeble spark seen on breaking the battery contact,—while the powerful secondary current already named is set up in W, violently convulsing the hands which grasp its terminals. The strong spark from the large flat coil or single wire in the first case, is then the equivalent of the current which would be produced in the second case, if such current were permitted. This reflux current induced on a conductor, and the outflow or recoil of which produces vivid sparks, is what Faraday calls the *extra current.* In powerful coils, this extra current produces sparks, the report of which resembles the explosion of a pistol, especially under the inductive influence of a powerful electro-magnet, as in the engine of Dr. Page, already noticed. The heavy coils of this apparatus produced sparks from the extra current from two to six inches in length, and having the same rotative action as the conductor itself. (Am. Jour. Sci. [2], XI., 191.) Many forms of electro-magnetic apparatus, in which two coils are combined, show the extra current in a striking manner, as in:—

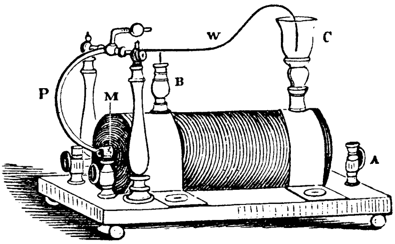
692



931. **Page's vibrating armature and electrotome.**—In this apparatus, fig. 693, the flow of the battery current is interrupted by the movements of the bent wire, P W C. At M is a bundle of soft iron

wires, forming the core of the inducing coil. Becoming magnetic, these attract a small mass of iron on the end of P to M. This movement raises the other end out of the mercury in the cup, C, with a brilliant spark, due to the flow of the extra current, the magnetism having disappeared by the break of the battery flow. Gravity then restores the wire to its original position, thus renewing the battery current and the magnetism, and with it the spark in C. A fine wire induction coil of two thousand or three thousand feet, wound about the inducing coil, develops the secondary currents already noticed, with powerful physiological and other inductive effects, resembling statical electricity.

693

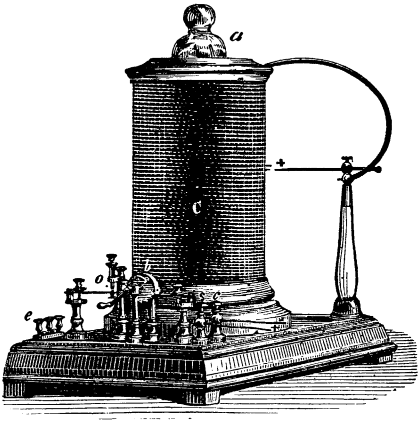


932. **Induced currents from the earth's magnetism.**—The earth's magnetism also induces electrical currents in metallic bodies in movement; another of the discoveries of Faraday. For this purpose, a helix in the form of a ring is made to revolve with its axis at right angles to the magnetic meridian, and, consequently, each point of the ring describes circles parallel to the plane of this meridian. A pole changer on the axis is so arranged as to keep the induced current moving always in the same direction; when so arranged, and its terminal wires are connected with a galvanometer, a deviation of the needle indicates the flow of a current to the east or the west, according to the direction of the rotation.

933. **Conversion of dynamic into static electricity.—The induction coil.**—By careful insulation of the secondary coil of fine wire —as well in itself as from the primary or magnetizing wire—electricity of high tension is produced, surpassing, in energy and abundance, that from machines of the greatest power. *Masson*, in 1842, first succeeded in obtaining these results, but in a very feeble manner compared with those we now know. *Ruhmkorff*, of Paris, in 1851, constructed the coils which bear his name. By careful insulation of the fine wire coil, he succeeded in producing sparks of about two inches in length between the electrodes, charging and discharging a Leyden jar with astonishing rapidity. No electrical instrument has, in modern times, been more celebrated.

Ritchie, of Boston, has so vastly improved this apparatus, as to deserve the highest praise from all interested in physical research. Ritchie's form of the induction coil is shown in fig. 694. The cause of the superiority in the American apparatus is due chiefly to the mode of winding the fine wire coil, by which it is possible to use with success a wire of eighty thousand feet in length, while the limit in the instruments made by Ruhmkorff was about ten thousand feet. The extreme length of spark obtained by the European instruments, was, for the French, about *two* inches (Jean's); and for the English, *four* inches (Hearder's); the American instruments have projected a torrent of sparks over *sixteen* inches in free air; while the one shown in fig. 694, is limited to about nine inches.

694



The chief parts of this apparatus are the two coils, an interruptor to the primary circuit, and the condenser. In the instrument here figured, over sixty-eight thousand feet of silk-covered copper wire, the softest and purest possible, twelve thousandths of an inch in diameter (No. 32 of the wire gauge), is wound upon the exterior bobbin, C. About two hundred feet of wire, one-seventh of an inch in diameter (No. 9), forms the inducing wire, whose ends + and — are visible in the binding screws on the base. A heavy glass bell, *a*, insulates the coils from each other, and its foot is turned outwards by a flange as wide as the thickness of the coil. The induction coil, for more perfect insulation, is also encased in thick gutta percha. The ends of this coil are carried by gutta-percha covered conductors, to two glass insulating stands (only one of which is visible in our figure), where they end in sliding rods pointed with platinum at one end, and having balls of brass at the other. The interruptor devised by Mr. Ritchie, is the toothed wheel, *b*, which raises a spring hammer, the blows of which fall upon the anvil, *o*, breaking contact between two stout pieces of platinum. The European machines are provided with a self-acting break-piece; but experience has shown, by comparative trials, that there is an advantage in varying the rapidity of the interruptions, according to the class of effects to be produced, and that a certain time is requisite for the complete charge and discharge of the soft iron wires (which form the core of the battery circuit), longer than the automatic break-piece allows.

The object of the *condenser* (which is due to Mr. Fizeau) is to destroy, by induction, the greater part of the force of the *extra current*, which, owing to the very powerful magnetism developed in the core of soft iron within the battery coil, would otherwise greatly impair the power of the apparatus, as it moves in a direction opposite to the primary current (931). The condenser consists, in the instrument figured, of one hundred and forty square feet of tin-foil, divided into three sections (two of 50, and one of 40 feet), whose termini are at *e*. The tin-foil of the condenser is carefully insulated by triple folds of oiled silk, and laid away in the base of the instrument, in a cell prepared for it, quite out of view.

The *battery force* needed to excite this apparatus, is only two or three large-sized cells of Bunsen's battery.

934. **Effects of the induction coil.**—The *physiological effects* are so distressing and even dangerous, that too great care cannot be taken to avoid them. M. Quet was confined to his bed for some time, after having accidentally received the shock. Small animals are instantly killed by its discharge.

The luminous effects.—When a series of sparks passes between the points of platinum, or between the balls, they are of a zigzag form, and accompanied by a loud noise and a strong odor of ozone. Their color is violet and yellowish, or greenish yellow. If the points are within an inch or two, the stream of sparks appears to be continuous, a fourth of an inch broad, surrounded by a violet areola, and crossed by numerous lines at right angles to its path. If it is blown by the breath, or by a bellows, it is deflected into a curve, and a bright flame is seen projected for some distance beyond the purple or violet stream of electric light. The color of the flame varies with the nature of the electrodes (885). If one of the electrodes is covered by a small glass flask, the power of the induction is such that a stream of violet electricity is seen, as it were, to pass directly through the glass, while the ball of the flask is covered with a magnificent net-work of violet light, spread out like the blood-vessels upon the eye-ball.

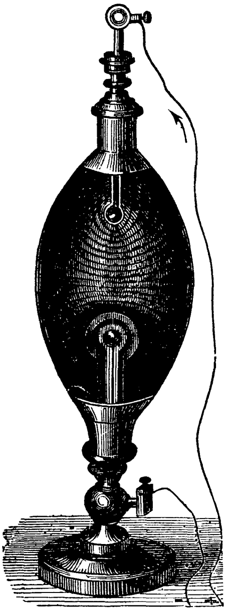
If an Æpinus condenser, or a Leyden jar, is put in the path of the current, the length of the spark is much diminished, but its intensity and splendor are increased twenty-fold. The electric light then becomes intensely white, and the sound of the explosion of the successive sparks, when these are drawn by a slow movement of the break-piece, is like the snap of fulminating mercury, or the sound of a pistol, while the electric stream appears continuous. If a Newton's chromatic disc is caused to revolve before it, each spark causes the colors of the revolving disc to appear stationary, although without this evidence of an intermittent character, the stream of electricity would appear to be unbroken.

Splendid phenomena of *fluorescence* with canary-colored glass—chemical decompositions, deflagrations of the leaf metals, discharges of flashes of lightning over the surface of a metallic mirror, a gilded board, or wet table, and numerous other most beautiful and instructive experiments, are made with this apparatus. Indeed, nearly all the phenomena of static electricity are shown by it, and some of them with a power which no frictional apparatus can approach. It is curious to observe, that the sparks of this kind of electricity pass freely from pointed wires (826). If two fine iron wires are used as the electrodes, the negative wire alone reddens and burns, unless the current is very energetic. All the apparatus used for showing the luminous effects of machine electricity, § 852, may be employed with this apparatus with vastly greater brilliancy.

The chemical effects of the induction coil are shown in the decomposition of water, &c., while a stream of sparks from it, passed through a tube containing air, soon causes the production of reddish vapors due to the formation of hyponitric acid from the union of the elements of the air. Mr. Gassiot has lately shown (Phil. Mag. Aug. 1860) that the intensity of the coil is such as to transmit electrolytic effects across glass, or apparently *through* the walls of a Florence flask.

935. **Light of these currents in a vacuum.**—The difference between the light from the positive and the negative electrodes has already been noticed. Owing to the absence of intense effects of heat in the currents from the induction coil, they are particularly adapted to illustrate this difference, especially *in vacuo.*

695

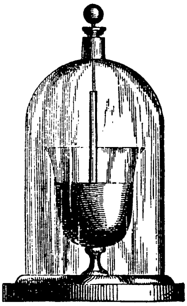


In a vacuum tube, or the electrical egg well exhausted, a torrent of rosy or violet fire falls, from the positive electrode above, toward the negative, which is surrounded with a blue and white light, extending down the stem, with splendid fluorescence (533). If the vacuum is made upon vapor of turpentine, or of phosphorus in the egg, or in an auroral tube, a most wonderful phenomena shows itself; *the stratification of the electrical light* in alternate bands of light and darkness, surrounding and depending from the positive pole, as indicated in fig. 695. This curious phenomenon was first observed by Mr. Grove. Vapor of alcohol, wood-naptha, biclorid of tin, or bisulphid of carbon, may be used, each with a different effect.

Mr. Gassiot has studied with great care the character of the spark in vacuums formed on various gases and vapors, and has established the curious fact, that in a perfect Torricellian vacuum, the spark will not pass, showing that an extremely tenuous vapor is essential to its passage. These facts bear in an important manner on the phenomena of the Aurora Borealis

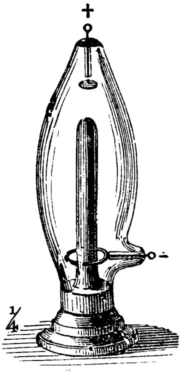
Gassiot's cascade in vacuo.—If we place in a vacuum a goblet coated with tin-foil in the manner of a Leyden jar, and carry the induction current to its bottom by means of a wire passing through the cap of the air-bell, as in fig. 696, the other electrode being in communication with the air-pump plate on which the whole apparatus stands, we are delighted to see, when the current is established from the induction coil, the vase overflow like a fountain with a gentle cascade of light, wavy and gauze-like, falling like an auroral vapor on the metallic base. This experiment requires a very good vacuum, and is certainly one of the most beautiful exhibitions in luminous electricity.

696



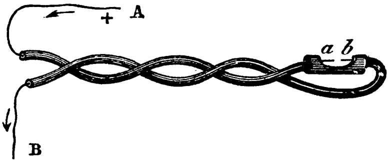
936. **Rotation of the electric light about a magnet.**—We here recall the early observation of Davy, § 883, on the influence of a magnet on the Voltaic arc. If an electro-magnet is enclosed in the electrical egg and a very perfect vacuum is made within it, when the induction current is caused to flow, the electrical stream is seen to revolve in a steady and easy manner about the magnet, the direction of its motion corresponding to the polarity of the magnet. In fig. 697 is shown such an apparatus. The magnet here is a bundle of soft iron wires enclosed in a glass tube, and passing through the foot, so that when the instrument is placed on one pole of an electro-magnet, the mass of wires may be magnetized inductively. Two platinum wires + and — pass in glass tubes hermetically through the walls of the vessel, into the vacuum and form the points of attachment for the electrodes.

697



937. **Applications** have been made of the induction current for firing blasts and sub-aqueous magazines, and also for lighting simultaneously all the gas burners in a large audience room or theatre.

698

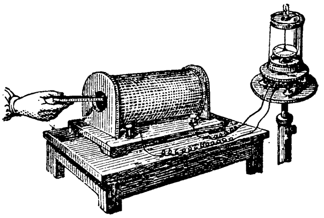


Electrical blasting by Ruhmkorff's coil is easily accomplished by the use of Stateham's fuse, fig. 698, which is only a gutta-percha covered conductor, A B, in which the discharge is interrupted at points, *a b*, buried in the gunpowder, producing its combustion, even at a distance of many miles, and in many distinct mines, or blast holes, su cessively, but almost at the same instant. Dr. Hare first employed his calorimotors for electrical blasting in 1831. (Am. Jour. Sci. [1] XXI. 139.) But the use of an extended battery, and all uncertainty, is avoided by using the induction coil. Ruhmkorff, in experiments made at Villette, inflamed gunpowder with this coil through about sixteen miles. In excavating the Napoleon docks at Cherbourg, lately, over 65,000 cubic yards of rock were thrown out by one series of blasts fired in this way.

II. MAGNETO-ELECTRICITY.

938. **Currents induced by magnets.**—If the helix in fig. 699 is connected with a galvanometer, and a bar magnet is quickly thrust into, and suddenly withdrawn from it, the needle of the galvanometer indicates the movement of a current of electricity opposite in the two cases, and whose direction in each case is opposite to that of a current which, on Ampère's theory, would produce a magnet like the one employed. It is hardly needful to say that reversing the ends of the bar magnet, reverses the movements of the galvanometer. This is a case of *magnetic electric induction.*

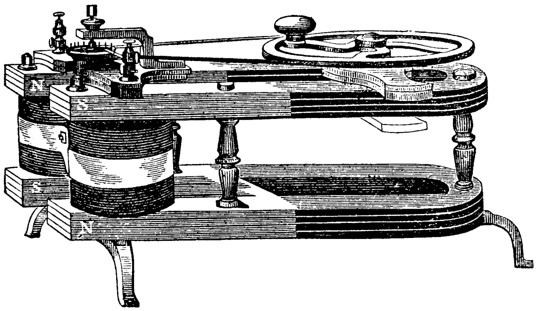
699



This fact is also illustrated in other modes, viz.:—

a. By revolving a circular plate of copper between the poles of a horse-shoe magnet (arranged in general like fig. 678), the axis of the copper being in connection with one pole, and the edge with the other, a series of sparks may be obtained, as in Faraday's original experiment, some device being inserted to interrupt the current during the revolution.

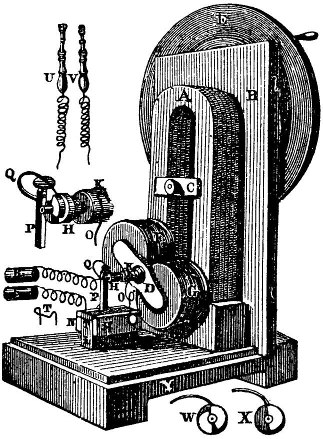
700



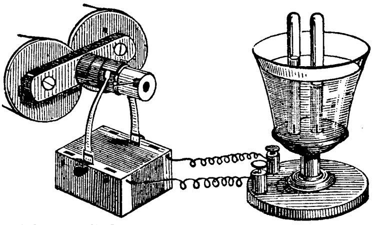
b. By a helix on the armature of a magnet, the ends of the helix being connected with the poles respectively, on suddenly sliding the armature from the poles of the magnet, a spark is seen, and if the fingers grasp the wires at the same time, a shock follows. This fact was first announced in December, 1831, by Srs. Nobili and Antinori. Saxton constructed the first magneto-electric machine in which the armature, wound with a helix, was made to revolve in front of the poles of a magnet, and so to reproduce all the phenomena of static and voltaic electricity from permanent magnets. Fig. 700 shows an improved form of Saxton's apparatus, where the double inducing coil revolves by means of a motor wheel and band between the poles of two powerful magnetic batteries. The magnetic-electric-induction is interrupted by the little crown-wheel seen on the upper end of the axis of the revolving coils.

Clarke's magneto-electric apparatus.—This apparatus is a modification of Saxton's, and consists of a powerful magnetic battery, A, fig. 701, clamped on the upright board, R, by the clamp C. The wheel, F, puts in motion two helices, H G, wound upon a rotating armature of soft iron. The electrical current induced in the coils is interrupted by the spring, or hook, Q, which rubs on the interrupted back piece, H, while the circuit is completed by the hook, O, passing upon the continuous part of the spindle, R. A stout wire, T, movable at pleasure, connects the two sides, M and N, otherwise insulated by the piece of dry wood, L. When the coils are rapidly rotated before the poles of the magnet, the current is interrupted twice in every revolution by the hook, Q, with the production of a brilliant spark. If the coils are composed of a long and fine wire, then powerful shocks will be experienced by one holding the handles R and S, but capable of a great graduation, by changing the position of the break-piece, H, with reference to the point of the revolution when it leaves Q. These shocks may be made quite intolerable.

701

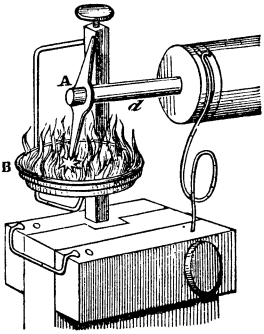


702



If the conducting wires of the intensity coil terminate in a decomposing apparatus, fig. 702, trains of minute gas bubbles are seen to rise from the platinum points under the tubes, showing the production of dynamic from magnetic electricity. Other effects of the intense class, as the decomposition of iodid of potassium, may also be produced with it. Substituting a coil of large wire not over two hundred feet long, for the small long wire, the *quantity armature* is produced, from which brilliant sparks, the deflagration of mercury, and setting fire to ether, as in fig. 703, may be produced; mercury, in a copper spoon, B, is touched by the revolving points, A, on the end of the axis, *d*, and with every disruption of the circuit, the extra current discharges with splendid effect. A platinum wire may also be ignited, and electro-magnets charged by the same armatures. Thus we see all the effects of electricity, physical and physiological, coming from a magnet.

703



939. **Identity of electricity from whatever source.**—It follows from all that has been said, that the phenomena of magnetic, static, and dynamic electricity, are all capable of being produced each by the other; and the conclusion seems warranted that electricity, from whatever source, is one and the same power.

Numerous and instructive forms of apparatus have been devised to demonstrate this point, as well also as to illustrate in detail, the principles we have, for want of space, been compelled to state in terms too concise. The student and teacher will find it useful to consult the figures of Davis's Manual of Magnetism for various forms of apparatus, due to the ingenuity of Faraday, Dr. Page, and many others. For works of standard authority, he is referred to Faraday's experimental researches, and De La Rive's treatise on electricity, each in three volumes.

§ 6. Other Sources of Electrical Excitement.

940. **Universality of electrical excitement.**—Every change in the physical or chemical condition of matter, seems to be attended with electrical excitement. This is evident from the phenomena attending the cleavage, or pulverizing, of many minerals and crystallized substances, as sugar, mica, zinc-blende, and numerous other substances which evolve light when suddenly cleaved. If precautions are taken to insulate these, as with mica it is easy to do, by sealing wax, they also show the effects of electrical excitement by the condenser. The production of crystals is often also accompanied by electrical light.

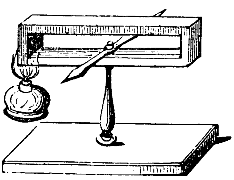
Combustion, evaporation, the escape of gas attending chemical transformations, chemical decompositions and combinations, have all been known to evolve electricity when properly observed; but in most such cases, the phenomena are too complicated to render it clear to which, if indeed to any single action of those enumerated, the excitement is due.

The electrical currents set up by heat (thermo-electricity), and those arising from the phenomena of life (animal electricity), are the most important of all sources of electricity not before dwelt on, and to them we will now briefly advert.

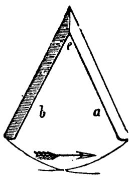
I. THERMO-ELECTRICITY.

941. **Thermo-electricity.**—The discovery of this source of electrical currents is due to Seebeck, of Berlin, in 1821. He found that if two metals of unlike crystalline texture and conducting power are united by solder, and the point of junction is either heated or cooled, an electrical current is excited, which in general flows from the point of junction to that metal which is the poorer conductor. Fig. 705 shows such an arrangement of two little bars of bismuth and antimony. When the junction, *e*, is heated, a current of positive electricity flows from the bismuth, *b*, to the antimony, *a*. If the form of a rectangle is given to this arrangement, as in fig. 704, an instrument resembling Schweigger's multiplier is formed (905), by which the magnetic needle is deflected. A twisted wire also produces a thermo-electric current when the twisted portion is gently heated, and, besides metals, other solids, and even fluids, give rise to this species of electricity. The order in which the metals stand in reference to this power is wholly unlike the Voltaic series, and appears related to no other known property of these elements. The rank of the principal metals in the thermo-electric series is as follows, as determined by Becquerel:—The numbers prefixed give the order of each metal in the table of specific heats as determined by Regnault. Those having the highest specific heat, as a general rule, being first in positive power (+) in the thermo-electric magnet: 6 antimony; 1 iron; 2 zinc; 4 silver; 7 gold; 3 copper; 5 tin; 9 lead; 8 platinum; 10 silver.

704



705

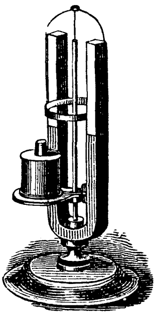


When the junction of any pair of these is heated, the current passes from that which is highest, to that which is lowest in the list, the extremes affording the most powerful combination.

Becquerel has found the intensity of the current in a thermo-electric combination to be proportional to the difference of temperature in the solderings up to 100° or 120° F., one of the points being at 32°. Above this limit, the increase of intensity is less and less, with an increase of heat. In a couple of copper and iron, the increase of current was insensible near 570° F.

In a compound thermo-electric series, intense effects, analogous to those of the Voltaic pile, are obtained only when half the solderings are heated, the alternates being cooled.

706

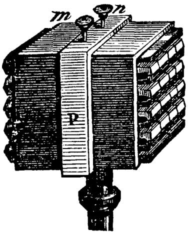


Thermo-electric motions.—If a compound ring of brass and German-silver is suspended within the poles of a magnet, as in fig. 706, when the soldering of the ring is heated, a revolution is set up, through the influence of the magnet on the electric current, quite analogous to similar electro-magnetic motions.

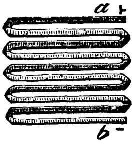
Cold produced by electrical currents.—If we pass a feeble current of electricity through a pair of antimony and bismuth, the temperature of the system rises, if the current passes from the former to the latter; but if from the bismuth to the antimony, cold is produced in the compound bar. If the reduction of temperature is slightly aided artificially, water contained in a cavity in one of the bars may be frozen. Thus we see that, as a change of temperature disturbs the electrical equilibrium, so, conversely, the disturbance of the latter produces the former.

942. **Melloni's thermo-multiplier.**—We have already alluded to this delicate metallic thermometer, § 588, and have shown its application in the phenomena of diathermancy (642). This instrument consists of a series of small bars of antimony and bismuth, *a* and *b*, fig. 707, soldered together at their alternate ends. Two wires connect the opposite members, *n* and *m*, fig. 708, of this battery, with a galvanometer. The needle of the galvanometer is suspended over a graduated circle, and moves in exact accordance with a thermo-electric current produced by the battery. The least difference in temperature between the opposite faces of this battery, produces a thermo-electric current, deflecting the needle of the galvanometer, fig. 658, as already explained in § 642.

708



707



II. ANIMAL ELECTRICITY.

943. **The galvanic current.**—We have already spoken of the dis-

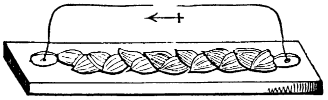
709

covery by Galvani of electrical currents in animals, living, or recently dead, flowing from the outer or cutaneous, to the inner or mucous surface. Thus, when contact is made between the muscles of the thigh and the lumbar nerves, by bending the legs of a vigorous frog, fig. 709, contractions immediately follow. Aldini, who was a zealous advocate of Galvani's views, during the controversy between the followers of Galvani and Volta, demonstrated the existence of such a current in other animals by the legs of a frog used as a galvanoscope. For this purpose, he brought the lumbar nerve of a frog, held as in fig. 710, in contact with the tongue of an ox lately killed, while the hand of the operator, wet with salt water, grasped an ear of the animal to complete the circuit. The legs were then convulsed as often as the nerves touch the mucous surface of the tongue. The same delicate electroscope also shows similar excitement when its pendulous ischiatic nerves touch the human tongue,—the toe of the frog being held between the moistened thumb and finger of the experimenter.

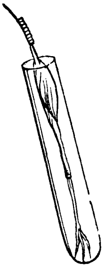
710

Matteucci, of Pisa, in 1837 (forty years after Galvani's result was obtained), has the merit of reviving Galvani's original and correct opinion as to the vital source of this electricity. He demonstrated, that a current of positive electricity is always circulating from the interior to the exterior of a muscle, and that, although the quantity is exceedingly small, yet, by arranging a series of muscles, having their exterior and interior surfaces alternately connected, he produced sufficient electricity to cause decided effects. By a series of half thighs of frogs, arranged as in fig. 711, he decomposed the iodid of potassium, deflected a galvanometer needle to 90°, and, by a condenser, caused the gold leaves of an electroscope to diverge. The irritable muscles of the frog's legs form an electroscope fifty-six thousand times more delicate than the most delicate gold-leaf electrometer. When the pendulous nerve of a single leg, arranged in a glass tube, as in fig. 712, is touched in the places where electrical excitement is suspected, the

711



712



muscles in the tube are instantly convulsed. Du Bois Raymond, of Vienna, has demonstrated the existence of these currents, in his own person, by the use of the galvanometer, for which purpose the muscles of the hands and arms are alternately contracted on a metallic bar in connection with a galvanometer.

944. **Electrical animals.**—In some marine and fresh-water animals, a special apparatus exists, adapted to produce, at pleasure, powerful currents of statical electricity, either as a means of defence, or of capturing their prey. Of these, the electrical eel, of Surinam, first described by Humboldt, and the *cramp-fish*, or torpedo, a flat fish found on our own coast, are the most remarkable. They have an alternate arrangement of cellular tissue and nervous matter in thin plates of a polygonal form, constituting a perpetually charged electrical battery, arranged in the manner of a pile. By touching their opposite surfaces, a very violent shock is received, such as to disable a very powerful man, or even a horse. Prof. Matteucci has shown us how to charge a Leyden jar, by placing the torpedo between two plates, arranged like the plates of a condenser; and Faraday has published an interesting account of his experiments with the eels of Surinam, from which he not only obtained shocks, but made magnets, deflected the galvanometer, produced chemical decompositions, evolved heat and electrical sparks. (Expt. Res. 1749–1795.) The student is also referred to Prof. Matteucci's interesting "Lectures on Living Beings," for further details on this very interesting subject, and to a memoir, on the American Torpedo (Dr. D. H. Storer, Am. Jour. Sci. [1], XLV., 164).

945. **Electricity of plants.**—Pouillet, in his researches on the origin of atmospheric electricity, made the interesting discovery of the disengagement of negative electricity during the germination of seeds, and the growth of plants. This observer estimates that a surface of 100 square yards covered with vegetation disengages, in a day, more electricity than is required to charge the most powerful Leyden battery.

Currents of electricity have also been detected in fruits, and in the bark, roots, and leaves of growing plants; the roots, and all internal parts of plants filled with juice, being, according to Buff, negative with relation to the exterior or less humid parts.

[*For Problems on Electricity, see end of Meteorology.*]

APPENDIX.

CHAPTER I.

METEOROLOGY.

946. **Meteorology** is that branch of natural philosophy which treats of the atmosphere and its phenomena. The subject may properly be divided into—1st, Climatology; 2d, Aerial phenomena, comprehending winds, hurricanes, and water-spouts; 3d, Aqueous phenomena, including fogs, clouds, rains, dew, snow, and hail; 4th, Luminous and electrical phenomena, as lightning, rainbows, and aurora borealis.

Our narrow limits of space restrict our remarks on this interesting subject to a very inadequate rehearsal of its principles. The student will refer to the works of Espy and Blodget, and the papers of Coffin, Henry, Loomis, Redfield and others, in the transactions of the American Philosophical Society, publications of the Smithsonian Institution, the United States Patent Office, and the American Journal of Science, for an exhibition of the American results in this science.

§ 1. Climatology.

947. **Climates, seasons.**—By *climate* is meant the condition of a place in relation to the various phenomena of the atmosphere, as temperature, moisture, &c. Thus, we speak of a warm climate, a dry climate, &c.

A *season* is one of the four divisions of the year, spring, summer, autumn, and winter. Astronomical seasons are regulated according to the march of the sun. In meteorology it is sought to divide them according to the march of temperature. Winter being the most rigorous of seasons, it is so arranged that its coldest days (about January 15th) fall in the middle of the season. Hence, winter consists of December, January, and February; spring of March, April, and May, &c. Few meteorologists have regard to the astronomical divisions, which make winter begin December 21st.

948. **Influence of the sun.**—The sun is the principal cause that regulates variations in temperature. In proportion as this luminary rises above the horizon, the heat increases; it diminishes as soon as it sets. The temperature, also, depends on the time it remains above the horizon. The sun, in winter, sends its rays obliquely upon the earth, and at this season, therefore, less heat is received than in summer, when its rays are more nearly perpendicular. Mathematicians have

in vain endeavored to deduce the temperature of days and seasons from the height of the sun above the horizon. This failure is owing to many accidental and local causes, which modify the result,—as elevation above the ocean, inclination of the surface, vicinity of seas, lakes, or mountains, prevalent winds, &c.

949. **Meteorological observations**, to be compared with each other, especially when made in different locations, should be made at certain fixed hours of the day. The hours regarded as most suitable for this purpose, are 6 A. M., 2 P. M., and 10 P. M. To these hours are sometimes added 9 A. M., and 6 P. M.

950. **Mean temperature.**—The mean or average daily temperature is commonly obtained by observing the standard thermometer at stated times during the day, and then dividing the sum of these temperatures, respectively, by the number of observations. It has been ascertained, that the mean temperature deduced from observations taken at 6 A. M., 2 P. M., and 10 P. M., corresponds almost exactly with the mean obtained from observations taken every hour in the 24; hence the three hours named are considered the proper hours for taking observations. The lowest temperature of the day occurs shortly before sunrise; the highest a few hours after noon. The mean daily temperature, at Philadelphia, is found to be one degree above the temperature at 9 A. M.

By taking the average of all the mean daily temperatures throughout the year, the mean annual temperature is obtained.

951. *Monthly variations in temperature.*—In the successive months of the year, there is a regular variation in temperature. From the middle of January, the temperature rises, at first slowly, in April and May rapidly, then less rapidly to the end of July, when it attains its maximum. It falls first slowly in August, rapidly in September and October, and reaches the minimum about the middle of January.

In the United States, the monthly variations of temperature are nearly as follows. (The figures attached to the signs + and — by the side of each month, signify the number of degrees colder or warmer it is than the one immediately preceding.)

January, the coldest; February, +2° to 4°; March, 8° to 10°; April, 10°; May, 9° to 12°; June, 7° to 9°; July, 4° to 6°; August, —1° to 3°; September, —5° to 8°; October, —8° to 10°; November, —10° to 14°; December, —10° to 15°. The coldest days are generally about the 15th of January; the warmest, near the 25th of July. The means of the months of April and October, are very near the annual mean.

952. *The range of temperature during the year*, is due to variations in the length of days and nights, and the height of the sun above the horizon at noon.

In January, when the days begin to increase in length, the sun acts with more force, because its angular height is greater, and because it remains longer above the horizon. This change is slow at first, and it is only towards the vernal equinox that the increase in temperature is considerable. The days being then longer than the nights, the earth receives more heat than it loses by radiation. The temperature increases more slowly as the summer solstice is approached, because the changes in the height of the sun and length of the day, are small. After the solstice, the temperature continues to increase, until about July 25th; the heat received through the day being still greater than the quantity lost during the night. As the days decrease in length, and the sun approaches the equator, the temperature falls, and attains its minimum near the middle of January. For the extremes of natural temperature, compare section 744.

953. **Variations of temperature in latitude.**—The mean temperature of different places on the same latitude, varies according to the height of the sun at mid-day above the horizon at these points. The highest temperature is, therefore, found at the equator; it diminishes either way to the poles.

The mean summer temperature of regions midway between the poles and the equator, may be as high as at the equator, because the sun is above the horizon a greater number of hours. At the poles, however, where the sun is above the horizon during six months of the year, the rays are directed so obliquely that their calorific action is very feeble.

The temperature is not the same for places in the same latitude in the two hemispheres, as is seen in the following table:—

Places.	Latitude.	Temp.	Places.	Latitude.	Temp.
Falkland Isles, .	51° S.	47°·23	London, . . .	51° 31′ N.	50°·72
Buenos Ayres, .	34° 36′ S.	62°·6	Savannah, . . .	32° 05′ N.	64°·58
Rio Janeiro, . .	22° 56′ S.	73°·96	Calcutta, . . .	22° 35′ N.	78°·44

This variation is owing to a variety of local causes, such as the elevation and form of the land, proximity to large bodies of water, the general direction of winds, &c.

954. **Variations of temperature in altitude.**—The average diminution in temperature in ascending from the sea level is 1° F. for every 300 feet. Supposing the average temperature of the air at the level of the sea, near the equator, to be 80°, and toward the poles 0°, the figures in the second and third column of the following table will express approximately the temperature at different elevations. (From Daniell.)

DECREASE OF TEMPERATURE IN THE ATMOSPHERE FROM ELEVATION.

Altitude in feet.	Equatorial temperature.	Arctic temperature.
0·	80°	0°
5,000·	64°·4	— 18°·5
10,000·	48°·4	— 37°·8
15,000·	31°·4	— 58°·8
20,000·	12°·8	— 82°·1
25,000·	— 7°·6	—109°·1
30,000·	—30°·7	—140°·3

955. **Limit of perpetual snow.**—It follows from what has just been stated, that in every latitude, at a certain elevation, there must be a point where moisture once frozen must ever remain congealed. The lowest point at which this is attained is called the limit of perpetual snow, or the snow-line. This point is generally highest near the equator, and sinks towards either pole. There are, however, numerous exceptions to this rule.

LIMIT OF PERPETUAL SNOW AT DIFFERENT PLACES.

Places.	Latitude.	Snow-lines.
Straits of Magellan, ...	54° S.	3760 feet
Chili, ...	41° S.	6009 "
Quito, ...	00°	15,807 "
Himalaya, North side, ...	30° 15′ N.	16,719 "
Alps, ...	42° N.	8220 "
W. Cordilleras, ...	14° 30′ S.	18,631 "
Mexico, ...	19° N.	14,763 "
Ætna, ...	37° 30′ N.	9531 "
Kamtschatka, ...	56° 40′ N.	5248 "

It has been observed that the different heights of perpetual frost decrease very slowly as we recede from the equator until we reach the limit of the torrid zone, when they decrease more rapidly. The average difference for every 5° of latitude in the temperate zone is 1318 feet, while from the equator to 30° the average is only 664 feet, and from 60° to 80° of latitude it is only 891 feet. The limit of perpetual snow presents remarkable and inexplicable phenomena at different points: thus it is much higher in the Himalayas, latitude 14° 15′ N., than at the equator. Humboldt remarks that the limit is not due alone to geographical latitude, that it is owing to a combination of many causes, such as differences in the temperature of each season, the direction of the winds, the habitual dryness or humidity of the atmosphere, the form of the mountain, its vicinity to other peaks, &c.

956. **Isothermal lines.**—If all the points whose mean temperature is the same are connected by lines, a series of curves are obtained, which Humboldt was the first to trace on charts, and which he has named *isothermes*, or *isothermal lines* (from ἴσος, equal, and θέρμη, heat). The latitude and longitude are the principal conditions which determine the temperature of any point upon the earth's surface, but the influence of these conditions is greatly modified by numerous accidental and local influences: hence, the isothermal lines present numerous sinuosities instead of passing around the earth parallel to any degree of latitude. The introduction of isothermes formed an important epoch in meteorological science, for by it have been established the great laws of the distribution of heat over the surface of the earth for the four

seasons. The chart of isoclinal lines, fig. 543, serves also to illustrate the *general* direction, and place of isothermal lines.

§ 2. Aerial Phenomena.

957. **General consideration of winds.**—Wind is air in motion. Winds are generally caused by variations in the temperature of the earth, produced in part by the alternation of day and night, and the change of the seasons. The air in contact with the hotter portion of the earth becomes heated, and being lighter than before, rises, while the surrounding air rushes in below to supply its place. The revolution of the earth on its axis, also comes in as an important modifying cause of the thermal conditions. Winds are also, sometimes, caused by the sudden displacement of large volumes of air, as in the fall of an avalanche. Winds are named from the points of the horizon from which they blow.

958. **Propagation of winds.**—Whether a wind is first felt in the country from which it comes, or in that to which it is directed, is still an unsettled question. It would seem, however, that often at least it is first felt in the region to which it is directed.

It has already been said that winds are caused by inequalities in the temperature of the air. If the air above a certain region, as in the tropics, becomes heated, it rises, and the air in the vicinity rushes into the space abandoned by the ascending column. This air becomes rarefied, and this rarefaction is communicated from point to point, just as the waves of sound expand. The wind is thus propagated in a direction opposite to that in which it blows. Such winds are called winds of *aspiration*. Winds which are propagated in the same direction from which they blow are called winds of *insufflation*. Franklin made some interesting observations on winds of aspiration. He noticed that a violent north-east wind, which arose about 7 o'clock, P. M., in Philadelphia, was not felt at Boston until 11 o'clock in the evening. Again, a violent south-west wind blew first at Albany, and afterwards at New York. But the existence of winds of insufflation is not less well proven. The terrible hurricane from the south-west, on the 29th of November, 1836, arrived at London at 10 o'clock in the morning, at the Hague at 1 o'clock in the afternoon, at Amsterdam at 1½, at Hamburg at 6, at Lubeck at 7, and at Stettin at half-past 9 o'clock in the evening.

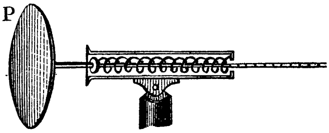
959. **Anemoscopes.**—The direction of currents of air blowing at great heights may often be determined by the direction in which the clouds move. The direction in which surface winds move is determined by means of anemoscopes (from ἄνεμος, wind, and σκοπός, one who watches).

The ordinary vane is the simplest anemoscope. From its position on the top of an edifice, observations of it are extremely inconvenient, and as ordinarily constructed, it is not sufficiently delicate to move with slight agitations of the air. Anemoscopes, suitable for accurate observation, are variously constructed. They may consist of two or more fan-wheels, whose axis supported horizontally is in connection with a vertical bar. The lower end of this bar is delicately supported, and has a needle attached to it at right angles, moving over a compass dial-plate. The needle points upon the dial the direction of the wind.

960. **Anemometers** (from ἄνεμος, wind, and μέτρον, measure) are instruments designed to measure the velocity of winds.

The velocity of winds is indicated by the force with which they move, *i. e.*, the pressure they exert. Some anemometers are constructed so as to exhibit the amount of pressure excited by a wind upon a plate placed perpendicular to its own direction. This plate may be supported on a spiral spring, the extent of its compression indicating the force of the wind. Fig. 713 represents a simple form of this class of instruments. Other anemometers indicate the velocity of the wind by the number of revolutions given to a fan-wheel in a given time. Such an one is Woltmann's anemometer. It consists essentially of a small wind-mill, to which is attached an index marking the number of revolutions per minute. The stronger the wind, the greater the number of revolutions made. The necessary data for ascertaining correctly with this instrument the velocities of winds are easily obtained as follows:—Nothing more is necessary, than on a calm day, to travel with the apparatus on a carriage or rail car, observing the number of revolutions made in going any known distance in a given time. The effect will be the same as if the air was in motion. A table is then constructed, indicating the velocity of a wind which turns the sails forty, fifty, sixty, or more times per minute.

713



Anemometers of very various forms have been designed. No one yet constructed indicates the velocity of winds with absolute precision.

961. **The velocity of winds** varies from that which scarcely moves a leaf to that which overthrows the staunchest oak.

VELOCITY AND POWER OF WINDS (Smeaton).

Velocity of the wind. Miles per hour.	Perpendicular force on one sq. ft. in lbs. avoirdupois.	Common appellation of such winds.
1	·005	Hardly perceptible.
4	·079	Gentle wind.
5	·123	
10	·492	Pleasant brisk gale.
15	1·107	
20	1·968	Very brisk.
25	3·075	
30	4·429	High wind.
35	6·027	
40	7·873	Very high.
50	12·300	Storm.
60	17·715	Great storm.
80	31·490	Hurricane.
100	49·200	Violent hurricane.

Formulæ.—The following formulæ for determining the velocity of winds from the observed pressures, have been deduced from Smeaton's table given above:—

If V represents the velocity per hour in miles, and P represents the pressure on a square foot of surface at right angles to the direction of the wind:—

$$V = \sqrt{\frac{P}{0{\cdot}00492}}, \text{ or } V = 14{\cdot}257\sqrt{P}.$$

The pressure in pounds avoirdupois, on a square foot of surface, when the wind moves one mile an hour, being $= 0{\cdot}00492$ lbs.

To obtain the velocity per second in feet, we multiply by 5280 (the number of feet in a mile), and divide by 3600, the number of seconds in an hour, and we have

$$v = V \times \frac{5280}{3600} = \frac{5280}{3600} \times 14{\cdot}257\sqrt{P} = 20{\cdot}91\sqrt{P}.$$

Winds are divided into three classes, viz., regular, periodical and variable.

962. **Regular winds** are those which blow continuously in a nearly constant direction, as the trade winds.

Trade winds occur in the equatorial regions, on both sides of the equator to about 30° of latitude. Those in the northern hemisphere blow from the north-east to the south-west; those in the southern hemisphere from the south-east to the north-west.

These winds are produced by the unequal distribution of heat upon the surface of the earth, and by the rotation of the earth on its axis. From the vertical position of the sun, the equatorial regions are intensely heated, the temperature gradually diminishing towards the poles. The heated air, above the equator, rises and blows off in the upper regions of the atmosphere towards either pole. At the same time, currents are established on the surface of the earth to supply to the equatorial regions the air which the upper currents have carried off. If the earth were at rest, these winds would blow due north and south. But the earth is revolving on its axis, from west to east, at the equator; therefore, the eastern velocity is greatest, but it gradually diminishes towards the poles. In consequence of this, the wind blowing from the north pole, towards the equator, acquires a westerly direction, and seems to come from the north-east, and for the same reason, the wind blowing from the south pole towards the equator, also acquires a westerly direction, and seems to come from the south-east.

These trade winds are not stationary, moving to the north in the summer of the northern hemisphere, and south as the sun withdraws to the southern tropic.

The general direction of these trade winds is no more altered by the form of continents, their elevation and headlands, than is the general course of the waters in a river by rocks in its bed, though abrading surfaces and irregularities may produce a thousand eddies in the main stream.

963. **Periodical winds** are those which blow regularly in the same direction, at the same seasons of the year, or hours of the day. The most interesting winds of this class are the monsoons, and the land and the sea breezes.

The monsoons occur within, or near the tropics; they blow from a certain quarter about one-half of the year, and from an opposite point during the other half. The cause of the monsoons is found in the effect produced by the sun in his annual progress from one tropic to another, successively heating the land on either side of the equator. The *simoon* is a periodical wind which blows over the deserts of Asia and Africa; it is noted for its high temperature, and the sand which it raises in the atmosphere, and carries along with it. This wind from the Great Sahara desert blows over Algeria and Italy, and reaches even the north shores of the Mediterranean, where it receives the name of *sirocco*.

On the coasts and islands within the tropics, and to some extent in temperate regions, a *sea breeze* daily occurs flowing from the sea to the land during the day; as it gradually subsides, it is succeeded by a *land breeze*, flowing from the land to the sea. In some places these breezes are scarcely perceptible beyond the shore; in others, they extend inland for miles.

The causes of the land and sea breezes are very apparent. During the day, while the sun shines, the land acquires a higher temperature than the water of the surrounding ocean. The air, above the land, becomes heated, and rises. To supply the place of that which has risen, air flows in from the sea, constituting the sea breeze. But when the sun descends, the land rapidly loses its heat, by radiation, while the temperature of the ocean is scarcely changed. In consequence of this, the air above the land becomes cooled, and therefore more dense, and flows towards the water, constituting the land breeze. At the same time, in the higher regions of the atmosphere, air flows in from the sea to the land.

964. **Variable winds** are those which blow sometimes in one direction, sometimes in another. The direction of winds is influenced by numerous causes, as the nature and form of the surface of the earth, the proximity of large bodies of water, &c. In these latitudes, the direction of the prevailing winds is from the north-west to the south-east.

965. **General direction of winds in the higher latitudes.**—In the table given below, the relative frequency of different winds is given. The total number of winds in each country is represented by 1000; the figures in the table represent their relative frequency.

FREQUENCY OF DIFFERENT WINDS.

Countries.	N.	N. E.	E.	S. E.	S.	S. W.	W.	N. W.
England, . . .	82	111	99	81	111	225	171	120
France, . . .	126	140	84	76	117	192	155	110
Germany, . . .	84	98	119	87	97	185	198	132
Denmark, . . .	65	98	100	129	92	198	161	156
Sweden, ...	102	104	80	110	128	210	159	106
Russia, ...	99	191	81	130	98	143	166	192
North America, .	96	116	49	108	123	197	101	210

In these countries there is a predominance of south-west winds, with the exception of Russia, where the greater proportion are from the north-west. In all the northern hemisphere there is a predominance of westerly winds. This is shown by the fact that the average length of the voyage from New York to Liverpool by packet is but 23 days, while that of the return voyage is 40. In the high southern latitudes the same thing is observed. Lieut. Maury remarks that at Cape Horn there are three times as many westerly as easterly winds.

966. **Physical properties of winds.**—Winds are hot, cold, dry, or moist, according to the direction from which they come, and the kind of surface over which they pass.

If they come over the sea, from lower latitudes, they are warm and moist; if across the land, and from the north, they are cold and dry. Our north-east winds are cold and moist, because they come from the north, over the Atlantic Ocean. South winds are here warm and humid; north winds cold and dry.

967. **Hot winds.**—Over the deserts of Asia and Africa, an intensely hot wind occasionally prevails. In Arabia, it is called *simoon*, signifying poisonous: in Egypt, *khamsin*, because it blows forty days. In the western part of the Great Desert, and in Guinea, it bears the name of *harmattan.*

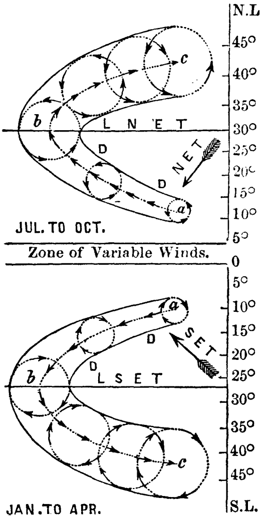
The soil of these countries is uniformly covered with a fine reddish sand, which becomes prodigiously heated by the sun's rays. As the wind passes over this surface, it becomes intensely heated; the fine particles of sand are raised in the air, giving a reddish or purplish tinge to the atmosphere; the sky becomes obscured, the sun loses its brilliancy, as the winds blow from the desert. The barometer falls very low, plants dry up, and evaporation takes place with great rapidity from the surface of the skin, giving rise to the greatest suffering. Whole caravans have been known to perish, the prey of a consuming thirst.

968. **Hurricanes, or cyclones**, are terrific storms, often attended by thunder and lightning; they are distinguished from every other tempest by their extent, their power, and the sudden changes in their direction. From numerous observations, "it appears that hurricanes are storms of wind, which revolve around an axis, upright or inclined to the horizon, while, at the same time, the body of the storm has a progressive motion over the surface of the earth." This law has been established by Redfield and Reid. Their progressive velocity varies from ten to thirty or forty miles per hour; the rotatory velocity is sometimes as much as a hundred miles per hour. The diameter of a hurricane is from a hundred to five hundred miles, though sometimes, as in the Cuban hurricanes, it is much more.

Fig. 714 shows the origin, rotation, and general course of hurricanes in both the northern and southern hemispheres. These terrible storms have never been known to sweep across the equator, although, in one case, similar hurricanes were raging at the same time, on both sides of the equator.

A northern hurricane commences with a violent rotary motion, as shown by the arrows at *a*, in latitude 10° or 15° north of the equator, corresponding very nearly with the sun's northern declination, and extending over an area of from 50 to 200 or 300 miles, the rotary motion of the storm tending inwards towards its centre. At the same time the general progress of the storm is obliquely west and northward until it reaches the limit of the north-east trade winds, where it sweeps around, taking a north-easterly direction, the vortex enlarging as it progresses, spreading over an area of 500 or 1000 miles, until at last it moves in a nearly easterly direction, and exhausts its force by its excessive enlargement, in latitude 40° or 50° N.

714



Southern hurricanes pursue a similar course in the southern hemisphere, as shown in the lower part of the figure. The circular arrows show the rotation of the air in the area of the cyclone, and the arrows in the parabolic curve, *a b c*, show the general course of the moving storm.

The arrow, N E T, shows the region of the north-east trade winds, and the parallel of latitude, L N E T, shows their northern limit. The arrow, S E T, and line, L S E T, also show the region and limit of the south-east trades.

Between the northern and southern hurricane regions, is the zone of variable winds, which the hurricanes are not known to pass. On either side of the zone of variable winds lie the zones of expansion, in which hurricanes originate. The letters, D D, indicate the most dangerous portion of the hurricane track.

It is now well ascertained that, in both hemispheres, the air in a cyclone or hurricane rotates in a direction contrary to that of the course of the sun. Thus, in the northern hemisphere, the course of the sun is from the east around to the south, then to the west and north, and the movement of the air in the hurricane is in the opposite direction; *i. e.*, from the north, by the west, south, and east, or in a direction contrary to the motion of the hands of a watch, lying with the face upward. In the southern hemisphere, the course of the sun is from the east, by north, west, and south, and that of the cyclone is from the north, by east, south, and west, or in the direction of the hands of a watch. East winds are, in both hemispheres, characteristic of a commencing hurricane (*i. e.*, an east wind is always found on the front of the advancing storm, as shown in the figure); while, in general, west winds occur only in the latter part of the storm, as decreasing winds; hence, in the northern hemisphere, the most dangerous part of a hurricane is in the advancing border of the right hand semicircle, or about the line D D; while, in the southern hemisphere, the dangerous limit, D D, is the advancing quadrant of the left hand semicircle.

The effect of the rotary movement of the hurricane is to accumulate the air around its outer margin, with a pressure increasing as it recedes from the centre; consequently, the barometer is lowest at the middle of a storm, and highest at its extremity. The barometer oscillates before and during a hurricane, rising and falling rapidly, owing to the inequality of the pressure which causes the storm; so that:—*Great barometric oscillations generally announce the approach of a tempest.*

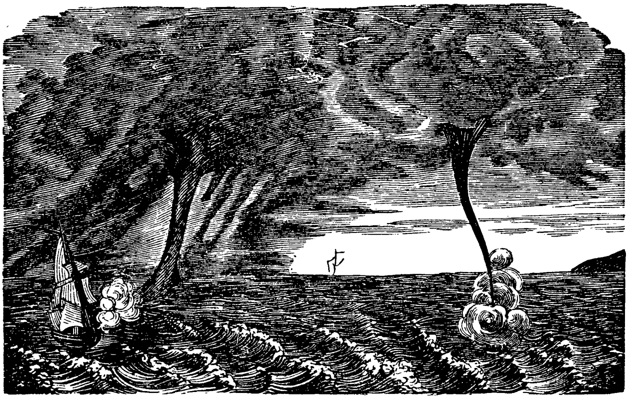
By careful observation of the barometer, the mariner anticipates the approach of a hurricane, or other dangerous storm. So, also, by careful attention to the course of the winds, and the sailing directions, based on the researches of Redfield, Reid, and Maury, he knows how to sail out of the track of a hurricane after it has commenced.

969. **Tornadoes or whirlwinds** are distinguished from hurricanes, chiefly in their extent and continuance. They are rarely more than a few hundred rods in breadth, and their whole track is seldom more than twenty-five miles in length. The continuance of tornadoes is but a few seconds at any one place. They are oftentimes of great energy, uprooting trees, overturning buildings, and destroying crops.

970. **Water-spouts** differ from whirlwinds in no other respect than that water, or vapor of water, is subjected to their action, instead of bodies upon the surface of the land.

Water-spouts first appear as an inverted cone, extending downward from a dark cloud. As the cone approaches the water, the latter becomes agitated, the spray rises higher and higher, and finally, both uniting, there is formed a continuous column from the cloud to the water, usually bent as in fig. 715, but sometimes erect. After a little time, the column breaks, and the phenomena disappear. As to the origin of water-spouts, philosophers are divided. Kaemtz, a distinguished German meteorologist, assumes that they are due principally to two opposite winds, which pass side by side, or when a very brisk wind prevails in the higher regions of the atmosphere, while it is clear below. Peltier, and other physicists, ascribe water-spouts to an electrical cause.

715



Water-spouts are, in great part, formed of atmospheric water, as is shown by the fact, that the water escaping from them is not salt, even in the open sea.

If the atmosphere is not moist, there is no condensation of vapor, and the only noticeable phenomenon is the violence of the wind and its rotatory motion.

971. **General laws of storms.**—Great storms, in temperate climates, are governed by general laws, presenting more or less analogy to the movements of hurricanes. As a general thing, the great storms which pass over North America have no connection with the storms of Europe; and the storms of both continents are modified in their course and general phenomena by the configuration and elevation of the land.

Great storms of rain and snow, and also lesser storms, which prevail in the United States from November to March, are governed by the following general laws, which have been condensed from the researches of Professors Espy (Reports to United States Senate), and Loomis (Smithsonian Contributions, 1860).

1. Storms travel from the region of the Mississippi eastward as far as the Gulf of St. Lawrence, and disappear in the Atlantic Ocean.

2. They are accompanied with a depression of the barometer, often amounting to an inch or more below the mean height, along a line of great extent from north to south, the line being curved with its convexity eastward. In the front and rear of the storm, there is a rise of the barometer, which, in some places, is more than an inch above the mean.

3. Great storms travel from the Mississippi to the Connecticut River in twenty-four hours; and from the Connecticut to Newfoundland in nearly the same time, or about thirty-six miles an hour; though sometimes they appear to remain nearly stationary for four or five days.

4. The area covered by a storm is nearly circular, often of great extent; frequently 1000 miles from east to west, and 2000 or 3000 miles from north to south.

5. For several hundred miles, on each side of the centre of a violent storm, the wind inclines inwards towards the area of least pressure, and, at the same time, it circulates around the centre in a direction contrary to the motion of the hands of a watch. Compare § 968.

6. On the borders of the storm, near the line of maximum pressure, the wind has but little force, and tends outwards from the line of greatest pressure.

7. The wind uniformly tends from an area of high barometer towards an area of low barometer.

8. In the northern parts of the United States, the wind generally, in great storms, sets in from the north of east, and terminates from the north of west; and in the southern states, the wind generally sets in from the south of east, and terminates from the south of west.

9. During the passage of storms, the wind generally veers from the eastward around by the south, and then towards the west.

10. In a great storm, the area of high thermometer frequently does not coincide with the area of low barometer, or with the centre of rain and snow. But in the United States, on the north-east side of a storm, at a distance of 500 miles from the area of rain and snow, the thermometer sometimes rises as much as twenty degrees above its mean height.

11. In Europe, storms are often modified and controlled by the Alps of Switzerland, which appear, for days together, to serve as the centre of great storms.

For the connection of barometric changes with changes of weather, see section 271; and for fuller discussions of the theory and laws of storms, see the works already referred to above, and also §§ 946, 968.

§ 3. Aqueous Phenomena.

972. **Humidity of the air.**—Vapor of water is always contained in the air. This may be demonstrated by placing a vessel filled with ice or a freezing mixture in the atmosphere; in a little time the vapor from the air will be condensed on the walls of the vessel, in the form of minute drops of water.

Air is said to be *saturated with moisture,* when it contains as much of the vapor of water as it is capable of holding up at a given temperature. That the capacity for moisture is greater as the temperature increases, is shown in the following table:—

A body of air can absorb
At 32° F., the 160th part of its own weight of watery vapor.
" 59° " " 80th " " " "
" 86° " " 40th " " " "
" 113° " " 20th " " " "

It will be noticed that for every 27° of temperature above 32°, the capacity of air for moisture is doubled. From this it follows, that, while the temperature of the air advances in an arithmetical series, its capacity for moisture is accelerated in a geometrical series.

Table XXI., Appendix, shows the weight of aqueous vapor in a cubic foot of saturated air, at temperatures from 0° to 100° F.

Absolute humidity,—relative humidity.—The term absolute humidity of the air has reference to the quantity of moisture contained in a given volume. Relative humidity refers to the dampness, or its proximity to saturation with aqueous vapor.

The absolute humidity is greatest in the equinoctial regions, and diminishes towards either pole; it diminishes, also, with the altitude, but the true ratio is not fully known. The absolute humidity is also greater on coasts than inland, in summer than in winter, and less in the morning than about midday.

Relative humidity is dependent upon the mutual influence of absolute humidity and temperature. The atmosphere is considered dry when water rapidly evaporates, or a wet substance quickly dries. The expressions wet and dry convey simply an idea of the relative humidity of the atmosphere, and have no reference to the absolute quantity of moisture present; for a damp air is rendered dry by raising its temperature, and a dry air damp by cooling it.

Near great bodies of water, the atmosphere generally contains a greater amount of moisture, both absolute and relative, than at inland places, or over arid plains and deserts.

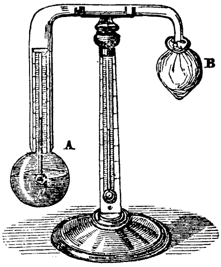
973. **Hygrometers** are instruments by which the humidity of the atmosphere is determined. They are of various kinds, and may be classified as follows:—chemical hygrometers, absorption hygrometers, condensation hygrometers, and psychrometers.

All hygroscopic substances (viz., those which have an affinity for water) are chemical hygrometers. The amount of moisture in the air is determined with these substances, by filling a tube with chlorid of calcium, for example, and passing a known volume of air through it; the increase in weight of the tube, after the experiment, indicates the weight of moisture present in the air. This method yields the best results, but is difficult of execution.

Saussure's hygrometer depends upon the elongation and contraction of a hair by increase or diminution of relative humidity; but such instruments afford no means of accurate comparison.

Daniell's hygrometer depends upon the condensation of moisture by means of artificial cold.

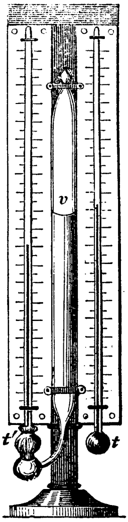
716



It consists of a glass tube, bent twice at right angles, having a bulb at either extremity. The bulb, A, fig. 716, is partly filled with ether, into which is inserted the ball of a delicate thermometer, enclosed in the stem of the instrument. The tube is filled with the vapor of ether, the air having been driven out. The bulb, B, is covered with fine muslin. Upon the supporting pillar, a second thermometer is placed. In order to determine the dew point, or hygrometric state of the atmosphere, by this instrument, a few drops of ether are allowed to fall upon the muslin-covered bulb, evaporation of the ether takes place, the bulb is cooled, and condenses the ethereal vapor within. In consequence of this effect, the ether in A evaporates, causing a reduction of temperature, indicated by the internal thermometer. At a certain point, the atmospheric moisture begins to form in a ring of dew upon the bulb A. The difference at this moment between the degrees indicated by the two thermometers, denotes the relative humidity of the atmosphere; the dryer the air, the greater is this difference.

August's psychrometer or hygrometer of evaporation depends for its action upon the rapidity of evaporation in the open air. It consists of two similar thermometers, *t t′*, placed side by side, fig. 717, supported on a frame. The bulb of *t′* is covered with fine muslin, the lower end of which dips into a small vessel, *v*, like a bird-glass, containing water; by this arrangement, the bulb is kept continually moist. Evaporation takes place from the moistened bulb, with a rapidity varying with the humidity of the atmosphere, and a corresponding depression in the temperature of the thermometer is produced. The hygrometric state of the atmosphere is determined from the observed difference in the two thermometers by the use of tables prepared for the purpose. (*Meteorological and Physical Tables,* Smithsonian Collection.)

717



This is a very convenient instrument to determine the condition of the air in dwellings heated by different methods. Observations with this instrument show how much our comfort and health depend upon preserving the proper state of humidity or dryness in our dwellings, or in the sick-room.

974. **Fogs, or mists,** are visible vapors that float in the atmosphere, near the surface of the earth. Fogs are produced by the union of a body of cool air with one that is warmer and humid. Many philosophers, as Saussure and Kratzenstein, consider that the globules or vesicles of which a fog is composed, are hollow, the water serving only as an envelope; it is probable this is true, in some cases: there are probably also mixed with the vesicles many minute drops containing no free air. According to Kaemtz, the average diameter of fog globules does not exceed $\frac{1}{1250}$ of an inch. Maille, of Paris, has computed that it would require 200,000,000 fog globules to make a drop of rain $\frac{1}{10}$ of an inch in diameter.

975. **Dew** is the moisture of the air condensed by coming in contact with bodies cooler than itself. The temperature at which this deposition of moisture commences, is called the dew point (674). The dew point varies according to the hygrometric state of the atmosphere; being nearer the temperature of the air, the more completely the air is saturated with moisture. In this climate, in summer, the dew point is often 30° or more below the temperature of the atmosphere. In India, it has been known to be as much as 61°.

Cause of dew.—Dr. Wells, of London (born in South Carolina), determined, by his researches, the cause of dew. It may be given briefly as follows:—During the day, the surface of the earth becomes heated by the sun, and the air is warmed by it. When the sun goes down, the earth continues to radiate heat without receiving any in return, and thus its temperature diminishes. The air loses its heat more slowly, and is cooled only when it comes in contact with the cooler earth. If this cooling reaches the dew point of the air, moisture is condensed in the form of small drops upon cold objects (good radiators), as the soil or vegetation.

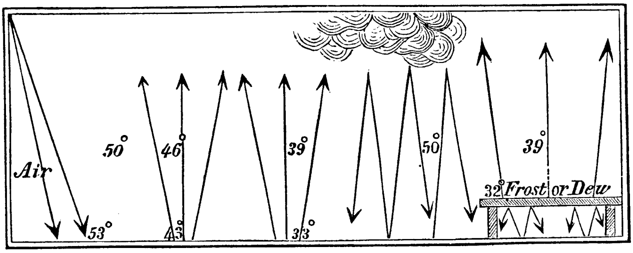
Circumstances influencing the production of dew.—The amount of dew deposited in a given time depends upon the humidity, tranquility, and serenity of the air. Since dew is the moisture of the atmosphere condensed, it is evident it must be affected by the amount the air contains. On a windy night, the air in contact with cold objects is so quickly changed, that it is not cooled down to the dew point; but gentle agitation of the air favors the production of dew, bringing more moist air to furnish dew to cold objects. The most copious deposits of dew take place on cool, clear nights. For, when there are clouds, these radiate back the heat which has escaped from the earth, and thus prevent its cooling, and therefore no dew is deposited. If the clouds separate only for a short time, dew is rapidly deposited.

Straw, mats, boards, &c., used by gardeners to protect delicate plants from freezing, act in the same manner as clouds, to prevent the deposit of dew or frost. See Fig. 718.

976. **Substances upon which dew falls.**—Dew does not fall upon all substances alike; in consequence of differences in radiating and conducting power, certain substances cool quicker and more perfectly than others. The dusty road, the rocks, and barren soil, cool slowly, receiving heat from the earth by conduction, and therefore on them but little dew falls. Trees, shrubs, grasses, and vegetation of every kind, radiate heat easily, and, on account of their peculiar structure, they receive but little heat from the earth, or other objects, by conduction; hence they become rapidly cooled, and abundance of dew is deposited upon them.

977. **Frost** is frozen dew. When the temperature of the earth sinks in the night to the freezing point, the aqueous vapor then deposited congeals in the form of sparkling crystals, known as hoarfrost. Fig. 718, from Stoeckhardt, in which the arrows indicate the movements of heat, and the numerals the temperature of the air, will render the phenomena of dew and frost more intelligible.

718



The sun's rays in winter, may, in the day, warm the soil to 53°, as in the figure, while the air above the ground is 50°. At night, the radiation into a cloudless sky, will reduce the temperature of the ground to 43°, or even 33°, while the air above the same points is 46° or 39°. But a cloud resting above the earth prevents radiation, and reflects the heat back to the earth. So dew or frost will be deposited on the upper surface of a platform when radiation takes place freely, while boards, like the cloud, reflect back the heat coming from the ground.

978. **Clouds** are masses of vapor that float in the upper regions of the atmosphere. They are distinguished from fogs only by their altitude; they always result from the partial condensation of the vapors that rise from the earth. As clouds often float in regions whose temperature is many degrees below the freezing point, they are sometimes, no doubt, composed of frozen particles.

Clouds being condensed moisture, are heavier than the air, and have a tendency to fall to the earth. They are kept suspended in the air, 1. By ascending currents during the day, the warmer air dissolving the cloud as fast as it falls into it. Such clouds are more elevated at midday than in the morning, and they descend towards the earth at evening.

2. Horizontal currents also oppose the fall of clouds. The minute vesicles or globules, whichever they may be, are carried forward and dissolved by the drier air on the advancing side of the cloud, while on the windward side of the cloud, vapor is constantly precipitated.

3. The resistance of the air opposes the rapid descent of clouds. This resistance is in the inverse proportion to the dimensions of the particles. For this reason, considerable time would be required for vapor to descend even a few hundred feet. If, as many writers suppose, the water of clouds exists in the form of minute vesicles containing air, the expansion of the enclosed air by heat would at once account for the buoyancy of clouds; for they would float like balloons in air of their own aggregate density, and every increase of heat would increase their buoyancy.

979. **Classification of clouds.**—Clouds are generally divided into four great classes, viz.: the *nimbus*, the *cumulus*, the *stratus*, and the *cirrus*, as shown in the diagram, fig. 719.

Intermediate forms of clouds are distinguished by the names of *cirro-stratus*, *cirro-cumulus*, and *cumulo-stratus*.

The *cirrus* (cirrus, *curl*) usually resembles a disheveled lock of hair, being composed of streaks or feathery filaments, assuming every variety of figure. The cirrus floats at a higher elevation than other clouds, and probably is often composed of snow-flakes. It is among the cirri that halos and parhelia are formed.

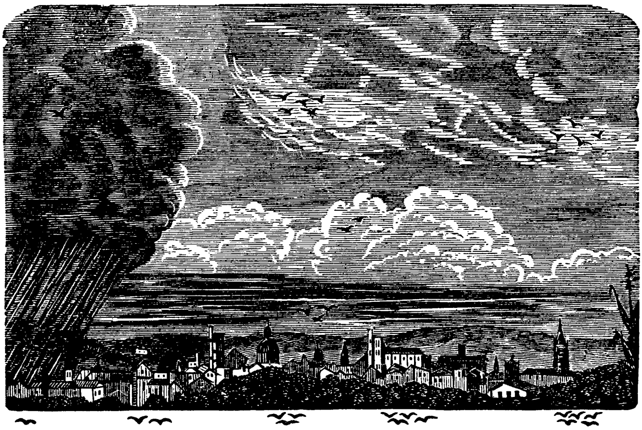
The *cumulus* (cumulus, *heap*) appears often in the form of a hemisphere, resting on a horizontal base; sometimes in detached masses, gathered in one vast cloud, near the horizon. When lighted up by the sun, they present the appearance of mountains of snow. The cumulus is the cloud of day; in the fine days of summer it is most perfect.

The name of cirro-cumulus is given to little rounded clouds.

The cumuli owe their existence to ascending currents; their height varies greatly, but it is always less than that of the cirri.

The *stratus* (stratus, *covering*) consists of sheets of cloud, or layers of vapor, stretching along and resting upon the horizon. It forms about sunset, increases

719



during the night, and disappears about sunrise. It floats at a moderate elevation above the earth.

The *cirro-stratus* partakes of the character of the cirrus and stratus. It is remarkable for its length and thickness. It appears, sometimes, as a long and narrow band; at other times, as composed of small rows of clouds or bars of vapor. Cumuli, heaped together, pass into the condition of *cumulo-stratus*, which consists of a horizontal stratum of vapor, from which rise masses of cumuli. These often assume, at the horizon, a dark tint, and pass into the nimbus state.

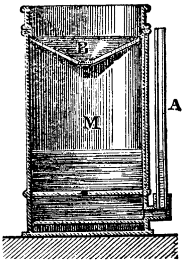
The *nimbus*, or rain-cloud (nimbus, *storm*). This has a characteristic storm-like form; it is distinguished from others by its uniform gray or blackish tint, and its edges fringed with light.

980. **Rain** is the vapor of clouds, or of the air, precipitated to the earth in drops. Rain is generally produced by the rapid union of two or more volumes of humid air, differing considerably in temperature; the several portions, when mingled, being incapable of absorbing the same amount of moisture that each would retain if they had not united. If the excess is great, it falls as rain; if it is of slight amount, it appears as cloud. The production of rain is the result of the law, that the capacity of air for moisture decreases in a higher ratio than the temperature.

Rain-gauges.—Instruments for determining the quantity of rain, are called *rain-gauges*, *ombrometers*, *hyctometers*, &c. They are of very various construction.

One of the simplest rain-gauges consists of a cylindrical copper vessel, furnished with a float; the rain falling into the vessel, the float rises. The stem of the float is accurately graduated, so that an increase in the depth of the water of one one-hundredth of an inch, is easily measured.

720



Another rain-gauge, a section of which is represented in fig. 720, consists of a cylindrical copper vessel, M, closed by a cover, B, shaped like a funnel, with an aperture in the centre, through which the water passes into the interior. This cover prevents loss by evaporation. A lateral glass tube, A, carefully graduated, rises from the base of the vessel. The water rises in the tube to the same height as in the copper cylinder. If the apparatus has been placed in an exposed situation, for a certain time, as a month, and the gauge shows three inches of water, this indicates that the rain that has fallen during the interval would cover the earth to the depth of three inches, if it were not diminished by evaporation, or infiltration.

From a series of experiments made at the Smithsonian Institution, and continued for several years, it is found that a small cylindrical gauge, of 2 inches in diameter, and about six inches in length, connected with a tube of half the diameter, to retain and measure the water, gives the most accurate results. In still weather, it indicates the same amount of water as the larger gauges; but when the wind is high, it receives more rain; for, on account of its small size, the force of the eddy which is produced, is much less in proportion to the drops of water. This gauge may be still further improved by cutting a hole of the size of the cylinder, in a circular plate of tin, of 4 or 5 inches in diameter, and soldering this to the cylinder like the rim of an inverted hat, three or four inches below the orifice of the gauge.

981. **Distribution of rain.**—Rain is not equally distributed over the surface of the earth. As a general rule, it may be stated that, the higher the average temperature of a country, the greater will be the amount of rain that falls upon it. Local causes, however, produce remarkable departures from this rule.

In the tropics, the average yearly rain fall is ninety-five inches; in the temperate zone it is thirty-five inches. Within the tropics, the greatest quantity of rain falls when the sun is at its zenith, that is, in the season corresponding to our summer. North of the tropics it rains more abundantly in winter.

In certain regions there is a periodical season, when rain is very abundant for six months, called the rainy season. During the remainder of the year, called the dry season, it seldom rains.

Many regions are destitute of rain. In Egypt, it scarcely ever rains. Along the coast of Peru, is a long strip of land upon which no rain ever descends. A similar destitution of rain occurs on the coast of Africa, and some parts of North America; the intervals between the showers being six or seven years.

In Guiana it rains during a great part of the year; this is also the case, according to Davison, at the Straits of Magellan. In the Island of Chiloe (S. lat. 43°), there is a proverbial saying, tha it rains six days of the week, and is cloudy on the seventh.

982. **Days of rain.**—The rainy days are more numerous in high than in low latitudes, as is seen in the following table, although the

annual amount of rain which falls is smaller. Consequently, the ordinary rains of the tropical regions are more powerful than those of the temperate regions.

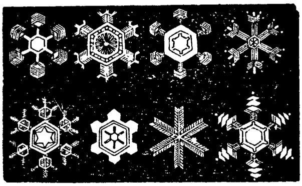
N. latitude.	Mean annual number of rainy days.
From 12° to 43°	78
" 43° " 46°	103
" 46° " 50°	134
" 50° " 60°	161

In the northern part of the United States there are, on the average about 134 rainy days in the year; in the southern part, about 103.

983. **Annual depth of rain.**—The greatest annual depth of rain occurs at San Luis, Maranham, 280 inches; the next in order are Vera Cruz, 278; Grenada, 126; Cape François, 120; Calcutta, 81; Rome, 39; London, 25; Uttenberg, 12·5. In our country, the annual average fall is 39·23 inches; at Hanover, N. H., 38; New York State, 36; Ohio, 42; Missouri, 38·265.

984. **Snow** is the frozen moisture that descends from the atmosphere, when the temperature of the air at the surface of the earth is near, or below, the freezing point. The largest flakes of snow are produced when the atmosphere is loaded with moisture, and the temperature of the air is about 32°; as the cold increases, the flakes become smaller.

721



The bulk of recently fallen snow is ten or twelve times greater than that of the water obtained from it. Snow flakes are crystals of various forms. Scoresby has enumerated six hundred forms, and figured ninety-six. Kaemtz has met with at least twenty forms not figured by Scoresby. Crystals of snow are not solid, else they would be transparent as ice; they contain air. It is to the reflection of light from the assemblage of crystals, that its brilliant whiteness is due. Snow crystals are produced with most regularity during calm weather, without fog. Fig. 721 represents a few of the beautiful forms assumed by snow crystals.

985. **Colored snow** is mentioned by Pliny. It occurs under two very different circumstances,—either while the snow is falling, or some time after its descent.

In 1808, rose-colored snow fell in the Tyrol and in Carinthia. Its color was owing to an earthy powder, composed of iron, silex, and alumina. Of a similar composition was a red snow that fell on the mountain of Toual, in Italy, in 1816. These facts prove conclusively that, at times, snow tinged with mineral ingredients, falls upon the earth. That the color is sometimes (and generally) produced by the presence of minute organisms, is no less conclusively demonstrated. Captain Ross, in 1819, discovered a crimson snow clothing the sides of the mountains at Baffin's Bay. It has been observed on the Pyrénées, Alps, and Apennines; in Scotland, Sweden, Norway, &c. Certain French meteorologists, at Spitzbergen, in 1838, passed over a field covered with snow, which appeared of a green hue, whenever pressed upon by the foot. Agassiz regards these colors as animal products, believing them to be the ova of a rotiferous animalcule. The more common belief is, that (generally, at least) these hues are owing to the presence of a certain class of microscopic plants, the different colors representing different stages of development. Martini gives, perhaps, the correct explanation: that this product is a vegetable cell, enclosing fluid, in which multitudes of infusoria find a nidus and support.

986. **Hail** is the moisture of the air frozen into globules of ice. Hail-stones are generally pear-shaped; they are formed of alternate layers of ice and snow, around a white, snowy nucleus. It is necessary for the production of hail, that a warm, humid body of air, mingle with another so extremely cold, that, after uniting, the temperature shall be below the freezing point. The difficulty of explaining the phenomena of hail-storms, consists in accounting for this great degree of cold.

Hail-storms are most frequent in temperate climates. They rarely occur in the tropics, except near high mountains, whose summits are above the snow-line. It is in great part during the summer, and in the hottest part of the day, that hail falls. Hail-storms rarely occur at night. Hail-stones are often of considerable size; the largest are frequently an aggregation of several frozen together. Sleet is frozen rain; it occurs only in cold weather; it falls only during gales, and when the weather is variable.

§ 4. Electrical Phenomena.

987. **Free electricity of air.**—The general laws of atmospheric electricity have been considered in a previous paragraph (861).

It is common to refer the free atmospheric electricity to several causes, always at work on the earth's surface, as, 1. Evaporation, especially of impure water; 2. Condensation; 3. Vegetation (945), 4. Combustion; and, 5. Friction; without doubt these are all causes of electrical excitement in the air. But far more important than them all is the powerful inductive influence of the *negatively* excited earth upon its gaseous envelope. The dense air near the earth's surface is like the dielectric of the Æpinus condenser, and the constant presence of *positive* electricity in the air is a fact not explicable on any other hypothesis than that of induction from the negative earth.

In addition to the laws already announced in § 861, may be added the fact that atmospheric electricity is more abundant in summer than in winter.

988. **Thunder-storms** are most frequent and violent in the equatorial regions. They decrease in frequency towards either pole, and are more frequent in the summer than in the winter months, and after mid-day than in the morning. They are produced in the same manner as ordinary storms; but they differ from them in their local character,

in the rapidity and extent of the condensation of the atmospheric vapor, and in the accumulation of electricity.

Thunder-storms are usually attended by an alteration in the direction of the wind. Of one hundred and sixteen thunder-storms recorded in the Meteorological Register of the Connecticut Academy, ninety-nine were either preceded or followed by an alteration in the direction of the wind.

Thunder-storms generally prevail in the lower regions of the atmosphere. They are, however, not unfrequently observed at great elevations. Kaemtz notices one on the mountains of Switzerland which rose to the height of more than 10,000 feet.

The geographical distribution of thunder-storms has been lately discussed by Prof. Loomis (Am. Jour. Sci. [2] XXX. 94), whose results confirm the general statement already made with reference to latitude, thus:—

Between	latitude	0°	and	latitude	30°	The average number of thunder-storms annually is	51·6
"	"	30	"	"	50		19·9
"	"	50	"	"	60		14·9
"	"	60	"	"	70		4·
Beyond	"	70					0·

Maury's storm and rain charts, however, show that the frequency of lightning depends on other circumstances than simply latitude, since throughout the western half of the Atlantic Ocean lightning is three times as frequent as over the eastern half of that ocean, and two and a half times as frequent in the North Atlantic as in the South Atlantic.

The origin of thunder-clouds appears, by both theory and observation to be due, in this country, to the rushing up of the lighter air to restore the normal equilibrium of the atmosphere, which had been disturbed or rendered unstable by the gradual introduction, next to the ground, of a stratum of warm and moist air. The upper end of such an ascending column of air, on the principle of Peltier (861). must be negatively electrized, as its lower end receives positive induction from the negative earth. As, by the principles established by Espy, the excess of watery vapor in such a cloud will be precipitated as it rises, it follows, that the ascending column becomes a conductor, and a series of electrical discharges will take place between the upper and lower parts of the cloud. The hour-glass form, which the aeronaut Wise asserts is the shape of a thunder-cloud, when seen from one side, in a balloon, confirms this view. (Consult Professor Henry's paper on Meteorology, in the Report of the Patent Office for 1859, *Agriculture.*)

989. **Thunder.**—As lightning passes through the air with amazing velocity, it violently displaces it, leaving void a space into which the air rushes with a loud report; this is *thunder*.

The rolling of thunder is generally ascribed to the reverberation of the sound from clouds and adjacent mountains. It is also considered that as the lightning darts to a great distance with immense velocity, thunder must be produced at every point along its course, and the sounds not reaching the ear at the same time that elapses between lightning and its thunder, we are enabled to calculate the distance of the former. According to Mr. Earnshaw, the sound of a thunder clap is propagated with much greater velocity than ordinary sounds. See Appendix, p. 668.

990. **Lightning.**—It has already been stated, that air subjected to compression emits a spark. The production of lightning is by some attributed to the energetic condensation of the atmosphere before the electric fluid, in its rapid progress from point to point. When lightning is emitted near the earth, the flashes are of a brilliant white color; when the storm is higher, and therefore in a rarefied atmosphere, their color approaches to violet. Clouds appear to collect and retain electricity. When a cloud overcharged with electricity approaches another less charged, the electric fluid rushes from the former to the latter. In the same manner the electric fluid may pass from the clouds to the earth. In such cases, elevated objects, as trees, high buildings, church steeples, &c., often govern its direction. It is unnecessary to dwell upon the powerful and destructive effects of lightning.

991. **Classes of lightning.**—Lightning has been divided by Arago into three classes, viz.: zigzag or chain lightning, sheet lightning, and ball lightning. We may add heat lightning and volcanic lightning. This classification is convenient, and is universally adopted.

Zigzag or chain lightning is supposed to owe its form to the resistance of the air compressed before it. The lightning takes the path of least resistance; then moves forward until it meets with a like opposition, and so continues glancing from side to side until it meets the object it seeks. Sometimes the flashes divide into two, and sometimes into three branches; it is then called forked lightning.

Sheet lightning appears during a storm as a diffuse glow of light illuminating the borders of the clouds, and occasionally breaking out from the central part.

Heat lightning as it is called, appears often in serene weather during summer, near the horizon; it is generally, if not always, unattended with thunder; heat lightning is the reflection in the atmosphere of lightning very remote, or not distinctly visible. By many, this phenomenon is supposed to be occasioned by the feeble play of electricity when the air is rarefied, and the pressure upon the clouds is so much diminished that the electric fluid cannot accumulate upon their surface beyond a certain point, and escapes in noiseless flashes to the earth.

Ball lightning appears in the form of globular masses, sometimes remaining stationary, often moving slowly, and which in a little time explode with great violence. This form of lightning is of very rare occurrence, and philosophers have not as yet been able to account for it.

Volcanic lightning.—The clouds of dust, ashes, and vapor, that issue from active volcanoes, are often the scene of terrific lightning and thunder. Volcanic lightning is probably caused by rapid condensation of the vast volumes of heated vapor thrown into the air.

The rapidity of lightning of the first two classes is probably not less than two hundred and fifty thousand miles per second. Arago has demonstrated that the duration of a flash of lightning does not exceed the millionth part of a second. The waving trees illuminated at night by a single flash of lightning during a storm appear motionless; the duration of the flash is so short, that, during its continuance, the trees have not sensibly moved.

992. **Return stroke.**—When a highly charged thunder-cloud approaches the earth, it induces the opposite kind of electricity upon the ground below, and repels that of the same kind. If the cloud is extended, and comes within striking distance of the earth, or of another cloud, a flash at one extremity is often followed by a flash at the other. This latter is called the return stroke, and sometimes is of such violence as to prove fatal, even at a distance of several miles from the point of the first discharge.

993. **Lightning-rods** were first introduced by Dr. Franklin. He was induced to recommend their adoption as a means of protection to buildings, from the effects of lightning, by observing that electricity could be quietly and gradually withdrawn from an excited surface by means of a good conductor, pointed at its extremity (826).

Lightning-rods are ordinarily made of wrought iron; but copper is preferable, being a better conductor of electricity, and less easily corroded. The size of the rod, if of iron, should not be less than three-quarter inch in diameter. The upper extremity of the rod should be pointed. Three points is the usual number used in the United States, but one is sufficient. The points should be tipped with silver, gold, or platinum, or copper gilded by electricity; these metals being unaffected by the air, which would corrode the copper or iron, and render them poorer conductors. The rod should be continuous from top to bottom, and securely fastened to the building. Glass or wooden insulators are often recommended, but when once wet by a shower, there is but little advantage in them over metallic supports. When there are surfaces of metal about the building, as gutters, pipes, &c., these should be connected with the conductor by strips of metal, as first recommended by Prof. Henry. The lower part of the rod, where it enters the ground, should be divided into two or three branches, and bent away from the building, penetrating so far below the surface of the earth as to reach water, or permanently moist soil. Charcoal is recommended to fill the hole in the centre as a means of effecting a better conduction. In a church, in New Haven, the lightning has twice penetrated a twenty inch brick wall at a point opposite a gas-pipe, 20 feet above the earth, through which the discharge has escaped to the earth, although the conductor of three-quarter inch iron was well mounted, but its connection with the earth was less perfect than that of the gas-pipe.

Protective power.—According to Mr. Charles, a lightning-rod protects a space around it, whose radius is equal to twice its height above the building. Thus, if a conductor extend ten feet above the house, it affords protection to a circular space forty feet in diameter, the rod being in the centre.

Conductors do not attract the lightning toward the building upon which they are placed. They simply direct the course and facilitate the passage of the electricity to the earth, which otherwise might have been effected in a powerful and destructive discharge through the building. It is indeed considered by Arago, that "lightning-rods not only render strokes of lightning inoffensive, but considerably diminish the chance of their being struck at all."

994. **Aurora borealis.**—Under this name are comprised the luminous phenomena seen frequently in the northern sky; and also, although more rarely, in the neighborhood of the south pole; they are then called aurora *australis.* They present, when in full display, a spectacle of surpassing splendor and beauty. The cause of the aurora borealis is yet involved in obscurity. Although it is, evidently, intimately connected with terrestrial magnetic electricity, it is impossible at present to say exactly what this connection is. It has been ascribed to the passage of electrical currents through the upper regions of the atmosphere, the different colors being manifested by the passage of the electricity through air of different densities.

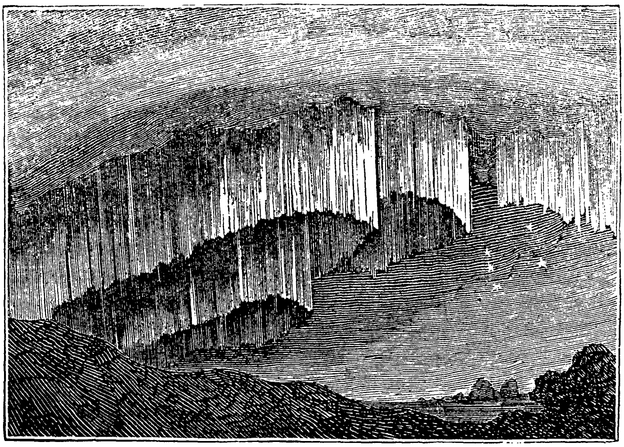
Appearance of auroras.—Before the aurora appears, the sky in the northern hemisphere usually assumes a darkish hue, which gradually deepens, until a circular segment of greater or less size is formed. This dark segment is bounded by a luminous arc, of a brilliant white color, approaching to blue.

The lower edge of this arc is clearly defined; its upper edge gradually blends with the sky. When this luminous arc is formed, it frequently remains visible for many hours, but it is always in motion. It rises, falls, and breaks in various places. Clouds of light are suddenly disengaged, separating into rays, which stream upwards like tongues of fire, moving backwards and forwards. When the luminous rays are numerous, and their palpitating lights pass to the zenith, they form a brilliant mass of light, called the *corona* or crown, whose centre is the point towards which the dipping-needle at the place is directed. The aurora is then seen in its greatest splendor; the sky resembles a fiery dome, supported by waving columns of different colors. When the rays are darted less visibly, the aurora soon disappears, the lights momentarily increase, then diminish, and finally disappear. It is asserted that sounds, like the rustling of silk, often accompany the display of auroras, but this is extremely problematical; the most celebrated polar navigators never heard any noises which they could certainly ascribe to the auroras.

995. **Remarkable auroras.**—The aurora is not a local phenomenon; it is often seen simultaneously in places far apart, as in Europe and America.

In 1796, a beautiful aurora was observed simultaneously in Pennsylvania and France. The aurora of January 7th, 1831, was observed in all central and northern Europe, and at Lake Erie. The aurora of November 17th, 1848, is one

722



Auroral display, seen at Bossekop, 70° N., 1838-9.

of the most remarkable previously recorded. It was seen from Odessa, on the Black Sea, lat. 46° 35′, long. 30° 35′ E. to San Francisco (California), 38° N. lat., 122° W. long., and as far south as Cuba. It seems everywhere to have had a prevailing red hue, mistaken in many places for a conflagration. (Am. Jour. Sci. [2] VII. 203.)

More remarkable than all, however, was the aurora of August 28th to September 4th, 1859, for the great extent of territory over which it was seen, for its long duration, and for the brilliancy of its colors, the intensity of its illumination, and the rapidity of its changes. It was equally remarkable for the accompanying magnetic disturbances, recorded not only by the usual magnetic instruments, but over the whole system of telegraph wires both in America and Europe. This aurora was seen as far west as the Sandwich Islands, lat. 20°, long. 157° W., and east as far as Barnaul, Russia, long. 83° 27′ E., a circuit of 240° about the earth. The observations seem to justify the inference that it was as vivid in the southern as in the northern hemisphere. It was seen off Cape Horn and in Australia, in the southern hemisphere, up to Concepcion, Chili (lat. 36° 46′ S.), and from about lat. 60° N., in North America, to San Salvador in 13° 18′ N. For full details of this aurora, see Am. Jour. Sci. [2] Vols. XXVIII., XXIX., and XXX.

996. **Height of auroras.**—Many astronomers have endeavored to determine the height of auroras, but the results of their calculations are not certain. Earlier philosophers computed their altitude at several hundred miles; a lower limit is assigned by later observers. A brilliant auroral arch was observed in the Northern and Middle States,

April 7th, 1847; from the observations made at Hartford and New Haven, Conn., its height was computed by Mr. E. C. Herrick, of the latter place, to be nearly one hundred and ten miles. Another, seen April 29th, 1859, is by the same observer estimated approximately at much over 100 miles in height. (Am. Jour. Sci. [2] XXVIII. 154.)

Prof. Loomis calculates the height of the base of the auroral curtain, August 28th, 1849, as about forty miles, but the same observer estimates the height of the belts of this aurora in other places as over one hundred and fifty miles.

Frequency of auroras.—Auroras are perhaps rather more frequently seen in winter than in summer; but this circumstance does not indicate that during the former season there are actually a greater number, for the increased length of night would render a greater number visible, even if they were equally distributed throughout the year. During the summer of 1860, auroras have been uncommonly frequent in the N. United States. About the period of the equinoxes they appear to be more frequent than at other times.*

In addition to the annual period, there appears to be another, a secular period, extending through a number of years. One of these periods was comprised between 1717 and 1790; its maximum was obtained in 1752. An increase in the frequency of auroras began again in 1820. Prof. Olmsted, in an important paper on this subject, in the Contrib. of Smithson. Inst., vol. 8, selects one of these secular periods between August 27th, 1827, and November, 1848, or a little later. The number of auroras, observed for a period of about sixteen years, at New Haven, by Mr. E. C. Herrick, is given in the following table:—

AURORAS OBSERVED AT NEW HAVEN DURING SIXTEEN YEARS.

	Number of auroras.		Number of auroras.
April 1837 to April 1838,	42	April 1845 to April 1846,	21
" 1838 " " 1839,	34	" 1846 " " 1847,	25
" 1839 " " 1840,	43	" 1847 " " 1848,	30
" 1840 " " 1841,	48	" 1848 " " 1849,	42
" 1841 " " 1842,	29	" 1849 " " 1850,	18
" 1842 " " 1843,	9	" 1850 " March 1851,	15
" 1843 " " 1844,	7	Oct. 1851 " Oct. 1852,	33
" 1844 " " 1845,	10	" 1852 " " 1853,	24

997. **Geographical distribution of auroras.**—Prof. Loomis has lately (Am. Jour. Sci. [2], XXX., 89) published a chart showing the distribution of auroras in the northern hemisphere. He shows from a tabular comparison of recorded observations, that near the parallel of 40° N., on the meridian of Washington, there are only 10 auroras annually; nearly 42° N., the average is 20 annually; near 45° it is

---

* See a paper on Huxham's Observations (Am. Jour. Sci. [1], XXXIII., 301), showing that the A. B. is as abundant in summer as in winter.

40; and near the parallel of 50° it is 80 annually. Between this and the parallel of 62°, auroras are seen almost every night, appearing high in the heavens, and as often to the south as to the north. Above 62° they are seldom seen, except in the south, and from this point they diminish in frequency and brilliancy as we advance towards the pole. On the meridian of St. Petersburg a similar comparison gives a like result, except that the auroral region is situated further north than it is in America, the zone of 80 auroras annually being from 66° to 75° N.

Prof. Loomis's chart (*loc. cit.*) shows that the region of greatest auroral activity is an elliptical belt, having one focus near the north pole, and the other near the pole of magnetism, and whose major axis crosses the meridian of Washington, near lat. 56°, and the meridian of St. Petersburgh, in lat. 71°. Accordingly, auroras are more frequent in the United States than in the same latitudes of Europe. Thus, on the line of 50°, we find in North America 40 auroras annually, but in Europe less than 10 on the same parallel.

998. **Magnetic disturbances during auroral displays.**—During the prevalence of auroras, all the magnetic elements show great disturbance, simultaneously, at the most distant stations. This statement is confirmed by comparing the observations at Toronto, Canada West, lat. 43° 59′ 35″ N., long. 79° 21′ 30″ W., with those at St. Petersburgh, Russia, lat. 59° 56′ 30″ N., lon. 30° 19′ E., on the 2d and 3d of September, 1859, during the great aurora already described, when, on several occasions, the magnets in the several instruments oscillated completely beyond their scales—equal to a total deflection of over 5½° of arc. (Am. Jour. Sci. [2], XXVIII., 390, and XXX., 80.)

The magnetic oscillations sympathize with the auroral streamers; when the arc is quiet, the needle rests. During the grand aurora of November 14, 1837, the range of oscillation, as observed at New Haven by Messrs. Herrick and Haile, was 6°.

999. **Effect of the aurora on telegraphic wires.**—This phenomenon, already alluded to (994), is entirely distinct from the induction of static electricity during thunder-storms from the atmosphere (860). During the aurora of August–September, 1859, several of the telegraphic lines in the United States were worked, for hours together, entirely by the magnetic current induced from the aurora, the batteries being detached. Chemical decompositions, and powerful heating and luminous effects, have been often observed from the currents induced during auroral disturbances. These facts were first noticed by Mr. G. B. Prescott, at New Haven, in 1847. In Europe, during the great aurora of 1859, the same disturbances of the telegraphic lines were observed as in this country.

While all the lines were more or less affected, whatever their direction, it appears that the disturbances were more marked on the north and south going lines, than on those going east and west: and in Tuscany, Prof. Matteucci observed, that where there were several parallel lines, one above the other, the upper wires were most affected, and those nearest the earth, least; and that the inductive effects were stronger on the longest lines.

1000. **Reversal of polarity in the auroral current.**—Mr. Prescott first determined, by observation on the aurora of July 19, 1852, that the auroral current invariably changes its polarity with every wave. First, a positive current, producing, on Bain's system, a deep blue mark, light at first, and then stronger, until, having attained the intensity of at least 200 Grove's cups, it subsided, and was followed by a current of reverse polarity, which bleached instead of coloring the paper. Sometimes a flame of fire followed the steel stylus, and burned through a dozen thicknesses of the prepared paper. Free or atmospheric electricity, when it is induced on the telegraph wires, produces no color on the paper. (Am. Jour. Sci. [2], XXIX., 92 and 391.)

Problems on Electricity.

243. Compare the force of electricity on two similar balls, of which one repels the needle of the torsion electrometer 45° and the other 100°.

244. The extreme plates of a voltaic battery, being placed in contact, there was no exterior resistance, and the electro-motive force manifested by the evolution of hydrogen was reckoned as unity, or, $E = 1$, $r = 1$, § 881. A pair of electrodes having then been united to the poles, and the bath arranged for electrotyping, the gas evolved was found to be only one-twentieth as much as before. Calculate the relative value of r and L, and also the intensity of the battery.

245. In the case of a Voltaic battery, so constructed that, when in use, the exterior resistance L is equal to nineteen times the resistance of the battery r, what would be the effect of doubling, trebling, and quadrupling the dimensions of all the plates in the battery?

246. With the same conditions as in the preceding case, $L = 19r$, how would the intensity of the current be changed by doubling the number of couples in the battery?

247. In a battery in which $L = r$, or the external resistance is equal to the resistance of the battery, how will the intensity vary by doubling the number of couples of the same dimensions in the battery?

248. When $L = 4r$, what advantage would be gained by uniting two similar batteries in a single series?

249. If in the use of a Voltaic battery of 100 pairs of plates, arranged in a series, the exterior resistance, L, is found to be six times the resistance of the battery, r, what change of intensity will be produced by so uniting the couples as to form only four groups, each having twenty-five times the previous extent of surface?

ADDENDA.

Note to § 369.—**Uniform musical pitch.**—A general congress, called together by the Society of Arts, at London, June 8, 1860, of musicians, amateurs, and others interested in music, have accepted the report of a committee appointed in 1859, to consider the subject of uniform musical pitch. This committee recommend a pitch of 528 full vibrations for C′ = 440 for A, basing their calculations on 33 single vibrations of an organ pipe 32 feet high, in place of 32 vibrations, the actual number. The following is the scale at this pitch—the only one yet proposed which gives all the sounds in whole numbers:—

C	D	E	F	G	A	B	C′
264	297	330	352	396	440	495	528

This pitch is but 16 vibrations per second higher than the normal Diapason, C′ = 512, or "Stuttgard pitch," and 18 vibrations lower than the present pitch of 546. It is therefore nearly half way between the two, being a quarter tone above one, and the same quantity below the other.

The commission recently appointed to report on the pitch in France have advised the following scale:—

C	D	E	F	G	A	B	C′
261	298⅝	326¼	348	391½	435	489⅜	522

The following is a list of the several pitches considered in this report:—

Handel's Tuning Fork (C. 1740)	A at 416	= C at 499⅕
Theoretical Pitch	A " 426⅔	= C " 512
Philharmonic Society (1812–42)	A " 433	= C " 518⅖
Diapason Normal (Paris, 1859)	A " 435	= C " 522
Stuttgard Congress (1834)	A " 440	= C " 528
Italian Opera, London (1859)	A " 455	= C " 546

(*Journal of the Society of Arts*, June 8, 1860.)

Note to § 343.—**The velocity of all sounds not the same.**—Rev. E. S. Earnshaw, of Sheffield, England, lately brings good evidence, both mathematical and physical, to show that the accepted views stated in § 343 are correct only for sounds having no very great difference of intensity. Every note in music may be formed by two kinds of vibrations of the same rapidity, but differing in wave-length and velocity of transmission. Only one variety of these waves is supposed in general to be sensible by human ears. The velocity of sounds of all kinds is a certain function depending upon the rapidity and length of vibration. In the case of violent thunder the numerical value of this function becomes much greater than for ordinary sounds. These and other remarkable conclusions are sustained by mathematical reasoning. The author of the memoirs also cites evidence to show that the crash of violent thunder-claps has been often heard almost simultaneously with the flash of lightning, although the stroke fell several miles distant. (*London, Edinburgh, and Dublin Phil. Mag.*, June, July, Sept. 1860.)

Note to § 392.—**Sounds produced by insects.**—Burmeister has shown that the usual opinion of naturalists (expressed in § 392) is erroneous, and that the sounds produced by insects are formed by the expansion and contraction of the air tubes, the sound being formed by the passage of air through the orifices of the tubes, which act like a whistle. (*Taylor's Scientific Memoirs*, Vol. I., p. 377.)

CHAPTER II.

PHYSICAL TABLES.

TABLE I.

MEASURES AND WEIGHTS.

ENGLISH MEASURES.

Measures of Length.

THE *inch* is the smallest lineal integer now used. For mechanical purposes it is divided either duodecimally or by continual bisection; but for scientific purposes it is most convenient to divide it decimally. The larger units are thus related to it:—

Mile.	Furlongs.	Chains.	Rods.	Fathoms.	Yards.	Feet.	Links.	Inches.
1 =	8 =	80 =	320 =	880 =	1760 =	5280 =	8000 =	63360
	1 =	10 =	40 =	110 =	220 =	660 =	1000 =	7920
		1 =	4 =	11 =	22 =	66 =	100 =	792
			1 =	2·75 =	5·5 =	16·5 =	25 =	198
				1 =	2 =	6 =	$9\frac{1}{11}$ =	72
					1 =	3 =	$4\frac{6}{11}$ =	36
						1 =	$1\frac{17}{33}$ =	12
·000125 =	·001 =	·01 =	·04 =	·11 =	·22 =	0·66 =	1 =	7·92

Measures of Surface.

Acre.	Roods.	Square Chains.	Square Yards.	Square Feet.
1 =	4 =	10 =	4840 =	43,560
	1 =	2·5 =	1210 =	10,865
		1 =	484 =	4,356
			1 =	9

Measures of Volume.

Cubic Yard.	Cubic Feet.	Cubic Inches.
1 =	27 =	46,656
	1 =	1,728

Imperial Measure.

The Imperial Standard Gallon contains ten pounds avoirdupois weight of distilled water, weighed in air at 62° Fahr. and 30 in. Barom., or 12 pounds, 1 ounce, 16 pennyweights, and 16 grains Troy, = 70,000 grains' weight of distilled water. A cubic inch of distilled water weighs 252·458 grains, and the imperial gallon contains 277·274 cubic inches.

Distilled Water..								
Grains.	Avoir. lb.	Cubic Inches.	Pint.	Quart.	Galls.	Pecks.	Bush	Qr.
8,750	1·25	34·659	1					
17,500	2·5	69·318	2	1				
70,000	10	277·274	8	4	1			
140,000	20	554·548	16	8	2	1		
560,000	80	2,218·192	64	32	8	4	1	
4,480,000	640	17,745·536	512	256	64	32	8	1

Apothecaries' Measure.

The gallon of the former wine measure and of the present Apothecaries' measure contains 58,333·31 grains' weight of distilled water, or 231 cubic inches, the ratio to the imperial gallon being nearly as 5 to 6, or as 0·8331 to 1.

Gallon.	Pints.	Ounces.	Drachms.	Minims.	Gr. of Dist. Wat.	Cub. Inch.
1	8	128	1024	61,440	58,333·31	231
	1	16	128	7,680	7,291·66	28·8
		℥ 1	8	480	455.72	1·8
			ʒ 1	60	56·96	0·2

ENGLISH WEIGHTS.

Avoirdupois Weight.

Pound.	Ounces.	Drachms.	Grains.
1	16	256	7000
	1	16	437·5
		1	27·34375

Apothecaries' Troy Weight.

Pound.	Ounces.	Drachms.	Scruples.	Grains.
1	12	96	288	5760
	1	8	24	480
		1	3	60
			1	20

FRENCH MEASURES.

Measures of Length.

1 Kilometre = 1000 Metres.
1 Hectometre = 100 "
1 Decametre = 10 "
1 Metre = 1 "

1 Metre = 1·000 Metre
1 Decimetre = 0·100 "
1 Centimetre = 0·010 "
1 Millimetre = 0·001 "

1 Kilometre = 0·6214 Mile.
1 Metre = 3·2809 Feet.

1 Centimetre = 0·3937 Inch.
2·539954 c. m. = 1 Inch.

Comparison of Standard Measures.

1 Metre = 3·28089917 English Feet, = 3·28070878 American Feet.
1 Metre = 3·07844400 Paris Feet, = 39·36850535 American Inches.

Measures of Volume.

1 Cubic Metre = 1000·000 Litres.
1 Cubic Decimetre = 1·000 "
1 Cubic Centimetre = 0·001 "

1 Litre = 0·22017 Gallon.
1 Litre = 0·88066 Quart.
1 Litre = 1·76133 Pints.

1 Cubic Metre = 35·31660 Cubic Feet.
1 Cubic Decimetre = 61·02709 Cubic Inches.
1 Cubic Centimetre = 0·06103 " "

FRENCH WEIGHTS.

1 Kilogramme = 1000 Grammes.
1 Hectogramme = 100 "
1 Decagramme = 10 "
1 Gramme = 1 "

1 Gramme = 1·000 Gramme
1 Decigramme = 0·100 "
1 Centigramme = 0·010 "
1 Milligramme = 0·001 "

1 Kilogramme = 2·67951 Pounds (Troy), = 2·20485 Pounds (Avoirdupois).
1 Gramme = 15·44242 Grains.

To convert French metrical quantities into English measures and weights consult Table II.

To convert Grains into Grammes.

Log. Grains + (— 2·8115680) = Log. Grammes.

To convert Cubic Inches into Cubic Centimetres.

Log. Cubic Inches + 1·2144993 = Log. Cubic Centimetres.

To convert Inches into Millimetres.

Log Inches + 1·4048337 = Log. Millimetres

TABLE II.

FOR CONVERTING FRENCH DECIMAL MEASURES AND WEIGHTS INTO ENGLISH MEASURES AND WEIGHTS.

A.—MEASURES OF LENGTH.

	1	2	3	4	5	6	7	8	9
Metre,									
English yards	1·09363	2·18727	3·28090	4·37453	5·46816	6·56180	7·65543	8·74906	9·84270
English feet .	3·28090	6·56180	9·84270	13·12360	16.40450	19·68539	22·96629	26·24719	29·52809
English inches	39·37080	78·74158	118·11236	157·48315	196·85394	236·22473	275·59552	314·96630	354·33709
Decimetre,									
Feet . . .	0·32809	0.65618	0·98427	1·31236	1·64045	1·96854	2·29663	2·62472	2·95281
Inches . . .	3·93708	7.87416	11·81124	15·74832	19·68539	23·62247	27·55955	31·49663	35·43371
Centimetre,									
Inches . . .	0·39371	0·78742	1·18112	1·57483	1·96854	2·36225	2·75596	3·14966	3.54337
Millimetre,									
Inches . . .	0·03937	0·07874	0·11811	0·15748	0·19685	0·23623	0·27560	0·31497	0·35434

B.—MEASURES OF CAPACITY.

	1	2	3	4	5	6	7	8	9
Cubic Centimetre,									
Cubic inch . .	0·06103	0·12205	0·18308	0·24411	0·30514	0·36616	0·42719	0·48822	0·54924
Litre,									
Eng. imp. galls.	0·22017	0·44033	0.66050	0·88066	1·10083	1·32100	1·54116	1·76133	1·98149
Eng. imp. qts.	0.88066	1·76133	2.64199	3·52266	4·40332	5·28398	6·16465	7·04531	7·92598
Eng. imp. pints	1·76133	2·52266	5.28399	7·04531	8·80664	10·56797	12·32930	14·09062	15·85195
Stere,									
Gallons . .	220·16643	440·33287	660·49930	880·66574	1100·83217	1320·99860	1541·16504	1761·33147	1981·49791

TABLE II.—(*Continued.*)

FOR CONVERTING FRENCH DECIMAL MEASURES AND WEIGHTS INTO ENGLISH MEASURES AND WEIGHTS.

C.—Weights.

	1	2	3	4	5	6	7	8	9
Kilogramme,									
Cwt.	0·01970	0·03939	0 05909	0·07879	0 09848	0·11818	0 13788	0·15758	0·17727
Pound (Avoird.)	2·20486	4 40971	6 61457	8·81943	11·02428	13·22914	15·43400	17·63886	19·84371
Kilogramme,									
Pound (Troy) .	2·67951	5·35903	8·03854	10·71805	13·39757	16·07708	18·75659	21·43610	24·11562
Gramme,									
Grains . . .	15·44242	30 88484	46.32726	61·76968	77·21210	92·65352	108 09694	123·53936	138·98178
Decigramme,									
Grains . . .	1·54424	3.08848	4·63273	6·17697	7·72121	9·26535	10·80969	12·35394	13·89818
Centigramme,									
Grains . . .	0 15442	0·30885	0·46327	0·61770	0·77212	0·92654	1.08097	1·23539	1·38982
Milligramme,									
Grains . . .	0 01544	0·03089	0·04633	0·06177	0·07721	0·09265	0·10810	0·12354	0·13898

The following example will illustrate the method of using this table:—

Let it be required to find how many grains are equal to 87·435 grammes.

By column 8, line 4, of Table C, we find : 80 grammes = 1235·3936 grains.
" 7, " 4, " " " 7 " = 108 0969 "
" 4, " 5, " " " .4 " = 6 1770 "
" 3, " 6, " " " .03 " = 0 4633 "
" 5, " 7, " " " .005 " = 0·0772 "

87·435 grammes = 1350·2080 grains.

TABLE III.

EXPANSION OF SOLIDS.

1,000,000 parts at 32° F.	At 212° F. become	Expansion. In length.	Expansion. In bulk.	Authority.
English Flint Glass	1,000,811	1 in 1248	1 in 316	Lavoisier & Laplace.
Glass tube (French)	1,000,861	1 in 1148	1 in 382	Dulong & Petit.
Platinum	1,000,884	1 in 1131	1 in 377	
Palladium	1,001,000	1 in 1000	1 in 333	Wollaston.
Tempered Steel	1,001,079	1 in 926	1 in 309	Lavoisier & Laplace.
Antimony	1,001,083	1 in 923	1 in 307	Smeaton.
Iron	1,001,182	1 in 846	1 in 282	Dulong & Petit.
Bismuth	1,001,392	1 in 718	1 in 239	Smeaton.
Gold	1,001,466	1 in 682	1 in 227	Lavoisier & Laplace.
Copper	1,001,718	1 in 582	1 in 194	
Brass	1,001,866	1 in 536	1 in 179	
Silver	1,001,909	1 in 524	1 in 175	
Tin	1,001,937	1 in 516	1 in 172	
Lead	1,002,848	1 in 351	1 in 117	
Zinc	1,002,942	1 in 340	1 in 113	Smeaton.

INCREASE OF MEAN EXPANSION BY HEAT.

	Expansion for each degree F.		
	Between 32° and 212°.	Between 32° and 392°.	Between 32° and 572°.
Glass	1 in 69660	1 in 65340	1 in 59220
Platinum	1 in 67860	. . . .	1 in 65340
Iron	1 in 50760	. . . .	1 in 40860
Copper	1 in 34920	. . . .	1 in 21060
Mercury	1 in 9990	1 in 9965	1 in 9518

1,000,000 parts at 62° F.	At 212° F.	At 662° F.	At Freezing Point.
Black lead ware	1,000,244	1,000,703	
Wedgewood ware	1,000,735	1,002,995	
Platinum	1,000,735	1,002,995	1,009,926 maximum, but not fused.
Cast iron	1,000,893	1,003,943	1,016,389
Wrought iron	1,000,984	1,004,483	1,018,378 to the fusing point of cast iron.
Copper	1,001,430	1,006,347	1,024,376
Silver	1,001,626	1,006,886	1,020,640
Zinc	1,002,480	1,008,527	1,012,621
Lead	1,002,323		1,009,072
Tin	1,001,472		1,037,980

TABLE IV.

EXPANSION OF LIQUIDS.

BETWEEN 32° AND 212° F.

1,000,000 parts mercury become . .	1,018,153	1 in 55	Regnault.
" " pure water become .	1,046,600	1 in 21·3	Dalton.
" " sulphuric acid become	1,058,823	1 in 17	Dalton.
" " chlorohydric acid become	1,058,823	1 in 17	Dalton.
" " oil turpentine become	1,071,428	1 in 14	Dalton.
" " sulphuric ether become	1,071,428	1 in 14	Dalton.
" " fixed oils become . .	1,080,000	1 in 12·5	Dalton.
" " alcohol become . . .	1,111,000	1 in 9	Dalton.
" " nitric acid become . .	1,111,000	1 in 9	Dalton.

EXPANSION OF LIQUIDS OF SIMILAR CHEMICAL COMPOSITION.

°C.	Aldehyde. $C_4H_4O_2$.		Butyric Acid. $C_8H_8O_4$.		Acetate of Ethyl. $C_8H_8O_4$.	
	Pierre. (B. P. 22°.)	Kopp. (20·8°.)	Pierre. (163°.)	Kopp. (157°.)	Pierre. (74·1°.)	Kopp. (74·3°.)
0	10000	10000	10000	10000	10000	10000
10	9817	9830	9872	9867	9846	9843
25	9567	9596	9688	9667	9629	9622
45	9284		9453	9439	9359	9352
60	9094		9288	9271	9172	9165
75			9128	9112	8996	8988
110			8781	8765	8633	

°C.	Chlorid of Ethylene. $C_4H_4Cl_2$.	Monochlorinated Chlorid of Ethyl. $C_4H_4Cl_2$.	Monochlorinated Chlor of Ethylene. $C_4H_3Cl_3$.	Bichlorinated Chlorid of Ethyl. $C_4H_3Cl_3$.	Formiate of Ethyl. $C_6H_6O_4$.		Acetate of Methyl.	
	Pierre. (84·9°.)	Pierre. (64·8°.)	Pierre. (114·2°.)	Pierre. (74·9°.)	Pierre. (52·9°.)	Kopp. (54·9°.)	Pierre. (59·5°.)	Kopp. (56·3°.)
0	10000	10000	10000	10000	10000	10000	10000	10000
25	9667	9669	9693	9648	9632	9631	9633	9631
55	9331	9300	9350	9267	9241	9243	9243	9243
80	9068	9003	9090	8988	8953		8955	

TABLE V.

EXPANSION OF GASES.

EXPANSION FOR A CONSTANT VOLUME.*

Air.			Carbonic Acid.		
Pressure at 32° F.	Pressure at 212° F.	Expansion for 180° F.	Pressure at 32° F.	Pressure at 212° F.	Expansion for 180° F.
m. m.	m. m.		m. m.	m. m.	
109·72	149·31	0·36482	758·47	1034·54	0·36856
174·36	237·17	0·36513	901·09	1230·37	0·36943
266·06	395·07	0·36542	1742·93	2387·72	0·37523
374·67	510·35	0·36587	3589·07	4759·03	0·38598
375·23	510·95	0·36572			
760·00	"	0·36650			
1678·40	2286·09	0·36760			
1692·53	2306·23	0·36800			
2144.18	2924·04	0·36894			
3655.66	4992·09	0·37091			

EXPANSION FROM 32° TO 212° F. AT A CONSTANT PRESSURE.*

Hydrogen.		Air.		Carbonic Acid.		Sulphurous Acid.	
m. m.		m. m.		m. m.		m. m.	
760	0·36613	760	0·36706	760	0·37099	760	0·3902
2545	0·36616	2525	0·36944	2520	0·38455	980	0·3980
		2620	0·36964				

TABLE VI.

RADIATING POWER ACCORDING TO PROVOSTAYE, DESAINS, AND MELLONI.

Lampblack being . . .	100	Rough silver (deposited on copper)	5·36
Pure rolled silver . . .	3·00	Burnished silver (pure) . .	2·25
Pure burnished silver . .	2·50	Burnished platinum . . .	9·50
Rolled platinum . .	10·80	Sheet copper	4·90
Gold in leaf	4·28		

---

* Cours de Physique. Par M. J. Jamin. Tome ii. p. 70.

TABLE VII.

CONDUCTING POWER OF METALS AND BUILDING MATERIALS

A.—CONDUCTING POWER OF METALS.

Name of Metal.	Despretz.	Wiedemann & Franz.	Becquerel.
Gold	1000·0	1000	1000
Platinum	981·0	158	124·91
Silver	973·0	1880	1451·37
Copper	898·2	1383	1383·61
Brass		444	
Steel		218	
Iron	374·3	224	188·3
Zinc	363·0		375·3
Tin	303·9	273	212·09
Lead	179·6	160	128·65
Palladium		118	217·08
Bismuth		34	
Marble	23·6		
Porcelain	12·2		
Brick clay	11·4		

B.—CONDUCTING POWER OF BUILDING MATERIALS.

Name of Substance.	Conducting power referred to slate = 100.	Name of Substance.	Conducting power referred to slate = 100.
Plaster and sand . .	18·70	Bath stone	61·08
Keene's cement . .	19·01	Fire brick	61·70
Plaster of Paris . .	20·26	Paniswick stone, H. P.	71·36
Roman cement . .	20·88	Malen brick . . .	72·92
Lath and plaster . .	25·55	Portland stone . . .	75·10
Fir wood	27·61	Lunelle marble . .	75·41
Oak wood	33·66	Balsover stone, H. P.	76·35
Asphalt	45·19	Norfal stone, H. P. .	95·36
Chalk (soft) . . .	56·38	Slate	100·00
Napoleon marble . .	58·27	Yorkshire flag . . .	110·94
Stack brick	60·14	Lead	521·34

TABLE VIII.

ABSORPTIVE POWER OF DIFFERENT BODIES.

Names.	Absorptive Power.	Reflective Power.
Smoke blackened surface	100	0
Carbonate of lead	100	0
Writing paper	98	2
Glass	90	10
China ink	85	15
Gum lac	72	28
Silver foil on glass	27	73
Cast iron, polished	25	75
Wrought iron, polished	23	77
Mercury	23	77
Zinc, polished	19	81
Steel	17	83
Platinum, thick coat, imperfectly polished	24	76
" on copper	17	83
" leaves	17	83
Tin	14	80
Metallic mirrors, a little tarnished	17	83
" " nearly polished	14	86
Brass, cast, imperfectly polished	11	89
" hammered, imperfectly polished	9	91
" " highly polished	7	93
" cast, " "	7	93
Copper, coated on iron	7	93
" varnished	14	86
" hammered or cast	7	93
Gold plating	5	95
Gold deposited on polished steel	3	97
Silver, hammered, and well polished	3	97
Silver, cast, and well polished	3	97

TABLE IX.

ABSORPTIVE POWER FOR HEAT FROM DIFFERENT SOURCES.

Name of Substance.	Incandescent Platinum.	Copper at 400°.	Copper at 100°.
Lampblack	100	100	100
Carbonate of lead	56	89	100
China ink	95	87	85
Isinglass	54	64	91
Lac	47	70	72
Metallic surface	13·5	13	13

TABLE X.

DIATHERMANCY OF DIFFERENT LIQUIDS.

Of 100 incident rays.

	Transmitted		Transmitted
Bisulphid of carbon (colorless)	63	Ether	21
Bichlorid of sulph. (red brown)	62	Sulphuric acid (colorless)	17
Terchlorid of phosphorus .	62	Sulphuric acid (brown) . .	17
Essence of turpentine . . .	31	Nitric acid	14
Colza oil (yellow)	30	Alcohol	15
Olive oil (greenish) . . .	30	Distilled water	11

TABLE XI.

[*Ratio of Specific Heat to Atomic Weight.*]

SPECIFIC HEAT.

A.—SOLIDS.

Water = 1.00.

Names.	Specific Heats. C.	Atomic Weights. p.	Product, C × p.
Aluminum	0·2143	13·7	2·94
Sulphur	0·2026	16	3·24
Iron	0·1138	28	3·19
Cobalt	0·1070	29·5	3·16
Nickel	0·1086	29·6	3·21
Copper	0·0952	31·7	3·02
Zinc	0·0956	32·6	3·12
Selenium	0·0762	40	3·04
Tin	0·0562	59	3·31
Platinum	0·0324	98·7	3·20
Lead	0·0314	103·7	3·26
Phosphorus	0·1887	31	5·85
Arsenic	0·0814	75	6·10
Silver	0·0570	108	6·16
Iodine	0·0541	127	6·87
Antimony	0·0508	120·3	6·11
Gold	0·0324	197	6·38
Bismuth	0·0308	208	6·41
B.—LIQUIDS.			
Mercury (liquid)	0·03331	100	3·33
Mercury (solid)	0·03241	100	3·24
Bromine (liquid)	0·11094	80	
Bromine (solid) 28° C. . .	0·08432	80	6·74

TABLE XI —(*Continued.*)

SPECIFIC HEAT.

C.—GASES AND VAPORS.

Name of Substance.	Capacity for equal weights. Water = 1.	Capacity for equal volumes. Water of equal weight being = 1.	Specific Gravity.
Atmospheric air*	0·2379		1·0000
Oxygen	0·2182	0·2412	1·1056
Nitrogen	0·2440	0·2370	0·9713
Hydrogen	3·4046	0·2356	0·0692
Chlorine	0·1214	0·2967	2·4400
Bromine	0·0552	0·2992	5·3900
Nitrous oxyd	0·2238	0·3413	1·5250
Nitric oxyd	0·2315	0·2406	1·0390
Carbonic oxyd	0·2479	0·2399	0·9674
Carbonic acid	5·2164	0·3308	1·5290
Sulphid of carbon	0·1575	0·4146	2·6325
Sulphurous acid	0·1553	0·3489	2·2470
Ammonia gas	0·5080	0·2994	0·5894
Olefiant gas	0·3694	0·3572	0·9672
Water vapor	0·4750	0.2950	0·6210
Alcohol vapor	0·4513	0.7171	1·5890
Ether vapor	0·4810	1·2296	2·5563
Chloroform	0·1568	0·8310	5·3000
Vapor of mercury			6·9760
Vapor of Iodine			8·7160

TABLE XII.

FREEZING MIXTURES.

Substances.	Parts by Weight.	Cooling in Degrees F.
Sulphate of soda	8	from + 50° to 0°
Hydrochloric acid	5	
Snow or ice	2	“ x “ — 5°
Common salt	1	
Sulphate of soda	3	“ + 50° “ — 3°
Dilute nitric acid	2	
Sulphate of soda	6	“ + 50° “ — 14°
Nitrate of ammonia	5	
Dilute nitric acid	4	
Snow or ice	3	“ + 20° “ — 14°
Chloride of calcium	4	

---

* De la Roche and Berard.

TABLE XIII.

DIATHERMANCY OF DIFFERENT SOLIDS.

Substance of Screens.	Sources of Heat.			
[Each plate was 2·62 m. m. (0·1 in.) in thick.]	Naked Flame.	Ignited Platinum.	Copper 750° F.	Copper 212° F.
Rock salt (limpid)	92·3	92·3	92·3	92·3
Silician sulphur (yellow) . . .	74	77	60	54
Fluor spar (limpid)	72	69	42	53
Rock-salt (cloudy)	65	65	65	65
Beryl (greenish yellow) . . .	46	38	24	20
Iceland spar (limpid)	39	28	6	0
Plate glass	39	24	6	0
Quartz (limpid)	38	28	6	3
Quartz (smoky)	37	28	6	3
White topaz	33	24	4	0
Tourmaline (dark green) . . .	18	16	3	0
Citric acid	11	2	0	0
Alum	9	2	0	0
Sugar candy (limpid)	8	1	0	0

TABLE XIV

TENSION OF VAPORS AT EQUAL DISTANCES ABOVE AND BELOW THE BOILING POINTS OF THEIR RESPECTIVE LIQUIDS.

Number of degrees above or below boiling.	Regnault.		Ure.		Ure.		Marx.		Avogadro.	
	Water.		Alcohol. Sp. Gr. 0·813.		Ether.		Sulph't Carbon.		Mercury.	
	Temp. °F.	Pressure inches.	Temp. °F.	Pressure inches.	Temp. °F.	Pressure inches.	Temp. °F.	Pressure inches.	Temp. °F.	Pressure inches.
+ 40°	252	63·14								
+ 20°	232	44·06			124	42·64	137	40·19		
Boiling p't.	212	30·00	173	30·00	104	30·00	117	29·87	680	30·00
— 20°	192	19·87	153	19·30	84	20·90	97	20·65		
— 40°	172	12·78	133	11·60	64	13·00	77	13·89	630	19·85
— 60°	152	7·94	113	6·70	44	8·10	57	9·07		
— 80°	132	4·67	93	3·67			37	5·73	590	14·08

TABLE XV.

MELTING POINTS AND LATENT HEAT OF FUSION OF DIFFERENT BODIES.

Substances.	Melting Point. °F.	Latent Heat.* °F.	Water = 1.
Mercury	— 39	5·11	0·035
Oil of vitriol	— 30		
Bromine	— 4		
Water		142·1	1·000
Phosphorus	+ 111	8·08	0·056
Potassium (about) . . .	131		
Yellow wax	143	78·32	0·551
Sodium	190		
Iodine	224		
Sulphur	239	16·51	0·116
Tin	455	25·74	0·180
Bismuth	518	22·30	0·156
Lead	630	9·27	0·065
Zinc	761	49·43	0·347
Antimony	963		
Silver	1873	37·92	0·265
Copper	†2143		
Gold	2016		
Cast iron (above) . . .	2786		
Wrought iron	3280		
Platinum	4591		
Nitrate of soda	591	113·36	·797
Nitrate of potash . . .	642	83·12	·584
Nitrate of silver . . .		113·34	·704

TABLE XVI.

BOILING POINT OF WATER UNDER DIFFERENT PRESSURES.

Boiling Point. °F.	Barometer. Inches.	Boiling Point. °F.	Barometer. Inches.
184	16·676	200	23·454
186	17·421	202	24·441
188	18·196	204	25·468
190	18·992	206	26·529
192	19·822	208	27·614
194	20·687	210	28·744
196	21·576	212	29·922
198	22·498	214	31·120

---

* The numbers in this column may be considered as the number of pounds of water that could be raised 1° F. by the heat emitted during the congelation of one pound of each of the substances included in the table.

† Plattner.

TABLE XVII.

BOILING POINTS OF LIQUIDS.

	Temperature. °F.		Temperature. °F.
Sulphurous acid .	17.6	Nitric acid, sp. gr. 1·42	248·0
Chlorid of ethyl .	51·9	Bichlorid of tin . .	240·2
Aldehide	69·4	Fousel oil	269·8
Ether	94·8	Terchlorid of arsenic	230·0
Bisulphid of carbon	118·5	Butyric acid . . .	314·6
Terchlorid of silicon	138·2	Naptha	320·0
Ammonia, sp. gr. 0·945	140·0	Sulphurous ether .	320·0
Bromine	145·4	Phosphorus . . .	554·0
Wood spirit . . .	149·9	Oil of turpentine .	568·5
Alcohol	173·1	Linseed oil . . .	597·0
Dutch liquid . . .	184.7	Sulp. acid, sp. gr. 1·843	620·0
Water	212.0	Mercury	662·0

TABLE XVIII.

BOILING POINT OF WATER AT DIFFERENT PLACES AND THEIR ELEVATION ABOVE THE SEA.

Names of Places.	Above (or below) the level of the sea.	Mean height of the Barometer.	Thermometer.
	Feet.	Inches.	Degrees.
Donkia (Himalaya)	+ 17,337	15·442	179·90
Donkia Pass (Himalaya) . . .	16,621	15·489	181·40
Farm of Antisana, S. A. . . .	13,455	17·870	187·30
Micuipampa (Peru)	11,870	19·020	190·20
Quito	9,541	20·750	194·20
Mexico	7,471	22·520	198·10
Hospice of St. Gothard . . .	6,808	23.070	199·20
Black Mountain, N. C. (highest point in the eastern U. S.)* .	6,702	22·502	†199·67
Mount Washington, N. H. . .	6,290	22·905	†200·43
Madrid	1,995	27·720	208·00
Salzburg	1,483	28·270	209·10
Plombieres	1,381	28·390	209·30
Moscow	984	28·820	210·20
Vienna	436	29·410	211·10
Rome	151	29·760	211·60
Dead Sea (*below* Mediterranean Sea)	— 1316·7	31·496	†214·44

* Guyot. † Estimated by Forbes' coefficient.

TABLE XIX.

BOILING POINT OF WATER AT DIFFERENT ATMOSPHERIC PRESSURES.—REGNAULT.

Pressure in atmospheres of 30 inches mercury.	Boiling Point of Water.	Pressure in atmospheres of 30 inches mercury.	Boiling Point of Water.
1	212 °F.	11	364·2 °F
2	249·5	12	371·1
3	273·3	13	377·8
4	291·2	14	384
5	306·0	15	390
6	318·2	16	395·4
7	329·6	17	400·8
8	339·5	18	405·9
9	348·4	19	410·8
10	356·6	20	415·4

TABLE XX.

LIQUEFACTION AND SOLIDIFICATION OF GASES.

Names of the Gases.	Melting Point. °F.	Pressure in Atmospheres.		
		At 32° F.	At 60° F.	°F.
Sulphurous acid . . .	— 105°	1·53	2·54	5·16 at 100°
Cyanogen	— 30	2·37		4·00 at 63
Hydriodic acid . . .	— 60	3·97	5·86	
Ammonia	— 103	4·4	6·90	10·00 at 83
Sulphuretted hydrogen .	— 122	10		14·60 at 52
Protoxid nitrogen . .	— 150	32		33·40 at 35
Carbonic acid	— 70	38·5		
Euchlorine	— 65			
Hydrobromic acid . .	— 124			
Fluorid of silicon . . .	— 220			
Chlorine		8·95	13·19	
Arseniuretted hydrogen				
Phosphuretted hydrogen				
Olefiant gas				26·90 at 0°
Fluorid of boron . . .				11·54 at — 62
Hydrochloric acid . .		26·20		40 at 50

TABLE XXI.

AQUEOUS VAPOR IN A CUBIC FOOT OF SATURATED AIR AT DIFFERENT TEMPERATURES.

Degrees Fahrenheit.	Degrees Fahrenheit.									
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
	Grains.	Grains.	Grains.	Grains.	Grains.	Grains.	Grains.	Grains.	Grains.	Grains.
0°	0·545	0·569	0·595	0·621	0.649	0·678	0·708	0·739	0·772	0·806
10°	0·841	0·878	0·916	0·957	0·999	1·043	1.090	1·138	1·190	1·243
20°	1·298	1·355	1.415	1·476	1·540	1·606	1·674	1·745	1·817	1·892
30°	1·969	2·046	2·126	2·208	2·292	2·379	2·469	2·563	2·659	2·759
40°	2·862	2·967	3·076	3·180	3·306	3·426	3.550	3·679	3·811	3·948
50°	4·089	4·234	4.383	4·537	4·696	4·860	5·028	5·202	5·381	5·562
60°	5·756	5·952	6·154	6·361	6·575	6·795	7·021	7·253	7·493	7·739
70°	7·992	8·252	8·521	8·797	9·081	9.372	9·670	9·977	10.292	10·616
80°	10·949	11·291	11·643	12·005	12·376	12.756	13·146	13·546	13·957	14·378
90°	14·810	15·254	15·709	16·176	16·654	17·145	17·648	18·164	18·693	19·235
100°	19·790	20·357	20·938	21·535	22·145	22·771	23·411	24·069	24·742	25·429

TABLE XXII.—LATENT AND SENSIBLE HEAT OF STEAM.

Temp.	Latent Heat.	Sum of Latent and Sensible Heat.	Temp.	Latent Heat.	Sum of Latent and Sensible Heat.
32°	1092·6°	1124·6°	248°	939·6°	1187·6°
68	1067·4	1135·4	284	914·4	1198·4
86	1054·8	1140·8	320	889·2	1209·2
104	1042·2	1146·2	338	874·8	1212·8
140	1017·0	1157·0	374	849·6	1223·6
194	979·2	1173·2	410	822·6	1232·6
212	966·6	1178·6	446	795·6	1241·6

REGNAULT'S RESULTS. § 683.

Pressure in Atmospheres.	Temperature.	Latent Heat.	Sum of Latent and Sensible Heat.
0·0014	0°	1114·0°	1114·0°
0·006	32	1091·7	1123·7
1·000	212	966·6	1178·6
8.000	339	877·3	1216·8

TABLE XXIII.—SPECIFIC GRAVITY OF SOLIDS AND LIQUIDS.

Substances.	Sp. Gravity.	Substances.	Sp. Gravity.
Platinum	21·	Sulphate of lime . .	2·32
Gold	19·24	Sulphur	2·03
Tungsten	17·	Bone	1·8–1·99
Mercury	13·60	Ivory	1·92
Rhodium and Palladium	11·	Caoutchouc	0·989
Silver	10·47	Sodium	0·97
Bismuth	9·82	Wax	0·97
Copper	8·78	Gutta-percha . . .	0·966
Arsenic	8·60	Ice	0·9175
Steel	7·81	Pumice-stone . . .	0·92
Iron	7·78	Potassium	0·86
Meteoric iron . . .	7·26–7·79	Pine wood	0·66
Cast iron	7·21	Cypress wood . . .	0·60
Zinc	6·86	Cedar wood	0·56
Antimony	6·71	Common poplar . .	0·38
Iodine	4·95	Lombardy poplar . .	0·36
Heavy spar	4·43	Cork	0·24
Oriental ruby . . .	4·28	LIQUIDS.	
Topaz	3·56	Sulphuric acid . . .	1·84
Diamond	3·50	Nitrous acid	1·55
English flint-glass .	3·33	Water from Dead Sea	1·24
Parian marble . . .	2·84	Nitric acid	1·22
Emerald	2·77	Milk	1·03
Pearl	2·75	Wine	0·99
Iceland spar . . .	2·72	Linseed oil	0·94
Common marble . .	2·70	Spirits turpentine . .	0·87
Coral	2·68	Absolute alcohol . .	0·79
Quartz	2·65	Naphtha, "light oil" .	0·733
Agate	2·61	Sulphuric ether . .	0·72
St. Gobain's glass . .	2·49	Eupion	0·655

TABLE XXIV.

VOLUME AND DENSITY OF WATER.—BY KOPP.

Temperature C.	Volume of Water (at 0° = 1).	Sp. Gr. of Water (at 0° = 1).	Volume of Water (at 4° = 1).	Sp. Gr. of Water (at 4° = 1).
0°	1·00000	1·000000	1·00012	0·999877
1	0·99995	1·000053	1·00007	0·999930
2	0·99991	1·000092	1·00003	0·999969
3	0·99989	1·000115	1·00001	0·999992
4	0·99988	1·000123	1·00000	1·000000
5	0·99988	1·000117	1·00001	0·999994
6	0·99990	1·000097	1·00003	0·999973
7	0·99994	1·000062	1·00006	0·999939
8	0·99999	1·000014	1·00011	0·999890
9	1·00005	0·999952	1·00017	0·999829
10	1·00012	0·999876	1·00025	0·999753
11	1·00021	0·999785	1·00034	0·999664
12	1·00031	0·999686	1·00044	0·999562
13	1·00043	0·999572	1·00055	0·999449
14	1·00056	0·999445	1·00068	0·999322
15	1·00070	0·999306	1·00082	0·999183
16	1·00085	0·999155	1·00097	0·999032
17	1·00101	0·998992	1·00113	0·998869
18	1·00118	0·998817	1·00131	0·998695
19	1·00137	0·998631	1·00149	0·998509
20	1·00157	0·998435	1·00169	0·998312
21	1·00178	0·998228	1·00190	0·998104
22	1·00200	0·998010	1·00212	0·997886
23	1·00223	0·997780	1·00235	0·997657
24	1·00247	0·997541	1·00259	0·997419
25	1·00271	0·997293	1·00284	0·997170
26	1·00295	0·997035	1·00310	0·996912
27	1·00319	0·996767	1·00337	0·996644
28	1·00347	0·996489	1·00365	0·996367
29	1·00376	0·996202	1·00393	0·996082
30	1·00406	0·995908	1·00423	0·995787
35	1·00570			
40	1·00753			
45	1·00954			
50	1·01177			
55	1·01410			
60	1·01659			
65	1·01930			
70	1·02225			
75	1·02541			
80	1·02858			
85	1·03189			
90	1·03540			
95	1·03909			
100	1·04299			

TABLE XXV.

COMPARISON OF THE DEGREES OF BEAUMÉ'S HYDROMETER WITH THE REAL SPECIFIC GRAVITY.

A.—FOR LIQUIDS HEAVIER THAN WATER.

Degrees.	Specific Gravity.	Degrees.	Specific Gravity.	Degrees.	Specific Gravity.	Degrees.	Specific Gravity.
0	1·000	20	1·152	40	1·357	60	1·652
1	1·007	21	1·160	41	1·369	61	1·670
2	1·013	22	1·169	42	1·381	62	1·689
3	1·020	23	1·178	43	1·395	63	1·708
4	1·027	24	1·188	44	1·407	64	1·727
5	1·034	25	1·197	45	1·420	65	1·747
6	1·041	26	1·206	46	1·434	66	1·767
7	1·048	27	1·216	47	1·448	67	1·788
8	1·056	28	1·225	48	1·462	68	1·809
9	1·063	29	1·235	49	1·476	69	1·831
10	1·070	30	1·245	50	1·490	70	1·854
11	1·078	31	1·256	51	1·495	71	1·877
12	1·085	32	1·267	52	1·520	72	1·900
13	1·094	33	1·277	53	1·535	73	1·924
14	1·101	34	1·288	54	1·551	74	1·949
15	1·109	35	1·299	55	1·567	75	1·974
16	1·118	36	1·310	56	1·583	76	2·000
17	1·126	37	1·321	57	1·600		
18	1·134	38	1·333	58	1·617		
19	1·143	39	1·345	59	1·634		

B.—FOR LIQUIDS LIGHTER THAN WATER.

Degrees.	Specific Gravity.	Degrees.	Specific Gravity.	Degrees.	Specific Gravity.	Degrees.	Specific Gravity.
10	1·000	23	·918	36	·849	49	·789
11	·993	24	·913	37	·844	50	·785
12	·986	25	·907	38	·839	51	·781
13	·980	26	·901	39	·834	52	·777
14	·973	27	·896	40	·830	53	·773
15	·967	28	·890	41	·825	54	·768
16	·960	29	·885	42	·820	55	·764
17	·954	30	·880	43	·816	56	·760
18	·948	31	·874	44	·811	57	·757
19	·942	32	·869	45	·807	58	·753
20	·936	33	·864	46	·802	59	·749
21	·930	34	·859	47	·798	60	·745
22	·924	35	·854	48	·794		

TABLE XXVI.

TENSION, VOLUME, AND DENSITY OF AQUEOUS VAPOR.

Temperature of Vapor. Degrees Centigrade.	Tension of Vapor expressed in Atmospheres.	Tension expressed by a column of mercury in metres.	Volume occupied by a kilogramme of vapor, in cubic metres.	Weight of a cubic metre of Aqueous Vapor in kilogrammes.
0°	$\frac{1}{166}$ or 0·0060	0·00460	205·222400	0·00487266
17·86	$\frac{1}{50}$ or 0·0200	0·01520	66·144960	0·01512517
29·37	$\frac{1}{25}$ or 0·0400	0·03040	34·364490	0·02909000
33·30	$\frac{1}{20}$ or 0·0500	0·03800	27·852530	0·03590307
37·38	$\frac{1}{16}$ or 0·0625	0·04750	22·578310	0·04430600
42·66	$\frac{1}{12}$ or 0·0833	0·06330	17·232270	0·05803375
46·25	$\frac{1}{10}$ or 0·1000	0·07600	14·515640	0·06892666
50·60	$\frac{1}{8}$ or 0·1250	0·09500	11·769500	0·08495800
53·35	$\frac{1}{7}$ or 0·1428	0·10857	10·391950	0·09622000
56·63	$\frac{1}{6}$ or 0·1666	0·12666	8·996440	0·11119230
60·40	$\frac{1}{5}$ or 0·2000	0·15184	7·582970	0·13185937
65·36	$\frac{1}{4}$ or 0·2500	0·19000	6·156670	0·16240770
81·72	$\frac{1}{2}$ or 0·5000	0·38000	3·227120	0·30983570
92·18	$\frac{3}{4}$ or 0·7500	0·57000	2·215120	0·45141000
100·	1 or 1·0000	0·76000	1·696000	0·59130000
106·33	1¼ or 1·2500	0·95000	1·380541	0·72430000
111·83	1½ or 1·5000	1·14000	1·167228	0·85670000
116·50	1¾ or 1·7500	1·33000	1·012619	0·98752500
120·64	2 or 2·0000	1·52000	0·895462	1·11571700
124·39	2¼ or 2·2500	1·72000	0·798033	1·25305700
127·83	2½ or 2·5000	1·90000	0·729463	1·37087500
130·98	2¾ or 2·7500	2·09000	0·668363	1·49620700
133·91	3 or 3·0000	2·28000	0·616697	1·62038400
136·72	3¼ or 3·2500	2·47000	0·573576	1·74340000
139·29	3½ or 3·5000	2·66000	0·534694	1·86582600
141·72	3¾ or 3·7500	2·85000	0·503167	1·98731800
144·	4 or 4·0000	3·04000	0·474320	2·10828500
146·28	4¼ or 4·2500	3·23000	0·448860	2·22780000
148·44	4½ or 4·5000	3·42000	0·426093	2·34683300
150·35	4¾ or 4·7500	3·61000	0·405507	2·46605500
152·26	5 or 5·0000	3·80000	0·386960	2·58417600
154·15	5¼ or 5·2500	3·99000	0·370170	2·70135200
155·94	5½ or 5·5000	4·18000	0·354778	2·81225050
157·64	5¾ or 5·7500	4·37000	0·340669	2·93460000
159·25	6 or 6·0000	4·56000	0·327779	3·05078500
165·40	7 or 7·0000	5·32000	0·284938	3·50930700
170·84	8 or 8·0000	6·08000	0·252423	3·97063600
175·77	9 or 9·0000	6·84000	0·226771	4·40770000
180·30	10 or 10·0000	7·60000	0·206248	4·84844400
184·60	11 or 11·0000	8·36000	0·189189	5·28325000
188·54	12 or 12·0000	9·12000	0·174952	5·71425000
190·	12·4250	9·44300	0·169437	5·91142800
195·	13·8160	10·52000	0·153660	6·50428600
200·	15·3560	11·68900	0·138717	7·31716600
230·	27·5340	20·92600	0·083115	12·03722000

ANSWERS TO PROBLEMS.

Prob. 1. Ans. (*a.*) 147640·46 yds.; (*b.*) 1·021 inches; (*c.*) 0·03937079 inch; (*d.*) 1·1811237.

Prob. 2. Ans. (*a.*) 1·39697 metres; (*b.*) 1131·5495 metres; (*c.*) 20·921 kilometres; (*d.*) 4·57 metres.

Prob. 3. Ans. (*a.*) 1 litre and 703·258 cubic centimetres; (*b.*) 1·1 gallons; (*c.*) 31·7936 litres; (*d.*) 0·01232931 pint.

Prob. 4. Ans. (*a.*) 0·63499 metre; (*b.*) 54·7286 Amer. inches; (*c.*) 22·86 metres; (*d.*) 5468·48 Amer. yards.

Prob. 5. Ans. (*a.*) 4258·1458 cubic centimetres; (*b.*) 45419·4486 cubic centimetres; (*c.*) 0·1618 gallon.

Prob. 6. Ans. 0·1515 foot per second.

Prob. 7. Ans. $60 \times \frac{a}{n}$ feet.

Prob. 8. Ans. Unit of time = 0·3896 second.

Prob. 9. Ans. At an angle of 36° 52′ 12″ with the component 4, and with a velocity = 5.

Prob. 10. Ans. Speed of A = $\frac{48}{49}$ of speed of B.

Prob. 11. Ans. Velocity = 180 feet per second; distance = 1800 yards.

Prob. 12. Ans. 20 feet per second.

Prob. 13. Ans. Retardation = 25 feet per second; distance = $312\frac{1}{2}$ feet.

Prob. 14. Ans. $25142\frac{6}{7}$ lbs.

Prob. 15. Ans. 31,250 feet per second or 5.9 miles.

Prob. 16. Ans. It would not.

Prob. 17. Ans. 144 feet, 9 inches.

Prob. 18. Ans. 3 seconds.

Prob. 19. Ans. If h represent the height of the tower, the velocity required = $\sqrt{gh}$.

Prob. 20. Ans. If v represents the vertical velocity of the balloon, the height = $\frac{1}{2g}(gt - v)^2$.

Prob. 21. Ans. The height = $\frac{g}{8} \times \left(t + \frac{2a}{gt}\right)^2$.

Prob. 22. Ans. 396·03 feet.

Prob. 23. Ans. $64\frac{1}{3}$ feet.

Prob. 24. Ans. Height of bridge = 100·52 feet; time required = 2·4 seconds.

Prob. 25. Ans. $402\frac{1}{12}$ feet.
Prob. 26. Ans. Velocity $= \frac{193}{12} \times t$ seconds of ascent and return.
Prob. 27. Ans. 208·44 feet.
Prob. 28. Ans. 6·2 seconds.
Prob. 29. Ans. 96·75 feet.
Prob. 30. Ans. 103·37 miles per hour.
Prob. 31. Ans. The train cannot ascend such a grade without more steam. It would require an initial velocity of 54.69 miles per hour to overcome such a grade.
Prob. 32. Ans. 265·099 lbs.
Prob. 33. Ans. 3·3256 lbs.
Prob. 34. Ans. Twelve times its present velocity.
Prob. 35. Ans. 8·45 revolutions per second.
Prob. 36. Ans. 1·74 seconds.
Prob. 37. Ans. $31\frac{1}{4}$ feet.
Prob. 38. Ans. 0·88 second.
Prob. 39. Ans. 1·003 seconds.
Prob. 40. Ans. At New York $g = \pi^2 l = 32{\cdot}155399$ feet.
At Cape Horn $g = \pi^2 l = 32{\cdot}205083$ feet.
At Boston $g = 32{\cdot}17076\,(1 - 0{\cdot}00259 \cos 2\lambda) = 32{\cdot}163064$ ft.*
At New Orleans $g =$ " " " " $= 32{\cdot}125757$ ft.*
At Stockholm $g =$ " " " " $= 32{\cdot}131002$ ft.*
Prob. 41. Ans. $g' = \frac{9}{10} g$.
Prob. 42. Ans. 13916·25 feet = 2·64 miles.
Prob. 43. Ans. 31088 feet = 5·89 miles.
Prob. 44. Ans. 24·87 seconds.
Prob. 45. Ans. $35^\circ\ 19'\ 43''{\cdot}5$ or $54^\circ\ 40'\ 16''{\cdot}5$.
Prob. 46. Ans. 736·35 feet per second.
Prob. 47. Ans. $4\frac{1}{16}$ times greater.
Prob. 48. Ans. 21·21 miles per hour.
Prob. 49. Ans. As 1 to $2\frac{1}{4}$.
Prob. 50. Ans. 50 feet.
Prob. 51. Ans. As 1 to $7\frac{1}{32}$.
Prob. 52. Ans. $2\frac{2}{7}$ feet from the smaller weight.
Prob. 53. Ans. Under the weight 7.
Prob. 54. Ans. 125 lbs. and 75 lbs.
Prob. 55. Ans. At one point $9\frac{2}{3}$ cwt., at the other $20\frac{1}{3}$ cwt.
Prob. 56. Ans. Pressure on A = 10·4 cwt.; pressure on B = 17·6 cwt.
Prob. 57. Ans. 40 lbs.
Prob. 58. Ans. Diameter of axle $2\frac{1}{30}$ inches.
Prob. 59. Ans. $92\frac{4}{73}$ lbs.
Prob. 60. Ans. 156 cwt.
Prob. 61. Ans. 1382·4 tons.
Prob. 62. Ans. 4970 lbs.
Prob. 63. Ans. $3\frac{3}{4}$ cwt.
Prob. 64. Ans. 60 lbs.
Prob. 65. Ans. 12 cwt.
Prob. 66. Ans. A power equal to 8 cwt. would balance the train, but some additional force is required to impart motion independent of friction, which is not considered.
Prob. 67. Ans. 67858·56 lbs.

---

* Approximate values. See § 90.

Prob. 68. Ans. 79·58 lbs.

Prob. 69. Ans. 10 lbs.

Prob. 70. Ans. A force of 18 cwt. in both cases.

Prob. 71. Ans. 476·22 horse-power.

Prob. 72. Ans. 1·5874 times greater.

Prob. 73. Ans. 11·94 miles per hour.

Prob. 74. Ans. At 14°, 12321·53 kilogrammes; at 50°, 12925·69 kilogrammes; at 212°, 13885.42 kilogrammes; at 392°, 12291·67 kilogrammes.

Prob. 75. Ans. At 50° a rod having a section of one square millimetre would be elongated one-fourth of an inch.

Prob. 76. Ans. Double the weight in the first case.

Prob. 77. Ans. Sixteen times as much as if the beam were secured at one end and the weight applied at the other extremity.

Prob. 78. Ans. Tempered steel, 5634·95 to 7546·76 lbs.; untempered steel, 5433·5 to 6238·7 lbs.

Prob. 79. Ans. 0·07 inch.

Prob. 80. Ans. 3498·86 lbs. to 8724·71 lbs.

Prob. 81. Ans. 15·474 tons to 18·548 tons of 2000 lbs.

Prob. 82. Ans. Considering the ends secured by union with the entire structure, the breaking weight = 2990·18 tons; considering the ends not secured, but merely supported on the piers, the breaking weight = 1495·09 tons.

Prob. 83. Ans. If the ends are securely fastened the working load = 532·69 tons; but if the ends are merely supported the working load = 233·67 tons.

Prob. 84. Ans. 280·69 tons. Since the entire structure forms a continuous tube, the ends of the middle span are securely fastened.

Prob. 85. Ans. 59·43 tons.

Prob. 86. Ans. $v = 5{\cdot}85579$ feet per second; $v' = 8{\cdot}67579$ feet per second; $x = 6\frac{17}{19}$ feet per second.

Prob. 87. Ans. $m = 7\frac{1}{2}m'$.

Prob. 88. Ans. $e = 0{\cdot}6$.

Prob. 89. Ans. 153 feet.

Prob. 90. Ans. $r = \dfrac{1}{2n - 1}$.

Prob. 91. Ans. 25·6 feet.

Prob. 92. Ans. 84·3 feet per second.

Prob. 93. Ans. Condensation = 0·001006; specific gravity = 1·001007.

Prob. 94. Ans. Specific gravity = 1·0487.

Prob. 95. Ans. 0.5723 of a cubic inch.

Prob. 96. Ans. The pressure $= \mathrm{P} \times \dfrac{\mathrm{B}^2}{\mathrm{A}^2}$.

Prob. 97. Ans. Pressure : Power = 4050 : 1.

Prob. 98. Ans. Pressure on the bottom = the weight of the liquid = half the sum of the pressures on the four sides.

Prob. 99. Ans. 0·5773 *a*, 0·2392 *a*, and 0·1835 *a*.

Prob. 100. Ans. $\mathrm{P} : \mathrm{P}' = 1 : 2$.

Prob. 101. Ans. As the height of the cylinder to its radius.

Prob. 102. Ans. Pressure of water = 1101·3 lbs.; pressure of mercury = 7·48888 tons.

Prob. 103. Ans. Let R = pressure on the triangles, the pressure on the base being reckoned as unity. Then:—$\mathrm{R} = \frac{1}{3}\sqrt{1 + \frac{4c^2}{b^2}} + \frac{1}{3}\sqrt{1 + \frac{4c^2}{a^2}}$.

Prob. 104. Ans. 7138·1 grs. = 1·0197 lbs.
Prob. 105. Ans. 1·83823 feet.
Prob. 106. Ans. 14·4 inches.
Prob. 107. Ans. 500 lbs.
Prob. 108. Ans. 0·03789 × given weight of the iron.
Prob. 109. Ans. 11·50228 ounces.
Prob. 110. Ans. Gold = 13·8102 ounces, silver = 8·1898 ounces.

Prob. 111. Ans. The addition of 10 lbs. of lead weights under water to produce equilibrium shows that 209·5 lbs. of iron are concealed in the commercial lead.

Prob. 112. Ans. 5·0752 lbs.
Prob. 113. Ans. 0·6028 diameter.
Prob. 114. Ans. 138·637 cubic inches.
Prob. 115. Ans. 64·1832 cubic feet.
Prob. 116. Ans. 473·538 tons.
Prob. 117. Ans. 67·698 tons.
Prob. 118. Ans. 0·9825 feet.

Prob. 119. Ans. Volume of B = 4795·34 cubic inches = 2·775 cubic feet; (both cases assume a depth of twenty feet below the surface).

Prob. 120. Ans. Specific gravity = 3·84.
Prob. 121. Ans. Specific gravity of granulated tin = 7·288.
Prob. 122. Ans. Specific gravity = 3·8093.

Prob. 123. Ans. Specific gravity of the first = 2·8; specific gravity of the second = 1·0946. The volumes of the two bodies are as 1 to 14·8.

Prob. 124. Ans. Specific gravity = 8·13.
Prob. 125. Ans. Specific gravity = ·832.
Prob. 126. Ans. 23·385 gallons. Theoretical discharge, 37·718 gallons.
Prob. 127. Ans. Actual range, 15·187 feet.
Prob. 128. Ans. Actual velocity = 0·3833 theoretical velocity.
Prob. 129. Ans. 10180·217 gallons.
Prob. 130. Ans. 38,450 gallons, or 610 hogsheads.

Note. In the formula, $D = 20{\cdot}8\sqrt{\dfrac{Hd^5}{l + 54d}}$, all the quantities, H, d, and l, are to be taken in metres, and the result gives D in cubic metres per second.

Prob. 131. Ans. 97·496 feet.

Prob. 132. Ans. If the actual height of the mercury is 1 inch, the height of the alcohol will be 22·68 inches, difference of level 21·68 inches.

Prob. 133. Ans. 63·68 miles. This problem involves the principles of § 172.
Prob. 134. Ans. 286·1855 lbs.
Prob. 135. Ans. 1156·3 lbs.
Prob. 136. Ans. 0·0796 grain.
Prob. 137. Ans. 460·00437 grains.

Prob. 138. Ans. Capacity of the globe = 148·626 cubic feet; specific gravity of gas = 0·0767.

Prob. 139. Ans. 31·7576 feet.
Prob. 140. Ans. 68 feet.
Prob. 141. Ans. 17·98 feet.
Prob. 142. Ans. 6655·85 feet.

Prob. 143. Ans. Ascensional force with illuminating gas, 358·4837 lbs.; ascensional force with hydrogen, 473·0793 lbs.

Prob. 144. Ans. The conditions of this problem require that the weight of the balloon should be nothing, or that the ballast should have an ascensional force of its own equal to the weight of the balloon, since, at the height indicated, if the enclosed gas could not expand, it would of itself be in equilibrium with the atmosphere. If one-half the gas were liberated the balloon would ascend; to make it remain stationary an amount of ballast must be added equal to the weight of the gas liberated, or equal to one-fourth the weight of air which would fill the balloon at the surface of the earth.

Prob. 145. Ans. The tube will admit no water by compression of the air.

Prob. 146. Ans. 0·016 inch.

Prob. 147. Ans. 45·10735 grammes.

Prob. 148. Ans. 76·306 centimetres.

Prob. 149. Ans. $\frac{16}{21}l$, $\frac{3}{4}l$, $\frac{2}{3}l$, $\frac{16}{27}l$, $\frac{8}{15}l$.

Prob. 150. Ans. 5 lbs. 10 oz.

Prob. 151. Ans. 0·09698 inch.

Prob. 152. Ans. Four times as long.

Prob. 153. Ans. 8105½ feet, or a little more than 1½ miles.

Prob. 154. Ans. Velocity at 90° F. = 1150·091 feet per second, and at —40° F. = 1007·091 feet per second.

Prob. 155. Ans. 11·2 seconds.

Prob. 156. Ans. In iron, 1·59 seconds; in wood, 1·03 to 1·65 seconds; in carbonic acid, 21·49 seconds; in hydrogen gas, 4·44 seconds; in vapor of alcohol at 140° F., 21·44 seconds; in vapor of water at 154° F., 13·72 seconds.

Prob. 157. Ans. 27 minutes, 9 seconds.

Prob. 158. Ans. 1677·45 feet in the most favorable position.

Prob. 159. Ans. 422 feet.

Prob. 160. Ans. 125·69 miles.

Prob. 161. Ans. 13 seconds.

Prob. 162. Ans. Distance = 2294 feet; velocity = 764 feet per second.

Prob. 163. Ans. $\frac{7}{37}$ of its length.

Prob. 164. Ans. $101\frac{1}{40}$ vibrations.

Prob. 165. Ans. 3·408 feet.

Prob. 166. Ans. E : D♯ = 25 : 24.

Prob. 167. Ans. $\frac{2048}{2025}$.

Prob. 168. Ans. It is higher by a *comma*.

Prob. 169. Ans. The chromatic semitone = $\frac{135}{128}$; the grave chromatic semitone = $\frac{25}{24}$.

Prob. 170. Ans. 1800 beats per minute.

Prob. 171. Ans. Vibrations per minute for one node, 66·56; for two nodes, 133·13; for three nodes, 199·695; for four nodes, 266·26 vibrations.

Prob. 172. Ans. 263·57.

Prob. 173. Ans. Reduce the length of the tube to 9·343 metres.

Prob. 174. Ans. For C 1, 8·0405 feet; D 1, 7·0939 feet; E 1, 6·3407 feet; F 1, 5·9262 feet; G 1, 5·2214 feet; A 1, 4·6618 feet; B 1, 4·1022 feet; C 2, 3·8326 feet.

Prob. 175. Ans. 11·39 in., 10·03 in., 8·95 in., 8·38 in., 7·37 in., 6·59 in., 5·8 in., 5·45 in.

Prob. 176. Ans. F 5 (too flat), A 5 (too high by half a semitone), and C 6 (also too high by half a semitone).

The formula $N = \dfrac{nV}{L + \frac{5}{3}nB}$ gives for

$n = 2$ $N = 2672{\cdot}5$ while F 5 = 2739·2.
$n = 3$ $N = 3538{\cdot}6$ while A 5 = 3424.
$n = 4$ $N = 4223{\cdot}2$ while C 6 = 4108·8.

Prob. 177. Ans. 8 minutes, 14·79 seconds.

Prob. 178. Ans. 46 years, 155 days, 10 hours, 8 minutes, and 45 seconds.

Prob. 179. Ans. As 1 to $2\frac{1}{3}$.

Prob. 180. Ans. $90\frac{1}{4}$ per cent.

Prob. 181. Ans. 22·85 candles.

Prob. 182. Ans. 21·43 candles.

Prob. 183. Ans. 60°.

Prob. 184. Ans. 18, including the object itself.

Prob. 185. Ans. $6\frac{6}{23}$ inches.

Prob. 186. Ans. 7·2 inches.

Prob. 187. Ans. $u = r\sqrt{\frac{1}{2}}$.

Prob. 188. Ans. Once and a half the distance of the object from the first surface.

Prob. 189. Ans. 39° 49′ 3″, when the eye is 5 feet above the water.

Prob. 190. Ans. 6 feet, 8 inches.

Prob. 191. Ans. $n = 2$.

Prob. 192. Ans. At a distance of 2·571 feet from the refracting surface, and on the same side as the radiant point.

Prob. 193. Ans. The surface is convex, and $r = 7{\cdot}2$ inches.

Prob. 194. Ans. On the opposite side of the lens at a distance of 3·134 inches

Prob. 195. Ans. $r : s = 10 : 242$, the surface of shorter curvature being turned towards parallel rays.

Prob. 196. Ans. 6·44 inches.

Prob. 197. Ans. A convex lens in which $f = 4$ inches.

Prob. 198. Ans. A double convex lens of crown glass $r = 2{\cdot}955$ inches, $s = 2{\cdot}667$ inches, and a concavo-plane lens of flint glass $r' = 2{\cdot}667$ inches, and $s' =$ infinity, i. e. the second surface is plane.

Prob. 199. Ans. They must converge toward a point between the lenses and distant $\frac{3}{2}f$ from the first.

Prob. 200. Ans. 2 diameters.

Prob. 201. Ans. $r = 0{\cdot}449$ inches, $s = -1{\cdot}235$ inches.

Prob. 202. Ans. 161280 times the light received by the unassisted eye.

Prob. 203. Ans. Illuminating power = 362880; penetrating power = 602·4.

Prob. 204. Ans. Illuminating power = 18225; penetrating power = 135.

Prob. 205. Ans. The illuminating power given by 150° aperture is $2\frac{1}{4}$ times, and the penetrating power $1\frac{1}{2}$ times as great as that given by 100° aperture.

Prob. 206. Ans. Crown glass, 56° 43′ 40″; plate glass, 56° 36′ 26″; flint glass, 57° 30′ 18″.

Prob. 207. Ans. Reflected by crown glass, 0·0726; by plate glass, 0·0738; by flint glass, 0·0869.

Prob. 208. Ans. 1·544.

Prob. 209. Ans.

Degrees F.	=	Degrees C.	=	Degrees R.
—40°	=	—40°	=	—32°
— 4	=	—20	=	—16
+158	=	+70	=	+56
+194	=	+90	=	+72
+442·4	=	+228	=	+182·4
+771·8	=	+411	=	+328·8
+977	=	+525	=	+420
+1832	=	+1000	=	+800
+9732	=	+5388·9	=	+4311·12

Prob. 210. Ans.

Degrees C.		Degrees F.		Degrees R.
3°·87	=	38°·966	=	—3°·096
—40	=	—40	=	—32
—10	=	+14	=	— 8
+75	=	+167	=	+60
+290	=	+554	=	+232
+360	=	+680	=	+288

Prob. 211. Ans. $18\frac{9}{25}$ times.

Prob. 212. Ans.

	At 10° F.	25° F.	75° F.	100° F.
Iron . . .	3 ft. 1·99002 in.	3 ft. 1·99376 in.	3 ft. 2·00623 in.	3 ft. 2·01248 in.
Brass . .	3 ft. 1·98425 in.	3 ft. 1·99016 in.	3 ft. 2·00984 in.	3 ft. 2·01968 in.
Copper . .	3 ft. 1·98550 in.	3 ft. 1·99093 in.	3 ft. 2·00906 in.	3 ft. 2·01812 in.
Glass . .	3 ft. 1·99032 in.	3 ft. 1·99578 in.	3 ft. 2·00403 in.	3 ft. 2·00670 in.
Platinum .	3 ft. 1·99253 in.	3 ft. 1·99533 in.	3 ft. 2.00466 in.	3 ft. 2.00933 in.
Silver . .	3 ft. 1·98387 in.	3 ft. 1·98992 in.	3 ft. 2·01060 in.	3 ft. 2·02014 in.

Prob. 213. Ans. 1·002672 gallons.

Prob. 214. Ans. 0·134 inch.

Prob. 215. Ans. 41°·51 Fahrenheit.

Prob. 216. Ans. 0·04728 in.

Prob. 217. Ans. At London, steel 92·93378 in.; brass 53·79322 in.
At Paris, steel 92·90854 in.; brass 53·77861 in.
At New York, steel 92·84311 in.; brass 53·74073 in.
At St. Petersburgh, steel 93.00482 in.; brass 53·83434 in.

Prob. 218. Ans. (1.) 30·0757 in.; (2.) 29·42076 in.; (3.) 27·8075 in.; (4.) 28·1778 in.; (5.) 23·158 in.; (6.) 24·581 in.; (7.) 17·4228 in.; (8.) 15·835 in.

Prob. 219. Ans. (1.) 24·0962 in.; (2.) 27·58513 in.; (3.) 28·74528 in.; (4.) 19·539 in.

Prob. 220. Ans. 112·588 grains. (Calculated by Table XXIV.)

Prob. 221. Ans. 122°·75, 245°·5, and 368°·25 above its previous temperature.

Prob. 222. Ans. 1220·357 cubic feet.

Prob. 223. Ans. Water, 5500 units; sulphur, 724·5 units; charcoal, 9617·7 units; alcohol, 525 units; ether, 922 units of heat.

Note. Specific heat of charcoal = 0·2415; of alcohol (Sp. Gr. 0·81) = 0·7; of ether (Sp. Gr. 0·76) = 0·66.

Prob. 224. Ans. 68°·529 Fahrenheit.

Prob. 225. Ans. 4⅔ lbs. at 200°, and 15⅓ lbs. at 50° F.

Prob. 226. Ans. 88°·39 F.

Prob. 227. Ans. 155° 88 F.

Prob. 228. Ans. 70°·79 F.

Prob. 229. Ans. 10·337 lbs.

Prob. 230. Ans. 0·628 lb.

Prob. 231. Ans. 9·26 units of heat (as in Table XV.).

Note. The temperature of the water was raised to 52°·96 F. instead of 20°·76 C.

Prob. 232. Ans. Required for air, 0·831 unit; for oxygen, 0·843 unit; for carbonic acid gas, 27·879 units; for hydrogen, 0·8235 unit of heat.

Prob. 233. Ans. 3993·64 units of heat. By table on page 451.

Prob. 234. Ans. 1174·4 units of heat.

Prob. 235. Ans. 28·088 inches.

Prob. 236. Ans. With alcohol, 26·742 in.; sulphuric acid, 29·00 in.; (at 75° F. the tension of vapor of sulphuric acid is too little to make any perceptible difference;) oil of turpentine, 28·79 inches.

Prob. 237. Ans. Tension of vapor of water at 50° F. = 0·358 in. mercury; at 75° = 0·884 in.; at 110° = 2·582 in.; at 175° = 13·673 in.; at 220° = 35·091 in.; at 265° = 78·619 in.; at 300° = 136·742 inches of mercury.

Prob. 238. Ans. Boiling points of water at the given pressures = 213°·802, 211°·709, 210°·794, 208°·679, 207°·608, 200°·51 F. Boiling points of ether at the same pressures = 95°·786, 93°·72, 92°·83, 90°·83, 89°·83, 81°·22. Boiling points of alcohol at the same pressures, 174°·33, 172°·24, 171°·34, 169°·31, 168°·31, 162°·13.

Prob. 239. Ans. If the temperature is not allowed to change, a part of the steam will be condensed and the tension will remain unchanged. In the second case the tension will be reduced to one atmosphere.

Prob. 240. Ans. 457° F. to 460° F. This tension exceeds the limits for which accurate data are given.

Prob. 241. Ans. 12 flues.

Prob. 242. Ans. 15 flues.

Prob. 243. Ans. The two forces are to each other as 1 to 10·974.

Prob. 244. Ans. The intensity equals $\frac{1}{20}$ of its original force; and $L = 19r$.

Prob. 245. Ans. The intensities are as 1, 1·026, 1·034 and 1·039.

Prob. 246. Ans. The intensity would be increased 1·9 times.

Prob. 247. Ans. The intensity is increased by one-third its original amount.

Prob. 248. Ans. The intensity is increased by two-thirds its original amount

Prob. 249. Ans. The intensity is increased to 1·16 what it was before.

INDEX.

[THE REFERENCES ARE TO SECTIONS, NOT TO PAGES.]

ABERRATION, chromatic, 465; of glass covers, 510; of lenses, 454; of mirrors, 437; of sphericity, 455.
Absolute strength, 170; zero, 666.
Absorptive power for heat, 637.
Accumulated electricity, 843.
Achromatic microscope, 508, 511; telescopes, 504.
Achromatism, 466.
Acoustics, 335.
Acoustic shadow, 350.
Action and reaction, 27; of a falling body, 77.
Action of a double convex lens, 447; of heat on matter, 566; of magnetism on light, 919; of surfaces upon heat, 635
Actual and theoretical velocities, 144.
Adaptation of eye to distance, 480; of power to weight, 110.
Addenda, page 668.
Adhesion distinguished from cohesion, 147.
Advantage of friction, 141.
Aerial phenomena, 957; waves, 332.
Air, buoyancy of, 258; impenetrability of, 259; inertia of, 260; pump, 287; vibrating in tubes, 379.
Amalgam, 834.
Amalgamation of plates, 868.
American electrical machine, 836; turbine, 231.
Amorphism, 152.
Ampère, discoveries and theory, 908.
Amusement with electricity, 840.
Analogy of light and heat, 765.
Analysis of central forces, 54; of colors, 458; of light, 456; of light by absorption, 458; by prisms, 456; of trains of wheel-work, 117.
Anemometers, 960.
Anemoscopes, 959.
Aneroid barometer, 164.
Animal electricity, 943; heat, cause of, 756; strength, 130.
Annealing, 178.
Anode, 882.
Aplanatic foci, 509.
Apparatus, Atwood's, 72; Bohnenberger's, 55; for condensation of gases, 690; for distillation, 687; illustrating barometer, 264; Melloni's, 642; Morin's, 72.
Appendix, meteorology, 946; addenda, page 668; physical tables, page 669.
Application of laws of falling bodies, 73; of levers, 115; of pendulum and measure of time, 84; of polarized light, 563; of reflection, absorption, and radiation, 640; of screw, 128; of wedge, 126.
Appreciation of colors, 490; of distance, 481.
Aqueous phenomena, 972; solutions, maximum density, 604.
Arago's experiment, 912; polariscope, 557.
Archimedes, theorem of, 205; demonstrated, 206.
Archimedes' screw, 298.
Are, 18.
Areometers, 212.
Artesian wells, 204.
Artificial magnets, 802; temperature, 727.
Ascent of liquids in tubes, 239.
Astatic needle, 787.
Astronomical telescope, 499.
Atlantic cable, 927.
Atmosphere, 256; free electricity in, 861.
Atmospheric electricity, 860; a source of heat, 747.
Atmospheric engine, 708; magnetism, 800; pressure, 257; measure of, 261; refractions, 538.
Atoms, 20.
Attraction and repulsion of light bodies, 831.
Attraction, electrical, 812; experiments, 842.
Attraction, magnetic, 780.
Atwood's apparatus, 72.
Auditory organs of man, 393.
August's hygrometer or psychrometer, 973.
Auroral current, reversal of polarity in, 1000.
Auroras, 994; effect on telegraph wires, 999; geographical distribution, 997; height and frequency, 996; magnetic disturbance, 998 remarkable, 995.

BABBAGE'S experiment on friction, 140
Back-ground, 476.
Bain's telegraph, 926.
Balance, spring, 37.
Ballistic curve, 145; pendulum, 104.
Balloons, 273.
Barker's mill, 217.
Bath, tempering by, 178.

Barometer, aneroid, 164; at different altitudes, 265; cistern, 266; construction of, 263; correction for temperature, 599; errors of, 269; Fortin's, 266; Gay Lussac's, 267; measuring heights, 272; metallic, 163; principles of, illustrated, 264; wheel, 268.
Barometric changes and the weather, 271; height, variations of, 270.
Batteries, Smee's, 871; trough, 870; voltaic, 869–881.
Beams, flexure of, 162; lateral strength, 172; of light, 401.
Beating, 376.
Beaumé's hydrometer, 212.
Bellows, 281.
Blasting by electricity, 937.
Body, defined, 1.
Bohnenberger's apparatus, 55.
Binocular vision, 484.
Boiler for Gold's steam-heater, 733.
Boiling point, 569; application in arts, 678; circumstances influencing, 676; heights measured by, 679.
Bourdon's metallic barometer, 163.
Boyden's American turbine, 231.
Brachystochrome, 76.
Bramah press, 190.
Breast-wheel, 230.
Breguet's metallic thermometer, 580.
Brightness of ocular image, 473.
Britannia tubular bridge, 172.
Brittleness, 177.
Bronze tempering, 178.
Bunsen's photometer, 414.
Buoyancy of air, 258.
Buoyancy of liquids, 205.

CAMBRIDGE TELESCOPE, 506.
Camera lucida, 518.
Capillarity, 232; general facts in, 233; a source of heat, 741; influenced by curve of surface, 236; laws of, 237.
Capstan, 116.
Calorimetry, 650.
Camels, 206.
Camera obscura, 517.
Carbon battery, 876.
Cartesian devil, 206.
Cathode, 882.
Catoptrics, 415.
Caustic curves, 437.
Centigrade thermometer, 570.
Centimetre, 18.
Central forces, analysis, 54.
Centre of gravity, 60, 62; in bodies of unequal density, 68; of regular figures, 64; without the body, 65.
Centre of hydrostatic pressure, 197.
Centre of oscillation, 83.
Centrifugal and centripetal forces, 52.
Centrifugal drying machine, 53.
Centrifugal forces, demonstration, 53.
Chain pump, 297.
Change of density, 169.
Chart of magnetic variations, 789.
Chart of isoclinal lines, 794.
Chemical affinity and molecular attraction, 900; combination, 748; effects of the pile, history, 888; force, 5; sources of heat, 748–757; union by electricity, 856.
Chemistry, relation of to physics, 9.
Chevreul's classification of colors, 491.
Chimneys draught in, 717.
Chromatic aberration, 465.
Chromatic diagram, 491.
Chromatic polariscope, 557.
Chromatics, 456.
Chronometers, compensating balance wheels, 596.
Cistern barometer, 266.
Clarke's magneto-electric apparatus, 938.
Cleavage, 157.
Climates, climatology, 947.
Clocks, electrical, 928.
Clothing, relations to heat, 625.
Clouds, 978; classification of, 979.
Coexistence of sound waves, 338.
Cohesion among solids, 147.
Cohesion and repulsion, 146.
Cohesion in liquids, gases, and solids, 148.
Cold, apparent radiation of, 634.
Cold by evaporation, 681.
Color blindness, 490.
Color dependent on temperature, 768.
Colored polarization, 555; rings in crystals, 558
Colors, analysis of, 458; Chevreul's classification of, 491; complementary, 459; of grooved plates, 535; of thin plates, 529; study of, 492
Columns, resistance to pressure, 171.
Combination of waves, 327.
Combustion, a source of heat, 749; cause of heat in, 750.
Comparison of different thermometers, 576.
Compass, mariner's, 787.
Compensating balance wheels, 596; pendulums, 596.
Complementary colors, 459.
Components and resultants, 44.
Composition of white light, 457.
Compound chords, 372; crystals, 156; lenses, 450; levers, 114; examples of, 115; microscope, 496; achromatic, 511; motion, 33; pendulum, 83; pulleys, 120.
Compressed gases, escape of, 283.
Compressibility, 21; of gases, 274; of liquids, 188.
Compressing machine, 288.
Compression a source of heat, 739.
Concave lenses, 448.
Concave mirrors, 426; foci of, 427–431; images by, 433.
Condensation of gases, 689, 690.
Condenser of Æpinus, 844.
Conductibility, clothing, 625; of crystals, 616; examples of, 622, 623; of gases, 620; of liquids, 619; of metals, 615; of powders or fibres, 624; relative, of solids, liquids and gases, 621; of solids, 614, 622; of wood, 617.
Conduction of heat, 613.
Conjugate mirrors, 635.
Constitution of liquid veins, 222.
Construction of barometers, 263; of musical instruments, 385; of thermometers, 568.
Convection of heat, 626; of heat in liquids, 627.
Convex lenses, 447.
Convex mirrors, 426, 433; images by, 435.
Cooling by radiation, 632.
Copper, tempering, 178.
Cords, vibration of, 308, 309.
Corollaries, on centre of gravity, 63.
Coronas, 537.
Correlation of forces, 758.
Coulomb on rolling friction, 139; on starting friction, 138.
Coulomb's electrical laws, 819; laws of torsion, 166.

Couples, 48.
Cowls, 725.
Crystalization by feeble currents, 893.
Crystalography, 151.
Crystals conduct heat, 616; forms of, 153, 158; positive and negative, 551.
Cubical expansion, 590.
Culinary paradox, 677.
Currents, in air and gases, 716; induced by magnets, 938; in the ocean, 628; of electricity, path and velocity of, 818; produced by ice, 724.
Curve, ballistic, 145; influence in capillarity, 236; of liquid surfaces, 234; of swiftest descent, 76.
Curvilinear motion, 51.
Cut-off, 712.
Cycloidal pendulum, 85
Cyclones, 968.
Cylinder electrical machine, 834.

Daniell's constant battery, 874; hygrometer, 973.
Dark lines in spectrum, 461, 462.
Declination of magnetic needle, 788.
Decimetre, 18.
Deep-sea thermometer, 581.
Deflagration, 886.
De La Rive's floating current, 909.
Demonstration of Torricellian theorem, 220.
Density, 98; changed by tension, 169; of gases, 611; of the earth estimated by experiment, 102; of vapors, 693.
Depth of waves, 322.
Depression of mercury in tubes, 238.
Descent on curves, 75; on inclined planes, 74.
Despretz's experiments, 275.
Destructive effects of impact, 112.
Determination of reflective power, 636.
Deviation of light twice reflected, 423.
Dew, 975; on what it falls, 976.
Dew-point, 674.
Diamagnetism, 920
Diamond jar, 852.
Diapason, 377; natural, 395.
Diathermancy, 641; applications of, 647; causes which modify, 645.
Differential thermometers, 587.
Diffraction, 532.
Diffused light, 410.
Dilatation by heat explained, 769.
Dimensions of the earth, 92.
Dioptrics, 438.
Dipping needle, 793.
Direction of force, 40; of osmotic current, 247; of terrestrial attraction, 60; of vibrations of light, 541.
Directive action of the earth, 911.
Dispersion of light, 406.
Displacement of zero-point, 573.
Dissecting microscope, 495.
Distance calculated by sound, 346; of distinct vision, 478; that sound can be heard, 351.
Distillation, 686.
Distilling apparatus, 687.
Divisibility, 19.
Double diatonic scale, 373; refraction, 550; polarization by, 552; vision, 483.
Downward pressure of liquids, 191.
Draught in chimneys, 717.
Drops of liquids in conical tubes, 241.
Dry piles, 873.
Drying machine for laundries, 53.
Duboscq's electric lantern, 884.
Dub's laws of electro-magnetism, 915.
Ductility, 175.
Duration of visual impressions, 487.
Dwellings, supply of fresh air, 726.
Dynamical electricity, 862; theory of heat, 762.
Dynamics, 39; Dynamometers, 37.

Ear, 393; sensibility of, 378; trumpet, 359.
Earth circuit, 922.
Earth's rotation, effect of, upon gravity, 94; demonstrated by the pendulum, 86.
Ebullition, 675.
Echo, 353; tone changed by, 355.
Echoes repeated, 354.
Effects of centrifugal force, 53.
Elastic balls transmit shock, 185; bodies, impact of, 182; fluids, 252.
Elasticity, limit of, 168; modulus of, 183; of flexure, 162; of liquids, 188; of metals, table, 161; of solids, 159; coefficient of, 161; of tension and compression, 160; of torsion, 165.
Electric battery, 849.
Electric currents, induced, 929; light in a vacuum, 935; mutual action, 909.
Electric discharge, effects of, 853; in vacuum, 852.
Electric light, properties of, 885; regulators of, 884; rotation about a magnet, 936.
Electric spark, color of, 852.
Electric telegraph, history of, 921; Morse's recording, 924; varieties of, 923.
Electricity, atmospheric, 860, 861; chemical effects, 858; chemical union by, 856; classification, 773; conductors of, 814; discharge in cascade, 850; disguised, 843; distribution of, 825; dynamical, 862; dynamical converted into statical, 933; earth a reservoir of, 815; floating current, 909; from all sources identical, 939; from steam, 839; loss of, in excited bodies, 827; magnetic, 774; mechanical effects, 857; only on outer surface, 824; of plants, 945; positive and negative, 813; of the air, 987; physiological effects, 854; theories of, 816; theory of voltaic, 863; universal discharger of, 851; velocity of, 818; vitreous and resinous, 813.
Electrical amusements, 840; animals, 944; attraction and repulsion, 812; bells, 842; blasting, 937; cascade in vacuo, 935; clocks, 928; condenser, discharge of, 845; effects, 811; egg, 852.
Electrical excitement, sources, 810; universality of, 940; various sources, 840.
Electrical fire alarm, 928; hail-storm, 842; helix, 910; induction, 828; lamp, Volta's, 856.
Electrical machines, 834–838; care of, 838; theory of, 841.
Electrical nomenclature, 882; pendulum, 812; phenomena, 987; power of points, 826; retardation, 880; wheel, 842; tension and currents, 817.
Electro-chemical telegraph, 926.
Electrode, electrolyte, 882.
Electro-dynamic spiral, 910.
Electro-dynamics, general laws, 902.
Electrolysis, laws of, 890; of salts, 891; of water, 889.
Electro-magnetic currents, motion of, 904
Electro-magnetic motion, 917.
Electro-magnets, 913; Page's revolving, 914; power of, 915.
Electrometers, 832.

Electrometer, gold leaf, 846; torsion, 820.
Electrophorus, 833.
Electro-positive and electro-negative, 867.
Electro-printing telegraph, 925.
Electroscopes, 813, 842.
Electroscope, Bohnenberger's, 873; Volta's condensing, 846.
Electrotype, 892.
Emerson's ventilators, 725.
Emissive power for heat, 638.
Elements are simple bodies, 1
Endless screw, 129.
Endosmose, 244; of gases, 250; theories of, 251.
Endosmometer, 245.
English and American weights, 101.
English units of length, 17.
Eolipile, 704.
Epipolic dispersion, 533.
Equatorial telescope, 505; Cambridge, 506.
Equilibrium, 38; conditions of, in liquids, 199; neutral, unstable, and stable, 207; of bodies supported in more than one point, 69; of machines, 107.
Equilibrium of liquids, between laminæ, 240; free from gravity, 200; in communicating vessels, 201; of different densities, 202.
Equilibrium of solids placed upon a horizontal surface, 67; supported by an axis, 66.
Equilibrium of the lever, 114.
Errors of barometer, causes of, 269.
Escape of compressed gases, 283; of liquids through tubes, 223, 224.
Essential properties of matter, 12.
Estimation of high temperatures, 586.
Eustachian tube, 393.
Evaporating power of fuel, 715.
Evaporation, causes influencing, 673; cold by, 681; mechanical force of, 684.
Examples of compound levers, 115.
Exosmose, 244.
Expansibility and compressibility, 610.
Expansibility, 22.
Expansion, amount of, in solids, 591; apparent and absolute, 598; coefficient of, 591; cubical, 590; curve of, for liquids, 602; force exerted by, 593; increases with temperature, 592; linear, 589; of crystals, 590.
Expansion of gases, 253, 605; Regnault's results, 607.
Expansion of liquids, 597; above boiling, 602; amount of, 601.
Expansion apparent of mercury, 600; of mercury, coefficient, 598; of solids, 589; of water, 604; phenomena of, 594; unequal, of solids, 595.
Experiment a source of knowledge, 2; hydrostatic pressure, 194.
Experiments of Despretz, 275; of Pascal, 194, 262; of Plateau, 200; of Regnault, 276.
Experiments on density of the earth, 102; on liquid surfaces, 235.
Explosions, cause of, 701.
Extension, 13.
Extremes of temperature, 744.
Eye, action of, on light, 469; adaptation to distance, 480; a polariscope, 562; structure of, 468.
Eye-piece, Tolles' solid, 512.
Eye-pieces, 500.

FACTS (in interference) versus theory, 528.
Fahrenheit's hydrometer, 212; thermometer, 570.
Fairbanks' scales, 115.
Falling bodies, laws of, 71.
Falling body, action and reaction of, 77; **space** described by, 71.
Faraday's nomenclature, 882.
Faraday **on** liquefaction of gases, 689
Fire-alarm, electrical, 928
Fire engine, 294.
Fire regulators, 594.
Fixed lines in spectra, 461, 462.
Fixed pulley, 118.
Flexure of beams, 162.
Floating bodies, 206; equilibrium of, 207.
Floating current, 909.
Floating docks, 206.
Flow from capillary tubes, 243.
Flow of liquids, 218.
Flow, theoretical and actual, 216.
Fluids, 186; resistance of, 143.
Fluorescence, 533.
Foci of lenses, compound, 450; concave, 448; convex, 447; principles, 445; rules for, 449.
Fog-bows, 537.
Fogs, or mists, 974.
Foot-pound a measure of heat, 759.
Force and heat, relations, 758.
Force, chemical or physical, 5; definition of, 35; developed by evaporation, 684; direction of, 40; of expansion, 593, 603; of gravity, 58; unit of, 37.
Forces are definite quantities, 36.
Forces, centrifugal and centripetal, 52; correlation of, 758; not parallel applied at different points, 49; measure of, 41; parallelogram of, 45; propositions in regard to, 42; resolution of, 50; statical and dynamical, 39; system of, 44.
Forcing pump, 292.
Forms of crystals, 153; of mirrors, 417; of vibrations, 307.
Formulæ, achromatism, 467.
“ altitudes by barometer, 272.
“ apparent expansion of mercury, 600.
“ central forces, 54.
“ change of volume in gases, 608.
“ compensating pendulum, 596.
“ compound lenses, 450.
“ concave lenses, 448.
“ concave mirrors, 428.
“ convex lens, 447.
“ convex mirrors, 432.
“ correction of barometric height, 599.
“ electric piles, 881.
“ endless screw, 129.
“ escape of liquids, 223, 224.
“ expansion of liquids, 598.
“ expansion of solids, 591.
“ flexure of beams, 162.
“ flow of liquids, 216.
“ inclined plane, 122–124
“ index of refraction, 440.
“ light twice reflected, 423.
“ magnifying power of lenses, 494.
“ microscopes, 513.
“ motion of projectiles, 103.
“ Newton's, for velocity of sound, 653
“ optical centre of lenses, 452.
“ pencils of light refracted at plane surfaces, 443.
“ pencils of light transmitted by plane glass, 444.
“ pendulum, 81, 82.
“ penetrating power of telescopes, 507.

Formulæ pulley, 119, 120.
“ refraction at spherical surfaces, 445.
“ refraction by parallel media, 442.
“ relation of volume, temperature, and pressure in gases, 609.
“ screw, 127.
“ specific gravity, 210, 211.
“ specific heat, 653.
“ spherical aberration of lenses, 454.
“ standard thermometer, 577.
“ strength of beams, 172.
“ uniform motion, 30.
“ variable motion, 32.
“ variation of gravity by rotation of the earth, 94.
“ variation of gravity in altitude, 95.
“ variation of gravity in latitude, 90.
“ velocity after impact, 184.
“ velocity of discharge, 220.
“ velocity of light, 404.
“ velocity of winds, 961.
“ vibration of air in tubes, 384.
“ visual power of telescopes, 507.
Fortin's barometer, 266
Foucault's apparatus, 404.
Franklin's kite, 860; pulse glass, 677.
Fraunhofer's dark lines, 461.
Freezing in red-hot crucibles, 699.
Freezing mixtures, 657.
Freezing point, 569.
Fresh air in dwellings, 726.
Fresnel lens, 521.
French system of measures, 18; weights, 100.
Friction, advantage from, 141; during motion, 138; heat from, 735, 736; sliding, 137; starting, 138.
Frictional electricity, 809.
Frost, 977.
Fuel, evaporating power, 715; relative value of, 753.
Functions of the ear, 394.
Furnace blowers, 282.
Furnaces for hot air, 729.
Fusion, laws and heat of, 658; peculiar in some solids, 659.

GALVANIC battery a misnomer, 864.
Galvanic current, 943.
Galvanism, discovery of, 862; contact theory, 863.
Galvanometer, 905; sine compass, 906.
Gamut, 366.
Gas illumination, products of, 721.
Gas jet, musical note, 382.
Gases, 252; and vapors, identity of, 688; compressibility of, 274–277; conductibility for heat, 620; density of, 611; expansion of, 253; expansion by heat, 605; laws of expansion, 606; liquid and solid, properties of, 691; mechanical condition of, 254; transmit pressure, 255; relation of volume, temperature, and pressure, 609; reduced to liquids, 689; specific heat of, 652, 653; volume of, 608.
Gassiot's cascade, 935.
Gay Lussac's barometer, 267; hydrometer, 212; laws for expansion of gases, 606.
Glass, temper of, 178.
Glottis, 388.
Gold's steam heaters, 732.
Graduation of thermometers, 569.
Graham's compensating pendulum, 596.
Gravity, 58; a source of motion, 70; affected by the earth's rotation, 94; influenced by the earth's figure, 91; intensity varies with latitude, 90; measured by the pendulum, 88; varied by altitude, 95.
Gridiron pendulum, 596.
Grove's battery, 875.
Gyrascope, 57.

HADLEY'S sextant, 425.
Hail, 986.
Halos, 537.
Hammering, 179.
Hardening, 178.
Hardness, 176.
Hare's calorimotor, 870; deflagrator, 870, electrometer, 846.
Harmony, melody, 365.
Harrison's compensating pendulum, 596.
Hearing, 341; of animals, 396.
Heat, action of, on matter, 566; amount by chemical action, 751; and force, relations of, 758; and light, analogy of, 765: and light, by chemical and mechanical action, 768; cause of, in animals, 756; causes which modify emissive, absorbent, and reflective power, 639; change of state in bodies, 764; coloration, 646; conclusions, 772; conduction of, 613; convection of, 626; developed by solidification, 663; diffraction and interference, 649; dynamical theory, 762; expansion of gases by, 606; expansion of liquids by, 597; expansion of solids by, 589; from magnetism, 918; latent, 655; mechanical equivalent, 758, 760; mechanical unit of measurement, 759; nature of, 564; of combustion, 749; modes of communication, 612, of capillarity, 741; of chemical action, 748; of compression, 739; of friction, 735–738; of fusion, 658; of humid combinations, 754; of percussion, 740; of voltaic arch, 886; of voltaic currents, 887; origin of terrestrial, 746; polarization of, 649; quantity and intensity, 771; quantity developed by friction, 736; quality of, how changed, 770; radiation of, 629; reflection of, 635; refraction of, 648; relation to cold, 565; specific, 651; transmission of radiant, 641; unit of 650; universal radiation, 633.
Heating by hot water, 730.
Height measured by barometer, 272.
Heights measured by boiling water, 679.
Helix, electrical, 910.
Hiero's fountain, 295.
High-pressure engine, 711.
High-pressure steam, 680.
Horse-power, 714.
Horse-power machines, 132.
Hot air furnaces, 729.
Hot water apparatus, 731.
House's telegraph, 925.
Humidity of the air, 972.
Hurricanes, 968.
Hydraulic ram, 296.
Hydraulics, 213.
Hydrodynamics, 186.
Hydrometers, 212.
Hydrostatic paradox, 195.
Hydrostatic press, 190.
Hydrostatics, 186.
Hygrometers, 973.
Hypothesis defined, 3.
Hypsometer, 679.

ICE, currents produced by, 724

Ice machine, Twining's, 681.
Illumination of railways, 520.
Illumination, products of, 721.
Illumination, sufficiency of, 477.
Illustration of vis viva, 111.
Images, by concave mirrors, 433; by convex mirrors, 435; by plane mirrors, 419; by small apertures, 412; by inclined mirrors, 422; by two plane mirrors, 421; distortion of, 455; formed by lenses, 453; in the eye, inverted, 470; multiplied by two surfaces, 420; virtual, 434.
Impact and its results, 112; as related to vis viva, 112; in relation to momentum, 112; of elastic bodies, 182.
Impenetrability, 14.
Impenetrability of air, 259.
Imponderables, 11.
Impressions of light and heat, 766.
Incandescence, 767.
Inch of water, 221.
Inclination map, isoclinal lines, 794.
Inclined plane, 121–124.
Index of refraction, 440.
Induced currents, different orders, 930.
Induced currents from earth's magnetism, 932.
Induction an act of contiguous particles, 830.
Induction coil of Ruhmkorff, 933; effects of, 934.
Induction, electro-dynamic, 929; of a current on itself, 930; of electricity, 828; of magnetism, 783.
Inductive philosophy, 4.
Inductive power of earth's magnetism, 797.
Inertia, 26; of air, 260.
Inflammation by electricity, 855.
Influence of the earth's figure upon gravity, 91.
Intensity of aerial waves, 332; of light, 413; of luminous, calorific, and chemical rays, 463; of many couples, 881; of radiant heat, 631.
Interference colors, 529; of light, 527; of sound, 349; of waves, 327, 328.
Intermittent fountain, 286; springs, 286; syphon, 285.
Internal ear, 393.
Internal reflection of light, 408.
Interval in music, 371.
Ions, 882.
Irregular reflection of light, 410.
Isochronism of the pendulum, 80.
Isochronous vibrations, 303.
Isodynamic lines, 796.
Isogonal lines, 789.
Isothermal lines, 956.

JETS of water, 225.
Johnson on strength of materials, 170.
Joule's experiments on heat, 760, 761.

KALEIDOSCOPE, 424.
Kalychromatics, 464.
Key-note of nature, 335.
Kilometre, 18.

LARYNX, 388.
Latent electricity, 843.
Latent heat, 655.
Latent heat of fusion, 658; of steam, 682.
Lateral strength of beams, 172.
Latour's law for vapors, 692.
Law, definition of, 3.
Law of cooling by radiation, 632; of Latour, 692; of universal gravitation, 59.
Laws of acoustics determine specific heat, 653.
Laws of Bernoulli, 384; of capillarity, 237; of capillarity between laminæ, 240; of electrical attraction and repulsion, 819–822; of electrical induction, 829; of electricity and chemical action, 898; of electrolysis, 890; of electro-dynamics, 902; of electro-magnetism, 915; of falling bodies, 71; application, 73; verification, 72; of fusion, 658; of Ohm, 880; of solidification, 662; of storms, 971; of tenacity, 170; of torsion, 166.
Length of luminous waves, 531.
Lenses, 438; compound, 450; concave, 448; convex, 447; images formed by, 453; optical centre of, 452; refraction of oblique pencils by, 451; rules for foci, 449.
Le Roy's dynamometer, 37.
Lever, 113; application of, 115; equilibrium of, 114.
Leyden jar, 847; electricity in, 848.
Life an unknown power, 10.
Light, action of eye upon, 469; amount reflected, 407; and heat, analogy of, 765; and heat by chemical and mechanical action, 768; changed by polarization, 543; color of, depends on temperature, 768; diffused, 410; direction of vibrations, 541; heat and electricity are forces in nature, 11; in a homogeneous medium, 403; influenced by magnetism, 919; internal reflection, 408; irregular reflection, 410; length of vibrations, 531; nature of, 398; pencils of, passing through plane glass, 444; pencils of, refracted at plane surfaces, 443; polarized by absorption, 545; polarized by reflection, 546; polarized by refraction, 547, 548; properties of, 406; total reflection of, 409; rays, pencils, beams, 401; refracted by parallel strata, 442; relation of bodies to, 400; sources of, 399; transmission of vibrations, 542; velocity of, 404.
Light-houses, 522, 523.
Lightning, 990; classes of, 991; return stroke, 992.
Lightning-rods, 993.
Limits of elasticity, 168; of [illegible], 173.
Linear expansion, 589.
Liquid surfaces, cause of curve, 234.
Liquid veins, constitution of [illegible].
Liquids, 186; amount of expansion, 601; ascent in capillary tubes, 239; conditions of equilibrium, 199; conductibility for heat, 619; convection of heat in, 627; curve of expansion, 602; downward pressure of, 191; elasticity of, 188; equilibrium of, 199, 200; equilibrium of, in communicating vessels, 201, 202; expansion of, 597; expansion above boiling, 602; force of expansion, 603; lateral pressure, 193; mechanical condition, 187; repelled by heated surface, 697; spheroidal state, 694; transmit pressure, 189; upward pressure, 192.
Liquefaction and solidification, 655; gradual, 656.
Liquefaction of gases, theory, 688; of vapors, 685.
Lister's aplanatic foci, 509.
Litre, 18.
Living force, 111.
Lodestone, 774.
Looming, 539.
Long-sightedness, 486.

Low pressure engines, 710.

Machine power and weight, 106.
Machines, 105; equilibrium of, 107.
Magic lantern, 515; squares, 852.
Magnetic attraction and repulsion, 780; curves, 777; dip of needle, 791; electricity, 774; figures, 778; force, distribution, 776; force, lines of, 799; induction, 783; intensity, 795; meridian, 788; needle, 775, 787; observations, 798; phantom, 777; polarity, 776; rotary polarization, 560.
Magnetism, action upon light, 919; atmospheric, 800; coercitive force, 786; by contact, 781; conversion into heat, 918; different bodies, 782; of steel by solar rays, 807; of the earth, action of, on dipping needle, 792; origin of the earth's, 801; terrestrial, 787; theories, 784, 785.
Magnetizing by the helix, 912.
Magneto-electric apparatus, 938.
Magneto-electricity, 938.
Magnets, anomalous, 779; artificial, 775; by electro-magnetism, 805; by touch, 803; compound, 806; deprived of power, 808; directive tendency of, 787; horse-shoe, 804; natural, 774; production of, 802; value how varied, 802.
Magnifying glasses, 493.
Magnifying power of lenses, 494.
Magnitude, 13; limits of, 173.
Malleability, 174.
Manometers, 278; with compressed air, 280; with free air, 279.
Map of isoclinal lines, 794.
Mariner's compass, 787.
Mariotte's law, 274.
Martin's compensating pendulum, 596.
Mathematical pendulum, 78.
Matter, 1; accessory properties, 19; changes in, 7; essential properties, 12; general properties, 6; three states of, 15; properties of, physical or chemical, 8.
Mass, 41, 96.
Maximum and minimum thermometers, 578.
Maximum density of aqueous solutions, 604; of water, 604.
Mean temperature, 950.
Measure of forces, 41.
Measures of capacity, 17, 101.
Mechanical condition of gases, 254; efficiency of the screw, 128; equivalent of heat, 758; force of evaporation, 684; illustration of vibrations, 301; sources of heat, 735.
Mechanism of the voice, 389.
Melloni's apparatus, 642.
Melloni's thermo-multiplier, 942.
Melody, harmony, 365.
Membranes, vibration of, 318.
Men, strength of, 131.
Mercurial rain, 24.
Mercurial thermometer, 568; limits of, 574; defects of, 576.
Mercury, depression of, in tubes, 238.
Meridian measurements, 91.
Metacentre, 208.
Metallic barometer, 163.
Metallic thermometers, 580.
Metals, change of properties, 180; change of structure, 180, conduct heat, 615.
Metastatic thermometer, 579.
Meteorological observations, 949.
Meteorology, 946.
Metre, 18.
Microscope, achromatic, angular aperture, defining power, illuminating power, magnifying power, penetrating power, visual power, 513; compound, 496; object-glasses, 508; Raspail's dissecting, 495; simple, 495; solar, 516; stand, 514.
Millimetre, 18.
Minute division, 19.
Mirage, 540.
Mirrors, 415; forms of, 417.
Mobility, 25.
Modified forms of crystals, 155.
Modulus of elasticity, 183.
Molecular motion, in heat, 763.
Molecules, 20.
Momentum, 43.
Monochord, 367.
Monthly variations in temperature, 951.
Morin's apparatus, 72.
Morse's telegraph, 924.
Motion and force, composition of, examples, 45.
Motion communicated by collision, 181.
Motion, compound, 33; curvilinear, 51; of projectiles, 103; uniform, 30; uniformly varied, 32; variable, 31; varieties of, 28.
Mountings for telescopes, 505.
Mouth-pipes, 380.
Movable pulley, 119.
Movement of drops in tubes, 241.
Music halls, 362.
Music, theory of, 363.
Musical instruments, 383.
Musical interval, 371.
Musical scale, 366; new, 373.
Musical sounds, 335; quality of, 363.
Musical tones from magnetism, 916.

Natural diapason, 395.
Natural history, 10.
Natural philosophy, 9.
Nature, key-note of, 335.
Nearsightedness, 485.
Negative eye-piece, 500.
Negretti & Zambra's thermometer, 578.
Neutral equilibrium, 207.
New musical scale, 373.
Newton's formulæ for velocity of sound, 653
Newton's rings, 530.
Nicholson's hydrometer, 212.
Nicol's prism, 553.
Night glass, 498.
Nitric acid battery, 875.
Nobili's galvanometer, 905.
Nobili's rings, 894.
Nodal figures, 317.
Nodal lines, 313; how delineated, 316; position of, 314.
Nodal points, 305.
Noise, 335.
Nomenclature, Faraday's electrical, 882

Object-glasses for the microscope, 508.
Objectives compound, 509.
Oblique pencils, 430.
Oblique pencils refracted by lenses, 451
Observation and experiment, 2.
Ocular image, brightness of, 473
Œrsted's discovery, 903.
Ohm's law of retarding power, 880.
Open fire, 728.
Opera-glass, 498
Optic angle, 471.

Optic axis, 471.
Optical centre of a lens, 452.
Optical instruments, 493.
Optical toys, 488.
Optics, 397.
Organ of Poole and Alley, 375.
Organic solutions, osmose in, 248.
Organs of hearing in animals, 396.
Origin of terrestrial heat, 746.
Origin of undulations, 299.
Oscillation, centre of, 83.
Oscillation defined, 78.
Osmose, 244; conditions of, 246.
Osmose of inorganic solutions, 249; of organic solutions, 248; direction of current, 247.
Overshot wheel, 229.
Ozone, 859.

PAGE'S revolving magnet, 914.
Page's vibrating armature, 931.
Parachute, 273.
Parhelia, 537.
Parallel forces, resultant of, 46.
Parallelogram of forces, 45; of rotations, 56; of velocities, 34.
Pascal's experiments, 194, 262.
Passive resistance, 136.
Paths of vibration, 311.
Pencils of light, 401; refracted at plane surfaces, 443; at spherical surfaces, 446.
Pendulum, 78; applied to measure time, 84; applied to study of gravity, 87; ballistic, 104; compensating, 596; cycloidal, 85; demonstrates rotation of the earth, 86; formulæ for, 81; isochronism of, 80; length beating seconds, 89; physical or compound, 83; propositions respecting, 82; simple, 79; used to measure force of gravity, 88.
Penetrating power of microscopes, 513; of telescopes, 507.
Penumbra, 411.
Percussion a source of heat, 740.
Perfect concord, 372.
Perkins' apparatus, 731.
Perpetual motion, 135.
Phases of undulations, 304.
Phenomena defined, 2; of expansion, 594.
Philosophical egg, 852.
Phosphorescence, 399, 534.
Photo-electric lantern, 884.
Photography, 519.
Photometers, 414.
Physical force, 5; pores, 23; optics, 527; sources of heat, 742; tables, page 369; theory of music, 363.
Physics and chemistry, 9.
Physiological effects of the pile, 895.
Piano strings friable, 180.
Plane glass, refraction by, 441.
Plants, electricity of, 945.
Plateau's experiment, 200.
Plates, vibration of, 312.
Platform balance, 115.
Pneumatic ink-bottle, 284.
Pneumatics, 252.
Polarity of compound circuit, 878.
Polarization and transfer of elements, 899.
Polarization, atmospheric, 561; by absorption, 545; by heat and by compression, 559; by reflection, 546; by refraction, 547, 548; colored, 555; of heat, 649; of light, 541; partial, 549; rotary, 556.
Polarizing instruments, 554.
Poole's musical scale, 373.
Pores, physical, 23; sensible, 24.
Porosity, 23.
Positive and negative crystals, 551.
Positive eye-piece, 500.
Pouillet's galvanometer, 906.
Power of points, electrical, 826.
Power of steam, 714.
Power, adaptation to weight, 110.
Power and weight, 106; relation of, 109.
Press, hydrostatic, 190.
Pressure, atmospheric, 257; centre of, 197; of liquid in motion, 226; of liquid downward, 191; of liquid on side of vessel, 193; of liquid upon containing vessel, 214; produced by impact, 112; transmitted by gases 255; transmitted by liquids, 189; varies with Sp. Gr., 198.
Primary colors, 456.
Prime seventh, 373.
Prince Rupert's drops, 178.
Printing telegraph, 925.
Prisms and lenses, 438.
Problems on acoustics, page 289; on elasticity and tenacity of solids, page 146; on electricity page 667; on gravitation, page 73; on heat, page 506; on hydrodynamics, page 199; on laws of vibrations, page 251; on theory of machinery, page 107; on optics, page 392; on pneumatics, page 234; on weights, measures, and motion, page 37.
Production of waves, 319.
Products of combustion and respiration, 718.
Progressive undulations, 300; in liquids, 320.
Properties of matter, general or specific, 6; physical or chemical, 8.
Properties of liquid and solid gases, 691; of solar spectrum, 460; of solids, 149.
Propositions in regard to forces, 42.
Projectiles, theory and laws of, 103
Proof plane, 823.
Pulley, compound, 120; fixed, 118; movable, 119.
Pumps, 289-293.
Pyrometers, 589.
Pyrometer, Daniell's, 584; Draper's, 585; Saxton's reflecting, 582; Wedgewood's, 583.
Pyrometrical heating effects, 752.

QUALITY of musical sounds, 363.
Quantity and intensity of electricity, 865.
Quantity of heat from friction, 736.

RAIN, 980; annual depth of, 983; days of, 982; distribution of, 981.
Radiant heat, intensity of, 631; partially absorbed, 630; transmission of, 641.
Radiating power for heat, 638.
Radiation, of cold, 634; of heat universal, 633; law of cooling by, 632; terrestrial, 745.
Radiators, steam, 732.
Railway illumination, 520.
Rainbow, 536.
Ramsden's electrical machine, 835
Range of human voice, 390.
Rays of light, 401.
Reaction of escaping liquids, 217.
Reaumur's thermometer, 570.
Recomposition of white light, 457.
Reed pipes, 381.
Regulators of electric light, 884

Reflecting telescope, 501.
Reflection at curved surfaces, 426; by plane mirrors, 418; by regular surfaces, 415; of circular waves, 326; of light, 406; of sound, 352; of waves, 323.
Reflective power, how determined, 636.
Refractory bodies, 660.
Refraction at curved surfaces, 445, 446; at regular surfaces, 438; atmospheric, 538; at plane surfaces, 439; by lenses, 447–451; by plane glass, 441; by prisms, 439; of heat, 648; of light, 406; of pencils of light, 443; of sound, 357.
Refrigerators, 724.
Regnault's experiments, 276.
Regnault on expansion of gases by heat, 607.
Relation of bodies to light, 400; of power to weight, 109; of specific heat and atomic weight, 654.
Relative value of fuel, 753.
Repulsion, 146; electrical, 812; magnetic, 780.
Repulsion of light floating bodies, 242.
Repulsive action of heated surface, 697.
Resistance of columns, 171; of fluids, 143; passive, 136.
Rest, absolute and relative, 25.
Resultant of opposite parallel forces, 47; of unequal parallel forces, 46.
Results of impact, 112.
Revolving magnet, 914.
Revolving lights, 523.
Reynier's dynamometer, 37.
Rheostat, 907.
Rigidity of ropes, 142.
Ritchie's electrical machine, 837.
Roberts' compensating pendulum, 596.
Roberval's balance, 115.
Rods, vibration of, 310.
Roget's oscillating spiral, 909.
Rolling friction, 139.
Ropes, rigidity of, 142.
Rosse's telescope, 503.
Rotascope, 57.
Rotation of electric light about a magnet, 936.
Rotation of the earth demonstrated by the pendulum, 86.
Rotations, parallelogram of, 56; right-handed or left-handed, 56.
Rotary polarization, 556; magnetic, 560.
Rotary pump, 293.
Rousseau's hydrometer, 212.
Ruhmkorff coil, 933.
Rule for position of images, 436.
Rumford's photometer, 414; thermoscope, 587.
Rutherford's thermometer, 578.

SAIGEY's table of length of pendulum, 90.
Saturated space, 669.
Saturation, 661.
Saussure's hygrometer, 973.
Savart's toothed wheel, 361.
Savary's steam engine, 706.
Saxton's deep-sea thermometer, 581; reflecting pyrometer, 582.
Scale beam, 115.
Scintillating tube, 852.
Screens, diathermanic power of, 643.
Screw, 127; efficiency of, 128; endless, 129.
Sea lights, 522.
Seasons, 947.
Secondary axes, 430.
Seconds pendulum, 89; formulæ for, 90; table of lengths in different latitudes, 90.
Self-registering thermometers, 578.
Sensible weight varies in different localities, 93.
Sensibility of the ear, 378.
Septum required for osmose, 246.
Series of elastic balls, 185.
Seventh, harmony of, 373.
Sextant, Hadley's, 425.
Shock transmitted by elastic balls, 185.
Silliman's photometer, 414.
Simple microscope, 495.
Simple pendulum, 79.
Simple pendulum, propositions respecting, 82
Simple vision with two eyes, 482.
Siren, 360.
Sliding friction, 137.
Smee's battery, 871.
Snow, 984; colored, 985; limit of perpetual, 955.
Solar microscope, 516.
Solar spectrum, properties of, 460.
Solenoid, 910.
Solid eye-piece, 512.
Solidification, change of volume by, 664.
Solidification liberates heat, 663.
Solidification of gases, theory, 688.
Solids, characteristics of, 149; conductibility for heat, 614; expansion of, 589; structure of, 150; undulations of, 306; unequal expansion of, 595.
Solution, 661.
Sonometer, 367.
Sonorous tubes, 379.
Sonorous waves, length of, 370.
Sound, 335; distances calculated by, 346; in all elastic bodies, 340; interference of, 349; not instantaneous, 342; not propagated in a vacuum, 339; propagated by waves, 337; reflected, 352; refracted, 357; velocity in air, 344; velocity in gases, 345; velocity in liquids, 347; velocity in solids, 348.
Sounds of birds, 392; of insects, 392; produced by animals, 392.
Sounding bodies vibrate, 336.
Source of heat influencing diathermancy, 644.
Sources of heat, 734–757.
Sources of light, 399.
Space described by a falling body, 71.
Speaking trumpet, 358.
Specific gravity, 209.
Specific gravity bottle, 211.
Specific gravity by balance, 210.
Specific heat, 651.
Specific heat and atomic weight related, 654.
Specific heat determined by laws of acoustics, 653.
Specific heat of gases, 652.
Specific weight, 99.
Spectrum, 456; dark lines in, 461.
Specula, 416.
Spherical aberration of lenses, 454; of mirrors 437.
Spherical mirrors, 426.
Sphericity, aberration of, 455.
Spheroidal state, 694; cause of, 698; cause of explosions, 701; familiar illustrations, 702; remarkable phenomena, 700.
Spirit level, 203.
Spirit thermometers, 575.
Spring balance, 37.
Springs intermittent, 286.
Stable equilibrium, 207.
Standard thermometer, 577.

Starting friction, 138.
Statical and dynamical forces, 39
Statical electricity, 809.
Statics defined, 39.
Stationary undulations, 302.
Stationary waves, 321.
Steam boiler, 713; explosions, 701; Gold's 733.
Steam cylinder, Papin's, 707
Steam, electricity from, 839.
Steam, heat latent and sensible at different temperatures, 683.
Steam, high pressure, 680.
Steam, latent heat of, 682.
Steam, power of, 714.
Steam-engine, 703; high pressure, 711; low pressure, 710; Newcomen's, 707; Watt's, 709.
Steam-heaters, 732.
Steam-power, 134.
Steamboat, the first, 705.
Steelyard, 115.
Stere, 18.
Stereomonoscope, 526.
Stereoscope, 525.
Stone's ventilating shaft, 723.
Storms, general laws, 971.
Stream measurers, 227.
Strength, animal, 130; of beams and tubes, 172; of materials, 170; of men, 131.
Structure of human eye, 468.
Structure of metals changed, 180.
Study of colors, 492.
Stuttering, 391.
Submarine telegraphs, 927.
Substance defined, 1.
Suction and lifting pump, 291.
Suction pumps, 290.
Sulphate of copper battery, 872.
Sun, a source of heat, 742; influence of, 948; quantity of heat from, 743.
Surface of discharging liquid, 215.
Syphon, 285.
Syphon barometer, 267.
System of forces, 44.
System of magnetic observations, 798
System of wheels, 117.
Systems of crystals, 154.

TABLES, Ascent of liquids in tubes, 239.
“ Absorptive power for heat from different sources, App. tab. IX.
“ Absorptive power of different bodies, App. tab. VIII.
“ Aqueous vapor in a cubic foot of saturated air at different temperatures, App. tab. XXI.
“ Barometric column at various altitudes, 265.
“ Boiling point of liquids, App. tab. XVII.
“ Boiling point of water at different places, App. tab. XVIII.
“ Boiling point of water at different pressures, App. tab. XVI.
“ Boiling point of water under 1, 2, 3, &c., atmospheres, App. tab. XIX.
“ Colors of tempered steel, 178.
“ Comparison of thermometers, 576.
“ Compressibility of gases, 276.
“ Compressibility of liquids, 188.
“ Conducting power of metals and building materials, App. tab. VII
TABLES, Decrease of temperature from elevation, 954.
“ Degree of elasticity, 183.
“ Depression of mercury in tubes, 238.
“ Diathermancy for heat from different sources, 645.
“ Diathermancy of different liquids, App. tab. X.
“ Diathermancy of different solids, App tab. XIII.
“ Elasticity of metals, 161.
“ Expansion of gases, 607.
“ Expansion of gases, App. tab. V.
“ Expansion of liquids, App. tab. IV.
“ Expansion of mercury, 598.
“ Expansion of solids, App. tab. III
“ Force required to heat a pound of water 1° F., 761.
“ Freezing mixtures, App. tab. XII.
“ French decimal measures and weights changed to English measures and weights, App. tab. II.
“ Frequency of different winds, 965.
“ Frequency of auroras, 996.
“ Friction, 138.
“ Hardness, 176.
“ Heat of combustion in air, 753.
“ Heat of combustion in oxygen, 751.
“ Hydrometer degrees and specific gravities, App. tab. XXV.
“ Hydrostatic pressure at various depths, 196.
“ Latent and sensible heat of steam, App. tab. XXII.
“ Laws of falling bodies, 71.
“ Limits of perpetual snow, 955.
“ Liquefaction and solidification of gases, App. tab. XX.
“ Measures and weights, App. tab. I.
“ Melting points and heat of fusion, App. tab. XV.
“ Radiating and absorbent power of bodies for heat, 638.
“ Radiating power, App. tab. VI.
“ Reflection of light, 407.
“ Specific gravity of solids and liquids, App. tab. XXIII.
“ Specific heat, App tab. XI.
“ Specific heat for constant pressure and constant volume, 653.
“ Strength of men and animals, 133.
“ Strength of materials, 170.
“ Temperature in different latitudes, 953.
“ Tension of vapors, App. tab. XIV.
“ Velocity of winds, 961.
“ Volume and density of water, App. tab. XXIV.
Telegraph, 921-928.
Telescope, 497; achromatic, 504; equatorial, 505; Galileo's, 498; Lord Rosse's, 503; reflecting, 501; Sir William Herschel's, 502.
Telescopes, penetrating power, 507; visual power of, 507.
Telestereoscope, 524.
Temper, 178.
Temperament in music, 375.
Temperature, 565; estimation of high, 586; extremes of, 744; mean, 950; monthly variations, 951; of vaporization, 672; of vegetables, 757; variations in altitude, 954 variations in latitude, 953.
Tempered steel, colors, 178.

Tenacity of different substances, 170.
Tenacity, laws of, 170.
Tension, 252; changes density, 169; maximum, of vapors, 669; of vapors, Dalton's law, 670; of vapors in communicating vessels, 671.
Terrestrial attraction, direction of, 60; point of application, 61.
Terrestrial eye-piece, 500; heat, origin of, 746; magnetism, 787; radiation, 745.
Thaumatrope, 488.
Theorem of Archimedes, 205; of Torricelli, 219.
Theoretical and actual flow, 216.
Theories of endosmose, 251; of light, 398; of light unsatisfactory, 405.
Theory defined, 3; of liquefaction and solidification of gases, 688; of undulations, 299.
Thermochrosy, 646.
Thermo-electricity, 941.
Thermo-electric motions, 941.
Thermometers, 567; air, 587; Breguet's metallic, 580; Centigrade, 570; comparison of scales of, 571; construction of, 568; defects in mercurial, 576; Fahrenheit's, 570; graduation of, 569; house, 572; Howard's differential, 587; Kinnersley's, 852; Leslie's differential, 587; limits of mercurial, 574; metastatic, 579; Negretti & Zambra's maximum, 578; Reaumur's, 570; Rutherford's maximum and minimum, 578; Saxton's deep sea, 581; self-registering, 578; sensibility of, 572; spirit, 575; standard, 577; tests of, 572; Walferdin's maximum, 578.
Thermometric scales compared, 571.
Thermo-multiplier, 588, 942.
Thermoscopes, 587.
Thilorier and *Bianchi's* apparatus, 690.
Three states of matter, 15.
Thunder, 989.
Thunder-storms, 988.
Time and velocity, 29.
Time required for vision, 489.
Tolles' solid eye-piece, 512.
Tone, 363; changed by echo, 355.
Tornadoes, 969.
Torricellian theorem, 219; demonstrated, 220.
Torricellian vacuum, 261.
Torsion, elasticity of, 165; electrometer, 820; laws of, 166; of rigid bars, 167.
Total hydrostatic pressure, 196.
Total reflection, 409.
Trains of wheelwork, 117.
Transposition in music, 374.
Transverse strength, 172.
Tubes, strength of, 172.
Tubular bridges, 172.
Tuning fork, 377.
Turbine wheel, 231.
Twining's ice machine, 681.
Tylerian electrical machine, 837.
Tympanum, 393.

UMBRA, 411.
Undershot wheel, 230.
Undulation in oceans, &c., 329; of elastic fluids, 330; of liquids, 319.
Undulations of a sphere of air, 331; of solids, 306; origin of, 299; phases of, 304; progressive, 300; progressive, in liquids, 320; stationary, 302.
Uniform motion, 30.
Uniform musical pitch, page 668
Unison, 364.
Unit of force, 37.
Units of measure, 16.
Universal discharger, 851; gravitation, law of, 59; radiation of heat, 633.
Unstable equilibrium, 207.
Upward pressure of liquids, 192.

VACUUM, Torricellian, 261; vapors formed in, 668.
Value of fuel, 715.
Value of g in pendulum experiments, 89.
Vaporization, 667.
Vaporization, temperature, and limits, 672.
Vapors, 252; and gases, identity of, 688; density of, 693; formed in a vacuum, 668; from the body, 719; Latour's law, 692; liquefaction of, 685; maximum tension, 669.
Variable motion, 31.
Variation chart, 789.
Variations of barometric height, 270; of magnetic needle, 788; annual, daily, 790.
Vegetables, temperature of, 757.
Velocities, actual and theoretical, 144; parallelogram of, 34.
Velocity after impact, 184; of all sounds the same, 343; of all sounds not the same, page 668; of aerial waves, 332; of electricity, 818; of light, 404; of rivers and streams, 227; of sound in air, 344; of sound in liquids, 347; in solids, 348; of sound, Newton's formula, 653.
Ventilation, 716-726; a practical problem, 722; necessity of, 718; quantity of air required, 720.
Ventilating shaft, 723.
Ventilators, 725.
Ventriloquism, 391.
Verification of laws of falling bodies, 72.
Vernier, 266.
Vibrating armature, 931.
Vibrating dams, 386.
Vibration of air in tubes, 379, 384; of cords, 308; laws of, of cords, 309; of rods, 310; of membranes, 318; of plates, 312; of plates, laws of, 315; paths of, 311.
Vibrations, 299; forms of, 307; from magnetism, 916; isochronous, 303; of different notes, absolute, 369; relative, 368; of heat and light, 765; of light, direction, 541; of light, length of, 531; of light, transmission of, 542; of light, resolution of, 544.
Victoria tubular bridge, 172.
Virtual focus, 429; images, 434; velocities, 105.
Visible bodies, 402.
Vision, 468; conditions of, 475; binocular, 484; double, 483; single, 482.
Visual angle, 472.
Visual impressions require time, 489.
Visual power of microscope, 513; of telescopes, 507.
Visual rays nearly parallel, 479.
Vis viva, 111.
Vitality, 10.
Vocal apparatus of man, 388.
Voice and speech, 387.
Voice, mechanism of, 389; range of human, 390.
Volta-electric induction, 929.
Voltaic arch, heat of, 886.
Voltaic batteries, 869-881; effects in various forms, 881.
Voltaic circuit, polarity of, 878.
Voltaic couple, 866.
Voltaic currents, heat of, 887.